Title: Frequency Shift Observer for SNS Superconducting RF Cavity

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FREQUENCY SHIFT OBSERVER FOR A SNS SUPERCONDUCTING RF CAVITY*

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Abstract

In contrast to a normal conducting RF cavity, a superconducting RF cavity is very susceptible to shifts in its resonance frequency. The main sources of the shift are Lorentz force detuning and microphonics. In SNS, to compensate for the frequency shift, a feedforward control is to be applied. In this paper, as an initiative step, a frequency shift observer is proposed which is simple enough to be implemented with a digital signal processor in real time. Simulation results of the proposed frequency shift observer show reliable performance and acceptable computational time for the real time implementation.

1 INTRODUCTION

The RF magnetic field in a SRF cavity interacts with the RF wall current resulting in a Lorentz force which is significant at high accelerating fields and for a pulsed accelerator such as SNS or TESLA facility [1]. The radiation pressure which is proportional to the square of magnetic field intensity and accelerating gradient causes a small deformation of the cavity shape resulting in a shift of its resonance frequency, called Lorentz force detuning [2]. Lorentz force detuning influences the performance of the low level RF control system due to the extra power needed to control an incorrectly tuned cavity. In the feedback loop, a klystron can be treated as an actuator with nonlinear saturation characteristics. The klystron should not be operated in its saturation region. In order to prevent the klystron from operating in the saturation region, a hardware limiter or a software limiter can be implemented in the low level RF control system. In this case, when the low level RF control system output reaches the upper bound of the limiter due to the increasing Lorentz force detuning, the control system does not supply enough output to compensate for the Lorentz force detuning. One way to avoid this actuator saturation is to make the klystron operate in such a way that it generates enough maximum output power to guarantee power control margin for the frequency shift, thus requiring a lot of klystron power overhead for the minimum detuning regime.

The Lorentz force detuning can be compensated by a pure feedforward control based on the disturbance estimation. In this paper, as an initiative step for a pure feedforward control, a frequency shift observer is proposed. The frequency shift observer yields the estimate of the frequency shift with measured outputs—cavity field In-phase and Quadrature, and measured inputs—klystron output In-Phase and Quadrature (or, low level RF controller output In-Phase and Quadrature). The computational time of the frequency shift estimator is small enough to be implemented with a digital signal processor (DSP) in a real time manner. Based on the estimated frequency shift, a pure feedforward controller can be designed in such a way that the (time varying) tuning frequency (frequency offset) which is the negative of the estimated frequency shift, is generated.

2 LORENTZ FORCE DETUNING MODELING

The Lorentz force detuning for a one mechanical mode can be is modelled as a second order differential equation. Detailed investigation has been performed by Ellis [3] and Mitchell [4]. For a mechanical mode frequency, \( \omega_{m1} \), a second order differential equation defines the Lorentz force detuning:

\[
\ddot{\omega}_L + 2 \zeta_\omega \dot{\omega}_L + \omega_m^2 \omega_L = 2 \pi k_{LF} \omega_m^2 E^2(t)
\]

(2.1)

where \( \omega_m = \frac{k}{m} \), \( m \) : general mass, \( k \) : stiffness constant.

Define \( x_1 = \omega_L \), \( x_2 = \dot{\omega}_L = \dot{x}_1 \). Then, (2.1) can be written as the formal second order state space equation describing the Lorentz force detuning due to a single mechanical mode vibration.

\[
\begin{bmatrix}
\dot{x}_1 \\
\dot{x}_2
\end{bmatrix} =
\begin{bmatrix}
0 & 1 \\
-\omega_m^2 - \alpha_1 & -\alpha_1
\end{bmatrix}
\begin{bmatrix}
x_1 \\
x_2
\end{bmatrix} + \begin{bmatrix}
0 \\
2 \pi k_{LF} \omega_m^2
\end{bmatrix} E^2(t)
\]

(2.2)

\[
y = \begin{bmatrix} 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}.
\]

(2.3)

When multi mechanical modes are considered, the modeling of the Lorentz force detuning is complicated. Cross coupling of mechanical mode vibrations must be considered. This means that the Lorentz force detuning constants \( k_{LF}^i \), \( i = 1,2,\ldots,m \), for each mechanical mode are distributed with certain conditions, and boundary conditions of the second order differential equations for each mechanical mode need to be assigned properly. Details are addressed in [3].

3 SRF CAVITY MODEL

A SRF cavity is given by the state space model [1], [5]

\[
\dot{z} = A_r (\Delta \omega) z + B_r \mu + B_s I
\]

(3.1)

\[
y = C_r z
\]

(3.2)
where

\[
A_1(\Delta \omega) = \begin{bmatrix}
-1 & -\Delta \omega \\
\frac{1}{\tau_t} & -1
\end{bmatrix}, \quad B_1 = \begin{bmatrix}
\frac{2}{Z_0} & 0 \\
\frac{2}{Z_0} & \frac{1}{\tau_t}
\end{bmatrix},
\]

\[
B_\phi = \begin{bmatrix}
-\frac{2}{Z_0} & 0 \\
0 & -\frac{2}{Z_0}
\end{bmatrix}, \quad C_1 = \begin{bmatrix}
1 & 0 \\
0 & 1
\end{bmatrix},
\]

\[
z = \begin{bmatrix} z_1 \\ z_2 \end{bmatrix}, \quad u = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}, \quad I = \begin{bmatrix} I_t \\ I_\phi \end{bmatrix},
\]

\[
\tau = 2Q_0 \omega_b, \quad \tau_t = \frac{2Q_0}{\omega_b}
\]

\(R_w\): Resistance of the Equivalent circuit of cavity transformed to RF generator
\(Z_0\): Transmission Line characteristic impedance
\(\zeta\): Transformation ratio
\(V_B, V_Q\): Forward In-phase (I) and Quadrature (Q)
\(I_t, I_\phi\): Beam current in In-phase (I) and Quadrature (Q)
\(V_I, V_Q\): Cavity Field In-phase (I) and Quadrature (Q).

In the above model, \(\Delta \omega\) is the sum of the predetuning, \(\Delta \omega_B\), the Lorentz force detuning, \(\Delta \omega_L\), and microphonics, \(\Delta \omega_{MC}\).

The objective of this paper is to design an observer such that the estimate \(\hat{\Delta \omega}\) yielded by the observer exponentially approaches the frequency shift \(\Delta \omega\). As mentioned in the previous section, when the Lorentz force detuning model includes all mechanical mode dynamics, the observer structure may be complicated and computational complexity increases. Instead of this complex higher order model, the frequency shift is modeled as

\[
\Delta \dot{\omega} = 0. \quad (3.3)
\]

The model (3.3) is widely used for constant or slowly varying disturbance.

\section{4 NONLINEAR OBSERVER}

Consider the state equation for the cavity field Quadrature (Q). It can be written by

\[
y_1 \Delta \omega = \dot{z}_2 + \frac{1}{\tau_L} z_2 - \frac{2}{Z_0} c_1 u_2. \quad (4.1)
\]

It is easily verified that the beam loading is cancelled out by the predetuning, \(\Delta \omega_B\) and (4.1) can be written as

\[
y_1 \Delta \omega = \dot{z}_2 + \frac{1}{\tau_L} z_2 - \frac{2}{Z_0} c_1 u_2
\]

where \(\Delta \omega = \Delta \omega_B + \Delta \omega_{MC}\). When beam is unloaded, (4.1) reduces to

\[
y_1 \Delta \omega = \dot{z}_2 + \frac{1}{\tau_L} z_2 - \frac{2}{Z_0} c_1 u_2
\]

where \(\Delta \omega = \Delta \omega_B + \Delta \omega_L + \Delta \omega_{MC}\).

For the frequency shift estimation, a disturbance observer is proposed as follow.

\[
\Delta \dot{\omega} = -L y_1 \Delta \dot{\omega} + L \left( \dot{z}_2 + \frac{1}{\tau_L} z_2 - \frac{2}{Z_0} c_1 u_2 \right). \quad (4.4)
\]

It follows from (3.3) and (4.4) that the observer error dynamics is given by

\[
\dot{e} = \Delta \dot{\omega} - \Delta \dot{\omega} = L y_1 e \quad (4.5)
\]

The observer gain \(L\) is determined so that the characteristic equation \(s + L y_1 = 0\) has a desired root in the left half plane of the complex domain. The observer error dynamics (4.5) shows that for a properly chosen gain \(L\), the estimate \(\hat{\Delta \omega}\) asymptotically converges to \(\Delta \omega_B + \Delta \omega_L + \Delta \omega_{MC}\) when beam is unloaded, and to \(\Delta \omega_B + \Delta \omega_{MC}\) when beam is loaded.

The observer (4.4) is difficult to implement practically because the derivative term \(\dot{z}_2\) is noisy and is hard to measure. A filter whose transfer function is \(\frac{s}{s + \epsilon}\) where \(\epsilon\) is a small constant can be used to approximate the differentiator. In this paper, a new variable is introduced.

\[
\dot{\hat{\omega}}_L = \Delta \dot{\omega} - LZ_2 \quad (4.6)
\]

where \(L = \frac{1}{y_1}\). The derivative of (4.6) with respect to time is

\[
\dot{\hat{\omega}}_L = -L y_1 \dot{\hat{\omega}}_L + L \left( \frac{1}{\tau_L} z_2 - \frac{2}{Z_0} c_1 u_2 - L z_2 \right) \quad (4.7)
\]

and the estimate of the frequency shift is

\[
\Delta \dot{\omega} = \hat{\omega}_L + L z_2. \quad (4.8)
\]

\section{5 SIMULATIONS AND DISCUSSION}

For SNS, an observer is to be used together with the piezoactuator in order to compensate for the frequency shift in a SRF cavity. The chosen observer is implemented with a DSP and so the observer should be as simple as possible for the real time implementation provided with satisfactory performance. In addition, the observer is turned on when the RF is turned on and is
turned off when RF is turned off. During the RF turn off period, the cavity field control is turned off and hence, the In-phase and Quadrature of cavity field are difficult to predict. If their behaviors during the RF off period are the solutions of the stable first order differential equations with zero inputs, then the proposed observer do not need to be turned off and it can estimate frequency shift. As mentioned in [3], the dominant frequencies of the mechanical modes for the medium-β SRF section exist up to 2.0 kHz and the dominant frequencies of microphonics are within a few hundred Hertz. For satisfactory performance of the observer, the sampling frequency of the observer must be at least 20 kHz. The simple Euler method is used for the discretization of the nonlinear first order observer. The observer is implemented in a TMS320C6201 evaluation module (EVM) [6], which includes an A/D converter (ADC) and a D/A converter (DAC). The clock speed of the DSP is 133 MHz and the sampling frequency of the ADC and DAC is 40 kHz. Currently, a prototype SRF cavity is being developed at Jefferson National Laboratory, so real data is not yet available. For the observer performance investigation, SIMULINK simulation data of the klystron output I/Q and cavity field I/Q were used.

Figure 1 shows the experiment result of the nonlinear first order observer where the Lorentz force detuning results from a 494.73 Hz single mechanical mode vibration [7]. Figure 2 shows the result of the nonlinear first order observer where the Lorentz force detuning results from 29 mechanical mode vibrations [3]. For one data sample, the computational time was 21 CPU clock cycles. When 200 MHz CPU clock is used for the DSP, the computational time is 0.11 μsec. Hence, the sampling frequency should be less than 9.5 MHz. With this sampling frequency, observer gains can be determined so as to guarantee fast response of observers.

6. CONCLUSIONS

In this paper, a deterministic disturbance observer has been proposed to estimate the frequency shift in a SRF cavity. The observer is simple and yields the satisfactory performance. The observer algorithm was implemented in TMS320C6201 EVM and the observer performance was investigated. The experiment shows that the proposed observer had reasonable computational time and was reliable, promising for frequency shift estimation of a SRF cavity.

References


