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Inclusive Scattering and Dynamics in Light Nuclei

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Abstract

Inclusive scattering offers a unique opportunity to study nuclear dynamics in the regime of moderate momentum and energy transfers. We show that realistic models of nuclear interactions and current operators provide a quantitative description of the α-particle longitudinal and transverse responses measured in electron scattering. A consistent picture of in-medium nucleon dynamics naturally emerges from an analysis of these (e, e') data and quasi-elastic data from hadronic reactions on heavier nuclei. Its essential features are: (1) a significant quenching of the longitudinal strength; (2) a substantial enhancement of the transverse strength due to the two-nucleon currents required by gauge invariance; and (3) a large shift to higher energies of the isovector strength as observed in the charge-exchange response to hadronic probes. Within this theoretical framework the interactions and currents due to pion exchange play an absolutely crucial role.

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Inclusive scattering in quasi-elastic kin directly probes nucleon propagation in the nuclear medium. In this paper w the longitudinal and transverse responses measured in electron scattering, as well as responses to a variety of idealized single-nucleon couplings. The former are quantitatively reproduced in our calculations based on realistic interactions and current operators, while the latter provide a qualitative understanding of the dynamics. The resulting picture of inclusive scattering is markedly different from that obtained on the basis of naive independent particle models or plane-wave-impulse-approximation calculations.

In particular, the charge-exchange components of the nucleon-nucleon interaction, specifically those associated with pion exchange, shift the longitudinal strength towards the high-energy portion of the spectrum, and consequently quench the response in the region of the quasi-elastic peak. Simple arguments show that an even more pronounced shift will exist for purely isovector single-nucleon couplings, an effect observed in recent (p,n) experiments [1]. In the electromagnetic transverse response, however, the two-nucleon currents required by current conservation play a crucial role over the entire spectrum, and in particular produce a large enhancement near the quasi-elastic peak and in the low energy regime, near threshold.

We address these issues in the framework of Euclidean response functions. In a recent letter we developed a method to calculate response functions in imaginary time (Euclidean response functions), and applied it to the Euclidean proton response of the α-particle [2]. Here we first extend our calculations to a variety of simple single-nucleon couplings, and later incorporate 'realistic' couplings to longitudinally and transversely polarized virtual photons. For these couplings we can directly compare to experimental results on ^{4}He from Bates [3] and Saclay [4].

The response of a quantum many-body system to a weakly coupled external probe is characterized by a function $S(k,\omega)$ defined as:

$$S(k,\omega) = \sum_n |< n|\rho(\mathbf{k})|0 >|^2 \, \delta(\omega + E_0 - E_n) \ ,$$

where E_0 and E_n are the energies of the initial and final states, respectively, and $\rho(\mathbf{k})$ is a suitably chosen coupling. The Euclidean response $E(k,\tau)$ is related to the $S(k,\omega)$ by a Laplace transform:

$$E(k,\tau) = \int_0^\infty \exp(-\tau\omega) S(k,\omega) d\omega = < 0|\rho^\dagger(\mathbf{k}) \exp[-\tau(H - E_0)]\rho(\mathbf{k})|0 > \ .$$

For a given k, $E(k,\tau = 0)$ gives the total strength of the response, while the full $E(k,\tau)$ measures its energy distribution. It is possible to calculate $E(k,\tau)$ straightforwardly in a path-integral representation. The method has been summarily presented in ref. [2], and will be discussed in more detail in a forthcoming paper. There is a close similarity between the present calculation and the Green's Function Monte Carlo simulation of α-particle ground-state properties described in ref [5].

The ^{4}He response functions for a variety of single-nucleon couplings are shown in Fig. 1. The nucleon, proton, isovector, spin-longitudinal, and spin-transverse couplings are defined, respectively, as:

$$\begin{aligned}
\rho_N(\mathbf{k}) &= \sum_i e^{i\mathbf{k}\cdot\mathbf{r}_i} , \\
\rho_p(\mathbf{k}) &= \sum_i e^{i\mathbf{k}\cdot\mathbf{r}_i} \frac{1+\tau_z(i)}{2} , \\
\rho_\tau(\mathbf{k}) &= \sum_i e^{i\mathbf{k}\cdot\mathbf{r}_i} \tau_+(i) , \\
\rho_{\sigma\tau L}(\mathbf{k}) &= \sum_i e^{i\mathbf{k}\cdot\mathbf{r}_i} (\sigma_i \cdot \hat{\mathbf{k}}) \tau_+(i) , \\
\rho_{\sigma\tau T}(\mathbf{k}) &= \sum_i e^{i\mathbf{k}\cdot\mathbf{r}_i} (\sigma_i \times \hat{\mathbf{k}}) \tau_+(i) .
\end{aligned}$$

For each coupling ρ_α there is an associated response E_α. Note that the spin response functions $E_{\sigma\tau L}$ and $E_{\sigma\tau T}$ defined here are purely isovector, and that neglecting isospin-breaking interactions one may replace τ_+ by τ_z when calculating $E(k,\tau)$ for an isoscalar target. The spin-independent isovector response E_τ is simply a weighted average of $E_{\sigma\tau L}$ and $E_{\sigma\tau T}$.

Most immediately apparent in Fig. 1 is the strong isospin dependence in the response. This arises naturally in any model incorporating charge exchange. For example the proton response $E_p(k,\tau)$ measures the propagation of charge in imaginary time in a nucleus, and can be written as

$$E_p(k,\tau) = \sum_{ij} \int d\mathbf{r}_{ij}\; j_0(kr_{ij})\; E_p(\mathbf{r}_{ij},\tau) ,$$

where $\mathbf{r}_{ij}$ is the distance between the initial position of proton i at time 0 and final position of proton j at time τ, $\mathbf{r}_{ij} \equiv \mathbf{r}_i - \mathbf{r}'_j$. In the limit $\tau \to 0$, the propagator $< \mathbf{R}|e^{-\tau H}|\mathbf{R}' > \to \delta(\mathbf{R}-\mathbf{R}')$. As τ increases the nucleons move, the imaginary-time free particle propagator is proportional to $\exp[-(m/2\tau)(\mathbf{R}-\mathbf{R}')^2]$. In addition, the charge exchange terms in the interaction shift the charge from one nucleon to another, substantially reducing the contribution of the incoherent ($i=j$) terms to the response. The difference between the nucleon and proton response indicates the importance of the charge-exchange mechanism in quasi-elastic scattering.

For short imaginary times τ, this difference can be described as a decreased effective mass for probes coupling to the nuclear charge. In a naive independent particle model, one would naturally be led to ascribe a bigger charge radius to the proton. In fact, this 'increased radius' is a simple consequence of nuclear dynamics.

The above dynamical mechanism becomes even more important in the purely isovector channel. Since ^{4}He is an isoscalar target, up to Coulomb effects $E_p = (E_N + E_\tau)/2$, implying a much more substantial shift of strength from the quasi-elastic peak towards higher energies in E_τ than in E_N. Indeed, this effect has recently been observed in comparisons of quasi-elastic spectra for $(p,p\prime)$ and (p,n) reactions measured for a variety of nuclear targets [1], see Fig. 2. As these hadronic probes do not couple weakly to the nucleus, care should be taken in interpreting the experimental results. Nevertheless, it is clear that the basic difference between the $(p,p\prime)$ and (p,n) reactions lies in the different isospin nature of the couplings, to which ρ_N and ρ_τ are only a rough approximation. Therefore, the essential feature of the empirical spectra, namely the signicantly stronger shift of strength for isovector couplings, has a simple dynamical interpretation: it is a manifestation of the charge-exchange character of the underlying nucleon-nucleon force. It should be noted that these same experiments have also measured spin polarization observables in an attempt to separate $S_{\sigma\tau,L}$ and $S_{\sigma\tau,T}$ [6]. In contrast to a naive interpretation of the experimental results, we do find excess strength in the longitudinal channel. However, the calculated enhancement $E_{\sigma\tau,L}/E_{\sigma\tau,T}$ is much smaller than that obtained in traditional random-phase-approximation calculations. We will address these questions thoroughly in a separate paper.

The electromagnetic transverse response is also predominantly isovector, and hence one might expect a strong quenching there as well. However, such an effect is not observed. In Fig. 3 we show the experimental longitudinal and transverse responses rescaled as:

$$R_L^*(k,\omega) = \frac{1}{Z} R_L(k,\omega) \ ,$$

$$R_T^*(k,\omega) = \frac{2m^2}{k^2} \frac{1}{Z\mu_p^2 + N\mu_n^2} R_T(k,\omega) \ ,$$

where μ_p and μ_n are the proton and neuton magnetic moments. In PWIA the ratio R_T^*/R_L^* is one, whereas it is found to be much larger experimentally. Obviously, one cannot expect the transverse response to be described adequately in terms of single-nucleon currents. Gauge invariance requires that the same charge-exchange interaction that quenches the longitudinal response produce large exchange-current corrections in the transverse response.

The calculated $E_L(k,\tau)$ and $E_T(k,\tau)$ are compared with the experimental data in Figs. 4 and 5. In each of them we present results for both one-body and one-plus-two-body charge and current operators. These have been described in detail in refs. [7-8]. Here we note only that, by far, the leading two-body contributions in both the longitudinal and transverse channels are those associated with pion exchange.

They are determined by the spin-spin and tensor charge-exchange components of the nucleon-nucleon interaction, and in this sense can be viewed as model-independent. Furthermore, the two-body part of the current operator also includes a small model-dependent contribution due to the virtual excitation of Δ-resonances. We should point out that the $1/r^2$ terms in the pion (and ρ-meson) two-body operators have been cut off at short distances in order to yield finite sum rules. This cut-off makes little difference for finite τ, though, as there the high-energy response is exponentially suppressed. It should be emphasized that these charge and current operators provide a very satisfactory description of the elastic form factors of the A=3 and 4 nuclei [9], and of the deuteron structure functions [10] in the momentum transfer range of interest here ($k \leq 600$ MeV/c). However, the present theory fails to reproduce the observed deuteron tensor polarization [11-12], although more accurate data are needed in order to firmly resolve the issue [13].

Our results can be directly compared to experiment only for $\tau \geq 0.015$-0.02 MeV^{-1}, since for these τ-values the unobserved strength at high energies is suppressed. It is possible to reliably estimate this strength by means of sum-rule techniques [14]. We show the effect of including this tail contribution to the the Laplace transform of the experimental response by the curve labelled 'extrapolated'. The points with error bars are obtained by integrating the response up to the maximum measured energy, we see that the extrapolation has essentially no effect for $\tau \geq 0.015$ MeV^{-1}. Calculations with both impulse and full charge operators are shown, the former includes only the proton and Darwin-Foldy contributions to the charge operator.

A similar situation exists in the transverse channel, where the response through the delta resonance is dominant at high ω. Here we simply show the truncated responses obtained from the Bates [3] and Saclay [4] experiments. Since the Saclay measurements extend to higher ω, they naturally lead to an increased response near $\tau = 0$. Again, though, the effects of this high energy strength are rapidly suppressed at finite τ, so that the two measurements are nearly identical by $\tau \sim 0.02$ MeV^{-1}. The difference between the Bates and Saclay measurements at large τ is associated with a ~ 5 MeV relative shift in the two responses.

Of course, the theoretical framework employed here can only be expected to provide a description of the quasi-elastic region of the transverse response. It does not include pion production nor a dynamic treatment of the Δ-resonance, and hence cannot explain the response in the Δ-peak region. However, for $\tau \geq 0.02$ MeV^{-1} a static parametrization of the currents associated with virtual delta production, such as the one used in the present work, should be adequate, since the product $\tau \Delta E \gg 1$, where ΔE is a typical $\Delta - N$ energy excitation.

As shown in the figures, the agreement between theory and experiment is very good. In order to achieve this agreement, it is absolutely essential to include both a realistic Hamiltonian for the treatment of initial and final states, as well as consistent and realistic current operators. Neglect of either aspect leads to erroneous conclusions. In particular, using one-body currents with a realistic Hamiltonian produces a drastic underestimation of the transverse response. A complete description of the calculations, along with a detailed discussion of the individual components of the response for a wider range of kinematics, will be presented separately.

Although the agreement between theory and experiment is satisfactory in this regime, a variety of important physics issues remain in inclusive scattering experiments. They include: microscopic calculations of response functions in heavier nuclei, description of the pion and delta electroproduction region, effects of final-state interactions and two-body currents on polarization observables, and response to other probes, including the weak interaction couplings probed in parity-violating electron scattering. Inclusive scattering remains an important area for studying nuclear dynamics, and a rich field for both theory and experiment.

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Figure Captions

- Fig. 1. The ^{4}He Euclidean responses at $k = 350$ MeV/c for the single-nucleon couplings given in the text. Each response has been normalized such that $E_\alpha(k \rightarrow \infty, \tau = 0) = 1$.
- Fig. 2. Centroids of (p,p′) and (p,n) quasielastic response functions. The solid line corresponds to the free particle energy $k^2/2m$.
- Fig. 3. The experimental longitudinal and transverse (e,e′) response functions at $k = 400$ MeV/c, scaled as explained in text.
- Fig. 4. ^{4}He longitudinal and transverse responses at $k = 300$ MeV/c. The GFMC calculations with (GFMC-Full) and without (GFMC-Impulse) two-body corrections to the electromagnetic couplings are compared with the Bates and Saclay data.
- Fig. 5. ^{4}He longitudinal and transverse responses at $k = 400$ MeV/c (as in Fig. 4).

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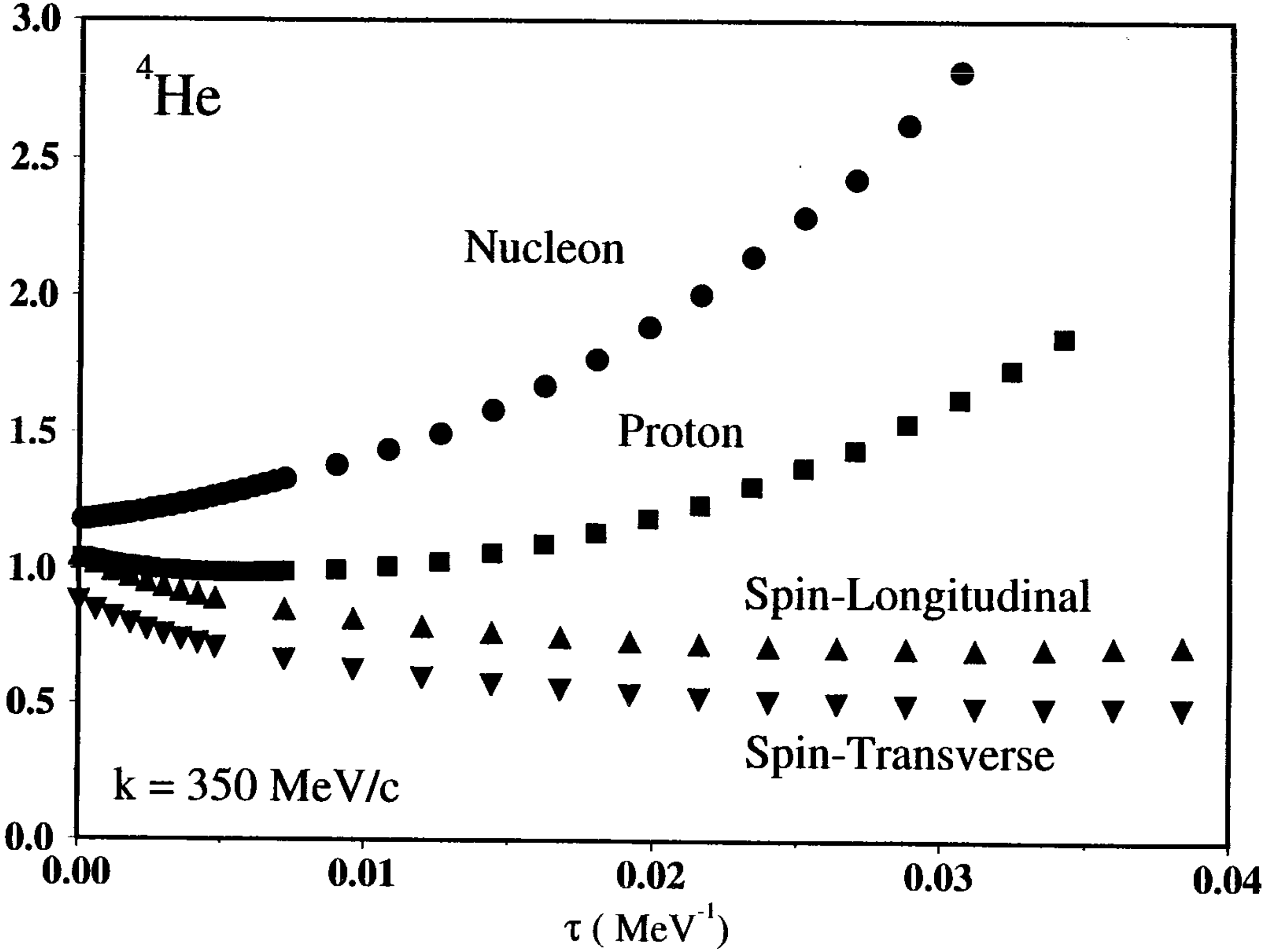
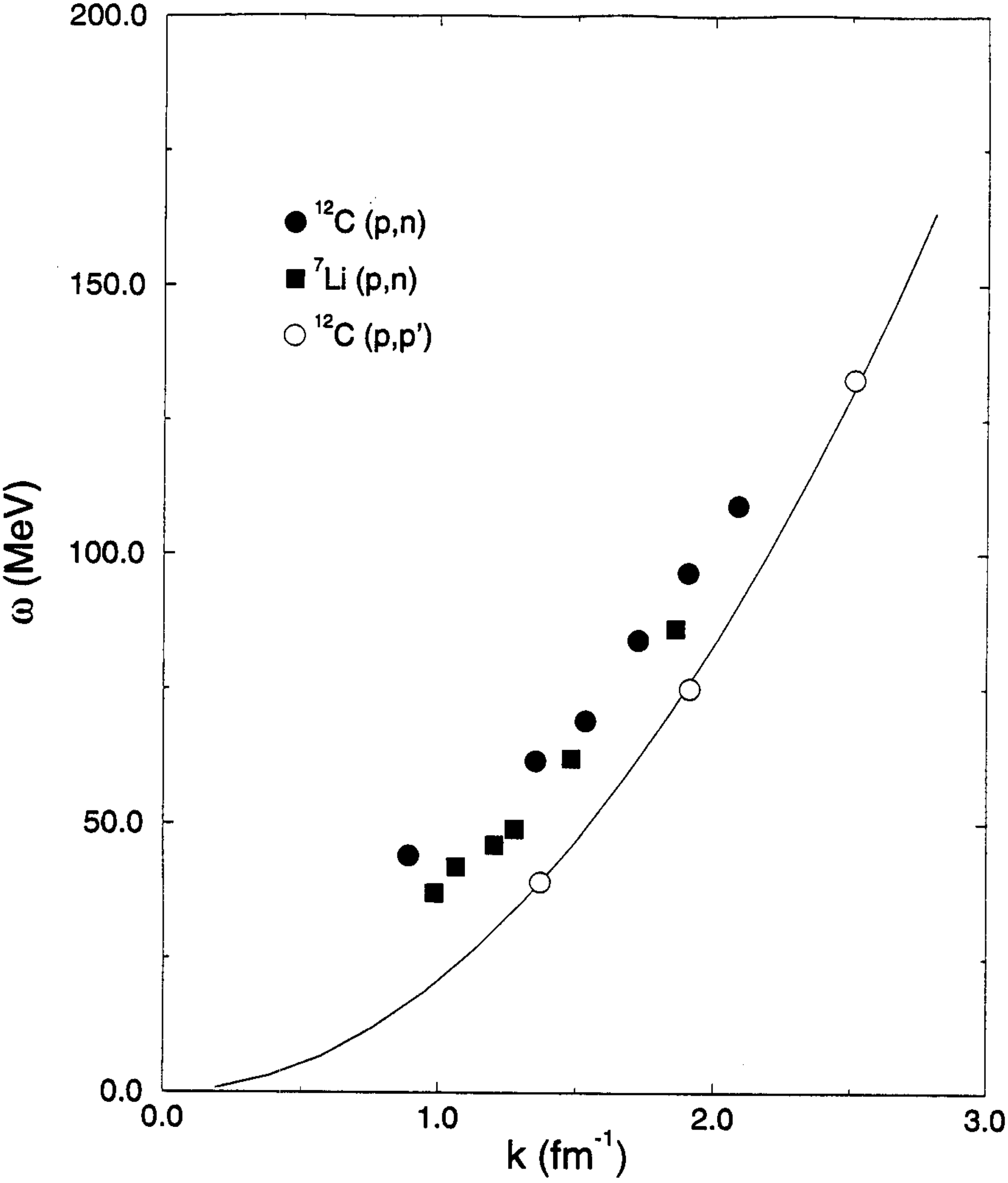
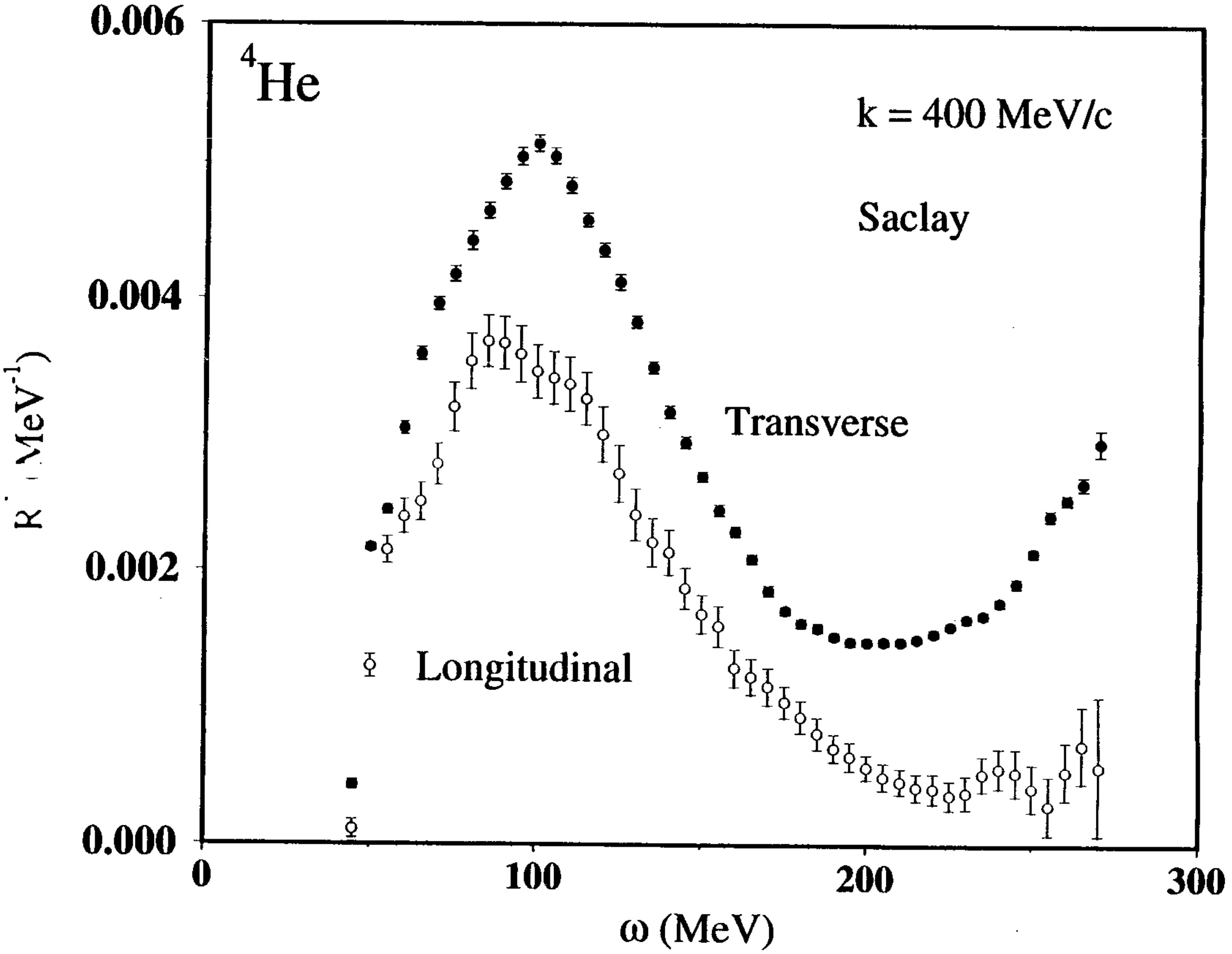


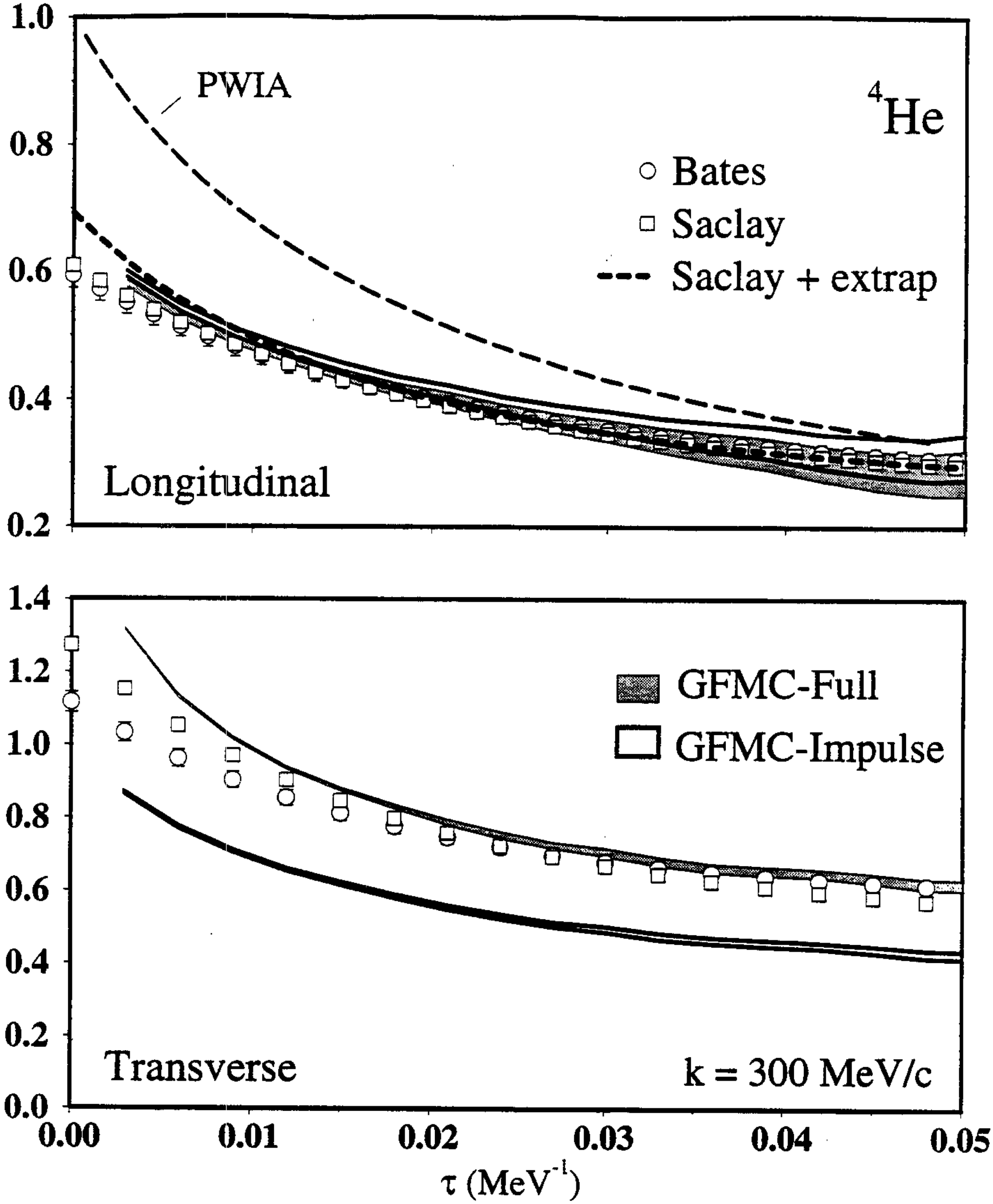
3.0
2.5
2.0
1.5
1.0
0.5
0.0
0.00
0.01
0.02
0.03
0.04
^{4}He
Nucleon
Proton
Spin-Longitudinal
Spin-Transverse
k = 350 MeV/c
τ (MeV^{-1})



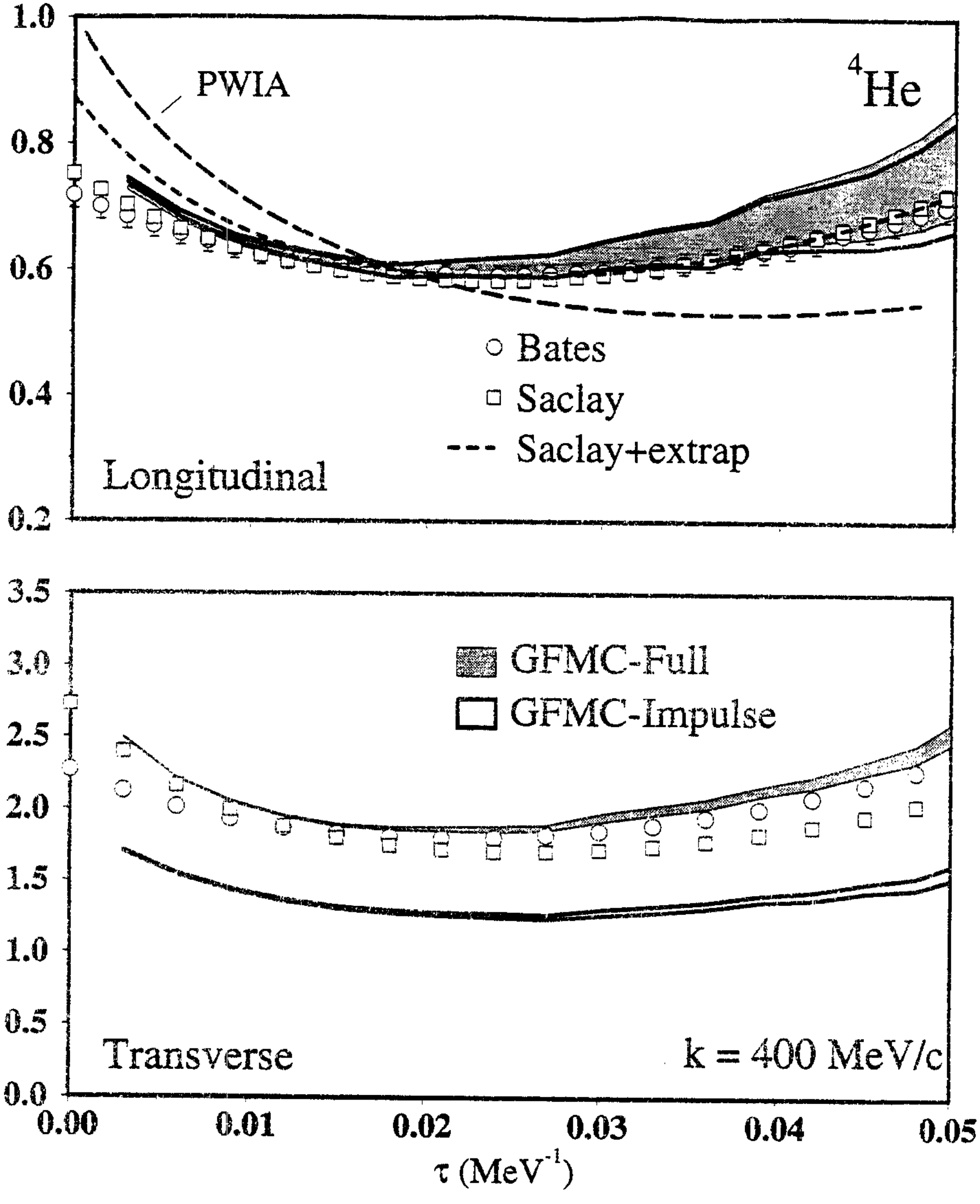
● ^{12}C (p,n)
■ ^{7}Li (p,n)
○ ^{12}C (p,p')
ω (MeV)
k (fm^{-1})
200.0
150.0
100.0
50.0
0.0
0.0
1.0
2.0
3.0



^{4}He
k = 400 MeV/c
Saclay
Transverse
Longitudinal
0.006
0.004
0.002
0.000
0
100
200
300
ω (MeV)
R (MeV^{-1})



1.0
0.8
0.6
0.4
0.2
PWIA
^{4}He
○ Bates
□ Saclay
--- Saclay + extrap
Longitudinal
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0.0
▨ GFMC-Full
☐ GFMC-Impulse
Transverse
k = 300 MeV/c
0.00
0.01
0.02
0.03
0.04
0.05
τ (MeV^{-1})


1.0
0.8
0.6
0.4
0.2
PWIA
^{4}He
○ Bates
□ Saclay
--- Saclay+extrap
Longitudinal
3.5
3.0
2.5
2.0
1.5
1.0
0.5
0.0
▨ GFMC-Full
☐ GFMC-Impulse
Transverse
k = 400 MeV/c
0.00
0.01
0.02
0.03
0.04
0.05
τ (MeV^{-1})