Fermilab

$\bar{p}$ Note #450

*Longitudinal Phase space in Booster to
Debuncher/Accumulator Beam Transfers*

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LONGITUDINAL PHASE SPACE IN BOOSTER TO DEBUNCHER/ACCUMULATOR BEAM TRANSFERS

(A) FORMULAE

β = velocity

ω_{rf} = angular rf frequency

E_s = beam energy

V = rf voltage

e = proton charge

h = harmonic number

η = momentum compaction factor

W_B = bucket height

A_B = bucket area

A = beam emittance

W = beam Δp

f_s = synchrotron frequency

f_0 = revolution frequency

then

$$A_B = \frac{8\beta}{\omega_{rf}} \left[\frac{2 E_s (\text{eV})}{\pi h \eta} \right]^{\frac{1}{2}}$$

$$W_B = \frac{A_B \omega_{rf}}{8}$$

where A, A_B are in eV-sec

$$W = W_B \sin \frac{\phi}{2}$$

and W, W_B are in eV

$$A = \frac{\pi W \phi}{\omega_{rf}} \Rightarrow \phi \sin \frac{\phi}{2} = \frac{A \omega_{rf}}{\pi W_B}$$

$$f_s = f_0 \left[\frac{h \eta (\text{eV} \cos \phi)}{2\pi E_s} \right]^{\frac{1}{2}}$$

(B) Booster Beam To Accumulator

Booster beam emittance = 0.08 per bunch (measured)
EXP IIIA

Booster r.f. voltage at extraction = 400 kV

In the booster $h=84$, $\eta=0.0226$, $\omega_{rf} = 2\pi 52.813 \times 10^6 = 3.3183 \times 10^8$
 $E_s = 8.89 \times 10^9$, $\beta \sim 1$

$$A_B = \frac{8\beta}{\omega_{rf}} \left[\frac{2 E_s \text{ eV}}{\pi h \eta} \right]^{\frac{1}{2}} = 0.8325 \text{ eV-sec}$$

$$W_B = \frac{A_B \omega_{rf}}{8} = 34.532 \text{ MeV}$$

$$A = 0.08 = \frac{\pi W \phi}{\omega_{rf}} ; W = W_B \sin \frac{\phi}{2}$$

$$\Rightarrow \phi = 0.71 \text{ radians} \quad W = 12.0 \text{ MeV}$$

$$= 0.226\pi$$

Since the Accumulator $\eta = 0.024$ is about the same as the Booster to make a matched bucket to bucket transfer we would need to make the same voltage in the Accumulator as the Booster. Clearly 400kV in the Accumulator is impossible. Alternatives are

❖ If nothing can be done in the Booster then and the maximum voltage in the accumulator is 100kV (pulsed) and 50kV (CW) then

At 100 kV in the Accumulator

$$A_B = 0.404 \text{ eV-sec}$$

$$W_B = 16.75 \text{ MeV} \quad (.508\pi)$$

$$\sin \frac{\phi}{2} = \frac{W}{W_B} = \frac{12}{16.75} = 0.716 \text{ radian} \left(\frac{\pi}{1.96} \text{ i.e. } \sim 90^\circ \right)$$

$$A = \frac{\pi W \phi}{\omega_{rf}} = 0.1817 \Rightarrow \text{dilution} = 2.27$$

We reduce to 50kV in the Accumulator adiabatically

then A remains the same : $A_B = 0.286$, $W_B = 11.84$

$$A = 0.1817 = \frac{\pi W \phi}{W_{rf}} \quad \text{and} \quad W = W_B \sin \frac{\phi}{2}$$

$$\Rightarrow \phi \sin \frac{\phi}{2} = \frac{0.1817 W_{rf}}{\pi \times 11.84 \times 10^6} = 1.62 \Rightarrow \phi \sim 0.62 \pi$$

another scenario is to snap down from 100 kV down to a matched bucket after a $\frac{1}{4}$ turn in longitudinal phase space.

If the beam spot can fit inside a 100 kV bucket then ideally there will be no dilution in the beam phase space

Coming out of the ~~Linac~~ Booster $W = 12 \text{ MeV}$
 $\phi = 0.71 \text{ radians}$

at 100 kV in the Accumulator

$$A_B = 0.404 \text{ eV-sec} , W_B = 16.75 \text{ MeV} \parallel A = 0.08 \text{ eV-sec}$$

$A \pm 12 \text{ MeV}$ beam can fit into this bucket.

What is the rf voltage that the v.f. should be reduced to, to form the matched bucket?
 at 100 kV

$$(1) W = 12.0 , \text{ then } \sin \frac{\phi}{2} = \frac{12}{16.75} \Rightarrow \phi = 0.508 \pi$$

$$(2) \phi = 0.71 \quad \text{then } W = W_B \sin \frac{\phi}{2} \Rightarrow W = 5.82 \text{ MeV}$$

$\Rightarrow$ Matched bucket is that which contain $\pm 5.82 \text{ MeV}$ in $\phi = 0.508 \pi$

$$\Rightarrow W_B = \frac{5.82}{\sin(\frac{0.508\pi}{2})} = 8.13 \text{ MeV} \quad eV = \frac{W_B^2 \pi h \eta}{2 E_s} = 24 \text{ kV}$$

$$\text{at 100 kV} \quad \text{synchrotron frequency} = f_0 \left[\frac{h \eta eV \cos \phi}{2 \pi E_s} \right]^{\frac{1}{2}} = 1140 \text{ kHz}$$

$\Rightarrow \frac{1}{4}$ turn happens in $\sim 220 \mu\text{seconds}$.

(C) Booster Beam To Debuncher

We would like to capture beam in the $h=4$ system (DRF3) in the debuncher

The maximum voltage that DRF3 can generate is 1.8 kV

For a ~~bunch~~^{beam} that is $\pm \frac{\pi}{2}$

$$A_B = \frac{8}{\omega_{rf}} \left[\frac{2E_s (\text{eV})}{\pi h \eta} \right]^{\frac{1}{2}} \quad \begin{array}{l} \omega_{rf} = 1.483 \times 10^7 \\ \eta = 0.0055 \\ h = 4 \end{array}$$

$$= 11.608$$

$$W_B = 21.52 \text{ MeV} \left(\frac{A_B \omega_{rf}}{8} \right)$$

$$W = 15.21 \text{ MeV} \left(W_B \sin \frac{\phi}{2} \right)$$

$$A = \frac{\pi W \phi}{\omega_{rf}} = 5.06 \text{ eV-sec.}$$

Incoming into the debuncher from the booster are bunches that are $\pm 12 \text{ MeV}$ and $\frac{0.71}{\pi} \times 18.93 \text{ ns}$ wide

$$= 4.28 \text{ ns wide} \quad \left(\text{at } A = 0.08 \text{ per } 53 \text{ MHz bunch} \right)$$

$V_{rf} = 400 \text{ kV}$
in the booster

For an ellipse

$$W' = W \cos \alpha \quad \text{to trace out the}$$

$$\phi' = \phi \sin \alpha \quad \text{Circumference}$$

$$\Rightarrow \cos \alpha = \frac{W'}{W} = \frac{12.0}{15.21} \div$$

$$\Rightarrow \phi' = \frac{\pi}{2} \sin \alpha = 0.965$$

$$\Rightarrow \# \text{ of bunches} = \frac{0.965 \times 22.5}{\frac{\pi}{2}} = 6.91$$

22.5 53 MHz =
1 h=4 bunch.

$$\text{Width of beam} = 18.93 \text{ ns} / 53 \text{ MHz bunch.}$$

$$= 130 \text{ ns}$$

If we can time the $h=4$ bucket so that the beam always arrives in time then we can capture about 7 r.f. buckets from the booster

If there is a ± 50 ns jitter in the timing of the $h=4$ bucket relative to the beam

then we can only use $130 - 100 = 30$ nsec of the beam

This corresponds to 2 r.f. (53 MHz) buckets.

(ie. $18.93 + 4.28$ ns < 30 ns)