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Multi-mode wavepath depth imaging for the SEG/EAGE salt model
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Summary
Elastic depth imaging of both P-wave and S-wave prestack seismic reflection data is formulated as a degraded form of Kirchhoff migration known as Wavepath Migration (WM). Applications to the SEG/EAGE salt model show that the method is sufficiently versatile and relatively inexpensive. It handles S-wave data with at least the same accuracy as P-wave data when local mode conversions are removed. WM also provides an understanding of multi-mode illumination.

Introduction
Prestack depth migration (PreSDM) is widely used in imaging many subsalt prospects. Over the past decade, several PreSDM techniques have been presented to improve the image of the PP reflections associated with salt overhangs, base of salt and subsalt horizons. In addition to enhanced PP images, S-wave data have been demonstrated to be beneficial to image geological structure beneath high-velocity layers (Purnell, 1992; Bevc et al., 2000; Wu et al., 2001). Specifically, O’Riien et al. (1999) have shown that PS-wave images can improve seismic continuity to delineate salt geometry successfully. In multi-mode processing, it is common practice to enhance individual wave modes prior to PreSDM. However, severe migration artifacts due to incomplete wavefield separation can occur. Moreover, multi-mode PreSDM is not cost effective because it smears all available trace samples along isochrones for various wave types. In order to balance S-wave image quality against computation cost, we introduce a multi-mode extension of the WM formula derived by Sun and Schuster (2001). The method is based on the stationary phase approximation applied to the elastic Kirchhoff integral. This leads to the concept of multi-mode wavepaths whose width corresponds to the size of a Fresnel zone for the given wave type. Numerical examples are performed using results of elastic modeling for the SEG/EAGE salt model (after House et al., 2000).

Method
In PreSDM, we define a grid of image points \( Q \) in the depth domain. Then, using the given interval velocity model, we compute the ray-trace or finite-difference traveltimes \( \tau(\mu)(S,Q) \) and \( \tau(v)(Q,R) \) along the raypaths \( SQ \) and \( QR \) for the sources \( S \) and receivers \( R \). Here, the superscripts \( \mu \) and \( v \) refer to selected wave modes. Assuming that the observed dataset is decomposed into its dominant wavefields, conventional PreSDM smears the trace sample \( u(S,R,t) \) over the isochrone surface \( \Gamma_t \) described by the equation \( t = \tau(\mu)(S,Q,R) \) for the fixed time value \( t \), where \( \tau(\mu)(S,Q,R) \) is the total traveltime along the composite Huygens’ raypath \( SQR \) given by

\[
\tau(\mu)(S,Q,R) = \tau(\mu)(S,Q) + \tau(v)(Q,R)
\]

(a)

(b)

Figure 1: 2-D profile from the SEG/EAGE salt model represented by a 20 m grid: (a) P-velocity \( V_p \) versus (b) S-velocity \( V_S \). The 2-D model dimensions are 13.5×3.4 km. The S-velocity model was obtained using simple empirical relationships between \( V_p \) and \( V_S \) (House et al., 2000).

The stationary phase principle picks out the point \( S_0 \) or \( R_0 \) on the acquisition surface where the direction of the specular (Fermat) raypath is coincident with the direction of the Huygens’ raypath \( SQ \) or \( QR \) for corresponding (common-shot or common-receiver) gathers. This principle...
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also states that the above isochrone surface must be tangent to the depth-migrated reflector at the reflection point \( Q_0 \). Assuming a sufficiently high source frequency yields the stationary phase approximation to the multi-mode PreSDM formula being referred to as the WM formula. WM smears a trace's energy along wavepaths defined by the condition

\[
\epsilon(\mu\nu)(S, Q, R) - \epsilon(\mu\nu)(S_0, Q_0, R_0) < T/2 ,
\]

where \( T \) denotes the dominant wave period which limits the bandwidth of the source spectrum. Since the major contributions to the migration formula occur at locations where the phase is stationary, this approximation is consistent with the elastic migration principle: when the decoupled downward continuation is carried out to the point \( Q \) at the location of the reflector, the phase of the reflected P- or S-wave matches those of the direct wave coming from the point \( S \) or \( R \). Therefore, the enhanced images are formed at these locations through the simultaneous migration of P- and S-waves. Otherwise, WM yields a negligible value.

\[\text{Figure 2: Example of stable upgoing ray tracing after optimal smoothing of the velocity model in Figure 1. Raypaths and wavefronts are shown. Optimal smoothing produces reasonable approximations of traveltimes (1) that can fill in shadow zones caused by salt irregularities.}\]

**Implementation**

As with other PreSDM techniques, the WM approach implicitly assumes that the input interval velocity model is known. Based on the method proposed, a WM algorithm can be implemented as follows:

1. Prestack wavefield separation (pre-processing);
2. Calculate traveltimes (1) for all observation and image points \( S, R \) and \( Q \);
3. Based on the stationary phase principle, find a full set of specular points \( S_0, R_0 \) and \( Q_0 \);
4. Apply the local diffraction stack along wavepaths according to condition (2).

The \( f-k \) or Radon-based dip filtering is suitable for P/S splitting and multiple removal at far-offset traces during step 1. To achieve better illumination, upgoing

\[\text{Figure 3: Time-domain exploding reflector models: (a) PP versus (b) multi-mode time processing. Multiples and local mode conversions are removed at the pre-processing stage.}\]

ray tracing can be undertaken for each image point (step 2). A common type of error is associated with the interface-sampling problem. In order to control accuracy, ray-trace traveltimes are compared with those obtained using existing eikonal solvers in the domains where later arrivals do not occur. The input data required to solve the eikonal equation and ray equations are the original and smoothed velocity models, respectively. According to the exploding reflector model, acoustic reverse-time migration builds a complete set of points \( Q_0 \) along reflectors at depth represented by the energy snapshot at \( Q_0 \) \((t = 0)\).

Following Sun and Schuster (2001), the arrival angle \( \alpha_0 \) is
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determined by performing a local slant stack over a window of traces centered at $S$ or $R$. It is associated with

$$\frac{\partial U}{\partial \omega} = 0$$

The stationary phase principle implies that $S = S_0$ or $R = R_0$ if $\alpha = \alpha_0$, where $\alpha$ specifies the direction of the ray $Q_0S$ or $Q_0R$. Finally, the amplitude value $U$ is spread along a portion of the isochrone surface $\Omega_1$ that surrounds the specular point $Q_0$. This portion is confined by the Fresnel zone of the reflector given by condition (2).

Results

The WM algorithm has been implemented on a 16-node Linux cluster. We have taken a 2-D slice from the SEG/EAGE salt model (Figure 1). Synthetic elastic wave seismograms for the model in Figure 1 (House et al., 2000) and ray-trace illumination studies demonstrate significant coupling between P and S modes. Traveltimes (1) were computed by upgoing ray tracing through the smoothed velocity model (Figure 2). Elastic synthetic data were processed to enhance strongly polarized wave modes prior to PreSDM (step 1). Figure 3 shows acoustic (PP) and elastic (multi-mode) time-domain exploding reflector sections. These sections idealize the best possible stack. It is obvious that most of subsalt structure cannot be reliably interpreted on both sections. In Figure 3, it is very difficult to relate subsalt reflection events with their counterparts in Figure 1. Ideally, migrating P-wave and S-wave data with separate velocity models should image reflectors at correct depths. Even though Figure 4a gives a clear view of the salt body, migration artifacts dominate both PP and SS subsalt images in Figure 4. These artifacts of acoustic Kirchhoff PreSDM are likely due to errors of travelt ime interpolation and frequency-dependent ellipticity of the trajectories of the arrival ellipses caused by strong lateral velocity variations. Recall that traveltimes were computed on coarse grids. Traveltimes on the fine grid were obtained by B-spline interpolation. To obtain a less contaminated image of subsalt reflectors, the same dataset has undergone multi-mode WM imaging tests. It appears that the base of salt image and the subsalt reflectors are defined better on Figure 5 than on Figure 4. By comparing our results with the PP Kirchhoff migration applied to the similar model (Roberts et al., 2001), we conclude that the latter does not have the same high quality as the present method. The reason for the improved WM images is three-fold: (1) accurate wavepath traveltimes can be computed without grid resampling; (2) WM enhances the contribution of the specular point and suppresses false contributions far from that point; (3) WM provides an understanding of illumination. See also Sun and Schuster (2001).

Conclusions

The multi-mode WM method combined with robust wavefield separation techniques is capable to improve subsalt quality from S-wave data alone. The numerical algorithm allows elastic migration to be done inexpensively on PC-based distributed memory clusters.

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Figure 5: Multi-mode WM depth images for the model in Figure 1: (a) PP, (b) SP, (c) PS, and (d) SS. For PS-waves, we use the P-wave velocity to downward continue the wavefield and S-wave velocity for upward propagation below the sea floor. In SP imaging, we assume that downgoing S-waves are converted to P-waves at every image point. Observe that PS WM produces the best image. This is consistent with field data examples by O'Brien et al. (1999). The overall spatial resolution of S-wave images is somewhat higher than that of the PP image.

References