Fermilab

PBAR NOTE #455

Transverse Beam Heating Distributions

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Transverse Beam Heating Distributions

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A beam is heated in one dimension with a dipole kicker which deflects the beam by an angle $\theta(t)$, at a location where the lattice functions are (α, β, γ) . θ has a power spectrum $P(\omega)$ in the sense that θ^2 represents power. We will use variables I, γ defined as

$$\begin{aligned} I &= (\gamma x^2 + 2\alpha x x' + \beta x'^2)/2 \\ \tan \gamma &= \alpha + \beta x'/x \end{aligned} \quad 1.$$

When a particle is deflected by an angle θ , I and γ change by

$$\begin{aligned} \delta I &= (2I\beta)^{1/2} \sin \gamma \theta + \beta \theta^2/2 \\ \delta \gamma &= (2\beta/I)^{1/2} \cos \gamma \theta \end{aligned} \quad 2.$$

Although γ will diffuse, we are only interested in amplitudes I . To use the Fokker-Planck (FP) equation, we need to calculate the sum over many revolutions of the δI 's ($=\Delta I$) for a particle, and find those parts of ΔI and $(\Delta I)^2$ which are proportional to time. The particle, which has (angular) revolution frequency ω_0 and betatron frequency $\omega_\beta = \omega_0 \nu$ passes the kicker at times t_k with betatron phase γ_k where

$$\begin{aligned} t_k &= t_0 + 2\pi k/\omega_0 \\ \gamma_k &= \gamma_0 + \omega_\beta t_k \end{aligned} \quad 3.$$

The final values of ΔI and $(\Delta I)^2$ are then averaged over t_0 and γ_0 . We find;

$$\begin{aligned} \Delta I/T &= \beta \omega_0 \int P(\omega) d\omega / 2\pi \equiv k_w \\ (\Delta I)^2/T &= \beta I \omega_0^2 \sum_{p=0}^{\infty} P(|p\omega_0 \pm \omega_\beta|) / 2\pi \equiv 2KI \end{aligned} \quad 4.$$

The FP equation is

$$\begin{aligned} \partial f / \partial t + \partial \Phi / \partial I &= 0 \\ \Phi &= k_w f - KI \partial f / \partial I = 0 \end{aligned} \quad 5.$$

For narrow band heating, the first term can be neglected, since its contribution to FP is of order BW/f_0 compared to that of the second term. For wideband heating $BW > f_0$, it changes the nature of the diffusion, and the subsequent distribution. Eq. 5 admits solutions, where C is a constant,

$$f = Ce^{-\alpha t} J_0(2\sqrt{(\alpha I/k)}) \quad 6.$$

Now f must be 0 at, as well as beyond, the aperture limit I_m ; otherwise its infinite derivative there would imply infinite flux. Then $J_0(2\sqrt{(\alpha I_m/k)}) = 0$, and the allowed values of α are

$$\alpha_i = k j_i^2 / 4 l_m \quad 7.$$

Here j_i is the i th zero of J_0 . Since the set of functions $J_0(j_i x)$ is complete in the interval $(0,1)$, the initial distribution can be expressed as a series of $J_0(j_i \sqrt{l/l_m})$. Each term then decays with its own lapse rate α_i . For our purposes, we note that all terms above $i=1$ decay very rapidly compared to $i=1$, so the distribution quickly approaches $J_0(j_1 \sqrt{l/l_m})$.

What one measures with the scraper is the integral distribution $F(y)$ as a function of y , where $y = \sqrt{l \beta_s}$, and β_s is the value at the scraper. $F(y)$ is given by

$$F(y) = y J_1(j_1 y / y_m) / [y_m J_1(j_1)] \quad 8.$$

This can be compared directly to the plots on the Lexidata.