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Nuclear Power – Ready, Steady, Go

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1 Introduction

There has been a long standing interest in developing numerical models for the analysis of fast burst critical experiments. For example, in an early model [1] the uncoupled thermomechanics of fissile materials was studied by inserting prescribed temperature fields into the elastic equations. More recently, a model was developed [4] which coupled thermomechanics of a fast burst reactor with point reactor kinetics. Here, we present a more realistic model which fully couples spatially-dependent neutron diffusion and thermomechanics in order to simulate transient behavior of a fast burst criticality excursion.

The problem involves solving a set of non-linear differential equations which approximate neutron diffusion, temperature change, and material behavior. With this equation set it is possible to model the transition from a supercritical to subcritical state and corresponding material response, e.g, possible mechanical vibration. For instance, the reactor is put into supercritical state leading to power rise (therefore resulting temperature rise). The rising temperature causes the material to expand leading to an increase in neutron leakage and thus to subcriticality.

We tested our new approach on a one-group, spherically symmetric diffusion model accounting for prompt neutrons only. We don't include delayed neutrons, since the time scales involved are too fast in order for delayed neutrons to contribute to the system.

2 Model Equations

Our model equations are formulated in a spherically symmetric coordinates [1, 4, 2]. In this case, the neutron diffusion is governed by

$$\frac{1}{v}\frac{\partial \phi}{\partial t} - \frac{1}{r^2}\frac{\partial}{\partial r}[r^2 \frac{1}{3N\sigma_{tr}}\frac{\partial \phi}{\partial r}] + [N\sigma_a - \nu N\sigma_f]\phi = 0. \quad (1)$$

The temperature field is evolved with

$$\rho c_p \frac{\partial T}{\partial t} - \omega N \sigma_f \phi = 0. \quad (2)$$

The material displacement is modeled by the following elastic wave equation;

$$\frac{1}{c^2}\frac{\partial^2 u}{\partial^2 t} - [\frac{\partial^2 u}{\partial^2 r} + \frac{2}{r}\frac{\partial u}{\partial r} - \frac{2}{r^2}u] + \frac{1+\upsilon}{1-\upsilon}\beta\frac{\partial T}{\partial r} = 0. \quad (3)$$

Finally, the material density is computed by considering the mass/particle conservation in a spherical domain;

$$\rho = \rho_0 [\frac{r}{r+u}]^3. \quad (4)$$

The unknowns and constant parameters that appear in these equations are defined as

ϕ : Neutron flux
T : Temperature
u : Material displacement
ρ : Material density
r : Spatial variable
t : Time variable
v : Average neutron speed
N : Number atom density
ν : Number of neutron produced per fission
σ_{tr} : Microscopic transport cross section
σ_a : Microscopic absorption cross section
σ_f : Microscopic fission cross section
ω : Amount of average energy released per fission
c_p : Specific heat
β : Linear thermal expansion coefficient
$c = [\frac{(1-\upsilon)\epsilon}{(1+\upsilon)(1-2\upsilon)\rho}]^{1/2}$: Wave speed
υ : Poisson's ratio
ϵ : Young's modulus

We note that the material density can be written as the product of the number atom density and the atomic mass, i.e, $\rho = NAm$.

3 Numerical Algorithm

Our numerical algorithm consists of an explicit and an implicit block. The reason why we develop this kind of approach is that the time steps are impractically small(e.g, due to the stiffness of the problem) if one wants to solve Eq. (1) explicitly. On the other hand, one has to follow characteristic wave speed to solve Eq. (3) due to numerical instabilities. Therefore, an explicit scheme is better choice for Eq. (3). We are currently using a first order coupling scheme, but will be moving to a second order predictor corrector scheme.

The numerical algorithm is executed as follows. First, Eq. (3) is advanced in time to obtain a new density. Then the updated density is inserted into the implicit loop to advance Eqs. (1) and (2). Our explicit block is based on a second order centered in time and space scheme. Our implicit block uses a Crank-Nicolson type time and space discretization. The non-linear solver within the implicit block is based on the Jacobian-Free Newton Krylov method [3].

4 Numerical Results

4.1 Results of a simplified model

In this section, we present the thermomechanical behavior of a simplified model. Here, the results are based on a simplified temperature model, i.e,

$$\frac{\partial T}{\partial t} = P(t)f(r), \tag{5}$$

where $P(t)$ represents a prescribed power, and $f(r)$ corresponds to the first spatial mode of the eigensolution of the linearized diffusion equation (e.g, the diffusion coefficient for Eq. (1) is assumed to be constant in space).

We tested three different power pulses and observed the material response accordingly. We note that the time scale for the power pulses is given by $\tau = [\frac{1}{P(t)}\frac{\partial P}{\partial t}]^{-1}$. And the elastic wave time scale is $\tau^{elastic} = R/c$ with R being the radius of the sphere. Figure 1 represents the material displacement resulting from a fast power pulse (e.g, $\tau < \tau^{elastic}$). In this case, the material doesn't have enough time to respond to the fast temperature rise. Consequently, it vibrates after expanding to a certain level. Figure 2 is the result of a relatively slower power pulse. We notice that the vibration is reduced significantly. Our last test (Figure 3) corresponds to a much slower power pulse (e.g, $\tau > \tau^{elastic}$). This time, we don't see vibrations since the material can respond to the slow temperature rise with non-vibrating expansion.

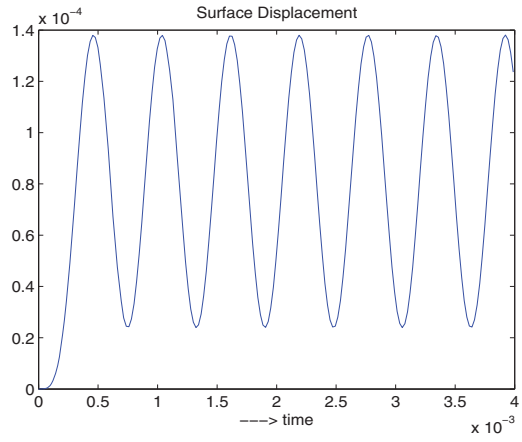

Figure 1: Displacement resulting from a fast power pulse.

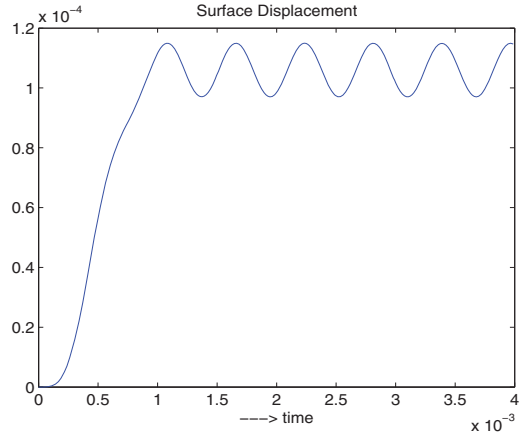

Figure 2: Displacement resulting from a medium pulse.

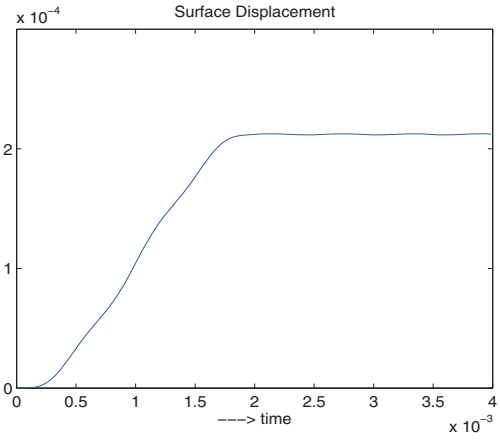

Figure 3: Displacement resulting from a slower pulse.

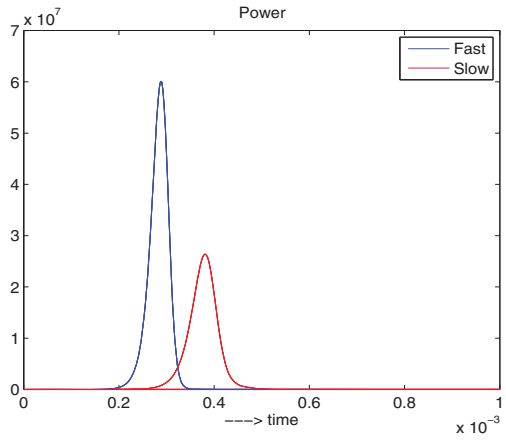

Figure 4: Power pulses from the coupled model.

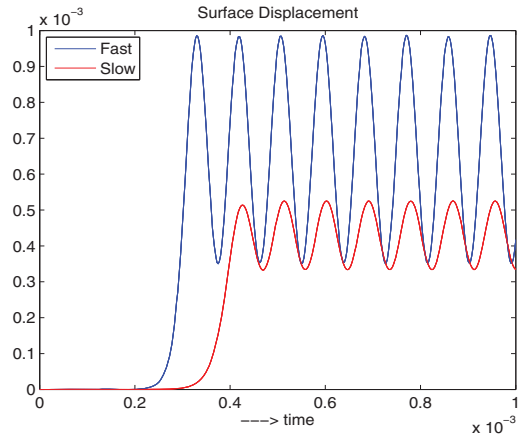

Figure 5: Displacements corresponding to the power pulses in Figure 4.

4.2 Preliminary results of our coupled model

In this section, we present our preliminary results from our coupled model. We intend to see the similar mechanical behavior as we have seen in the simplified model. Figure 4 shows two different power pulses one being faster than the other. The corresponding material response is shown in Figure 5. We notice that material vibration reduces when we have slow temperature rise (slower pulse). This is consistent with our simplified model. Again we observe an increase in vibration when the temperature rises faster.

We would like to make some initial remarks about the performance of our algorithm. Our implicit solver converges to a given tolerance with on average *three* Krylov iterations and *one* Newton step in smooth regions. On the other hand, in high gradient regions (e.g, where there is a steep power rise), the convergence takes on average 35 Krylov iterations and *two* Newton steps. We note that the code performance can be improved by preconditioning the Krylov block.

5 Conclusion and Future Work

We presented a preliminary study for the coupling of neutron diffusion and thermomechanics. We illustrated the mechanical response of the material to different power (temperature) settings. We showed that if there is a fast temperature increase in the system, then the material expands to a certain level and starts vibrating. On the other hand, if the temperature slowly increases, then the material expands with significantly less vibration. Our future work is to eliminate the time splitting error by introducing a predictor corrector methodology. Also, we are developing a semi-analytical solution to the coupled system for benchmarking purposes.

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