A Tiling Game and Its Properties in the Plane
Discover the Power of Mathematics!

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Scholars Day: April 14, 2011

1Financial support provided by a grant from the Honors College.
Definition: Combinatorial Game

A combinatorial game is a two person game of complete information, that is, it has no chance elements. Examples of combinatorial games are:
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Definition: Tiling

A tiling of the plane is a pattern of figures that covers the entire plane with no overlaps or gaps.
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The Game

The Rules
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Blocker moves first by playing a $2 \times 1$ tile anywhere on the board.
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- The Game is played on a board with a “chessboard-like” pattern.
- There are two players called **Blocker** and **Tiler**.
- Blocker moves first by playing a $2 \times 1$ tile anywhere on the board.
- Tiler and Blocker then alternate placing tiles adjacent to previously placed tiles until no more tiles can be placed on the board. Alternatively, we can say play continues until a maximal partial tiling is created.
The Game

The Rules

- The Game is played on a board with a “chessboard-like” pattern.
- There are two players called Blocker and Tiler.
- Blocker moves first by playing a 2×1 tile anywhere on the board.
- Tiler and Blocker then alternate placing tiles adjacent to previously placed tiles until no more tiles can be placed on the board. Alternatively, we can say play continues until a maximal partial tiling is created.
- Blocker wins if there exists less than a certain proportion of tiled squares to total squares when the game ends. i.e. The density of the maximal partial tiling is less than a value D.
The Game

The Rules

- The Game is played on a board with a “chessboard-like” pattern.
- There are two players called **Blocker** and **Tiler**.
- Blocker moves first by playing a $2 \times 1$ tile anywhere on the board.
- Tiler and Blocker then alternate placing tiles adjacent to previously placed tiles until no more tiles can be placed on the board. Alternatively, we can say play continues until a **maximal partial tiling** is created.
- Blocker wins if there exists less than a certain proportion of tiled squares to total squares when the game ends. i.e. The **density** of the maximal partial tiling is less than a value $D$.
- Tiler wins if that proportion of tiled squares to total squares or more is tiled. i.e. The density is greater than or equal to $D$. 
The Game (D=1)

A Random Game
The Game (D=1)

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A Random Game

The density is $\frac{8}{9}$. Blocker Wins!
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[Grid showing a game board with red and green blocks]
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The Game (D=1)

A Random Game
The Game \( (D=1) \)

A Random Game
The Game (D=1)

A Random Game
The Game \((D=1)\)
The Game (D=1)
The Game (D=1)

A Random Game

Blocker Wins!
The Game (D=1)

A Random Game

The density is $\frac{8}{9}$. Blocker Wins!
Who has the winning strategy \((D=1)\)?

**Problem (6x6 Game)**

*From the random game, we saw that blocker created a hole after 7 turns:*
Who has the winning strategy ($D=1$)?

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**Problem (6x6 Game)**

*From the random game, we saw that blocker created a hole after 7 turns:*

![Diagram of a 6x6 game board with holes created by a green and red blocker]
Who has the winning strategy \((D=1)\)?
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Who has the winning strategy ($D=1$)?

Problem (6x6 Game)

*From the random game, we saw that blocker created a hole after 7 turns:*
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Problem (6x6 Game)

*It turns out that no matter the size of the game, blocker can create a hole:*
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Problem (6x6 Game)

*It turns out that no matter the size of the game, blocker can create a hole:*

![6x6 Game Diagram]
Who has the winning strategy \( (D=1) \)?

**Problem (6x6 Game)**

*It turns out that no matter the size of the game, blocker can create a hole:*

![Diagram of a 6x6 grid with some cells shaded green and red]
Who has the winning strategy \( (D=1) \)?

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![Diagram of a 6x6 game board with a green block creating a hole.](image)
Who has the winning strategy \((D=1)\)?
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Problem (6x6 Game)

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Who has the winning strategy ($D=1$)?

Problem (6×6 Game)

*It turns out that no matter the size of the game, blocker can create a hole:*

![Diagram of a 6x6 grid with colored cells demonstrating a blockage](image.png)
Who has the winning strategy \((D=1)\)?

Problem (6x6 Game)

*It turns out that no matter the size of the game, blocker can create a hole:*

![Diagram of a 6x6 grid with a hole]
Who has the winning strategy \( (D=1) \)?

**Problem (6x6 Game)**

*It turns out that no matter the size of the game, blocker can create a hole:*

![Diagram of a 6x6 grid with a hole created by blocker and a red L-shaped piece at the bottom left corner.](image-url)
Problem (6x6 Game)

*It turns out that no matter the size of the game, blocker can create a hole:*
Who has the winning strategy \((D=1)\)?

**Problem (6x6 Game)**

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Who has the winning strategy \((D=1)\)?
Who has the winning strategy (D=1)?

Problem (6x6 Game)

It turns out that no matter the size of the game, blocker can create a hole:

Even if the game is infinite!
Who has the winning strategy \( (D=1) \)?
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**Problem (2x8 Game)**

*The only exception is when the game has exactly two rows or two columns.*

![2x8 Game Board](image)
Who has the winning strategy \((D=1)\)?

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Who has the winning strategy (D=1)?

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![Game Board](image)
Who has the winning strategy \((D=1)\)?

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*The only exception is when the game has exactly two rows or two columns.*

![Diagram showing the 2x8 game with colored cells and empty cells. Red and green cells are arranged in a specific pattern to illustrate the game's structure.](image)
Who has the winning strategy \((D=1)\)?
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Problem (2x8 Game)

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Problem (2x8 Game)

*The only exception is when the game has exactly two rows or two columns.*

What if \(D<1\)?
What is the minimum density of the game in the plane?
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Problem

*We can achieve a minimum density of $\frac{2}{3}$ in the game. However, the actual density we will discover for the game is much higher.*
More about the Game
More about the Game
More about the Game

The Density

The density here is $\frac{4}{5}$ which amounts to blocker creating a hole on every one of his turns in the limit. Therefore, we can expect the game to have a density this high or higher, since tiler can always move so that he doesn’t create a hole.
Where do we go from here?

The Future
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The Future

- Find the density of the game in the plane when both blocker and tiler are playing optimally.
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- Find the density of the game in the plane when both blocker and tiler are playing optimally.
- We can look at tiles of length greater than 2 and generalize for tiles of length n.
Where do we go from here?

The Future

- Find the density of the game in the plane when both blocker and tiler are playing optimally.
- We can look at tiles of length greater than 2 and generalize for tiles of length n.
- Look at these ideas in three dimensions or more.
The End

Thank you for coming.