Title: DAMAGE DETECTION USING FREQUENCY DOMAIN ARX MODELS AND EXTREME VALUE STATISTICS

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ABSTRACT

Structural health monitoring (SHM) is fast becoming a field of great importance as engineers seek for new ways to ensure the safety of structures throughout their designed lifetime. Current methods for analyzing the dynamic response of structures often use standard frequency response functions to model linear system input/output relationships. However, these functions do not account for the nonlinear response of a system, which damage often introduces. In this study, an auto-regressive model with exogenous inputs (ARX) in the frequency domain is used to extract damage sensitive features, explicitly considering the nonlinear effect in the frequency domain. Furthermore, because of the non-Gaussian nature of the extracted features, extreme value statistics (EVS) is employed to develop a robust damage classifier. The applicability of the ARX model combined with EVS to nonlinear damage detection is demonstrated using vibration data obtained from a laboratory experiment of a three-story building model.

1. INTRODUCTION

Many aerospace, civil and mechanical systems continue to be used despite aging and the potential for damage accumulation and unpredicted failure. There are currently many nondestructive methods for identifying damage in structural connections. However, current methods are costly visual procedures or localized experimental methods such as acoustic or ultrasonic methods, magnetic field methods, radiograph, eddy-current methods and thermal field methods. These approaches are limited in usage, as the vicinity of the damage must be known before testing and that area must be easily accessible. For a more complete literature review of current SHM methods please consult [1].

If a damage detection method based on measured vibration response can be developed, it can then be combined with current micro-electrical mechanical systems (MEMS) or fiber optic sensing technology, constituting a more economical and quantifiable damage detection method than other commonly available methods. Such a damage identification scheme can potentially provide significant economic and life-safety benefits by preventing unforeseen catastrophic failures.

The focus of this study is to demonstrate the feasibility of a vibration-based damage detection system for structural connections. In recent years, vibration-based damage detection techniques have come to the foreground as a legitimate method to determine local damage from global vibration characteristics. Many techniques have been investigated in this area. However, none has worked well enough to be considered for use in real world applications. Most techniques have problems being applied to various structures and the analysis of vibration data received can be a time intensive process. This study will attempt to investigate a damage detection technique that has not been extensively explored. This approach uses ARX frequency domain model coefficients originally proposed by Adams and Allemang [2] as the damage sensitive feature. This feature is then analyzed using a statistical method known as extreme value statistics. The approach taken in this study is unique in that it uses nonlinear analysis and identification of the system to distinguish between linear damage, nonlinear damage, and inherent structural nonlinearities.

2. DESCRIPTION OF TEST STRUCTURE

The test structure (shown in Figures 1 and 2) was a simulated three-story frame structure, constructed of unistrut columns and aluminum floor plates. Floors were 0.5-in-thick (1.3-cm-thick) aluminum plates with two-bolt connections to brackets on the unistrut columns. Floor heights were adjustable. The base was a 1.5-in-thick (3.8-cm-thick) aluminum plate. Support brackets for the columns were bolted to this plate. Four Firestone airmount isolators, which allowed the structure to move freely in horizontal directions, were bolted to the bottom of the base plate. The isolators were inflated to 10 psig (69 kPag). The shaker was connected to the structure by a 6-in-long (15-cm-long), 0.375-in-dia (9.5-mm-dia) stinger connected to a tapped hole at the mid-height of the base plate. The shaker was attached 3.75-in (9.5-cm) from the corner on the 24-in (61-cm) side of the structure, so that both translational and torsional motion would be excited.

3. STRUCTURAL HEALTH MONITORING
The aforementioned test structure is analyzed using a damage detection process that is the focus of this study. The implementation of a damage detection scheme is known as structural health monitoring (SHM). SHM consists of the following four-part processes based on a statistical pattern recognition paradigm.

### 3.1 Operational Evaluation

Operational evaluation determines the conditions under which the system to be monitored functions. The first step in this assessment is to define and, to the extent possible, quantify the damage that is to be detected. Limitations on data able to be retrieved for the damage detection process are also strictly defined during this stage. Because the test structure is located in a controlled laboratory environment, variability must be carefully considered and the features extracted for damage detection should be insensitive to local damage.

Variability in the simulated building test is introduced in three forms: environmental, operational and testing variability. These sources include clamping various masses to the structure, as well as placing a hand-held shaker on the top floor of the structure in order to simulate an electrical generator or other equipment that might be present inside the building. Varying levels of shaker input to the structure were also used in order to show the sensitivity of the method to excitation levels. Different joints, along with multiple joints, were also damaged in order to test the robustness of the method. Each of these sources of variability must be carefully considered and the features extracted for damage detection should be insensitive to all of them.

### 3.2 Data Acquisition and Cleansing

Data acquisition in a SHM process begins with the selection of the types of sensors to be used, placement of the sensors, the number of sensors to be used and the hardware used to transmit the data from the sensors into storage. Intervals at which data is taken must be explored as the amount of data necessary depends on the specific structure as well as the type of damage to be detected. In addition, because data can be collected under varying sets of conditions it becomes necessary to institute a normalization procedure so that, for instance, different amplitudes of vibration input to the structure do not adversely affect the damage detection procedure.

The sensor and cabling setup was verified by sending a low frequency sine wave into the structure and visually inspecting the read-outs for each channel. The acceleration time histories are analyzed in the feature extraction and statistical modeling portion of the study. For this study, 8-second time histories were sampled at a rate of 512 samples/sec. A matrix of baseline undamaged data sets was recorded before damage was introduced to the structure. For each damage case and condition, three separate time histories were recorded in order to insure the accuracy of any results achieved. Before acquiring each data set, the pressure in the air mounts was inspected, the bolt torques throughout the structure were verified and the accelerometers were also inspected for proper mounting.

### 3.3 Feature Extraction

Feature selection involves the extraction of certain kinds of information from the data that allow a distinction to be made between a damaged and an undamaged structure. This selection involves the condensation of the large amount of available data into a much smaller data set that can be analyzed in a statistical manner. Most of the articles in the technical journals focus on this aspect of structural health monitoring.

The features that are analyzed in this study are drawn from frequency domain analysis of the time histories obtained during experimentation on the test structure. Frequency response is important in structural dynamics because it relates inputs and outputs to the structure at various frequencies. Analyzing these responses can lead to helpful information regarding the structural health of a building. Conventional frequency response function estimators are based on a linearity assumption of the system. Though many large buildings can be approximated as behaving in a linear fashion, there are always local nonlinearities within the structure. Damage to a joint in a building will almost certainly be nonlinear in nature, and any method that seeks to identify damage location and severity will be enhanced by taking into account this nonlinear behavior. Therefore, the features to be examined in this study are the exogenous and auto-regressive model coefficients in a frequency domain transmissibility model. The concept behind this model is that the response of a nonlinear vibrating system at a particular frequency is related not only to the input at that frequency, but also to the response of the system at the sub/super harmonics of that frequency. The differentiation of these two features is important: While
the exogenous coefficients describe the linear transmissibility effects, the auto-regressive coefficients describe any nonlinear effects that may be present in the system. More details on frequency domain analysis of data using an auto-regressive exogenous (ARX) input model can be found in Adams and Allemang [2] and Adams [3].

There are many possible forms of the frequency domain ARX model, with each depending on how many subharmonics and superharmonics are to be considered. In this case, a first order model will be used to account for the effects of nonlinearities in the system. This ARX model in the frequency domain is as follows:

\[
Y(k) = B(k)U(k) + A_1(k)Y(k-1) + A_2(k)Y(k+1) - A_3(k)Y(k) = 0
\]

where \(Y(k)\) is the magnitude of the response at \(k\)th frequency, \(U(k)\) is the amplitude of the input at \(k\)th frequency, and \(Y(k-1)\) and \(Y(k+1)\) are the amplitudes of the response at \(k-1\) frequency and \(k+1\) frequency, respectively. \(A(k)\)'s are the frequency domain auto-regressive coefficients, and \(B(k)\) is the exogenous coefficient used to simulate the response. These coefficients are used as features to differentiate between damaged and non-damaged cases. It should be noted that this model is derived using nonlinear and non-hysteretic assumptions, whereas damage to a building will most certainly be nonlinear and hysteretic. However, this method can still be implemented and effective despite the different assumptions made in its derivation.

In order to determine the ARX coefficients, many sets of data taken while the structure is in the same condition are needed. Because only three 4096-point time histories are available for each damage condition, each time history is split up into five separate 2048-point blocks, with 75% overlap. A Hanning window is then applied to each block to minimize the leakage problem before applying a Fast Fourier Transform (FFT). The necessary condition is that the data to be transformed must be periodic within the window. A Hanning window achieves this condition by multiplying the data in the window by a sine curve so that the amplitude of the data is zero at the beginning and end of the window. A FFT is then applied to each block of data in order to transfer the time history information into the frequency domain. This process is performed once for each pair of accelerometer channels considered in this model.

\(B(k)\) and both \(A(k)\)'s are then determined by solving a least-square problem for every frequency \(k\). Equation (1) can then be used to predict what the response of one accelerometer will be given the response of the second accelerometer as well as the response of the harmonics of the first accelerometer. That is, one accelerometer response is treated as an input and the other accelerometer response as an output. Coefficients for each of the damage cases are determined using all three available time histories of each damage case. Coefficients for the undamaged baseline condition are determined using only two of the available files and are calculated three separate times. The files used in these three cases are the first and second, second and third, and first and third. This procedure is necessary because damage is diagnosed through examination of differences in coefficients between the undamaged state and the damaged state.

### 3.4 Statistical Model Development

Statistical model development is the area of SHM that is currently least developed. Very few of the available SHM techniques have incorporated an algorithm that analyzes the extracted features from the data and unambiguously determines the damage state of the structure. Examination of the aforementioned features using rigorous statistical procedures should yield information that allows a diagnosis of damage state in the structure being monitored.

The focus of this study is to demonstrate the feasibility of a vibration-based damage detection system for structural connections. Because the information being sought is a measure of the nonlinearity of the data, the auto-regressive coefficients are used instead of the exogenous coefficients for analysis of the results. Because of the symmetry in the ARX frequency domain model, it is not necessary to analyze both auto-regressive coefficients in order to obtain a result. Therefore, in this study only the \(A_1(k)\) coefficient is analyzed. The feature that is statistically analyzed is the difference between the auto-regressive coefficients of a known undamaged state \(A_1(k)\) and the coefficients from a state that is to be determined \(A_1(k)\). Damage in the structure causes the auto-regressive coefficients to differ from the undamaged coefficients for various frequencies. Certain frequencies are more sensitive to damage in the joint and cause a greater difference between auto-regressive coefficients than other frequencies. Therefore, the extracted feature will be at a maximum (or a minimum) at these certain frequencies. This result shows that the most useful data for identifying damage to the structure will come in the tails of the feature distribution.

If this new set of features, which will now be referred to as \(G(k)\), have a Gaussian distribution, then a standard control chart [4] could be applied to monitor the status of the system. However, by plotting the features on a normal probability chart (See Figure 3) it is seen that the tails of the distribution vary widely from that of the normal distribution; if the data were normally distributed, they would plot as a straight line. This normal probability chart clearly indicates that the data is not normally distributed and, therefore, any control chart based on the normal assumption of the data will show an inflated amount of outliers for a given confidence limit, which can lead to false-positive indication of damage. The outliers will be a result of the tails of the actual distribution being much longer than that of the normal distribution. This result can be seen in Figure 4. The non-Gaussian nature of the data suggests that a different method of statistical analysis should be used.

Extreme Value Statistics (EVS) are used in this analysis to accurately model the behavior of the feature distribution tails. The basis of this branch of statistics stems from the following situation [5]. If a moving window is taken along a vector of samples and the maximum value is selected from each of these windows, then the induced cumulative density function on the maxima of the samples, as the
number of vector samples tends to infinity, can only take three forms, Weibull, Frechet, or Gumbel:

\[
\text{Frechet: } F(x) = \begin{cases} 
\exp\left[-\left(\frac{x}{\lambda}\right)^\beta\right] & \text{if } x \geq \lambda \\
0 & \text{otherwise}
\end{cases} \quad (2)
\]

\[
\text{Weibull: } F(x) = \begin{cases} 
\exp\left[-\left(\frac{x}{\lambda}\right)^\beta\right] & \text{if } x \geq \lambda \\
1 & \text{otherwise}
\end{cases} \quad (3)
\]

\[
\text{Gumbel: } F(x) = -\exp\left(\frac{-x}{\delta}\right) \quad -\infty < x < \infty, \delta > 0 \quad (4)
\]

where \(\lambda\), \(\delta\) and \(\beta\) are the model parameters that are estimated from the data. Similarly, there are only three different types of distributions for minima.

The appropriate distribution is then chosen by plotting the extracted vector of maxima on the probability paper for each distribution. The vector will plot in a linear fashion for the correct distribution and will have an associated curvature if the wrong distribution is chosen. A parametric model is then fit using the chosen distribution and produces the model parameters. For the Frechet and Weibull distributions the location parameter \(\lambda\) must be estimated \textit{a priori} to running the model estimation program. While there are a few statistical manners by which to choose this parameter, in this study it was chosen by using an initial guess based on the parameter’s limits and then choosing the final value by observing the plot of the analytical model along with the actual vector of data.

Once the model parameters are chosen, it is possible to generate true confidence limits that can be applied to the distribution. These limits are far more accurate than those given by assuming a simple Gaussian distribution. The equations for the threshold values corresponding to a chosen confidence limit are given by the following equations:

\[
\text{Frechet: } x_{\max} = \lambda + \frac{\delta}{\beta} \left( -\ln\left[1 - \frac{n\alpha}{2}\right]\right)^\frac{1}{\beta} \quad (7)
\]

where \(n\) is the window size used to extract the maxima and \(\alpha\) is the associated Type I error of the confidence limit. It should be noted that the upper confidence limit is calculated from the associated maxima distribution and the lower confidence limit is calculated from the associated minima distribution. For more information on extreme value statistics, please see [4].

In this study, the parent distribution of \(G(k)\) has 800 data points. A window of 10 samples is moved along the parent vector and the maximum (minimum) of each window is then extracted. This process generates a maxima (minima) vector of 80 points to be analyzed by EVS. Once a distribution is chosen, the model parameters must be estimated. Only a portion of the data points in the maxima (minima) vector are used to compare to the fitted model, because agreement with the upper and lower ends of the extracted maxima and minima vectors, respectively, is more important than agreement with the entire vector. An example of this result can be seen in Figure 5, which shows an accepted fit of maxima in a Frechet distribution.

Once a model has been fit for both the maxima and minima of a parent distribution the confidence limits are calculated using the previous equations for the threshold values. In this study, confidence intervals of 99%, 99.5%, and 99.9% were tested to see which gave the best results. It was determined that a confidence interval of 99.5% \((\alpha = 0.005)\) was best suited to the data used in this analysis.

### 4. EXPERIMENTAL RESULTS

In this project, damage is simulated in joints through the loosening of the preload applied by the bolts at the joints of the structure. A “healthy” joint is held together by bolts that are torqued to a value of 220 inch-pounds (25 Nm). Multiple damage cases are then used so that the sensitivity of the damage detection method can be tested. The first damage case is simulated by loosening the preload on the bolts at a particular joint to 15 inch-pounds (1.8 Nm). The next case has the preload being loosened to 5 inch-pounds (0.6 Nm). Bolts on the selected joint are then completely removed to simulate a crack in the joint for the final damage case. Vibration input is applied by an electrodynamic shaker attached to the base of the structure. The input system is a random waveform with a uniform energy content between the frequencies of 0 and 200 Hz. The damage cases investigated in this study are summarized in Table 1.

<table>
<thead>
<tr>
<th>Damage Case</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Damage Case 1</td>
<td>No clamped mass, joint 2a has induced damage</td>
</tr>
<tr>
<td>Damage Case 2</td>
<td>Clamped mass on level a, joint 4b has induced damage</td>
</tr>
<tr>
<td>Damage Case 3</td>
<td>Clamped mass on level b, joints 4b and 2a have induced damage</td>
</tr>
<tr>
<td>Damage Case 4</td>
<td>Clamped mass on level c, induced damage at joint 4b, loose masses on level a and level b</td>
</tr>
<tr>
<td>Damage Case 5</td>
<td>Clamped mass on levels a &amp; b, induced damage at joints 2a and 4b, handheld shaker emitting sign wave on level a</td>
</tr>
</tbody>
</table>

\[
\text{Gumbel: } x_{\max} = \lambda - \delta \ln \left[ -\ln\left(1 - \frac{n\alpha}{2}\right)\right] \quad (5)
\]

\[
\text{Weibull: } x_{\max} = \lambda - \delta \ln \left[ -\ln\left(1 - \frac{n\alpha}{2}\right)\right]^\frac{1}{\beta} \quad (6)
\]

For the main analysis the suitability of the statistical model is extended for the most basic damage case. Having done that, the results will be expanded to consider more complicated damage scenarios. Therefore, the bulk of the analysis shown will involve Damage Case 1 (from Table 1) and the highest excitation level of the base shaker. Five
sets of data are used in each analysis. The first is the baseline undamaged data that is used to set the confidence limits. These limits are tested against data sets from all three damage levels (15 in-lb, 5 in-lb, and no bolts) and against another undamaged case to be sure that false positives are not a problem.

For this case, and for all other damage cases, it was determined that the Frechet distribution, for either maxima or minima, was the most appropriate extreme value distribution to use for the analysis. After the location parameter Λ is found through trial and error, the other model parameters are found by fitting the parametric model to data. Upper and lower 99.5% confidence limits are then calculated from the known parameters. In this damage case the lower confidence limit is -0.3674 (-0.2224 for the normal assumption) and the upper confidence limit is 0.3918 (0.2059 for the normal assumption). For a sample size of 800 points and 99.5% confidence interval, one should expect 2 outliers on each side of the confidence interval. Table 2 illustrates the effectiveness of the limits found using extreme value statistics versus those calculated using the normal assumption. All numbers displayed in parentheses are associated with the normal assumption.

Table 2: Number of outliers for particular damage cases at the damaged joint

<table>
<thead>
<tr>
<th></th>
<th>Lower outliers</th>
<th>Upper outliers</th>
</tr>
</thead>
<tbody>
<tr>
<td>Baseline 1</td>
<td>1 (15)</td>
<td>1 (10)</td>
</tr>
<tr>
<td>15 in-lb torque</td>
<td>88 (166)</td>
<td>20 (64)</td>
</tr>
<tr>
<td>5 in-lb torque</td>
<td>110 (202)</td>
<td>20 (64)</td>
</tr>
<tr>
<td>Bolt removed</td>
<td>9 (30)</td>
<td>38 (111)</td>
</tr>
<tr>
<td>Baseline 2</td>
<td>2 (11)</td>
<td>0 (12)</td>
</tr>
</tbody>
</table>

* Results from the normality assumption are shown in parentheses.

Clearly, the confidence limits derived using extreme value statistics are much nearer the actual 99.5% limits than those derived using the normal assumption. This result can also be expressed graphically. Figures 6-8 show plots of data and confidence limits for the baseline undamaged case, the 5 in-lb damage case, and then the alternate undamaged case.

Figure 6 shows only one outlier on each side of the confidence interval. That result is slightly less than the expected outcome of two outliers on each side, but does not produce a false positive result so it is acceptable. It is clear that if confidence limits based upon the normal assumption are used there are many false positives given, which negates any usefulness that the analysis method sought.

Figure 7 shows the effectiveness of using coefficients from the frequency domain ARX model. There is a clear graphical aberration from the data taken during the undamaged state. Both the EVS and normal confidence limits correctly indicate damage has taken place. However, the EVS confidence limits show the frequency range in which the accelerometer response is truly affected by the damage in the system.

Figure 8 indicates that false positives are not a problem with this analysis method. These data were not used to calculate the confidence limits and yet there are no more than 2 outliers on a side, which clearly indicates that the system is in an undamaged state. So, we can see that in the most general case, the frequency domain ARX model combined with extreme value statistics does an excellent job of indicating the damage state of the joint.

Using the same data, if another joint in the building that is not damaged is evaluated the results are not as favorable. Table 3 mirrors the format of the table from the damaged joint: Once again, the EVS confidence limits perform much better than those associated with the normal assumption. However, in each of the damage cases the EVS confidence limits still have a false indication of damage. While this may seem to show a severe limitation for this analysis method, the result is more likely caused by the construction of the test structure. While the structure is meant to be a scale model of a three-story building, the dynamics are considerably different than an ordinary building. In an actual structure, damage at one joint will mostly likely not affect accelerometer readings at another joint to the magnitude that the test structure displays. Therefore, the result shown in Table 3 might be explained as a result of the dynamics of the test structure.

Table 3: Number of outliers for particular damage cases at an undamaged joint

<table>
<thead>
<tr>
<th></th>
<th>Lower outliers</th>
<th>Upper outliers</th>
</tr>
</thead>
<tbody>
<tr>
<td>Baseline 1</td>
<td>2 (8)</td>
<td>1 (7)</td>
</tr>
<tr>
<td>15 in-lb torque</td>
<td>13 (33)</td>
<td>25 (58)</td>
</tr>
<tr>
<td>5 in-lb torque</td>
<td>9 (29)</td>
<td>26 (62)</td>
</tr>
<tr>
<td>Bolt removed</td>
<td>12 (42)</td>
<td>31 (75)</td>
</tr>
<tr>
<td>Baseline 2</td>
<td>1 (10)</td>
<td>0 (7)</td>
</tr>
</tbody>
</table>

* Results from the normality assumption are shown in parentheses.

To date, analysis has not been done for every damage case, including the most severe damage case (Damage Case 5). However, for the few cases that were investigated, the analysis method based on the 99.5% confidence interval produced no false positives on damaged joints and no false negatives for each damage level. The method also appears to be largely insensitive to excitation level, so long as the excitation is sufficient enough to excite nonlinearity into the system at damaged joints.

5. SUMMARY

Coefficients from a frequency domain ARX model show promise as a powerful feature for damage discrimination. The addition of extreme value statistics as a means for establishing true confidence limits greatly enhances this extracted feature. The next step in the process is to fully analyze all of the data taken from the test structure. This analysis will highlight any sources of variability to which the outlined analysis is sensitive. A more rigorous method of choosing the location parameter in the extreme value analysis would also strengthen the statistical background of the analysis.
The experiment design could also be simplified by exciting the structure in only one direction and applying symmetrical damage to the structure so that the analysis is reduced to a one-dimensional problem. This approach may provide insight that was not available in the previous experiment because of its overall complexity. It may also be helpful to normalize the data in a different fashion so that damage in one joint does not so easily show up in readings taken at another joint. A very important choice that needs to be made is whether the first order frequency domain model is an adequate one, or if substantially better results can be obtained by analyzing the data using a higher-order model.

ACKNOWLEDGEMENT

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REFERENCES


Figure 1: A side view of the assembled test structure

Figure 2: A top view of the assembled test structure

Figure 3: Normal probability plot of the feature G(k)

Figure 4: Probability density function of the extracted feature G(k) vs. the normal assumption
Figure 5: Probability density function of the extracted feature $G(k)$ vs. the normal assumption.

Figure 6: 99.5% Confidence Interval of baseline undamaged data with legend (solid line: $G(k)$, dash-dotted line: EVS confidence limit, solid inner line: normal confidence limit).

Figure 7: 99.5% Confidence Interval of 5 in-lb damage case with legend (solid line: $G(k)$, dash-dotted line: EVS confidence limit, solid inner line: normal confidence limit).

Figure 8: 99.5% Confidence Interval of alternate undamaged data with legend (solid line: $G(k)$, dash-dotted line: EVS confidence limit, solid inner line: normal confidence limit).