ABSOLUTE BETA COUNTING USING THICK SOURCES

APPROVED:

Ira C. Roberts
Major Professor

K. Hanson
Minor Professor

L. R. Miller
Director of the Department of Physics

Jack Johnson
Dean of the Graduate School
ABSOLUTE DATA COUNTING USING THICK SOURCES

THESIS

Presented to the Graduate Council of the North
Texas State College in Partial
Fulfillment of the Requirements

For the Degree of

MASTER OF SCIENCE

By 179804

Miles Edward Anderson, B. S.

Waxahachie, Texas
August, 1950
179804

TABLE OF CONTENTS

LIST OF TABLES ........................................ iv
LIST OF ILLUSTRATIONS ................................ v

Chapter
I. INTRODUCTION ........................................ 1

II. A THEORY OF DEUTERON PARTICLE ABSORPTION AND
    SCATTERING ........................................ 6

III. EXPERIMENTAL PROCEDURE ............................ 18

IV. DISCUSSION OF RESULTS ............................... 31

BIBLIOGRAPHY .......................................... 40
LIST OF TABLES

<table>
<thead>
<tr>
<th>Table</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Initial Counting Rates</td>
<td>23</td>
</tr>
<tr>
<td>2. Data for Calculating $U$</td>
<td>35</td>
</tr>
</tbody>
</table>
LIST OF ILLUSTRATIONS

Figure | Page
--- | ---
1. Beta Flux in Plane Medium | 6
2. Simple Beta Absorber | 8
3. Infinite Absorber-scatterer | 9
4. Absorption of Beta Flux from Source Mounted on an Infinite Backing | 11
5. External Backscattering from an Infinite Absorber-scatterer | 14
6. Radioactive Sample Holder and Counter Assembly | 20
7. Correction Factor for Absorption of Indium Resonance Neutrons in 0.04 Inch Cadmium | 25
8. \( \lambda \) as a Function of Foil Thickness | 27
9. Increase in Counting Rate Due to External Backscattering as a Function of Foil Thickness | 28
10. Backscattering Coefficient \( \beta \) as a Function of Foil Thickness | 29
11. The Product of Bothe's Factors \( \alpha \) and \( \delta \) as a Function of Thickness for 2.5 cm Diameter Indium Foils | 34
12. Saturation Counting Rate as a Function of Foil Thickness | 37
CHAPTER I

INTRODUCTION

Most beta-particle counting with Geiger-Müller tubes involves only the measurement of relative counting rates. There are, however, a number of applications in which absolute beta disintegration rates are required. The determination of neutron activation cross sections is an example. The two principal experimental uncertainties in this determination are (1) the absolute magnitude of the neutron flux and (2) the absolute beta disintegration rate of the activated material.

Burtt\textsuperscript{1} and Zumwalt\textsuperscript{2} have both given expressions relating the observed counting rate of a beta emitter to its absolute disintegration rate. The following expression is the one used by Burtt:

\[ \text{cpm} = \text{dpm} \cdot f_w \cdot f_c \cdot f_a \cdot f_B \cdot f_h \cdot f_s \cdot E, \]

where \text{cpm} is the observed number of counts per minute, \text{dpm} the absolute number of disintegrations per minute, \( f_w \) a factor for the effect of the absorption of beta particles by the counter tube window and the air space between the


\textsuperscript{2}L. R. Zumwalt, \textit{MDDC-1346}, 1.
window and the source, $f_c$ a factor taking into account the finite resolving time of the Geiger-Müller tube and its associated circuit, $f_a$ a factor for the effect of air in scattering beta particles into the counter, $f_b$ a factor for the effect of backscattering by the source holder, $f_h$ a factor for the effects of the walls of the counter housing in scattering beta particles into the counter, $f_s$ a factor for the effect of the mass of the source itself in causing both scattering and absorption of the beta particles, and $E$ the corrected efficiency, i.e., the ratio of counting rate to disintegration rate for a hypothetical weightless source in which there is no backscattering and no absorption between the source and the sensitive volume of the counter.

In order to determine absolute disintegration rates each of the above terms must be evaluated. It should be noted, however, that for a given counter arrangement and using a particular beta emitter only $f_b$ and $f_s$ of the above quantities are not constants. Since the backscattered beta-particles must pass through the source if they are to be counted, the quantity $f_b$ must certainly depend upon the source thickness. Obviously the effect of the mass of the source itself in causing absorption and scattering is a function of the source thickness. The problem with which we shall concern ourselves in this paper is the self-scattering and self-absorption of beta particles by the source.
Although the problem of self-scattering has received very little attention in the literature, a number of methods have been advanced for handling the problem of self-absorption. Three of the earliest methods\(^3\) of approach were to use (1) very thin sources, (2) standard thickness sources, and (3) "infinitely thick" sources. Provided a source is thin enough, absorption may certainly be neglected. This method is not practical, however, with materials of low activity. Efforts have been made to reproduce standard size sources, but the constructional difficulties involved have, in general, outweighed the advantages. The use of "infinitely thick" sources has also been suggested, but here, too, there arises a constructional problem, the problem of obtaining the source material in sufficient quantities. It should be emphasized that the last two methods neither eliminate nor correct for the self-absorption; they merely standardize it. Thus it is clear that unless extremely thin sources are employed, some method must be devised to correct for self-absorption if absolute disintegration rates are to be obtained.

Perhaps the most common method employed to correct for self-absorption is to assume that a simple exponential absorption law holds within the source, which is assumed to be uniformly active. This leads to the following expression:

\[ \frac{I}{I_0} = (1 - e^{-ad})/ad. \]

\(I/I_0\) is the ratio of the beta flux intensity with self-absorption to the beta flux intensity with no absorption, \(d\) is the source thickness, and \(a\) is the absorption coefficient for the source beta-particles in the source material. This expression has been used with varying degrees of success by a number of investigators.\(^4\) This method presents two very evident difficulties, however. No attempt is made to explain the effects of internal scattering, and the method of absorption correction is valid only over a rather limited range of source thicknesses. One observer\(^5\) has been able to obtain satisfactory agreement with experiment for sources as thick as 50 mg/cm\(^2\), but the average range of close agreement seems to be up to about 30 mg/cm\(^2\). This simple method of correction, then, becomes entirely unsatisfactory for


\(^5\) F. C. Larson et al., op. cit., 1207.
"thick" sources, that is, sources with thicknesses in the neighborhood of 100 mg/cm².

More recently a number of purely empirical methods of approach have been presented. Aten⁶ has presented the simple exponential absorption correction in graphical form, but results are quoted only up to source thicknesses of 5 mg/cm². Gueben⁷ has suggested a semi-log plot of observed specific activity against source thickness with the idea that a straight line would result. Extrapolation to zero source thickness would then be simple. Collie, Shaw, and Gale⁸ have recently shown the resultant plot to be very non-linear, however. They have further observed that very thin external absorbers tend to raise rather than lower the apparent specific activity and postulated that this increase is due to beta-particle scattering. It is this question of a combination of absorption and scattering within the source which we shall now analyze theoretically.

⁶ A. H. Aten, op. cit., 70.


CHAPTER II

A THEORY OF BETA-PARTICLE ABSORPTION
AND SCATTERING

Consider a plane medium of thickness $d$ which is beta radioactive with an activity $A$ per unit area as illustrated in Figure 1. If we call the absorption coefficient $\mu$ and the backscattering coefficient $\beta$, then we may write the differential equations for the beta flux inside the medium as follows:

\[ \frac{d\Phi_r}{dx} = -\frac{A}{2d} + \mu \Phi_R - \beta \Phi_L \quad (1) \]

and

\[ \frac{d\Phi_l}{dx} = +\frac{A}{2d} - \mu \Phi_L + \beta \Phi_R \quad (2) \]

---

1C. W. Tittle, unpublished research.
Here we define $\phi_L$ as the beta flux to the left inside the medium and $\phi_R$ as the beta flux to the right inside the medium. (Notice that the co-ordinate $x$ is measured positively toward the left from the right hand surface of the medium.) The variables in the above equations in $\phi_L$ and $\phi_R$ may be separated by differentiating (1) and (2) with respect to $x$ and substituting (1) and (2) into the resultant equations in place of the first order derivatives. Thus we have the two symmetrical equations

$$\frac{d^2 \phi_L}{dx^2} - (\mu^2 - \beta^2) \phi_L = - \frac{A}{2d} (\mu + \beta) \quad (3)$$

$$\frac{d^2 \phi_R}{dx^2} - (\mu^2 - \beta^2) \phi_R = - \frac{A}{2d} (\mu + \beta). \quad (4)$$

If we now define a new quantity $\lambda$ such that

$$\lambda^2 = \mu^2 - \beta^2$$

then the solutions to equations (3) and (4) are

$$\phi_R = c_1 e^{\lambda x} + c_2 e^{-\lambda x} + \frac{A}{2d(\mu - \beta)} \quad (5)$$

$$\phi_L = c_3 e^{\lambda x} + c_4 e^{-\lambda x} + \frac{A}{2d(\mu - \beta)}. \quad (6)$$

The four constants of integration, however, are not independent of one another. On substituting the solutions (5) and (6) back into the original equations (1) and (2) we find that the following relationships exist between the constants:

$$c_3 = \frac{\beta}{\mu + \lambda} c_1 \quad (7)$$

$$c_4 = \frac{\beta}{\mu - \lambda} c_2. \quad (8)$$
Thus the solutions to the differential equations (1) and (2) involve only two arbitrary constants. Consequently, only two boundary conditions on $\phi_L$ and $\phi_R$ combined are required in each medium.

Suppose we now consider two elementary problems. The first problem is to let a diffuse beta-particle flux $\phi_o$ (Figure 2) be incident on the left-hand face of an inactive absorber of thickness $d$ and to find the transmitted flux $\phi_T$.

Since the absorber is inactive, we have $A = 0$. The equations (5) and (6) may then be rewritten as

$$\phi_R = c_1 e^{\lambda x} + c_2 e^{-\lambda x} \quad (5)$$

$$\phi_L = \frac{\beta c_1}{\mu + \lambda} e^{\lambda x} + \frac{\beta c_2}{\mu - \lambda} e^{-\lambda x} \quad (6)$$

In this problem we have the following boundary conditions:

at $x = 0$, $\phi_L = 0$; at $x = d$, $\phi_R = \phi_o$. Thus,
\[ O = \frac{\beta e^1}{\mu + \lambda} + \frac{\beta e^2}{\mu - \lambda} \]  \hspace{1cm} (9)

\[ \phi_0 = c_1 e^{\lambda d} + c_2 e^{-\lambda d}. \]  \hspace{1cm} (10)

Solving equation (9) for \( c_2 \) and substituting into (10) we get

\[ c_1 = \frac{\phi_0 (\mu + \lambda)}{2(\mu \sinh \lambda d + \lambda \cosh \lambda d)}. \]  \hspace{1cm} (11)

Likewise we obtain

\[ c_2 = \frac{-\phi_0 (\mu - \lambda)}{2(\mu \sinh \lambda d + \lambda \cosh \lambda d)}. \]  \hspace{1cm} (12)

Evidently, the transmitted flux equals the flux to the right at \( x = 0 \). Therefore

\[ \phi_t = c_1 + c_2 = \frac{\phi_0 \lambda}{\mu \sinh \lambda d + \lambda \cosh \lambda d}. \]  \hspace{1cm} (13)

For the second problem consider a diffuse flux \( \phi_0 \) incident on the face of an infinite absorber-scatterer and find the backscattered flux \( \phi_\beta \) (Figure 3).
Since $A$ is again zero, we have the same equations of flux as in the first problem. For the infinite absorber-scatterer we have the following set of conditions. At $x = 0$, $\phi_r = \phi_o$, whence

$$\phi_o = \frac{\beta c_1}{\mu + \lambda} + \frac{\beta c_2}{\mu - \lambda}.$$ 

At $x = \infty$, $\phi_r = \phi_l = 0$, whence

$$c_1 = c_2 = 0.$$ 

Hence we have

$$\phi_o = \frac{\beta c_2}{\mu - \lambda},$$

$$c_2 = \phi_o \frac{\mu - \lambda}{\beta}.$$ 

In order to evaluate the backscattered flux we need only to determine the flux to the right at the surface, at $x = 0$. Therefore,

$$\phi_\theta = c_2 = \phi_o \frac{\mu - \lambda}{\beta} = \phi_o \frac{\beta}{\mu + \lambda}.$$ 

(14)

The backscattered flux in problem one is also easily obtained by evaluating $\phi_\theta$ at $x = d$. $\phi_\theta$ is given by equation (6'), where the arbitrary constants have already been evaluated in equations (11) and (12). We have, then, the following expression for the flux backscattered by an absorber-scatterer of finite thickness:

$$\phi_\theta = \phi_o \frac{\beta \sin \lambda d}{\mu \sin \lambda d + \lambda \cosh \lambda d}.$$ 

(15)
Numerical values for the quantities \( \lambda, \beta, \) and \( \mu \) may be obtained from two simple experiments, the first to determine \( \lambda \), the second to determine \( \beta \). Let us consider an active foil mounted on an infinite backing of the same type material. Call the emergent flux \( \phi_e \). Then an absorber of the same type material and of thickness \( d \) is placed over the source as shown in Figure 4. A different flux \( \phi_a \) will now be emitted by the absorber.

![Diagram of absorption of beta flux](image)

**Fig. 4--Absorption of beta flux from source mounted on an infinite backing.**

The flux incident on the absorber at \( x = d \) is greater than \( \phi_0 \) because of repeated backscattering in the absorber and the backing. As a first approximation, though, call it \( \phi_0 \). Then from equation (15) the flux scattered back into the source will be

\[
\phi_b = \phi_0 \frac{\beta \sinh \lambda d}{\mu \sinh \lambda d + \lambda \cosh \lambda d}.
\]  

(16)
This backscattered flux will again be scattered back by the source and backing (acting as an infinite absorber-scatterer) to give a doubly backscattered flux incident on the absorber. From equation (14) we have

\[ \phi_{bb} = \phi_0 \frac{\beta}{\mu + \lambda}. \]

This flux adds to \( \phi_0 \). So as a second approximation, the incident flux on the absorber is

\[ \phi_i = \phi_0 + \phi_{bb} \]

or

\[ \phi_i = \phi_0 \left[ 1 + \frac{\beta^2 \sinh \lambda d}{(\mu + \lambda)(\mu \sinh \lambda d + \lambda \cosh \lambda d)} \right]. \]

The additional flux \( \phi_{bb} \) will also be doubly backscattered so that a better approximation to the true flux incident on the absorber is

\[ \phi_i = \phi_0 + \phi_{bb} + \phi_{bbbb} \]

\[ = \phi_0 + \phi_{bb} \left[ 1 + \frac{\beta^2 \sinh \lambda d}{(\mu + \lambda)(\mu \sinh \lambda d + \lambda \cosh \lambda d)} \right] \]

\[ = \phi_0 + \phi_0 \frac{\beta^2 \sinh \lambda d}{(\mu + \lambda)(\mu \sinh \lambda d + \lambda \cosh \lambda d)} \left[ 1 + \frac{\beta^2 \sinh \lambda d}{(\mu + \lambda)(\mu \sinh \lambda d + \lambda \cosh \lambda d)} \right]. \]

Now define a new quantity

\[ z = \frac{\beta^2 \sinh \lambda d}{(\mu + \lambda)(\mu \sinh \lambda d + \lambda \cosh \lambda d)}. \]

Then we may write

\[ \phi_i = \phi_0 \left[ 1 + z + z^2 \right], \]

and continuing the backscattering process outlined above we obtain the complete expression for the flux incident on the
absorber,

\[ \phi_i = \phi_0 \left[ 1 + \frac{1}{2} + \frac{1}{3} + \ldots \right] = \frac{\phi_0}{1 - \frac{1}{2}}. \]

We can now find the flux transmitted by the absorber from equation (13). The emergent flux is

\[ \phi_e = \phi_0 \cdot \frac{\beta \cosh \lambda d}{1 - \frac{1}{2} \beta \sinh \lambda d \cosh \lambda d + \lambda \cosh \lambda d} \]

which, through the use of equation (17) and the definition of \(\lambda\), may be rewritten as

\[ \phi_e = \phi_0 e^{-\lambda d}. \quad (18) \]

Thus, to determine \(\lambda\) we place an active source on an infinite backing and measure the counting rate with and without an absorber, the ratio of the two counting rates being a simple exponential function of the absorber thickness. It will be noted that the quantity \(\lambda\) is analogous to the mass absorption coefficient commonly referred to in the literature, with the stipulation here that the emitting source must either be infinitely thick or mounted on an infinite backing, and the source, backing, and absorber must all be made of the same material.

Consider now the flux \(\phi_0\) from a source in mid-space (no backing, no absorber). If an infinitely thick backing made of the same material as the source (but inactive) is placed in position against the source as in Figure 5, find the new emergent flux \(\phi_e\).

Assume the flux incident on the backing to be \(\phi_0\) as a first approximation. The analysis is then exactly like that
in the previous problem. Thus, we get the flux incident on the backing to be

\[ \phi_i = \frac{\phi_0}{1 - \bar{z}} \]

where \( \bar{z} \) is given by equation (17). The total backscattered flux at \( x = d \) is then given by equation (14) to be

\[ \phi_B = \frac{\phi_0 \beta}{(1 - \bar{z})(\mu + \lambda)} \]

or

\[ \phi_B = \phi_0 \beta e^{-\lambda d} \left[ \frac{\mu \sinh \lambda d + \lambda \cosh \lambda d}{\lambda (\mu + \lambda)} \right]. \quad (19) \]

This \( \phi_B \) is the flux incident on the source from the left. It is transmitted according to equation (13) so that

\[ \phi_T = \phi_0 \frac{\beta}{\mu + \lambda} e^{-\lambda d}. \]

This transmitted flux, however, is the emergent flux in addition to \( \phi_0 \). Therefore, the total emergent flux is

\[ \phi_E = \phi_0 \left[ 1 + \frac{\beta}{\mu + \lambda} e^{-\lambda d} \right]. \quad (20) \]
If the ratio \( \phi_e/\phi_o \), which is a readily measurable quantity, is defined to be \((1 + k)\), we then have

\[
\kappa = \frac{\beta}{\mu + \lambda} e^{-\lambda d}.
\] (21)

Now write \( \mu \) in terms of \( \lambda \) and \( \beta \) and define \( ke^{\lambda d} \) to be \( m \) to get

\[
\beta = \frac{2m\lambda}{1 - m^2}.
\] (22)

Here it should be noted that \( \beta \) is not a constant, but rather it is a function of the source thickness \( d \), since \( m \) is an exponential function of \( d \).

We have now derived an expression (20) relating the observed flux to the actual flux in so far as external backscattering by the source mounting is concerned. Equation (20), then, corresponds to Burtt's factor \( f_b \). We shall now derive an expression corresponding to Burtt's \( f_s \), a factor taking into account the effects of absorption and scattering within the source itself.

Consider a uniformly active source alone in mid-space. The general equations of flux within the source have already been derived, equations (5) and (6), and the interdependence of the constants of integration shown in equations (7) and (8). The resultant equations are

\[
\phi_e = c_1 e^{\lambda x} + c_2 e^{-\lambda x} + \frac{A}{2d(\mu - \beta)}.
\] (23)

and

\[
\phi_s = \frac{\beta c_1}{\mu + \lambda} e^{\lambda x} + \frac{\beta c_2}{\mu - \lambda} e^{-\lambda x} + \frac{A}{2d(\mu - \beta)}.
\] (24)
To simplify the mathematical manipulations define

\[ Q = \frac{A}{2d(\mu - \beta)}. \]

For the single source suspended in mid-space the boundary conditions on \( \Phi_R \) and \( \Phi_L \) are: \( \Phi_L = 0 \) at \( x = 0 \) and \( \Phi_R = 0 \) at \( x = d \). Substituting these values in equations (23) and (24) gives

\[ Q = c_1 e^{\lambda d} + c_2 e^{-\lambda d} + Q \]  \hspace{1cm} (25)

\[ Q = \frac{\beta c_1}{\mu + \lambda} + \frac{\beta c_2}{\mu - \lambda} + Q. \]  \hspace{1cm} (26)

Solving (26) for \( c_1 \), substituting into (25), and solving for the constant \( c_2 \) we get

\[ c_2 = \frac{Q(\mu - \lambda)[1 - \frac{\mu + \lambda}{\beta} e^{\lambda d}]}{2(\mu \sinh \lambda d + \lambda \cosh \lambda d)}. \]

Similarly

\[ c_2 = \frac{Q(\mu + \lambda)[(\mu - \lambda) e^{-\lambda d} - 1]}{2(\mu \sinh \lambda d + \lambda \cosh \lambda d)}. \]

Now evidently the flux \( \Phi_E \) emerging from the source equals the flux to the right at the surface \( (x = 0) \) of the source. Therefore,

\[ \Phi_E = c_1 + c_2 + Q \]

\[ \Phi_E = \frac{A}{2d(\mu - \beta)} \left[ 1 - \frac{\lambda + \beta \sinh \lambda d}{\mu \sinh \lambda d + \lambda \cosh \lambda d} \right]. \]

Thus we see that our theoretical \( f_s \) relating the counting rate to the real activity is

\[ f_s = \frac{1}{d(\mu - \beta)} \left[ 1 - \frac{\lambda + \beta \sinh \lambda d}{\mu \sinh \lambda d + \lambda \cosh \lambda d} \right]. \]  \hspace{1cm} (27)
If we neglect scattering, then $\beta = 0$ and $\mu = \lambda$. In this case equation (27) reduces as follows:

$$f_s = \frac{1}{\lambda d} \left[ 1 - \frac{\lambda}{\lambda \sin \lambda d + \lambda \cos \lambda d} \right]$$

$$= \frac{1 - e^{-\lambda d}}{\lambda d}.$$  \hspace{1cm} (28)

Equation (28) is comparable with the familiar expression which is derived without considering scattering and assuming a simple exponential absorption law within the source.

We have now derived expressions to be used in correcting for external backscattering (20) and for self-scattering and self-absorption by the source (27). We have also pointed out how simple experiments may be performed to evaluate the unknown quantities involved, (18) and (22). The following chapter will deal with the experimental approach to the problem.
CHAPTER III

EXPERIMENTAL PROCEDURE

General

In all the experiments to be described metallic indium was used as the beta-radioactive source. This indium was obtained in sheets of approximately 100 mg/cm² thickness. Thinner samples were prepared by rolling the commercial indium; thicker samples, by using a combination of thin ones. All foils were cut to uniform dimensions with a 2 cm circular steel punch. Two and one-half centimeter foils were used exclusively throughout the experiments. Average thickness determinations were obtained from a knowledge of the foil areas and weights. Aten¹ has observed that for thin foils only about a 7 per cent change in measured activity occurs in the extreme case in which one-half the source is 50 per cent thicker than the average, the other half 50 per cent thinner than the average. Hence, slight variations in thickness over the surface were ignored for foils used as sources. Care was exercised, however, in selecting uniform foils as absorbers.

The indium sources used were artificially activated using a 50 millicurie radium-beryllium neutron source.

¹A. H. Aten, op. cit., 70.
In order to obtain a thermal neutron flux the activations were carried out in a water tank. The set-up for activating the foils was constructed entirely from lucite and aluminum to minimize neutron absorption in the supports. To keep the neutron source dry, it was housed in the bottom of a vertical aluminum tube. Activations were begun and ended by lowering the neutron source into, and removing it from, the tube. A plunger was used to assure uniformity in the position of the source in the bottom of the tube. Attached to this tube was a small, rigid lucite frame designed to fix the indium foil holders at a specified distance from the source, about 5 cm for most of the experiments. Two types of foil holders were constructed. Each consisted of a flat sheet of material with a circular indentation to aid in centering the foil. One holder was made of lucite, the other of cadmium. The cadmium holder was arranged to slip inside a cadmium shield so that the foil was entirely enclosed by 0.04 inches of cadmium.

All beta counting was done with a Tracerlab TGC-2 thin-window Geiger-Muller tube in conjunction with a Tracerlab 6H-Scaler. The tube was mounted vertically in a standard Tracerlab unshielded sample holder, type SC-10A. The counter arrangement is shown in Figure 6. The backing E and backing holder F are removable. The foil D rests on four small teeth projecting from the sides of the hole in the foil holder C. These teeth have a combined surface area of less than 0.1 per cent of the area of the foils. Thus by either
Fig. 6--Radioactive sample holder and counter assembly. A, Geiger-Muller tube; B, collimating baffle; C, sample holder; D, radioactive sample; E, removable backing; F, backing support.
leaving the backing in position or removing it, the conditions of a foil with no backing and a foil with "infinite" backing can be simulated. The "infinite" backing consists of 200 mg/cm² of indium mounted on one-eighth inch of brass.

Indium has two activities which are induced by the capture of thermal neutrons. One activity has a half-life against beta decay of about 13 seconds, the other about 5 1/4 minutes. All measurements in these experiments made use of the 5 1/4 minute activity. No measurements were made until at least 3 minutes after the end of the activations so that the effects of the 13 second activity were negligible. Since source decay with the 5 1/4 minute activity was appreciable even with short counting periods, all counting data was corrected for decay; that is, all data was corrected back to the initial counting rate at the end of the activation. For this purpose the value of 53.93 ± 0.13 minutes was used for the half-life.² All the data shown has also been corrected for the background counting rate.

**Initial Counting Rate as a Function of Foil Thickness**

In order to obtain a quantitative idea of the way in which saturation counting rate depends upon foil thickness, the following experiment was performed. A set of eight indium foils was prepared with thicknesses ranging from

roughly 20 mg/cm² up to about 250 mg/cm². Each foil was in turn activated in the lucite holder at a distance of 5 cm from the Ra-Be neutron source in water. Activations were not carried to saturation, however. Instead, an activation time of 50 minutes was taken as standard. This short activation time was selected so that all eight foils could be activated and counted during one period of operation. This procedure was adopted to avoid the necessity of correcting for natural day-to-day drift in the counter sensitivity.

Each active foil was mounted on the "infinite" backing and counted for a total of 40 minutes, 20 minutes with each side facing the counter tube. All measurements were corrected for decay and those for a given foil averaged. The results are shown in Table I under the heading C₀ Thermal plus Resonance.

Since our interest was in the activity produced by thermal neutrons, it was necessary to correct for the activity produced by the absorption of indium resonance neutrons (energy at resonance peak = 1.41 electron volts³). This correction was made by re-activating each foil in the cadmium holder. The neutron absorption characteristic of cadmium is such that it absorbs all thermal neutrons but practically none of the indium resonance neutrons. The counting

<table>
<thead>
<tr>
<th>(d) (mg/cm²)</th>
<th>(C_0) Resonance plus Thermal</th>
<th>(C_0) Resonance only</th>
<th>Cd Factor</th>
<th>(C_0) Resonance only Corrected</th>
<th>(C_0) Thermal only</th>
</tr>
</thead>
<tbody>
<tr>
<td>19.7</td>
<td>378.2 ± 3.8</td>
<td>45.6 ± 1.7</td>
<td>1.091</td>
<td>49.7 ± 1.9</td>
<td>328.5 ± 4.3</td>
</tr>
<tr>
<td>35.7</td>
<td>531.1 ± 3.1</td>
<td>55.7 ± 1.4</td>
<td>1.106</td>
<td>61.6 ± 1.5</td>
<td>469.5 ± 3.4</td>
</tr>
<tr>
<td>53.4</td>
<td>638.7 ± 3.3</td>
<td>56.7 ± 1.4</td>
<td>1.119</td>
<td>63.4 ± 1.6</td>
<td>575.3 ± 3.7</td>
</tr>
<tr>
<td>70.4</td>
<td>673.0 ± 3.3</td>
<td>56.5 ± 1.5</td>
<td>1.131</td>
<td>63.5 ± 1.7</td>
<td>609.5 ± 3.7</td>
</tr>
<tr>
<td>98.2</td>
<td>708.7 ± 3.5</td>
<td>55.4 ± 1.4</td>
<td>1.147</td>
<td>63.5 ± 1.6</td>
<td>645.2 ± 3.8</td>
</tr>
<tr>
<td>141.7</td>
<td>681.5 ± 3.4</td>
<td>58.7 ± 1.5</td>
<td>1.163</td>
<td>68.3 ± 1.7</td>
<td>613.2 ± 3.8</td>
</tr>
<tr>
<td>192.5</td>
<td>648.6 ± 3.4</td>
<td>54.3 ± 1.4</td>
<td>1.174</td>
<td>63.7 ± 1.6</td>
<td>584.9 ± 3.8</td>
</tr>
<tr>
<td>249.4</td>
<td>620.3 ± 3.3</td>
<td>53.2 ± 1.4</td>
<td>1.183</td>
<td>62.9 ± 1.6</td>
<td>557.4 ± 3.7</td>
</tr>
</tbody>
</table>
rates due to the absorption by the indium of resonance neutrons only is also shown in Table I as $C_0$ Thermal Only.

The fact that a few resonance neutrons were absorbed by the cadmium was taken into account through the use of a correction factor, which depends upon the indium foil thickness. This factor is plotted in Figure 7. Data for the first three points were obtained from the literature as noted in the graph. The three points beyond 100 mg/cm$^2$ were obtained by an empirical argument concerning the effective energy of indium resonance neutrons for cadmium absorption for various indium foil thicknesses, assuming that such effective energy approaches 1.0 electron volt asymptotically for very large indium foil thicknesses.\textsuperscript{4} Table I also shows the initial resonance counting rates multiplied by the appropriate correction factor obtained from Figure 7.

The final column in Table I shows the initial counting rates induced by thermal neutrons only for each foil thickness used. The uncertainties shown are statistical probable errors. Since the curve of initial counting rate vs. foil thickness passes through a maximum in the neighborhood of 100 mg/cm$^2$, there exists an obvious reason for using thick indium foils as neutron detectors.

\textsuperscript{4}C. W. Tittle, unpublished research.
Fig. 7--Correction factor for absorption of indium resonance neutrons in 0.04 inch cadmium.\footnote{Data for this point obtained from: R. L. Walker, NICC-114, 43.}

\begin{itemize}
  \item \textbf{c.} Data for this point obtained from: J. Kunstadter, \textit{Phys. Rev.}, 78, 484 (1950).
\end{itemize}
Evaluation of $\lambda$

Since we shall presently attempt to correlate the above obtained data with the theories derived in Chapter II, it is necessary to have numerical values for the quantities $\lambda$, $\beta$, and $\mu$.  They may be obtained as indicated in equation (18), Chapter II.

The following procedure was followed to evaluate $\lambda$: Three foils with thicknesses roughly 50, 100, and 150 mg/cm$^2$ were each in turn activated to saturation.  Each foil was counted for a total of 14 hours, 2 hours with an absorber in place and 2 hours without an absorber.  Thus sufficient counts were obtained in all cases to insure a statistical probable error of less than 0.5 per cent.  The absorbers used were 3 cm indium foils 100 mg/cm$^2$ thick.

The results obtained for $\lambda$ are shown in Figure 8.  It should be noted that $\lambda$ is not quite a constant, but rather it depends upon the foil thickness.

Evaluation of $\beta$

The backscattering coefficient $\beta$ was measured in the following manner.  Three foils of thicknesses 30, 50, and 150 mg/cm$^2$ (approximately) were in turn activated.  Then each foil was counted for 2 hours, 1 hour each with and without the "infinite" backing.  All counting rates were corrected for decay and the ratio of initial counting rates with and without backing determined.  This ratio was defined
Fig. 8—$\lambda$ as a function of foil thickness
Fig. 9--Increase in counting rate due to external backscattering as a function of foil thickness.
Fig. 10--Backscattering coefficient $\beta$ as a function of foil thickness.
as \((1+k)\) in Chapter II. The dependence of \((1+k)\) on the foil thickness is shown in Figure 9. Values for the quantity \(\beta\) were computed using equation (22), Chapter II, and the data in Figures 3 and 9. Figure 10 shows \(\beta\) as a function of foil thickness.
CHAPTER IV

DISCUSSION OF RESULTS

The manner in which self-absorption and self-scattering depend upon foil thickness has been studied in both Chapter II and Chapter III. Up to now, however, it has not been pointed out that the quantity $A$, the activity per unit area, is also dependent upon the foil thickness. For artificial radioactivity induced by neutron capture, there are a number of factors governing the activity produced. Neutron flux, exposure time, and the capture cross section of the source material for the neutrons are three such factors. Bothe,\textsuperscript{1} however, has worked out two expressions dependent upon foil thickness which relate the induced activity to the actual neutron flux. The first of these expressions $\alpha$ is the average probability that a neutron will be absorbed on one traversal of the foil. The following expression gives $\alpha$ as a function of $\mu d$, where $\mu$ is the neutron absorption coefficient for the source material and $d$ is the foil thickness:

$$\alpha = 1 - e^{-\mu d} (1 - \mu d) + (\mu d)^2 \frac{\gamma}{(\pi d)}$$  \hspace{1cm} (1)

The quantity $\mu$ is given by

$$\mu = \frac{N_a S}{M}$$  \hspace{1cm} (2)

\textsuperscript{1}W. Bothe, Zeits. f. Phys., 120, 437 (1943).
where \( N_0 \) is Avagadro's number, \( \bar{M} \) is the molecular weight of the foil material, and \( \sigma \) is the total capture cross section of the source material for thermal neutrons. Equation (2) gives a value for \( \mu \) of 0.892 using 170 barns\(^2\) as the total capture cross section of indium for neutrons of 0.032 ev energy. This value of \( \mu \) is used exclusively in the calculation of \( \alpha \).

The second expression advanced by Bothe concerns the depression of the neutron flux in the presence of an absorbing medium. Qualitatively this means that the average neutron flux incident on an absorber is less than the flux at that point in the medium when the absorber is removed. That is, the absorbing medium upon capturing a neutron removes entirely the possibility of this given neutron ever passing through the foil again. Thus, the average flux is depressed. One of Bothe's factors for flux depressions, the one we shall use exclusively, is as follows:

\[
\frac{\kappa}{\kappa + \frac{1}{2} \left( \frac{L}{\lambda} \cdot \frac{3L}{2R + 5L} - 1 \right)}
\]

(3)

where \( R \) is the foil radius, \( \lambda \) is the transport mean free path for thermal neutrons in the moderating medium, \( L \) is the diffusion length for the neutron in the moderator, and \( \alpha \) is given by equation (2). A value of 2.76 \pm 0.03 cm was taken for \( L \) and a value of 0.040 cm was used for the transport mean free path of thermal neutrons in water.\(^3\)


\(^3\)J. G. Beckerley, AECD-2664, 74.
Since $\alpha$ is the probability of neutron capture and $\delta$ is the flux depression factor, we may write that the absolute activity $A$ per unit area is proportional to the product $\alpha \delta H$, where $H$ is the magnitude of the neutron flux with no absorbers present. Thus we see that we are interested in the product $\alpha \delta$ rather than in each individual factor. Figure 11 is a graph of $\alpha \delta$ versus foil thickness for 2.5 cm foils and using the data for water listed above.

We may now re-write Burtt's equation from Chapter I to read

$$cpm = G \cdot \alpha \delta \cdot f_5 \cdot f_6$$

where the constant $G$ includes all those quantities which are independent of foil thickness. Hence, we may write that the observed counting rate of a foil of any thickness is proportional to the four quantities $\alpha$, $\delta$, $f_5$, and $f_6$. This means that the shape of the curve of the product $\alpha \delta f_5 f_6$ versus foil thickness should match that of the curve of observed initial counting rate as a function of foil thickness. We shall now investigate this possibility.

Let us define $\tilde{H}$ to be the product of the four terms $\alpha \delta f_5 f_6$ and then evaluate $\tilde{H}$ as a function of foil thickness. The results are shown in Table II. Values of $\lambda$ are taken from Figure 8, extrapolating along the dotted lines for very thick and very thin foils. $\beta$ is read directly from Figure 10. The product $\alpha \delta$ is obtained from Figure 11, and the factors $f_5$ and $f_6$ computed using the known values for $\lambda$, $\beta$, and $\mu$. 
Fig. 11--The product of Bothe's factors $\alpha$ and $\gamma$ as a function of thickness for 2.5 cm diameter indium foils.
<table>
<thead>
<tr>
<th>( d )</th>
<th>( \lambda )</th>
<th>( \beta )</th>
<th>( \mu )</th>
<th>( f_b )</th>
<th>( f_s )</th>
<th>( u )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.025</td>
<td>20.5</td>
<td>27.45</td>
<td>34.2</td>
<td>1.300</td>
<td>0.906</td>
<td>0.041</td>
</tr>
<tr>
<td>0.050</td>
<td>20.4</td>
<td>14.32</td>
<td>24.9</td>
<td>1.115</td>
<td>0.742</td>
<td>0.079</td>
</tr>
<tr>
<td>0.075</td>
<td>20.0</td>
<td>13.41</td>
<td>24.1</td>
<td>1.068</td>
<td>0.633</td>
<td>0.110</td>
</tr>
<tr>
<td>0.100</td>
<td>19.3</td>
<td>11.20</td>
<td>22.5</td>
<td>1.040</td>
<td>0.534</td>
<td>0.139</td>
</tr>
<tr>
<td>0.125</td>
<td>18.5</td>
<td>9.05</td>
<td>20.6</td>
<td>1.023</td>
<td>0.488</td>
<td>0.165</td>
</tr>
<tr>
<td>0.150</td>
<td>17.5</td>
<td>7.61</td>
<td>19.1</td>
<td>1.014</td>
<td>0.420</td>
<td>0.189</td>
</tr>
<tr>
<td>0.200</td>
<td>15.0</td>
<td>4.27</td>
<td>15.6</td>
<td>1.006</td>
<td>0.358</td>
<td>0.231</td>
</tr>
<tr>
<td>0.250</td>
<td>12.5</td>
<td>1.71</td>
<td>12.6</td>
<td>1.003</td>
<td>0.336</td>
<td>0.267</td>
</tr>
</tbody>
</table>
In order to examine the theory of Chapter II in the light of experimental results, both $U$ and the observed initial saturation counting rates have been plotted as a function of foil thickness in Figure 12. The dark, solid circles represent the experimental results. The small, open squares indicate $U$. The two curves have been arbitrarily normalized at a foil thickness of 75 mg/cm$^2$. As may be seen, the agreement is excellent below about 90 mg/cm$^2$. Above this thickness, however, there is obviously absolutely no correlation between theory and experiment. Nevertheless, it should be remembered that the theoretical curve depends for its existence upon experimental values for $\lambda$, $\beta$, and $\mu$.

Looking back at Figure 10, $\beta$ as a function of foil thickness, we find a reasonable curve, $\beta$ being large for small source thicknesses and small for large thicknesses. This finding is in agreement with the work of Collie, Shaw, and Gale.\textsuperscript{4} Figure 8, $\lambda$ as a function of foil thickness, shows a peculiar characteristic, however. As indicated in the graph $\lambda$ is decreasing rapidly with foil thickness, whereas, intuitively this cannot possibly be so. At very large foil thicknesses $\beta$ becomes a small number and $\lambda$ and $\mu$ are approximately equal. That is, $\lambda$ is essentially the absorption coefficient at large thickness, and it is

Fig. 12—Saturation counting rate as a function of foil thickness.
inconceivable to think that $\lambda$ should get small indefinitely. The curve shown for $\lambda$ would be much more reasonable physically if it were concave upward rather than as shown. The above line of reasoning predicts that $\lambda$ approaches a constant value for large foil thicknesses. We can immediately verify this assumption by evaluating $\psi$ using a larger value of $\lambda$ than that shown in Table II for a thick foil. Taking $\lambda$ to be 17 cm$^2$/gm at 200 mg/cm$^2$ we obtain the point shown as a cross in Figure 12. Thus, it is evident that the divergence of theory and experiment for large foil thicknesses may very well be due to inaccurate measurements of the parameter $\lambda$.

The obvious procedure to follow now is to set up a careful experiment to evaluate $\lambda$ as a function of foil thickness. More data would be useful in preparing a more accurate curve for $\beta$ as a function of foil thickness also. Much work is yet needed to verify Bothe's $\xi$ factor. In fact, it has been suggested$^5$ that quite a different expression may give a better approximation to the foil depression.

An interesting possibility to consider would be the selection of a set of constant values for $\lambda$, $\beta$, and $\mu$ which would give reasonable agreement with experiment over a range of perhaps 200 mg/cm$^2$. The process, however, is tedious, and results obtained to date agree with experiment only to within about 10 per cent.

$^5$C. W. Tittle, unpublished research.
In general the evidence to date is inconclusive. Certainly the theory of Chapter II is completely compatible with experiment up to source thicknesses of about 100 mg/cm². This in itself is far better than obtainable with the simple exponential absorption theory. For very thick sources, however, further investigations must be made into the validity of the theory.
BIBLIOGRAPHY


Beckerley, J. G., ANCD-2661.

Bothe, W., Zeits. f. Phys., 120, 437 (1943).


Zumwalt, L. R., MDDC-1346.