NONLINEAR UV LASER BUILD-UP CAVITY: AN EFFICIENT DESIGN

Nicholas Henry Rady, B.A.

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APPROVED:

David Shiner, Major Professor
Jerry Duggan, Committee Member
José Pérez, Committee Member
Samuel Matteson, Program Coordinator
Chris Littler, Physics Chair
Michael Monticino, Interim Dean of Robert B. Toulouse School of Graduate Studies

Using the concept of the build-up cavity for second harmonic generation to produce 243nm laser light, an innovative cavity is theoretically explored using a 15mm length CLBO crystal. In order to limit the losses of the cavity, the number of effective optical surfaces is kept to only four and the use of a MgF$_2$ crystal is adopted to separate the harmonic and fundamental laser beam from each other. The cavity is shown to have an expected round trip loss of five tenths of a percent or better, resulting in a conversion efficiency greater than 65%.
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**TABLE OF CONTENTS**

**ACKNOWLEDGEMENTS** .......................................................................................................................... iii

**LIST OF FIGURES** ............................................................................................................................... vi

**INTRODUCTION** ...................................................................................................................................... 1

**THEORETICAL DESIGN** ......................................................................................................................... 3
  Second Harmonic Generation .................................................................................................................. 3
  Uniaxial Crystals ....................................................................................................................................... 4
  Walk Off Phenomena ................................................................................................................................. 6
  Efficiency ................................................................................................................................................ 6
  Buildup Cavities ..................................................................................................................................... 7
  Low Loss Cavity Design ............................................................................................................................ 9
  MgF₂ Prism ............................................................................................................................................. 10
  Angular and Temperature Bandwidths .................................................................................................... 13

**EXPERIMENTAL DESIGN** ....................................................................................................................... 15
  Cavity Size and Eigenwaist ...................................................................................................................... 15
  15mm CLBO for 486-243 SHG ............................................................................................................... 16
  Losses – Reflection, Fresnel, and Absorption .......................................................................................... 17
  Impedance and Mode Matching ............................................................................................................ 17
  Total Conversion Efficiency .................................................................................................................... 19

**CONCLUDING REMARKS** ..................................................................................................................... 20
  Current Status ....................................................................................................................................... 20
LIST OF FIGURES

Figure 1 – A traditional bow-tie cavity with a Brewster cut crystal. Two of the mirrors are curved so that propagating beam will come to a focus inside the crystal. .......... 9

Figure 2 – The layout of the optical cavity with an angular deviation to compensate for the appropriate location of the waist at the center of the crystal. ......................... 10

Figure 3 – The eigenmode spot size as a function of propagation distance within the cavity. The red/blue curve represents the tangential plane mode and the yellow/green represents the sagittal plane mode. The beam starts at the halfway point on the return path and the two peaks are located at the input coupler and the return mirror. The color of the beam changes as it enters into a different medium. 16

Figure 4 – A graph of the Total Conversion Efficiency verses the Input Coupler. This graph assumes a one-half percent roundtrip loss, $\mu = 1.3 \times 10^{-4}$, and $P_{in} = f[W]$. .......... 19

Figure 5 – Two partially reflective mirrors arranged parallel to each other. If the incident light satisfies the standing wave equation, then the light will pass through the cavity as if it were not there. ................................................................. 22

Figure 6 – A computer-aided rendition of an optical mount used in the cavity set-up. A small ball bearing is placed opposite the setscrew hole for so as not to score the surface of the plane it pushes against. ................................................................. 30
INTRODUCTION

In the last few decades, the development of continuous wave UV lasers has been a focus of study with a wide range of applications, from commercial applications (e.g. the production of fiber-bragg gratings\textsuperscript{1}) and scientific studies (e.g. measuring the Rydberg constant and electron/proton mass ratio more accurately).\textsuperscript{2} The use of nonlinear medium combined with the newly discovered ruby LASER in 1960\textsuperscript{3} was quickly realized and the first SHG laser was built in 1961 using crystalline quartz.\textsuperscript{4} Since that time, it has been found that by doping borates and phosphates with light metals, materials with exceptionally high nonlinear effects can be created artificially. Some of the more commonly used materials for SHG today are BBO (beta barium borate), KTP (potassium titanyl phosphate), and CLBO (cesium lithium borate).

Recently, with the use of a CLBO crystal in a bowtie build-up cavity, a 244nm laser has been made through the process of SHG with 33% total conversion efficiency.\textsuperscript{5} It is desirable to develop a multiple hundred-milliwatt single mode continuous wave 243nm laser. An easy solution to obtain an adequate laser would be to exchange the tunable pump laser of 488nm used by the group from Arizona with one of 486nm and circulate the light through the widely used bowtie cavity with a Brewster-cut CLBO crystal to achieve the desired laser wavelength. However, there are number of improvements in the bowtie cavity design that would result in a highly efficient conversion of the 486nm light into 243nm light. Namely, these adjustments must achieve the following requirements:
• Have an internal round-trip loss of 1% or less
• Not incorporate a multi-wavelength AR coating
• Have a large Free Spectral Range
• Separation of pump laser from generated harmonic
THEORETICAL DESIGN

Second Harmonic Generation

In order to understand the process of SHG, one must look at the polarization of a molecule in a dielectric due to an applied electric field. Expanding the polarization as a function of the electric field, we get

\[ P(t) = \varepsilon_0 E(t) \left( \chi^{(1)}(t) + \chi^{(2)}(t) + \chi^{(3)}(t)^2 + \cdots \right) \tag{1} \]

where \(\varepsilon_0\) is the permittivity of free space and \(\chi^{(i)}\) is the \(i\)th susceptibility constant. For a pure order of magnitude approximation of \(\chi^{(2)}\), we take that the second order susceptibility is comparable to the first order susceptibility when the applied electric field is on the order of the atomic electric field. Thus,

\[ \chi^{(2)} \approx \frac{\chi^{(1)}}{E_{at}} \approx 10^{-12} \left[ \frac{m}{V} \right] \tag{2} \]

where, the characteristic atomic field strength, \(E_{at} = \frac{e}{4\pi\varepsilon_0 a_0^2}\). This implies that the second order polarization will be negligible unless the applied electric field is on the order of \(10^{12} \left[ \frac{W}{m^2} \right]\). An electric field of this strength is easily obtainable with the use of a laser.

If we have an electric field incident on a medium with a nonzero second order susceptibility constant such that
\[ E(t) = \frac{1}{2} \left( E_0 e^{i \omega t} + E_0^* e^{-i \omega t} \right) \]  \hspace{4cm} (3)

where \( \omega \) is the angular frequency of the laser, the second order polarization becomes

\[ P(t) = \frac{\varepsilon_0 \chi^{(2)}}{4} \left( 2E_0 E_0^* + |E_0|^2 e^{2i \omega t} + |E_0|^2 e^{-2i \omega t} \right). \]  \hspace{4cm} (4)

The first term in (4) indicates a constant induced electric field in the medium. The second term is an oscillatory term with twice the incident frequency. From a physical point of view, one can imagine from the conservation of energy that as two photons of frequency \( \omega \) interact with the medium, they are destroyed and one photon of frequency \( 2\omega \) is created.

Now consider a length of crystal in which two photons of frequency \( \omega \) are converted to one of frequency \( 2\omega \). As the two waves propagate down the length of the crystal, they must stay in phase with each other. This can be accomplished if they travel at the same velocity within the crystal. This gives us the condition known as phase matching for nonlinear optics. For most medium, the index of refraction of the medium changes as a function of frequency. However, uniaxial crystals can be cut in such a way that

\[ n(\omega) = n(2\omega). \]  \hspace{4cm} (5)

Uniaxial Crystals

Anisotropic crystals have a property such that the index of refraction changes not only as a function of frequency, but also as a function of polarization and direction of wave propagation within the crystal. This phenomenon is known as birefringence. For
uniaxial crystals, a subset of anisotropic crystals, there is a principle plane defined by the plane containing the optical axis and the wave vector of the propagating wave. Light propagating within this plane is defined as extraordinary and light propagating perpendicular to this plane is defined as ordinary. When the wave vector lies on the optic axis, birefringence is not observed, and all light propagating though the crystal is ordinary.

One might then imagine that the index of refraction for the extraordinary ray changes as a function of the angle between the wave vector and optic axis. It has been shown that\(^7\)

\[
\frac{1}{n(\omega, \theta)^2_e} = \frac{\cos^2(\theta)}{n(\omega)^2_o} + \frac{\sin^2(\theta)}{n(\omega)^2_e} \tag{6}
\]

where \(n(\omega, \theta)^e\) is the extraordinary index of refraction, \(n(\omega)^o\) is the ordinary index of refraction, and \(\theta\) is the angle between the wave vector and the optic axis. The index of refraction for the crystal at varying wavelengths can be calculated using the Sellmeier Equation discussed in the Appendix. As expected, (6) shows that the index of refraction of the extraordinary ray is equal to that of the ordinary when \(\theta = 0^\circ\) and the extraordinary when \(\theta = 90^\circ\).

The phase matching requirement expressed in (5) can be meet by finding a value for \(\theta\) such that

\[
n_o(\omega) = n_e(2\omega). \tag{7}
\]

This condition is called Type I phase matching where two photons in the ordinary plane interact to produce one photon with twice the frequency in the extraordinary plane. This
dictates that the fundamental frequency must be linearly polarized in the ordinary plane of the crystal, and the light created by SHG is linearly polarized perpendicular to the fundamental light. By using the trigonometric identity $\cos^2(\theta) + \sin^2(\theta) = 1$ and the phase matching requirement in (7), you can solve (6) for the phase matching angle, $\theta_m$,

$$\sin^2(\theta_m) = \frac{n(\omega_o)^2 - n(2\omega_o)^2}{n(2\omega_e)^2 - n(2\omega_o)^2}.$$  

(8)

Walk Off Phenomena

There is a small effect that occurs to the generated harmonic called walk off. This effect causes the harmonic to be transformed from a Gaussian beam into an elliptical shape, which is undesired for most laser applications. The origin of this phenomenon is from the Poynting vectors of the fundamental and its harmonic being slightly misaligned. This walk off angle, $\rho$, has been shown to be equal to\textsuperscript{8}

$$\rho = \arctan\left[\left(\frac{n_o^{\text{crystal}}(\omega)}{n_e^{\text{crystal}}(\omega)}\right)^2 \tan(\theta_m)\right] - \theta_m.$$  

(9)

Efficiency

The goal of SHG is to convert as much of the fundamental frequency into its second harmonic. The best measure of this is in the efficiency, $\eta$, defined as

$$\eta = \frac{P_{2\omega}}{P_{\omega}}.$$  

(10)
where $P$ is the power of the corresponding frequency. It has been shown that (10) can be written as

$$\eta = \frac{4\pi^2 \chi^{(2)} L h}{\epsilon_0 c \lambda^3}$$

(11)

where $L$ is the length of the crystal and $h$ is the Kleinmann factor. Although the Kleinmann factor can be calculated directly, its integral form is very cumbersome to compute, and the approximation, good within 10%, can be used for $\xi << 1$,

$$h = \frac{2\pi\xi}{t} \text{erf}\left(\frac{t}{\sqrt{2}}\right) - \left(\frac{2}{t^2}\right)\left(1 - e^{-\frac{\xi^2}{2}}\right)$$

(12)

where $t$ is what I call the Walk-Off parameter and is defined as

$$t = \frac{\sqrt{2} \rho L}{\omega_0}$$

(13)

and $\omega_0$ is the focal waist of the Gaussian beam, not to be confused with the fundamental frequency. When you use the approximation that $\xi << 1$, you will have a Rayleigh range for your waist that is large when compared to the length of the crystal; this is referred to as weak focusing.

**Buildup Cavities**

Due to the relatively small single pass conversion efficiency of non-linear crystals, high conversion efficiency can be difficult to achieve with a continuous wave laser. Because of this limitation, the use of build-up cavities has become a popular solution. The most basic build-up cavity is a Fabry-Perot cavity. A full description of the Fabry-Perot Cavity is given in the appendix. A light beam hitting the surface of the
first mirror will allow a portion of the light to enter the cavity. If the distance between
the mirrors is such that it satisfies the standing wave equation, then the incident mirror
will become transparent to the light beam, and allow all the light to enter the cavity,
despite the fact that it is partially reflective. The light that enters the cavity will have a
greater intensity than the incident beam, and the build-up factor, $Z$, is defined as

$$Z \equiv \frac{P_c}{P_0}$$

(14)

where $P_c$ is the power circulating inside the cavity. In order to satisfy the conservation
of energy, all of the energy from the incident beam must be accounted for. If the incident
electric field amplitude is proportional to $\sqrt{P_0}$, then the input electric field in the cavity
must be proportional to $\sqrt{P_0} \sqrt{T}$ where $T$ is the transmittance of the incident mirror. As
the beam propagates through the cavity, the electric field will drop in intensity by
approximately a factor of $\sqrt{1 - \tilde{l}}$ where $\tilde{l}$ is the loss of the fundamental beam, including
conversion of the fundamental into its harmonic, in one round trip of the cavity. The
reflection of the return beam will add with the transmitted electric field amplitude, giving
an equation of the self-consistent circulating amplitude of

$$\sqrt{P_c} = \sqrt{P_0} \sqrt{T} + \sqrt{P_c} \sqrt{1 - \tilde{l}} \sqrt{1 - T}.$$  

(15)

Substituting (14) for $P_c$, (15) can be rearranged such that the build-up factor can be
written as

$$Z = \frac{T}{\left(1 - \sqrt{1 - \tilde{l}} \sqrt{1 - T}\right)^2}.$$  

(16)

Assuming that the internal losses of the cavity, $l_\omega$, are low, (16) can be rewritten as
\[ Z = \frac{T}{\left(1 - \sqrt{1 - l_\omega - l_{2\omega} \sqrt{1 - T}}\right)^2} \]  

(17)

where the loss due to conversion, \( l_{2\omega} \), has been shown to be \(^{10}\)

\[ l_{2\omega} = \tanh^2\left(\sqrt{\eta Z P_\omega}\right) = \eta Z P_\omega \]  

(18)

where the approximation is valid for \( \eta \ll 1 \), which is in agreement with (10). Inserting (18) into (17) gives the final form of the build-up factor

\[ Z = \frac{T}{\left(1 - \sqrt{1 - l_\omega - \tanh^2\left(\sqrt{\eta Z P_\omega}\right) \sqrt{1 - T}}\right)^2}. \]  

(19)

This relates the build-up factor to fixed cavity constants, \( \eta, T, l \), and the variable \( P_\omega \). Taking (10) and inserting the buildup power, \( P_Z \), as

\[ P_c = Z P_\omega, \]  

(20)

you find that the generated power of the harmonic is

\[ P_{2\omega} = \eta P_c^2. \]  

(21)

Low Loss Cavity Design

As shown in (16), as you decrease the intrinsic losses of the cavity, you increase the build-up, and thus, an increase in the conversion of the fundamental into its harmonic. Traditionally, the cavity of choice for SHG has been the bow-tie cavity. This arrangement has a total of six surfaces for the light to interact per round trip. The addition of a beam splitter is also necessary in order to extract the harmonic from the fundamental, adding an additional two surfaces per round trip. In order to reduce losses, it has become common practice to place the nonlinear crystal in the cavity with parallel
cut Brewster surfaces. A more efficient cavity design has been demonstrated using a trapezoidal Brewster cut prism with only two mirrors. This design is not only more compact, but reduces the number of optical surfaces. However, this design uses a different type of crystal in which the phase matching condition of (7) does not have to be met. Therefore, the index of refraction of the fundamental and its harmonic are slightly different, allowing the refraction at the crystal-air interface to act as a beam splitter and sending the harmonic on a path outside the cavity. Unfortunately, this is not the case when phase matching must be met.

**MgF₂ Prism**

In order to separate the fundamental from the harmonic, I propose adding a MgF₂ prism to the system with an optical contact between the nonlinear crystal and the MgF₂ prism. The advantage of this addition to the cavity is two-fold. One, assuming that you have the ability to clean the surface of the crystals so that they are free from debris on the order of a quarter a wavelength and that the surface of the crystal can be polished to a
flatness of a quarter of a wavelength or better over the area of the face, then you avoid having two extra optical interfaces to the system. The second advantage is that fundamental and its harmonic refract at the MgF₂-air interface at different angles, thus, eliminating the need to use a beam-splitter.

Let’s follow the beam path starting from the input coupler. The length of the beam from the input coupler to the crystal surface I will call the input beam, \( l_1 \). The input beam must hit the crystal surface at Brewster’s angle and propagate through the crystal parallel to the crystals sides, \( l_2 \). I will call the beam inside the crystal the action beam, as this is where the action of SHG takes place. The action beam dictates that

\[
\gamma_1 = \arctan\left( n_{\text{crystal}}^{\omega} \right) \tag{22}
\]

For a minimal loss at the crystal-MgF₂ interface

\[
\beta = \arctan\left( \frac{n_{\text{crystal}}^{\omega}(\omega)}{n_{\text{MgF}_2}^{\omega}(\omega)} \right) \tag{23}
\]

The beam will refract at the crystal-MgF₂ interface, \( l_3 \), with an angle above its original propagation path of

\[
\alpha_1 = \arcsin\left( \frac{n_{\text{crystal}}^{\omega}(\omega)}{n_{\text{MgF}_2}^{\omega}(\omega)} \right) \sin(\beta) - \beta \tag{24}
\]

where \( \bar{\beta} \) is the complimentary angle of \( \beta \). As the beam leaves the MgF₂, the output beam let’s call it, must hit the surface at Brewster’s angle. This mandates that

\[
\gamma_2 = \arcsin\left( n_{\text{MgF}_2}^{\omega}(\omega) \right) - \alpha_1 \tag{25}
\]

The difference in (22) and (25) is due to the upward deflection of the beam when it refracts at the crystal-MgF₂ interface. This, along with the lower index of the MgF₂,
results in the need of (38). If you were to place the secondary mirror such that the return beam, the beam between the input coupler and reflector mirror, was parallel to the action beam, the angle between the return beam and the input beam, $\varepsilon_1$, would be

$$\varepsilon_1 = 2\gamma_1 - 90^\circ$$ \hspace{1cm} (26)

and the angle between the return beam and the output beam, $\varepsilon_2$, would be

$$\varepsilon_2 = 2\gamma_2 + \alpha_1 - 90^\circ$$ \hspace{1cm} (27)

If you define the distance between the return beam and the action beam as $h$, then the following distances are measured as follows:

$$l_1 = \frac{h}{\sin(\varepsilon_1)}$$ \hspace{1cm} (28)

$$\tilde{l}_1 = \frac{h}{\tan(\varepsilon_1)}$$ \hspace{1cm} (29)

$$l_2 = \tilde{l}_2 = L_{\text{crystal}}$$ \hspace{1cm} (30)

$$l_3 = \frac{\left(W(\tan(\beta)+\tan(\gamma_2)) + 4w\right) \sin(\gamma_2)}{4 \sin(\gamma_2 + \alpha)}$$ \hspace{1cm} (31)

$$\tilde{l}_3 = \frac{\left(W(\tan(\beta)+\tan(\gamma_2)) + 4w\right) \cos(\gamma_2)}{4}$$ \hspace{1cm} (32)

$$\bar{h} = l_3 \sin(\alpha_1)$$ \hspace{1cm} (33)

$$l_4 = \frac{h + \bar{h}}{\sin(\varepsilon_2)}$$ \hspace{1cm} (34)

$$\tilde{l}_4 = \frac{h + \bar{h}}{\tan(\varepsilon_2)}$$ \hspace{1cm} (35)
Ideally, the waist of the eigenmode for the cavity will be located at the center of the crystal. In order to assure that this is the case, a new variable, $\delta$, will be introduced. This will create a new return path by placing the reflecting mirror on a different spot on the output path. This variable changes the following lengths:

$$l' = \frac{\sin(\varepsilon_2) \sum_{i=1}^{4} \tilde{l}_i}{\sin(\varepsilon_2 + \delta)} \quad (36)$$

$$\tilde{l}_4 = l_4 - \frac{\sin(\delta)}{\sin(\varepsilon_2 + \delta)} \quad (37)$$

In order for the eigenwaist to be centered on the crystal, the following equality must hold,

$$\frac{l_4 + \frac{l_3}{n_{\text{MgF}_2}(\omega)} - l_1}{\sum_{i=1}^{4} \tilde{l}_i} = \frac{\sin(\delta)}{\sin(\varepsilon_2 + \delta)} \quad (38)$$

(38) can be easily solve by numeric techniques to find a value of $\delta$ to satisfy the equality.

Replacing $n_{\text{MgF}_2}(\omega)$ in (9) with $n_{\text{MgF}_2}(2\omega)$, you get that

$$\alpha_2 = \arcsin\left(\frac{n_{\text{crystal}}(\omega)}{n_{\text{MgF}_2}(2\omega) \sin(\beta)}\right) - \beta \quad (39)$$

and the angular deviation of the rays leaving the MgF$_2$ prism, $\Delta\theta$, is

$$\Delta\theta = \arcsin(n_{\text{MgF}_2}(2\omega)\sin(\bar{\theta} - \alpha_2)) - \arcsin(n_{\text{MgF}_2}(\omega)\sin(\bar{\theta} - \alpha_1)). \quad (40)$$

Angular and Temperature Bandwidths

An important factor to consider with nonlinear optical conversions is the angular...
and temperature bandwidths. Care must be taken into the design of a cavity so that the
beam divergence at the crystal and the temperature fluctuations on the crystal are within
the respectful bandwidths, or else, conversion will not occur. The angular bandwidth for
this case is given as

$$\Delta \theta_{\text{max}} = \frac{0.433 \lambda L \tan(\theta)}{\left(1 + \tan^2(\theta) \left(\frac{n_o(2\omega)}{n_e(2\omega)}\right)^2\right)^{1/2}}$$

(41)

and the temperature bandwidth for this case is given as

$$\Delta T_{\text{max}} = \frac{0.433 \lambda L}{n_e^2(\omega) \frac{\partial n^2_o(\omega)}{\partial T} - n_{2\omega}^2(\theta) \frac{\partial n_{2\omega}^2(\omega)}{\partial T}}$$

(42)

where $n_{2\omega}^c$ can be expressed as

$$n_{2\omega}^c(\theta) = \sqrt{\frac{1 + \tan^2(\theta_c)}{1 + \tan^2(\theta) \left(\frac{n_o(2\omega)}{n_e(2\omega)}\right)^2}}.$$  

(43)
EXPERIMENTAL DESIGN

Cavity Size and Eigenwaist

An eigenwaist of $76 \, \mu m$ has been chosen for the cavity design over the more commonly used confocal waist. The confocal waist is a waist size corresponding to a Rayleigh range of half the crystal length. The larger eigenwaist will decrease the single pass efficiency of the SHG, but will reduce heating in the crystal due to absorption of the generated UV light within the crystal. Any fluctuations in the crystals temperature may cause the conversion to stop as seen in equation (42). The larger cross-sectional area of the beam will more easily allow for heat conduction flow from the crystal to the thermoelectric cooler attached to the crystal.

Using the ABCD matrix formulation of Gaussian beam propagation, it is found that the $h$ parameter of the cavity is 6mm with the $\delta$ parameter equal to 4.9 degrees. An applied mathematical introduction to ABCD matrix calculations for Gaussian beams is given in the appendix. Due the both the crystal and the MgF$_2$ having a Brewster cut and the input coupler and return mirror tilted at an angle in respect to the propagating beam, the eigenmode of the cavity is different in the sagittal and the tangential planes. Therefore, the desired eigenwaist is the average of the two eigenmodes.
Figure 3 – The eigenmode spot size as a function of propagation distance within the cavity. The red/blue curve represents the tangential plane mode and the yellow/green represents the sagittal plane mode. The beam starts at the halfway point on the return path and the two peaks are located at the input coupler and the return mirror. The color of the beam changes as it enters into a different medium.

15mm CLBO for 486-243 SHG

For the purposes of producing an adequate laser for the two-photon absorption of Hydrogen, let’s take a look at a 15mm length crystal of CLBO kept at a temperature of 150°C with a waist of 76 [µm]. CLBO is hygroscopic in nature, and a length longer than 15mm is not feasibly producible; this is also the reason of the elevated temperature of the crystal. From the previously defined equations, referenced on the left hand side of the page, the following values apply to our crystal:

\begin{align*}
\text{(7)} & & n_o^{\text{crystal}}(\omega) &= 1.5007 \\
\text{(8)} & & \theta_m &= 76.6^\circ \\
\text{(9)} & & \rho &= 1.03^\circ \\
\text{(11)} & & \mu &= 1.3 \times 10^{-4} \left[ \frac{1}{W} \right]
\end{align*}
Losses – Reflection, Fresnel, and Absorption

There are three types of losses that can contribute to the depletion of a mode-matched fundamental frequency into the cavity not due to nonlinear effects. Because the crystals are cut at Brewster’s angle, the Fresnel losses at these surfaces are small. It is possible to have the angles $\gamma_1$, $\gamma_2$, and $\beta$ polished within $\frac{1}{2}$ degree of the calculated value. This gives a maximum a loss of one-tenth of a percent. The absorption of the fundamental in the crystal system is on the order of a tenth of a percent. The largest loss of the cavity would be from the return mirror. Because our fundamental frequency is close to that of the Ar-Ion gas laser, high reflective dielectric mirrors are readable available with losses of three-tenths of a percent or better. However, practical experience has shown that these mirrors actually operate closer to losses on the order of one or two tenths of a percent loss.

Impedance and Mode Matching

Now that we have a maximum loss for the round trip of the fundamental within the cavity, we can use (21) and find the value of the input coupler that will give the maximum power conversion to the second harmonic. However, there are two factors to consider when selecting the input coupler, the actual loss of the system and the incident power. When the power lost per round trip in the cavity equals the power incident on the cavity, $P_c(l_\omega + l_{2\omega}) = P_\omega$, resulting in zero reflection from the input coupler, then an
Impedance match is meet. It would be ideal to find an input coupler that is within 90% the peak impedance match of the extreme values of these two variables. Unfortunately, in all practicality, obtaining an input coupler of a precise value can be difficult and costly. Figure 4 shows the total conversion efficiency vs. input coupler with a peak efficiency of an input coupler of 1.4%. Within acceptable manufacturing tolerances an input coupler of $1\frac{1}{2} \pm \frac{1}{2}$ percent transmittance, there is only a drop in efficiency of up to 4%. At lower input powers, the curve’s peak will shift to the right. Considering this, with the projected losses of this cavity design, and an input power ranging between 1 watt and $\frac{1}{2}$ watt of the fundamental frequency, an input coupler of $1\frac{1}{2} \pm \frac{1}{2}$ transmittance will allow for no more than a 10% drop at perfect impedance matching.

In addition to the correct input coupler, the waist size of the input fundamental laser must be the same size and located at the same spot as the eigenmode beam. This condition is called mode matching. Due to the Brewster’s cut on the crystals, optimal mode matching occurs at the average of the two eigenwaists of the cavity, $\omega_{\text{ave}}$. It can be
shown from an overlap integral that the mode matching efficiency, \( \eta_{mm} \), can be expressed as\(^{13} \)

\[
\eta_{mm} = \left( \frac{2\omega_{ave}\omega_{x,t}}{\omega_{ave}^2 + \omega_{x,t}^2} \right), \tag{44}
\]

where \( \omega_t \) is the waist in the tangential plane and \( \omega_s \) is the waist in the sagittal plane. Focusing the 486nm laser light from a fiber to the average of these two waist sizes will allow for optimal mode matching efficiency of 99.9\% for the projected setup with \( \omega_t = 78[\mu m] \) and \( \omega_s = 74[\mu m] \). A turning mirror is introduced so that the location of the waist can be adjusted for the other component of optimal mode matching. See the appendix for a discussion on the design of the mirror and lens mounts used in this experiment.

**Total Conversion Efficiency**

Assuming a round trip loss of five-tenths a percent, and an input coupler of one and one-half percent with one watt of a 486nm laser incident on the cavity, (19) shows that there is a build-up factor of 71 within the cavity, corresponding to a total conversion efficiency of 65\% into the desired 243nm wavelength, resulting in 650 milli-watts of UV laser light. The effective useful UV laser light power is only 569 milli-watts due to a total of 12.5\% Fresnel losses at the crystal-MgF\(_2\) (0.5\%) and the MgF\(_2\)-air (12\%) interfaces.
CONCLUDING REMARKS

Current Status

Before the build-up cavity is constructed for operational conditions, all the optical components are undergoing quality testing to ensure that they meet the requirements of the low-loss design. The crucial component of the cavity design is the optical contact junction between the CLBO and MgF₂ crystals. Due to the hygroscopic nature of CLBO, a crystal made of Bk7 has been polished to the specifications of the CLBO crystal for testing of this interface. Because the index of refraction is close to that of CLBO for 486nm light, the Bk7 crystal can be easily replaced with the CLBO with only a small adjustment to the cavity geometry.

Future Work

Once the build-up cavity has been set-up and tested with the CLBO crystal, the cavity can be integrated with the 486nm laser source. This will allow for a compact design and a more integrated laser system.
APPENDIX A

THEORETICAL DESCRIPTION OF THE FABRY-PEROT CAVITY
A Fabry-Perot cavity is nothing more than two parallel, partially reflecting mirrors. Three assumptions will be made for the following derivations:

- The mirrors will not absorb energy
- Both mirrors will be perfectly flat, parallel, and will not produce internal interference
- Light will enter the Fabry-Perot cavity parallel to the optical axis

One thing to note about the light that enters the cavity, is that it must fit the standing wave condition

\[ 2d = m\lambda \]  \hspace{1cm} (45)

where \( d \) is the distance between the mirrors, \( m \) is an integer, and \( \lambda \) is the wavelength of the light entering the cavity. Also, Figure 5 suggests that light that travels from one end of the cavity to the other and back, has a phase shift, \( \delta \), such that

\[ \delta = \frac{4\pi d}{\lambda} \]  \hspace{1cm} (46)

The relationship between the transitivity, \( T \), and the reflectivity, \( r \), of the mirrors is represented by

\[ T = 1 - r \]  \hspace{1cm} (47)
The electric field of the light, $E$, incident on the mirror is a complex number

$$E = E_0 e^{-i\omega t},$$  \hspace{1cm} (48)

where $E_0$ is the magnitude of the electric field, $\omega$ is the angular frequency, and $t$ is the time. As shown in Figure 1, when light represented by (48) is incident on the first mirror with a reflectivity $r$, and the second mirror with a reflectivity $r'$, the sum of the reflected energy is

$$E_R = E \left[ -r + T^2 r' e^{-i\delta} \sum_{n=0}^{\infty} \left( r r' e^{-i\delta} \right)^n \right].$$  \hspace{1cm} (49)

This is a geometric series that converges to

$$E_R = E \left[ -r + T^2 r' e^{-i\delta} \frac{1 - T^2 r'^2 e^{-i\delta}}{1 - r r' e^{-i\delta}} \right] \forall \begin{array}{c} r \end{array}, \begin{array}{c} r' \end{array}.  \hspace{1cm} (50)$$

Using Stoke’s Relations of reflected light on a medium,

$$T^2 = 1 - r^2$$  \hspace{1cm} (51)

(50) reduces to

$$E_R = -E \left[ r + r' e^{-i\delta} \frac{1 - T^2 r'^2 e^{-i\delta}}{1 - r r' e^{-i\delta}} \right].$$  \hspace{1cm} (52)

Because we are more interested in the intensity of the light, and intensity is the square of the energy, the intensity of the reflected light, $I_R$, is the square of the absolute value of (50)

$$I_R = E_R^2 = E^2 \left[ \frac{r + r' e^{-i\delta}}{1 - r r' e^{-i\delta}} \right] \left[ \frac{r + r e^{i\delta}}{1 - r r e^{i\delta}} \right] = I_0 \left[ \frac{r^2 + (r')^2 - 2rr' \cos(\delta)}{1 + (rr')^2 - 2rr' \cos(\delta)} \right].$$  \hspace{1cm} (53)

Similarly, the transmitted intensity is
\[ I_r = I_0 \left[ \frac{(1-r)^2(1-r')^2}{1+(rr')^2-2rr'\cos(\delta)} \right] . \]  

(54)

The coefficient of Finesse, \( F \), is defined as the ratio of \( I_R \) to \( I_T \). This yields

\[ F \equiv \frac{I_R}{I_T} = \frac{r^2 + (rr')^2 + 2rr'}{(1-r)^2(1-r')^2} \]  

(55)

The transmittance, also known as the Airy Function after the man who first derived it \( T_{AF} \), of the Fabry-Perot is defined as the ratio of \( I_T \) to \( I_0 \),

\[ T_{AF} \equiv \frac{I_T}{I_0} = \frac{(1-r)^2(1-r')^2}{1+(rr')^2-2rr'\cos(\delta)}. \]  

(56)

Using the trigonometric identity \( \cos(\delta) = 1 - 2\sin^2\left(\frac{\delta}{2}\right) \) and simplifying, (56) becomes

\[ T_{AF} = \frac{1}{1 + F\sin^2\left(\frac{\delta}{2}\right)}. \]  

(57)

Using the small angle approximation, \( \sin(\theta) \approx \theta \), (57) becomes

\[ T_{AF} = \frac{1}{1 + F\left(\frac{\delta}{2}\right)^2}. \]  

(58)

If we look at (58), we see that \( I_T \) is dependent on \( \delta \). If we change the mirror spacing, and thus change \( \delta \), so that we have a standing wave condition inside the cavity such that

\[ \delta = \frac{2}{\sqrt{F}}, \]  

(59)

then half of the maximum intensity is transmitted through the cavity. Let’s assume that we can detect a change in wavelength when the intensity of the transmitted light drops
half of the maximum value. This implies that the smallest change in $\delta$ allowed for by the Fabry-Perot in order to detect a change in wavelength of incident light, $(\delta)_{\text{min}}$, is twice

\[(\delta)_{\text{min}} = \frac{4}{\sqrt{F}}. \tag{60}\]

By taking the derivative of (46) in respect to $\lambda$

\[
\frac{\partial \delta}{\partial \lambda} = \frac{4\pi d}{\lambda^2}, \tag{61}
\]

and approximating that

\[
\delta \delta \equiv \Delta \delta \tag{62}
\]

\[
\delta \lambda \equiv \Delta \lambda \tag{63}
\]

then

\[
\Delta \delta = \frac{4\pi d}{\lambda^2} \Delta \lambda = \frac{2\pi m}{\lambda} \Delta \lambda. \tag{64}
\]

In order to find the smallest detectable change possible for the Fabry-Perot cavity, $(\lambda)_{\text{min}}$, we set

\[
\Delta \delta = (\Delta \delta)_{\text{min}}, \tag{65}
\]

and plug into (64), which yields

\[
(\Delta \lambda)_{\text{min}} = \frac{\lambda^2}{\pi d \sqrt{F}} = \frac{2\lambda}{\pi m \sqrt{F}}. \tag{66}
\]

Resolution, $\Re$, for any system is defined as the ratio of $\lambda$ to $(\Delta \lambda)_{\text{min}}$. For the Fabry-Perot, this is

\[
\Re = \frac{\lambda}{(\Delta \lambda)_{\text{min}}} = \frac{\pi d}{\lambda} \sqrt{F} = \frac{\pi m}{2} \sqrt{F}. \tag{67}
\]
The other aspect of the Fabry-Perot that must be analyzed is the Free Spectral Range (fsr). When comparing two wavelengths, \( \lambda_1 \) and \( \lambda_2 \), a distinction cannot be made between the two wavelengths when the \( m \)th order of \( \lambda_2 \) falls on the \( m + 1 \) order of \( \lambda_1 \). In other words,

\[
m\lambda_2 = (m + 1)\lambda_1. \tag{68}
\]

As a result of

\[
\lambda_2 = \lambda_1 + \Delta \lambda, \tag{69}
\]

(68) becomes

\[
m(\lambda_1 + \Delta \lambda) = (m + 1)\lambda_1 \tag{70}
\]

Solving for the change in wavelength,

\[
(\Delta \lambda)_{fsr} = \frac{\lambda}{m} = \frac{\lambda^2}{2d} \tag{71}
\]

There is an inverse relationship between \( (\Delta \lambda)_{min} \) and \( d \) and a proportional relationship between \( \Re \) and \( d \). Also, the ratio of \( (\Delta \lambda)_{fsr} \) to \( (\Delta \lambda)_{min} \) is a constant,

\[
\frac{(\Delta \lambda)_{fsr}}{(\Delta \lambda)_{min}} = \frac{\pi\sqrt{F}}{2}. \tag{72}
\]

Let’s see how this compares to the Finesse of the Fabry-Perot, which is defined as the ratio of the separation of adjacent fringe maxima to \( (\Delta \delta)_{min} \),

\[
\Im = \frac{2\pi}{(\Delta \delta)_{min}} = \frac{\pi\sqrt{F}}{2}, \tag{73}
\]

which is the same as (72).

Now that a theoretical basis for the Fabry-Perot has been established, we need a way to measure the unique properties of the cavity. In order to write the previous
equations in terms of a frequency, an easily measurable quantity, we must use the well known expression,

\[ f = \frac{c}{\lambda}. \]  
(74)

If we take the derivative of (74), we obtain,

\[ \frac{df}{d\lambda} = \frac{c}{\lambda^2}, \]  
(75)

and by approximating

\[ df \equiv \Delta f \]  
(76)

\[ d\lambda \equiv \Delta \lambda, \]  
(77)

(75) becomes

\[ \frac{\Delta f}{\Delta \lambda} = \frac{c}{\lambda^2}. \]  
(78)

(78) is always positive, so we can ignore the absolute value bars. And by rearranging the (78) so that \( \lambda \) is a function of \( f \),

\[ \frac{\Delta \lambda}{\lambda^2} = \frac{\Delta f}{c}, \]  
(79)

we have an equation that relates \( \Delta \lambda \) to \( \Delta f \). By using the relationship developed in (79) and applying it to (71), the \( fsr \) is then in terms of \( f \) instead of \( \lambda \),

\[ (\Delta f)_{fsr} = \frac{c}{2d}. \]  
(80)

By solving (80) for \( d \),

\[ d = \frac{c}{2(\Delta f)_{fsr}}. \]  
(81)

The Finesse of the cavity can be expressed in the simple formula
\( \frac{(\Delta f)_{fr}}{(\Delta \lambda)_{fr}} = \frac{(\Delta f)_{half-max}}{(\Delta \lambda)_{half-max}}. \) \tag{82}

A simple spectrometer designed for 972nm was made and used to test these relations. The distance between the 99.5% reflective mirrors was about 34 thousands of an inch. \((\Delta f)_{frs}\) was measured at 194GHz \((\Delta f)_{half-max}\) at 2GHz. This corresponds to a measured Finesse of 490 and a measured mirror distance of 30 thousands of an inch.
APPENDIX B
SELLMEIER EQUATION
To find the index of refraction of the CLBO crystal as a function wavelength and polarization, the Sellmeier equation was used for the calculations of this paper.

\[ n_{\omega,e}^2(\lambda) = A_{\omega,e} + \frac{B_{\omega,e}}{\lambda^2} - D_{\omega,e} \lambda^2 \] (83)

Because the Sellmeier coefficients are different from each manufacture of CLBO, an average was used from many sources based on a temperature of 20°C. In order to extrapolate the index of refraction at other temperatures, a linear interpolation constant, \( \alpha \), was used such that \( n_{\omega,e}(\omega,T) = n_{\omega,e}(\omega)[1 + \alpha_{\omega,e}T] \) from empirical data, also averaged from the same manufactures as the Sellmeier coefficients.

<table>
<thead>
<tr>
<th></th>
<th>Ordinary</th>
<th>Extraordinary</th>
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</thead>
<tbody>
<tr>
<td>A</td>
<td>2.2104</td>
<td>2.0588</td>
</tr>
<tr>
<td>B</td>
<td>0.01018</td>
<td>0.00838</td>
</tr>
<tr>
<td>C</td>
<td>-0.01424</td>
<td>-0.01363</td>
</tr>
<tr>
<td>D</td>
<td>-0.01258</td>
<td>-0.00607</td>
</tr>
<tr>
<td>( \alpha )</td>
<td>-0.0019</td>
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APPENDIX C

DESIGN OF THE OPTICAL INTERFACES AND HOUSING STRUCTURE
There are many concerns that need to be addressed from a mode-locked cavity such as the one presented in this paper. Among others, considerations for thermal, vibration, and atmospheric problems need to be addressed. With preliminary experiments, it was noticed that the slightest breeze, such as exhaling near the cavity, would cause a dramatic fluctuation in the stability of the resonator. In order to isolate the optical components from the surrounding atmosphere, a sealed container will surround the entire apparatus. A sixteenths of an inch gasket made of red silicon rubber will cushion the lid and base of the container from each other and allow the input of the 486nm fiber laser to penetrate into the container. To alleviate unnecessary stress on the fiber, the seal is fabricated in the plane of the optical path within the cavity. This will also allow for the possibly of keeping the resonator in a vacuum if warranted in the future.

Enclosing the resonator inside a container also aids in keeping thermo-stability of the resonator. Because the CLBO crystal is hygroscopic, the temperature of the crystal is kept at 150°C. Surrounding atmospheric air will aid in cooling the crystal, an undesired result. The closure will also allow for more of the apparatus to be held at this temperature, thus, allowing for thermal deformations of the cavity to be less of an issue that
would ultimately lead to misalignment of the input laser.

As well as temperature stability over the entire optical cavity, the rigidity of the optical mounts aid in a reduction of vibrational instabilities. There are a total of five optical mounts in the system, four of them are made from a Beryllium-Copper alloy. BeCu is a softer metal that allows for the bending at a hinge, but with a slight springiness so that the metal will return to its original shape if deformed. Pressure between two planes of the metal, connected by a small bridge left in the block that it was machined out from, is applied by a small set screw, increasing the distance between one side of the two planes, and thus, introducing a tilt. Early experiments with the mount found that turning the setscrew a set number of degrees one way, and then bringing it back to the original position, was irreproducible after a short time. This led to the introduction of a ball bearing pressed into the mount that the setscrew pushes against. This addition eliminates scratching of the BeCu by the setscrew, and a very reproducible adjustment. Also, the mounts are attached to a copper plate by precision ground pins and a countersunk setscrew. This allows for the placement of the mounts within a few mils of their correct alignment before the actual alignment of the cavity has started. Once the cavity is aligned, the removal of a mount is trivial and can be replaced to within one mil of its original configuration. If necessary, because both the mounting plate and the mounts are copper, solder can used to make the mounts permanently attached to the mounting plate.
APPENDIX D

ABCD MATRICES
In non-Gaussian optics, the propagation of an optical beam is rather straight forward, being idealized as straight lines that change direction only at optical surfaces. Knowing the position and angle relative to the optic axis, any arbitrary beam can be traced through a set of optical components with ease. It is possible to state the equations of propagation in the form of matrices, and for any arbitrary system, set the constants of the matrices equal to that of your system. The resulting matrices are known as ABCD matrices.\(^{14}\)

For a freespace region of length \(L\) and an index of refraction \(n_0\):

\[
\begin{pmatrix}
1 & \frac{L}{n_0} \\
0 & 1
\end{pmatrix}
\]

where \(L\) goes to \(\frac{Ln_0^3(1 - \sin^2(\theta))}{(n^2 - \sin^2(\theta))^{\frac{3}{2}}}\) in the tangential plane and \(\frac{Ln_0}{(n^2 - \sin^2(\theta))^{\frac{1}{2}}}\) in the sagittal plane for a tilted plane of angle \(\theta\) in respect to the optic axis.\(^{15}\)

For a thin lens of focal length \(f\):

\[
\begin{pmatrix}
1 & 0 \\
\frac{1}{f} & 1
\end{pmatrix}
\]

For a curved mirror with a radius of curvature \(R\):

\[
\begin{pmatrix}
1 & 0 \\
\frac{2}{R} & 1
\end{pmatrix}
\]

where \(R\) goes to \(R\cos(\theta)\) in the tangential plane and \(\frac{R}{\cos(\theta)}\) in the sagittal plane.
Gaussian beams, however, cannot be idealized as straight rays, but the same matrix elements still apply to them. Instead of the position and angle the beam makes in respect to the optic axis, the radius of curvature and the spot size of the beam are used to propagate the beam through the matrix elements. In order to do this, the complex parameter $\tilde{q}$ must be introduced,

$$\frac{1}{\tilde{q}} = \frac{1}{R} - i \frac{\lambda}{\pi \omega^2}. \quad (87)$$

This new parameter, when inserted into the generalized one-dimensional Huygens’ integral, will produce

$$\frac{\tilde{q}_2}{n_2} = \frac{A\tilde{q}_1 + B}{n_1} + \frac{C\tilde{q}_1 + D}{n_1}. \quad (88)$$
REFERENCE


7. Ibid. p. 83.


