Heavy Quark Symmetry and $D_s(2420) \rightarrow D^*\pi$ Decay

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Abstract

Heavy quark symmetry relates the D-wave amplitude for $D_s(2420) \rightarrow D^*\pi$ to the amplitudes for the decays $D_s(2460) \rightarrow D^*\pi$ and $D_s(2460) \rightarrow D\pi$. We discuss the extraction of the S and D partial wave amplitudes for $D_s \rightarrow D^*\pi$ and their significance as tests of the applicability of heavy quark symmetry for charm quarks.
The physics of hadrons containing a single heavy quark \( Q \) (by "heavy", we mean \( m_Q \gg \Lambda_{QCD} \)) is greatly simplified by going over to an effective theory where the mass of the heavy quark goes to infinity with its four-velocity fixed\(^1\)\(^-\)\(^12\). For \( N \) heavy quarks, this effective theory has an \( SU(2N) \) spin-flavor symmetry group that acts on heavy quarks with the same four velocity.\(^1\)\(^-\)\(^13\) The heavy quark symmetry can be used to predict many nonperturbative properties of hadrons containing a heavy quark. The most important of these predictions are for exclusive semileptonic \( B \)-meson decays. They are expected to play an important role in determining the magnitudes of the elements \( V_{ub} \) and \( V_{cd} \) of the Cabibbo-Kobayashi-Maskawa matrix.

The weakest feature of this approach to determining \( |V_{ub}| \) and \( |V_{cd}| \) is treating the charm quark as heavy. In this paper we examine a system which bears on the issue of whether the charm quark mass is sufficiently large for the limit \( m_c \to \infty \) to be a good approximation.

In addition to predicting weak matrix elements, heavy quark symmetry relates the masses and strong decay amplitudes of excited hadrons containing a single heavy quark \( Q \) (Ref. 13). In the limit \( m_Q \to \infty \) the spin \( \hat{S}_Q \) of the heavy quark and the spin \( \hat{S}_t \equiv \hat{S} - \hat{S}_Q \) of the light degrees of freedom are separately conserved by the strong interactions; hadrons containing a single heavy quark can therefore be simultaneously assigned the quantum numbers \( s_Q, m_Q, \bar{s}_Q, m_t, \bar{s}_t, \) and \( \bar{s}_q \), where \( s_Q = \pm 1 \) and \( \bar{s}_q \) are the parities of the heavy quark and light degrees of freedom, respectively. Since the dynamics of the light degrees of freedom are independent of the mass and spin of the heavy quark it is convenient, in the limit \( m_Q \to \infty \), to classify states by \( s_t \) and \( \bar{s}_q \). Then associated with each such state of the light degrees of freedom will be a degenerate doublet of hadrons with total spin parity

\[
\frac{1}{2} - \frac{1}{2} \pm \frac{1}{2} = \frac{1}{2} \pm \frac{1}{2} = \frac{1}{2}^{+} \quad \text{or} \quad \frac{1}{2}^{-} \quad \text{or} \quad \frac{3}{2}^{+} \quad \text{or} \quad \frac{3}{2}^{-},
\]

unless \( s_t = 0 \) in which case a single spin 1/2 state is obtained. Since the state of the light degrees of freedom is independent of the heavy quark mass, the spectrum is identical for each heavy quark flavor, up to an overall shift associated with the mass of the heavy quark.

Since the heavy quark flavor symmetry applies to the complete set of \( n \)-point functions of the theory, not only mass splittings but also strong decay amplitudes arising from emission of light states like \( \pi, \eta, \rho, \omega, \) etc., are independent of the heavy quark flavor\(^13\). For a given flavor, because \( \hat{S}_Q \) generates a symmetry, the two states with spin \( s_+ \) must have the same total widths. This equality between total widths typically arises in a nontrivial way: the two states of a given multiplet can decay to both states of every available multiplet with distinct partial widths whose sum must be identical. The spin symmetry predicts the ratios of these partial widths.

At the present time, only for \( Q = c \) have excited hadrons containing a single heavy quark been observed. They are the charmed mesons \( D_s^* (2460) \) and \( D_s^* (2420) \). For mesons containing a single heavy quark both the constituent quark model and experiment tell us that the ground states have \( s_0^* = 1/2^- \) giving \( s_+^* = 0^- \) and \( s_+^* = 1^- \) states. In the case \( Q = c \) these are the \( D \) and \( D^* \) mesons. The constituent quark model suggests that the lowest-lying excited states are likely to be those that correspond to the spin 1/2 constituent antiquark having a unit of orbital angular momentum, giving \( s_0^* = 1/2^+ \) and \( 3/2^+ \) multiplets.
Heavy quark spin symmetry predicts\(^{13,14}\) that the \(s^+_c = 2^+\) state of the \(s^+_c = 3/2^+\) charm meson multiplet has decay amplitudes in the proportion \(\sqrt{2}/3 : \sqrt{3}/3\) to the states \(|D\rangle\) and \(|D^*\rangle\), respectively, while its multiplet partner with \(s^+_c = 1^+\) decays at the same total rate exclusively to \(|D^*\rangle\). Note that in the heavy quark limit, \(m_c \to \infty\), this latter state does not decay to \(|D^*\rangle\) even though this is an allowed channel. (Here the subscript \(S\) or \(D\) denotes the partial wave of the pion.) For the \(s^+_c = 1/2^+\) multiplet of charmed mesons heavy quark symmetry predicts\(^{13,14}\) that the \(s^+_c = 1^+\) state decays exclusively to \(|D^*\rangle\) even though \(|D^*\rangle\) is an allowed channel) and that its \(s^+_c = 0^+\) partner decays to \(|D\rangle\) with the same amplitude.

These predictions are not inconsistent with existing experimental information\(^{13}\). If one interprets the two confirmed states \(D_s^0(2460)\) and \(D_s(2420)\) as members of the \(s^+_c = 3/2^+\) multiplet, then their mass difference is consistent with being a \(\Lambda_{QCD}/m_c\) correction to the limiting theory. Moreover, the \(D_s^0 \to D\pi\) and \(D_s^+ \to D^*\pi\) decays (which are necessarily \(D\)-wave) have amplitudes that are in the ratio \(0.8 \pm 0.1\), which is close to the prediction \(\sqrt{2}/3\). (Here, and in what follows, we quote amplitudes with a phase space and "typical" barrier-penetration factor of \(p^3 \exp(-p^3/(1\text{GeV}^2))\) removed.

This amounts to including the effects of those \(\Lambda_{QCD}/m_c\) corrections which are enhanced in importance because \(p\), the magnitude of the pion three-momentum, is not very large compared with the \(D^* - D\) mass splitting.) The ratio of the \(D_s(2420) \to D^*\pi\) and \(D_s^0(2460) \to D^*\pi\) decay amplitudes is \(2.3 \pm 0.6\). Given the substantial error, it is not clear whether this result is in disagreement with the \(m_c \to \infty\) prediction of \(\sqrt{2}/3\). The examination of the \(D_s \to D^*\pi\) decay in more detail is the main purpose of this paper.

While heavy quark symmetry does not relate \(S\)-wave to \(D\)-wave decay amplitudes, the constituent quark model suggests that the \(S\)-wave decay amplitudes are very strong\(^{14}\). This may explain the difficulty so far in seeing the members of the \(s^+_c = 1/2^+\) multiplet which consequently are expected to have large widths. This observation suggests that the \(D\)-wave decay of \(D_s(2420)\) may be contaminated via \(\Lambda_{QCD}/m_c\) corrections by an \(S\)-wave component. (For example, such corrections could mix the \(s^+_c = 3/2^+\) and \(s^+_c = 1/2^+\) \(D_s\) states. Since the quark model predicts that the \(S\)-wave widths are an order of magnitude larger than the \(D\)-wave widths, even a weak mixing could produce a substantial effect.) To rigorously test the predictions of heavy quark symmetry, it will therefore be necessary to separately determine the \(D\)- and \(S\)-wave amplitudes of the decay \(D_s(2420) \to D^*\pi\).

Heavy quark symmetry must then pass two tests: the \(D\)-wave amplitude must be correctly related to those of the \(D_s^0(2460)\), and the \(S\)-wave amplitude must be small compared to other "typical" \(S\)-wave amplitudes, e.g., the decay amplitudes for the members of the \(s^+_c = 1/2^+\) multiplet. (Note, however, that it need not be small compared with the \(D_s \to (D^*\pi)\) decay amplitude: a small grapefruit can be larger than a typical apple.) It already seems likely that the second test will be met. The total width of the \(D_s(2420)\) is only about 20 MeV, and this is much smaller than typical \(S\)-wave widths (e.g., \(D_1 \to (p\pi)s\), which is the light quark analog of the \(D_s\) decays, has a width of about 350 MeV).

The \(D_s \to D^*\pi\) decay matrix element can be written, in the rest frame of the \(D_s\), as

\[
M(D_s \to D^*\pi) = \left( \frac{S e^{i\phi}}{\sqrt{2}} \right) \frac{3}{\sqrt{2}} D(\hat{r} \cdot \hat{p})(\hat{r} \cdot \hat{p})
\]

In Eq. (2), \(S\) and \(D \equiv \hat{D}(p/d)^2\) are real constants corresponding to the \(S\) and \(D\)-wave
amplitudes, $\phi_{SD}$ is their relative phase, $\hat{p}$ is a unit vector in the direction of the pion's three momentum $\vec{p}$, $\beta$ is a momentum scale characteristic of the decay, $\vec{r}$ is the polarization vector for the $D^*$ state, and $\vec{r}_1$ is the polarization vector for the $D_1$ state. A factor of $3/\sqrt{2}$ and the centrifugal barrier factor $(p/\beta)^2$ have been inserted into the definition of $D$ for convenience (see, e.g., Eq. (5)). Since the polarization of the $D_1$ depends on its production mechanism (i.e., generally it will be a mixed state), we use the density matrix formalism to describe it. We assume that the density matrix depends only on the direction of the $D_1$, which we call $\vec{z}$ (Ref. 17). Then

$$\rho^{ij} = \left(1 - f_L \right) \delta^{ij} + \left(3f_L - 1 \right) \vec{r}_z \cdot (\vec{r}_z \times \vec{z}) \delta^{ik} \delta^{jk},$$

(3)

where $f_L$ is the probability that the $D_1$ is in state $|0\rangle$ with zero helicity ($0 \leq f_L \leq 1$) and $-1 \leq f_L \leq 1 - f_L$ is the net polarization of the $D_1$ with respect to the direction $\vec{z}$.

Combining Eqs. (2) and (3) yields

$$\frac{d\Gamma}{d\Omega} = \frac{1}{2} D^2 - 3/2 D \cos \phi_{SD} \left[ \left(1 - f_L \right) + \left(3f_L - 1 \right) \left| \vec{r}_z \cdot (\vec{r}_z \times \vec{z}) \right| \right]$$

$$+ 3\sqrt{2} \Re \left\{ D (S^* e^{i\gamma} D^*) \right\} \left[ \left(1 - f_L \right) + \left(3f_L - 1 \right) \left| \vec{r}_z \cdot \vec{p} \right| \right]$$

$$\frac{1}{2} D^2 \left[ \left(1 - f_L \right) \left| \vec{r}_z \cdot \vec{p} \right|^2 + \left(3f_L - 1 \right) \left| \vec{r}_z \cdot \vec{p} \right| \right]$$

(4)

We can derive\textsuperscript{14} from Eq. (4) all of the results we need to determine $D$ and $S$. Several distinct types of measurements are required because in addition to the magnitudes of $S$ and $D$, which are of immediate interest, measurements typically will depend on the relative phase $\phi_{SD}$ as well as the parameters $f_L$ and $P$ describing the density matrix. We can eliminate dependence on $P$, thereby reducing the required number of measurements to three, by concentrating on unpolarized rates and polarization effects involving helicity zero $D^*$'s (or ones longitudinally polarized with respect to $\vec{z}$). See Eq. (4) with $\vec{r} = -\vec{p}$ (or $\vec{r} = \vec{z}$).

We first sum over the possible polarizations of the $D^*$ (using $\sum_{\epsilon} e^{i\epsilon} = \epsilon(\epsilon)$. Eq. (4) then yields

$$\Gamma = 4\pi \left| D^2 + S^2 \right|$$

(5)

and

$$\frac{4\pi}{\Gamma} \frac{d\Gamma}{d\Omega} = 1 \left( \frac{3f_L - 1}{2} \right) \left[ \frac{D^2 + 2\sqrt{2} \Re D \cos \phi_{SD}}{D^2 + S^2} \right] \left( \frac{3\cos^2 \theta - 1}{2} \right).$$

(6)

Here $\theta$ is the angle between the direction of the $D_1$ and the direction of the pion (in the $D_1$ rest frame). Note that if $f_L = 1/3$, the distribution is isotropic, while if $f_L = 0$ the angular distribution for a pure $D$-wave is proportional to $\sin^2 \theta$. Instead of averaging over the $D^*$ spins, we can also easily calculate from Eq. (4) the rate for $D^*$'s with zero helicity, i.e., with $\vec{r} = -\vec{p}$. This gives

$$\frac{4\pi}{\Gamma} \frac{d\Gamma}{d\Omega} = \left(1 + \left[ \frac{D^2 + 2\sqrt{2} \Re D \cos \phi_{SD}}{D^2 + S^2} \right] \left( \frac{3f_L - 1}{2} \right) \cos^2 \theta \right),$$

(7a)

Note that for $f_L = 1/3$ the angular distribution is once again isotropic, while now for $f_L = 0$ it is proportional to $\cos^2 \theta$, independent of the $D/S$ ratio. The total rate for $D_1$ decay to zero helicity $D^*$'s is independent of $f_L$:

$$\frac{\Gamma_{D^*_0}}{\Gamma} = \frac{1}{3} \left[ \frac{D^2 + 2\sqrt{2} \Re D \cos \phi_{SD}}{D^2 + S^2} \right].$$

(7b)
Since Eqs. (6) and (7) all depend on the same combination \( \frac{D^{*+} \Delta D_{2+}}{(D^+ S^2)} \), additional constraints are required. We therefore turn to the (P-independent) decay rate to \( D^*\)'s polarized longitudinally along \( \hat{z} \), which is

\[
\frac{4\pi}{\Gamma} \frac{d\Gamma}{dL} = f_L + \frac{2\sqrt{3} f_L S_D D_{2+}}{S^2 + D^2} \left( \frac{3\cos^2 \theta - 1}{2} \right) + \frac{D^2}{2(S^2 + D^2)} \left( f_L + 3\frac{3f_L - 3}{2}\cos \theta - 9\frac{3f_L - 1}{2}\cos^3 \theta \right) \tag{8a}
\]

The total decay rate to \( D^*\)'s polarized longitudinally along \( \hat{z} \) is

\[
\frac{\Gamma}{\Gamma} = f_L - \frac{3D^2}{5(S^2 + D^2)} \left[ f_L - \frac{3f_L - 1}{2} \right] \tag{8b}
\]

The measurements (5)-(8) over determine \( D, S, \phi_{2D}, \) and \( f_L \), so that many strategies can be devised for extracting the interesting amplitudes \( D \) and \( S \) from them. As an extreme example, we note that a fit to the single angular distribution in (8a) can in principle simultaneously determine \( f_L, S/D, \) and \( \phi_{2D} \), so that it and the total rate determine all four parameters. A more realistic strategy might be to use the angular distribution in (7a) to determine \( f_L, \) (8b) to determine \( S/D, \) and then (5) to determine \( D \) and \( S \) separately. This strategy avoids \( \phi_{2D} \).

It should be noted that no strategy based on these measurements will work if the \( D_1\)'s are produced with \( f_L = 1/3 \); in this case Eqs. (6) and (8b) are independent of \( S/D \), while Eqs. (7a), (7b), and (8a) depend only on \( \frac{D^{*+} \Delta D_{2+}}{(D^+ S^2)} \). This limitation is not, however, a serious one since there are specific production mechanisms for which the \( D_1\)'s are aligned. For example, a \( D_1 \) produced in the decay \( \bar{B} \to D_1 \pi \) is in a zero helicity state, \( f_L = 1 \), to conserve angular momentum. Also, assuming factorization\(^{19}\) of the weak nonleptonic decay amplitude, in \( \bar{B} \to D_1 \rho \) decay the \( D_1 \) is mostly emitted with zero helicity. Thus, if necessary, a selection of \( D_1\)'s can be made which will have \( f_1 \neq 1/3 \). Of course in practice the optimal strategy will depend on experimental conditions and must be devised by those doing these measurements.

In this comment we have seen how, in the decay \( D_1 \to D_1 \pi \), such measurements as the angular distribution of the pion with respect to the \( D_1 \) direction, the longitudinal helicity of the \( D^* \), and the longitudinal polarization of the \( D^* \) along the direction of the \( D_1 \), allow a determination of the fraction of the \( D_1 \) width that arises from the \( D_1 \to (D^* \pi)_D \) amplitude. This can be used to test a prediction based on heavy quark symmetry that relates the \( D_1 \to (D^* \pi)_D \) decay amplitude to the \( D\)-wave \( D'_1 \to D \pi \) and \( D'_1 \to D^* \pi \) decay amplitudes. Such tests are important for defining the extent to which a charm quark may be sufficiently heavy for the predictions of heavy quark symmetry to be applicable not only in hadron spectroscopy, but also in the prediction of weak matrix elements.

References

1. Research supported in part by the U.S. Dept. of Energy under Contracts DE-AC03-81ER40650 and DE-AC05-84ER40150 and by the Natural Sciences and Engineering Research Council of Canada.

5. E. Eichten, in Proceedings of the International Symposium on Field Theory on the


14. This result was obtained earlier by J. Rosner, Comments Nucl. Part. Phys. 16, 109 (1988) by taking the $m_c \rightarrow \infty$ limit of a constituent quark model calculation. Heavy quark symmetry shows that these results are model-independent consequences of QCD in that limit.


17. This assumption must be examined on a case-by-case basis. However, for $D_1$'s produ-

duced in the decay of $D$'s from the $\Upsilon(4S)$ it is a safe assumption.

18. To average over $\hat{p}$, the direction of the pion, one may use

\[ \left( \frac{1}{4\pi} \right) \int d\Omega \hat{p} \hat{p}' = \frac{1}{3} \delta \cdot \delta' \]

\[ \left( \frac{1}{4\pi} \right) \int d\Omega \hat{p} \hat{p}' \hat{p}' = \frac{1}{15} \left[ \delta \cdot \delta' + \delta' \cdot \delta + \delta \cdot \delta \right] \]