Longitudinal Phase space in Booster to Debuncher/Accumulator Beam Transfers

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LONGITUDINAL PHASE SPACE IN BOOSTER TO
DEBUNCHER/ACCUMULATOR BEAM TRANSFERS

(A) FORMULAE

\[ A_B = \frac{8\beta}{\omega_{rf}} \left[ \frac{2E_s (eV)}{\pi\eta} \right]^{\frac{1}{2}} \]

\[ W_B = \frac{A_B \omega_{rf}}{8} \]

\[ W = W_B \sin \frac{\phi}{2} \]

\[ A = \frac{\pi W \phi}{\omega_{rf}} \Rightarrow \phi \sin \frac{\phi}{2} = \frac{A \omega_{rf}}{\pi W_B} \]

\[ f_s = f_0 \left[ \frac{\hbar \eta (eV \cos \phi)}{2\pi E_s} \right]^{\frac{1}{2}} \]

\( \beta = \) velocity
\( \omega_{rf} = \) angular rf frequency
\( E_s = \) beam energy
\( V = \) rf voltage
\( e = \) proton charge
\( h = \) harmonic number
\( \eta = \) momentum compaction factor
\( W_B = \) bucket height
\( A_B = \) bucket area
\( A = \) beam emittance
\( W = \) beam \( \Delta \phi \)
\( f_s = \) synchrotron frequency
\( f_0 = \) revolution frequency

where \( A, A_B \) are in \( eV \)-secs
\( W, W_B \) are in \( eV \)
Booster Beam To Accumulator

Booster beam emittance = 0.08 per bunch (measured)

Booster r.f. voltage at extraction = 400 kV

In the booster

\[ h = 8 \text{f}, \quad \eta = 0.0226, \quad \omega_{rf} = 2\pi \times 52.813 \times 10^6 = 3.3183 \times 10^8 \]

\[ E_s = 8.89 \times 10^9, \quad \beta \sim 1 \]

\[ A_B = \frac{8\beta}{\omega_{rf}} \left[ \frac{2 E_s \text{ ev}}{\pi h \eta} \right]^{\frac{1}{2}} = 0.8325 \text{ ev-sec} \]

\[ W_B = \frac{A_B \omega_{rf}}{8} = 34.532 \text{ MeV} \]

\[ A = 0.08 = \frac{\pi W \phi}{\omega_{rf}} \quad \Rightarrow \quad W = W_B \sin^2 \frac{\phi}{2} \]

\[ \Rightarrow \quad \phi = 0.71 \text{ radians} \quad W = 12.0 \text{ MeV} \]

Since the Accumulator \( \eta = 0.024 \) is about the same as the Booster to make a matched bucket to bucket transfer we would need to make the same voltage in the Accumulator as the Booster. Clearly 400 kV in the Accumulator is impossible. Alternatives are

(a) If nothing can be done in the Booster than and the maximum voltage in the accumulator is 100 kV (pulsed) and 50 kV (CW) then

At 100 kV in the Accumulator

\[ A_B = 0.404 \text{ ev-sec} \]

\[ W_B = 16.75 \text{ MeV} \]

\[ \sin \frac{\phi}{2} = \frac{W}{W_B} = \frac{12}{16.75} = 0.70 \text{ radian} \quad \left( \frac{\pi}{1.96} \approx \sim 90^\circ \right) \]

\[ A = \frac{\pi W \phi}{\omega_{rf}} = 0.1817 \quad \Rightarrow \text{dilation} = 2.127 \]

We reduce to 50 kV in the Accumulator adiabatically
then $A$ remains the same: $A_B = 0.286$, $W_B = 11.84$ eV-sec

$$A = 0.1817 = \frac{\pi W \phi}{W_B} \quad \text{and} \quad W = W_B \sin \frac{\phi}{2}$$

$$\Rightarrow \phi \sin \frac{\phi}{2} = \frac{0.1817 W_B}{11.84 \times 10^6} = 1.62 \Rightarrow \phi \approx 0.62 \pi$$

another scenario is to snap down from 100 kV down to a matched bucket after a $\frac{1}{4}$ turn in longitudinal phase space.

If the beam spot can fit inside a 100 kV bucket then ideally there will be no dilution in the beam phase space.

Coming out of the Hadron Booster $W = 12$ MeV $\phi = 0.71$ radians

at 100 kV in the Accumulator

$A_B = 0.404$, $W_B = 16.75$ MeV $|\parallel| A = 0.08$ eV-sec

A ±12 MeV beam can fit into this bucket.

What is the r.f. voltage that the r.f. should be reduced to, to form the matched bucket?

at 100 kV

1. $W = 12.0 \quad \text{then} \sin \frac{\phi}{2} = \frac{12}{16.75} \Rightarrow \phi \approx 0.508 \pi$
2. $\phi = 0.71 \quad \text{then} \quad W = W_B \sin \frac{\phi}{2} \Rightarrow W = 5.82$ MeV

⇒ Matched bucket is that which contain ± 5.82 MeV $\phi = 0.508 \pi$

⇒ $W_B = \frac{5.82^2}{\sin^2 (0.508 \pi)} = 8.13$ MeV eV $= \frac{W_B^2 \pi \hbar \gamma}{2 E}$

at 100 kV $\text{Synchronous frequency} = f_0 = \left[ \frac{\hbar \gamma E \cos \phi}{2 \pi E} \right]^{\frac{1}{2}} = 1140$ Hz

⇒ $\frac{1}{4}$ turn happens in $\approx 220$ microseconds.
Booster Beam To Debuncher

We would like to capture beam in the $h=4$ system (DRF3) in the debuncher.

The maximum voltage that DRF3 can generate is 188 kV.

For a bucket that is $\pm \pi$

$$A_B = \frac{8}{\omega_0} \left[ \frac{2E_s (eV)}{\pi h \eta} \right]$$

$$\omega_0 = 1.483 \times 10^7$$

$$\eta = 0.0055$$

$$h = 4$$

$$= 11.608$$

$$W_B = 21.52 \text{ MeV} \left( \frac{A_B \omega_0}{8} \right)$$

$$W = 15.21 \text{ MeV} \left( W_B \sin \phi \right)$$

$$A = \frac{\pi W \phi}{\omega_0} = 5.06 \text{ eV-sec}.$$  

Incoming into the debuncher from the booster are bunches that are $\pm 12 \text{ MeV}$ and $\frac{0.71 \times 18.93}{\pi}$ ns wide.

$$= 4.28 \text{ ns wide} \quad \left( \text{at } A = 0.08 \text{ per bunch} \right)$$

$$V_{ref} = 400 \text{ eV}$$

in the booster.

For an ellipse

$$W' = W \cos x$$

$$\phi' = \phi \sin x$$

Circumference

$$1) \cos x = \frac{W'}{W} = 12.0 \div 15.21$$

$$\Rightarrow \phi' = \frac{\pi}{2} \sin x = 0.965$$

$$\Rightarrow \# \text{ of bunches } = 0.965 \times 22.5 = 21.9$$

$$22.5 \text{ MHz} \div 1 \text{ h}=4 \text{ bunch}.$$  

$$\text{Width of beam } = 19.93 \text{ ns} / \text{53 MHz bunch}.$$  

$$= 130 \text{ ns}$$
If we can time the $h=4$ bucket so that the beam always arrives in time, then we can capture about 7 rf. buckets from the booster.

If there is a $\pm 50$ ns jitter in the timing of the $h=4$ bucket relative to the beam, then we can only use $130 - 900 = 30$ usec of the beam.

This corresponds to 2 rf. ($53$ MHz) buckets.

(i.e. $18.93 + 4.28 \text{ ns} \leq 30 \text{ ns}$)