LBNL-48987

A Constrained Standard Model: Effects of Fayet-Iliopoulos Terms

Riccardo Barbieri[a], Lawrence J. Hall[b,c], Yasunori Nomura[b,c]

[a] Scuola Normale Superiore and INFN, Piazza dei Cavalieri 7, I-56126 Pisa, Italy
[b] Department of Physics, University of California, Berkeley, CA 94720, USA
[c] Theoretical Physics Group, Lawrence Berkeley National Laboratory, Berkeley, CA 94720, USA

This work was supported in part by the Director, Office of Science, Office of High Energy Physics, of the U.S. Department of Energy under Contract No. DE-AC03-76SF00098.

UCB-PTH-01/39
LBNL-48987

A Constrained Standard Model: Effects of Fayet-Iliopoulos Terms

Riccardo Barbieri[a], **Lawrence J. Hall**[b,c], **Yasunori Nomura**[b,c]

[a] *Scuola Normale Superiore and INFN, Piazza dei Cavalieri 7, I-56126 Pisa, Italy*
[b] *Department of Physics, University of California, Berkeley, CA 94720, USA*
[c] *Theoretical Physics Group, Lawrence Berkeley National Laboratory, Berkeley, CA 94720, USA*

Abstract

In [1] the one Higgs doublet standard model was obtained by an orbifold projection of a 5D supersymmetric theory in an essentially unique way, resulting in a prediction for the Higgs mass $m_H = 127 \pm 8$ GeV and for the compactification scale $1/R = 370 \pm 70$ GeV. The dominant one loop contribution to the Higgs potential was found to be finite, while the above uncertainties arose from quadratically divergent brane Z factors and from other higher loop contributions. In [3], a quadratically divergent Fayet-Iliopoulos term was found at one loop in this theory. We show that the resulting uncertainties in the predictions for the Higgs boson mass and the compactification scale are small, about 25% of the uncertainties quoted above, and hence do not affect the original predictions. However, a tree level brane Fayet-Iliopoulos term could, if large enough, modify these predictions, especially for $1/R$.

In ref. [1] we obtained the 1 Higgs doublet standard model from a 5D supersymmetric theory with both the standard model gauge particles and the Higgs boson in the bulk. The Scherk-Schwarz (SS) mechanism is employed to remove unwanted particles and symmetries: the entire superpartner spectrum, as well as other particles implied by 5D Lorentz invariance, are raised to the compactification scale, $1/R$. In ref. [2] we demonstrated that this is the unique such construction in 5D, up to a small deformation in the orbifold boundary condition.

A relevant property of the model is that the Higgs potential is calculable in terms of the compactification scale 1/R, up to small effects from supersymmetric counterterms. The Fermi constant determines $1/R \approx 370$ GeV, so that the Higgs mass is predicted. The leading 1 loop diagrams for the Higgs potential are exponentially insensitive to physics at energies above the compactification scale, but UV sensitivities can arise through the supersymmetric counterterms, which must therefore be studied. In ref. [1] the leading counterterms were found to be brane Z factors for the top quark superfields. These are quadratically divergent, and affect the masses of the Kaluza-Klein (KK) modes of the top quarks and squarks, which enter the radiative diagrams for the Higgs potential. The Higgs mass and compactification scale therefore have a sensitivity to unknown UV physics via a quadratic divergence at the two loop level, introducing uncertainties in the predictions for the Higgs mass and $1/R$ of 1% and 20%, respectively. Further uncertainties in the Higgs boson mass of about 6% arise from other higher loop contributions. In ref. [3] a quadratically divergent Fayet-Iliopoulos (FI) term was noticed, associated with standard hypercharge, which escaped our attention. This introduces a quadratic sensitivity to unknown UV physics at the one loop level, and its consequences are studied in this note.

It is indeed immediate to see that the diagram of fig. 1, properly calculated for the different KK components of the hypercharge D-term, gives rise to an effective Lagrangian term

$$\mathcal{L}_{\text{eff}} = \frac{\xi}{\sqrt{2}} \left(\delta(y) + \delta(y - \pi R/2)\right) D_Y, \tag{1}$$

where

$$\xi \simeq \frac{g'}{2} \int \frac{d^4p}{(2\pi)^4} \frac{1}{p^2} \simeq \frac{g'}{2} \frac{\Lambda^2}{16\pi^2}, \tag{2}$$

and

$$D_Y = \frac{D_0}{\sqrt{2}} + \sum_{n=1}^{\infty} D_n \cos \frac{2ny}{R}, \tag{3}$$

with D_0 and D_n canonically normalized in 4D. Λ is an ultraviolet cutoff and g' is the $U(1)$ hypercharge coupling. In the loop of fig. 1, only the Higgs zero-mode contributes to the zero-mode of D_Y, whereas the $2n$-th KK modes of D_Y receive contributions from the n-th KK modes of the Higgs field and of its charge conjugate. The matter hypermultiplets do not contribute to a FI term on the brane due to $\mathrm{Tr}Y = 0$ over the standard matter multiplets.

In the appendix, we comment on the theoretical issues about the generation of the FI term.

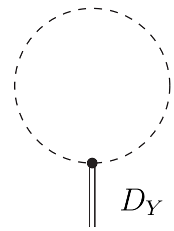

Figure 1: One-loop diagram generating the FI D-term.

It is important to notice that $\mathcal{L}_{\rm eff}$ is perfectly compatible with the residual supersymmetries of the full Lagrangian after the orbifold projection. This Lagrangian, other than the fully supersymmetric term in 5D, $\mathcal{L}_5$, must include the most general 4D Lagrangians at $y = 0$ and $y = \pi R/2$ compatible with the (different) $N = 1$ supersymmetries at each of the fixed points. The FI terms in eq. (1) can indeed be there, as can be supersymmetric kinetic terms for the different fields or any other $N = 1$ supersymmetric operator of higher dimension. If not inserted from the start, one has to expect them from the loop expansion. In turn, their effect on the calculation of the Higgs potential has to be discussed along the lines of ref. [1].

The model, being based on a non-renormalizable Lagrangian, is defined in terms of a cutoff scale M, which we take to be the scale at which the perturbative expansion in the top Yukawa coupling ceases to make sense. It is $M \simeq 5/R$, as can be seen from an actual perturbative calculation or from naive dimensional analysis. We assume that perturbativity is maintained up to M, even after the inclusion of all possible other terms in the Lagrangian. This limits the effects of the various counterterms mentioned above on the Higgs potential. The closeness of M to $1/R$ should not be viewed as an obstacle. In the chiral Lagrangian the relation of m_ρ to f_π is not very different and yet the usefulness of the chiral Lagrangian itself is not disputable.

Suppose that we use

$$\Lambda \simeq M \simeq \frac{5}{R}, \tag{4}$$

to estimate the size of the ξ term in eq. (2). We get $\xi \simeq 0.03/R^2$, which gives effects well within the uncertainties already discussed in ref. [1] both on the Higgs potential and, a fortiori, on the superpartner spectrum. Radiatively generated brane kinetic terms, and higher loop corrections, give larger effects, as quoted above.

The symmetries of the theory allow a tree level D-term as in eq. (1), although with differing magnitudes on the two branes. This introduces a correction to the Higgs squared mass parameter

$$\delta m^2_{\phi_H}(\xi) = \frac{g'}{2}\xi, \tag{5}$$

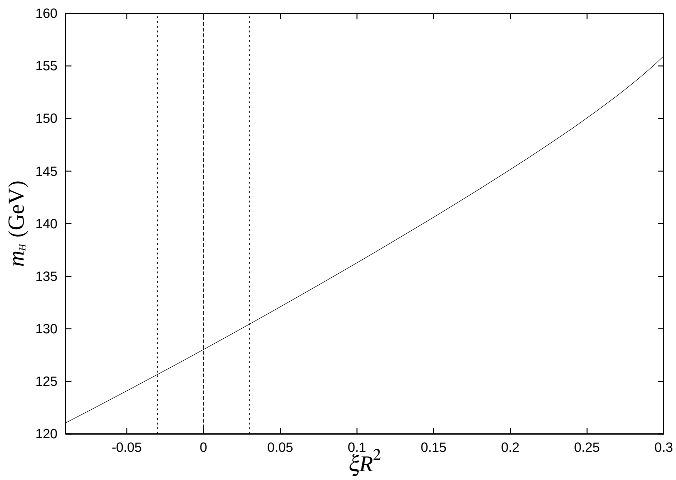

Figure 2: The physical Higgs boson mass m_H as a function of ξR^2.

where ξ is now the average value of the D-term on the two branes. The sign of such a term is unknown. It should be compared with the finite top loop contribution to the Higgs potential, which gives a mass squared

$$\delta m^2_{\phi_H}(\text{top}) = -\frac{63\zeta(3)}{8\pi^4}\frac{y_t^2}{R^2} \simeq -\frac{0.08}{R^2}. \tag{6}$$

If $\delta m^2_{\phi_H}(\xi)$ were positive and bigger than $|\delta m^2_{\phi_H}(\text{top})|$ its presence would prevent symmetry breaking. For other values of ξ, we have minimized the Higgs potential and predict the Higgs mass and $1/R$ as a function of the dimensionless parameter ξR^2, as shown in figs. 2 and 3. Vertical dashed lines at $\xi R^2 = \pm 0.03$ show the effects to be expected from the radiative FI term. The experimental limit on the top squark mass requires ξR^2 to be larger than about -0.1, and electroweak symmetry is broken only if ξR^2 is less than about 0.5. The figures are not extended to values of ξR^2 above 0.3 because corrections to the top KK masses from ξ have not been included in the calculation of the Higgs potential, and in this region they become important.

We note that a partial cancellation can take place between $\delta m^2_{\phi_H}(\text{top})$ and $\delta m^2_{\phi_H}(\xi)$, with a corresponding increase in $1/R$ and m_H^2 itself. As ξR^2 approaches the maximum value consistent with electroweak symmetry being broken, larger values of $1/R$ result, but at the price of an increasingly precise cancellation among the two contributions to the Higgs mass. Very large values of $1/R$ are therefore disfavored, although even in this case there is a strict upper limit on the Higgs mass in the region of 180 GeV. The hypercharge gauge interaction is weakly coupled at the cutoff scale M, so that ξ cannot be estimated using strong coupling arguments. A totally naive guess, $\xi \simeq g' M^2$, would be clearly excluded. The theory above M must lead to suppressed tree-level values for certain brane interactions, including the FI term and flavor-dependent kinetic terms for the light generations.

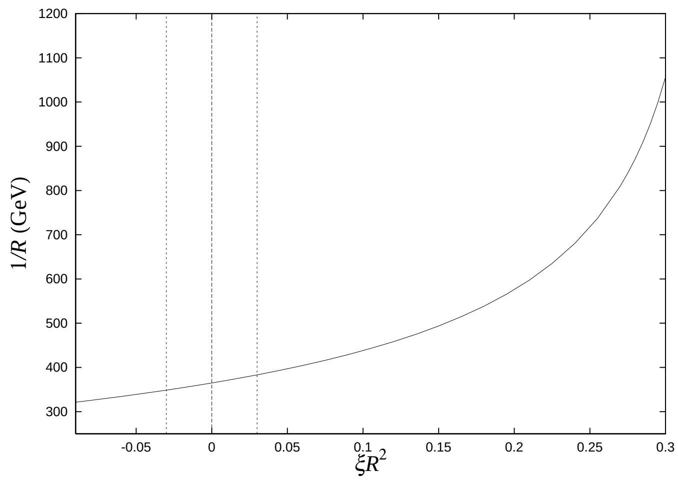

Figure 3: The compactification scale $1/R$ as a function of ξR^2.

A value of $\xi R^2 \simeq 0.3$ raises the compactification scale by about a factor of 3, thereby removing the fine tuning needed to satisfy the experimental constraint on the ρ parameter found in ref. [1]. Such a value of ξR^2 is an order of magnitude larger than the radiative contribution, but over an order of magnitude smaller than a naive guess in 5D, and does not require fine tuning in the Higgs mass squared parameter.

For values of ξ which are not too large we find analytic approximations:

$$m_H^2(\xi) \simeq m_H^2(0) + M_Z^2\Big(\cos[\pi R(0) m_t] - \cos[\pi R(\xi) m_t]\Big) + g'\xi\Big(1 - \cos[\pi R(\xi) m_t]\Big), \tag{7}$$

and

$$\frac{1}{R(\xi)} \simeq \frac{1}{R(0)}\left(1 + \frac{g'\xi}{M_Z^2}\right)^{\frac{1}{4}}. \tag{8}$$

Furthermore, for ξ sufficiently small, as in the one loop calculation, the predictions made in ref. [1] still hold:

$$m_H = 127 \pm 8 \text{ GeV}, \tag{9}$$

$$1/R = 370 \pm 70 \text{ GeV}. \tag{10}$$

The role of the FI term in the Higgs potential is to provide an additional contribution to the Higgs mass squared parameter. In the case that this contribution is positive, the effect is equivalent to a deformation in the translation orbifold boundary condition discussed in ref. [2]. Indeed, one can ask what measurement will provide a distinction between these theories. The answer lies in the details of the scalar superpartner spectrum with masses near $1/R$. The boundary condition deformation leads to a universal shift in the scalar masses, while the FI term leads to shifts that depend on the hypercharge of the scalar.

The D_Y field has a kinetic coupling to the real scalar field σ of the hypercharge chiral adjoint field: $D_Y \partial_y \sigma$. On performing a KK expansion one discovers that the effects of the FI term coupled to D_n, for $n \neq 0$, are canceled by vacuum expectation values (VEVs) of σ_n. The physical effects of the FI term discussed above all result from the coupling to the zero mode D_0. The VEV of σ_n leads to mass mixing amongst KK modes from the gauge interaction $g'[X\Sigma X^c]_{\theta^2}$ for any hypermultiplet (X, X^c). This leads to a violation of momentum in the fifth dimension allowing single production of excited KK modes, such as $g_0 g_0 \rightarrow \bar{q}_0 q_2$, and new decay modes of the excitations, such as $q_2 \rightarrow Z_0 q_0$, where subscripts label the KK modes.

In the constrained standard model introduced in ref. [1], electroweak symmetry is broken radiatively via a finite 1 loop contribution involving the top quark and its superpartners and KK resonances. Corrections to this picture arise from supersymmetric brane interactions. There is a quadratically divergence brane FI term, as pointed out in ref. [3], but this leads to only a 2% correction in the Higgs mass, which is smaller than other corrections. It is perhaps surprising that a 1 loop quadratic divergence is so mild relative to the finite 1 loop top quark contribution. This results from several factors: the top Yukawa coupling is larger than the hypercharge coupling, there is a color factor of 3, the Yukawa couplings of the KK towers are $\sqrt{2}$ times larger than that of the zero mode, and finally the cutoff of the theory is only about a factor of 5 above the compactification scale. A tree level brane FI term could be present. Since the hypercharge coupling is highly perturbative even at the cutoff, it seems likely to us that this tree contribution from physics at the cutoff is comparable to the quadratically divergent radiative correction and therefore also negligible. However, if it is larger by an order of magnitude it could lead to significant changes in the predictions of the theory as shown in figs. 2 and 3.

Acknowledgements

Y.N. thanks the Miller Institute for Basic Research in Science for financial support. This work was supported by the ESF under the RTN contract HPRN-CT-2000-00148, the Department of Energy under contract DE-AC03-76SF00098 and the National Science Foundation under contract PHY-95-14797.

Appendix

In this appendix we comment on theoretical issues about the generation of the FI term. We have seen that brane-localized operators, eq. (1), are radiatively generated in the model of ref. [1]. The coefficients of the two FI terms on $y = 0$ and $\pi R/2$ branes are the same, so that in the 4D picture the Lagrangian is given by

$$\mathcal{L}_{\rm 4D} = \sqrt{2}\,\xi \left(\frac{1}{\sqrt{2}} D_0 + D_2 + D_4 + \cdots \right), \tag{11}$$

where D_n is the n-th KK mode of D_Y with "mass" $2n/R$. The question is what happens for the

FI term if we take the supersymmetric limit of the theory. Does the FI term remain non-zero?

To answer this question, let us describe our model using $Z : y \to -y$ and $T : y \to y + \pi R$ rather than $Z : y \to -y$ and $Z' : (y - \pi R/2) \to -(y - \pi R/2)$. Then, our model corresponds to taking Z as the "standard" Z_2 parity reducing 5D $N = 1$ supersymmetry to 4D $N = 1$ supersymmetry, and T as a direct product of the SS rotation with the SS parameter $\alpha = 1/2$ and the overall negative sign for the Higgs hypermultiplet. In the notation of ref. [2], it is written as

$$Z = \Sigma_3 \otimes 1, \tag{12}$$

$$T = e^{2\pi i \alpha \sigma_2} \otimes -1, \tag{13}$$

where $\alpha = 1/2$ corresponds to the model of ref. [1].

Suppose we take SS parameter α to be equal to zero. Then, 4D $N = 1$ supersymmetry remains unbroken, and both Higgs and Higgsino KK towers have mass $(2n+1)/R$. In this case, one might conclude that no FI term is generated since the matter content is completely vector-like; we have full hypermultiplet states, $h, h^c, \tilde{h}, \tilde{h}^c$, at each KK level. However, the situation is not so simple. Although the matter content is vector-like, the interactions are not; D_n (n:odd) interactions do not have a charge conjugation symmetry. As a consequence, non-vanishing brane-localized FI terms are generated even in this supersymmetric case, $\alpha = 0$. Indeed, a simple calculation shows that the terms of the form

$$\mathcal{L}_{\rm eff} = \frac{\xi}{\sqrt{2}} \left(\delta(y) - \delta(y - \pi R/2) \right) D_Y, \tag{14}$$

are generated radiatively. In the 4D picture, this is

$$\mathcal{L}_{\rm 4D} = \sqrt{2}\, \xi \left(D_1 + D_3 + D_5 + \cdots \right). \tag{15}$$

It is important to realize that this special form of the FI terms is guaranteed by a symmetry; in the 5D picture there is a charge conjugation symmetry which is accompanied by a spacetime reflection with respect to $y = \pi R/4$, and it allows only brane-localized FI terms with opposite coefficients at $y = 0$ and $\pi R/2$. Incidentally, if α takes some arbitrary values, the situation is between the two extreme cases $\alpha = 0$ and $1/2$; the size of the FI terms on the two branes are different in general.

Thus, we conclude that, if we have only one hypermultiplet in the bulk, brane-localized FI terms are always generated (even if supersymmetry is not broken). However, there needs to be a slight care for this statement. In the supersymmetric case of $\alpha = 0$, the generated terms do not contain the FI term for the unbroken (zero-mode) $U(1)$ hypercharge, in contrast with the case of $\alpha \neq 0$ (see eqs. (11, 15)). This is a reasonable result since the appearance of a FI term is deeply related to the $U(1)$-(grav.)^2 anomaly in the usual 4D supersymmetric theories. In 4D theories with $U(1)$-gravitational anomaly canceled, the FI term is never generated unless we break either supersymmetry or the $U(1)$. In the present case with $\alpha = 0$, the "FI term" appeared in the supersymmetric limit. However, it is a brane-localized term and not a true FI term in the 5D sense. In other words, in the 4D picture the generated FI terms are only for higher KK modes and not for the zero mode. Since the $U(1)$ symmetries corresponding to the higher KK towers are non-linearly realized (spontaneously broken), the generation of these FI terms do not conflict with the above general theorem in the 4D supersymmetric theories, nor break supersymmetry since the FI terms for D_n $(n > 0)$ are completely absorbed by the expectation values for the physical scalar field σ_n coming from the gauge multiplet in 5D.

References

[1] R. Barbieri, L. J. Hall and Y. Nomura, Phys. Rev. D **63**, 105007 (2001) [arXiv:hep-ph/0011311].

[2] R. Barbieri, L. J. Hall and Y. Nomura, arXiv:hep-th/0107004.

[3] D. M. Ghilencea, S. Groot Nibbelink and H. P. Nilles, arXiv:hep-th/0108184.