Scaled simulations of a 10 GeV accelerator

Estelle Cormier-Michel*, C.G.R. Geddes*, E. Esarey*,†, C.B. Schroeder*, D. L. Bruhwiler**, K. Paul**, B. Cowan** and W.P. Leemans*,†

*LOASIS program, Lawrence Berkeley National Laboratory, Berkeley, CA 94720
†University of Nevada Reno, Reno, NV 89557
**Tech-X corporation, Boulder, CO 80303

Abstract. Laser plasma accelerators are able to produce high quality electron beams from 1 MeV to 1 GeV. The next generation of plasma accelerator experiments will likely use a multi-stage approach where a high quality electron bunch is first produced and then injected into an accelerating structure. In this paper we present scaled particle-in-cell simulations of a 10 GeV stage in the quasi-linear regime. We show that physical parameters can be scaled to be able to perform these simulations at reasonable computational cost. Beam loading properties and electron bunch energy gain are calculated. A range of parameter regimes are studied to optimize the quality of the electron bunch at the output of the stage.

Keywords: Laser plasma accelerator, 10 GeV stage, PIC simulation
PACS: 52.38.Kd, 41.75.Jv, 52.65.Rr

INTRODUCTION

Laser plasma accelerators are able to produce very high gradient electric fields which make them suitable to accelerate particles to high energies in a very short distance. Production of high quality electron beams has been demonstrated, with energies up to 1GeV reached in a few centimeters [1]. Until recently, acceleration of these electron beams relied on the self-trapping of background plasma electrons. Experiments have shown that it is now possible to control the quality of the electron beam by controlling the injection process and then further accelerating the beam [2, 3, 4]. In this paper we examine the design of a 10 GeV accelerator stage where we assume that a high quality, mono-energetic electron beam is externally injected in a “dark current free” (i.e., where there is no self-trapping) plasma accelerating structure. Parameters of the accelerating stage are studied in order to optimize charge, energy and energy spread of the bunch.

SCALING LAWS

The acceleration length, and hence the energy gain, in a laser plasma based accelerator, is limited by the dephasing of the electrons, i.e., the electrons stop being accelerated when they outrun the wake [5]. To increase the dephasing length and increase the energy gain up to 10 GeV, the density must be decreased to the order of 10^{17} cm^{-3} giving an acceleration length of about a meter [6]. Using a fully self-consistent Particle-In-Cell (PIC) algorithm to simulate such a stage is still impractical with state of the art computing facilities. The use of reduced models [7, 8, 9] is therefore required to model acceleration of particle beams to these high energies as it can greatly reduce the required number of simulation hours. Here we use the fully self-consistent PIC algorithm, implemented in the code VORPAL [10], to simulate the evolution of an externally injected electron bunch over the accelerating stage by scaling the physical quantities with the plasma density. In this way, it is possible to use short scaled simulations at high density and deduce the properties of the accelerated electron bunch by verifying and using scaling laws. This also allows us to fully simulate the laser pulse evolution up to depletion.

In this paper we study a laser plasma accelerator stage in the standard regime [5]. The normalized laser intensity profile is of the form $a_0^2 \exp(-2x^2/L^2)\exp[-2(y^2+z^2)/w_0^2]$ where $a_0^2 = 7.32\times 10^{-19}(\lambda_0[\mu\mathrm{m}])^2 I_0[\mathrm{W/cm^2}]$ with λ_0 the laser wavelength and I_0 the laser intensity, x is the longitudinal direction, L the laser pulse length, and w_0 the laser spot size. The laser is guided by a transverse parabolic plasma channel whose depth is adjusted, when the power of the laser is close to the critical power, to compensate for the self-focusing such that the laser beam radius undergoes minimal variation. The injected electron bunch profile is $n_b \sim \exp(-x^2/2\sigma_L^2)\exp[-(y^2+z^2)/2\sigma_r^2]$, with σ_L the bunch length and σ_r the bunch radius.

In the scaled simulations the dimensions of the problem are kept constant compared to the plasma wavelength λ_p, i.e, k_pL, k_pw_0, $k_p\sigma_L$ and $k_p\sigma_r$ are constant while the density is varied, where $k_p = 2\pi/\lambda_p \propto \sqrt{n_0}$ is the plasma wave number and n_0 the plasma density. The normalized laser strength a_0 is also fixed, fixing the ratio of the laser power over the critical power. Using the fixed parameters $k_pL = 2$, $k_pw_0 = 5.3$ and $a_0 = 1$, and $n_0 = 10^{19}$ and 10^{18} cm^{-3} we verify that the peak accelerating electric field scales as $\sqrt{n_0}$ as expected [11]. The laser evolution is also consistent. Although the laser betatron oscillation and Rayleigh length are not constant compared to λ_p, this issue is overcome by the use of the plasma channel and indeed we verify that the spot size stays constant along the guiding structure and that the same amount of energy is depleted after a dephasing length. Because of this, the wakefield structure also scales with the density, i.e., contours of the accelerating field overlay within a few percent, when normalizing the lengths by λ_p; this has also been confirmed using $n_0 = 10^{17}$ cm^{-3} and reduced models. We also verify that the dephasing length scales as $\lambda_p^3/\lambda_0^2 \propto n_0^{-3/2}$ [6]. Note that the wake phase velocity depends on the plasma density for a constant laser wavelength. This confirms the scaling laws predicted in the linear regime and extends them to multi-dimensions and to the quasi-linear regime.

The energy gain of a stage is given by $L_{\mathrm{acc}}E_x \propto 1/n_0$, where L_{acc} is the accelerating length which is equal to the dephasing length in the linear regime. The linear theory predicts that, for the parameters given above, an energy gain of 100 MeV is obtained at $n_0 = 7\times 10^{18}$ cm^{-3} and 10 GeV is achieved at $n_0 = 7\times 10^{16}$ cm^{-3}. A gain of 8 GeV has been demonstrated with test particles using a simulation in the boosted frame in 1D at 10^{17} cm^{-3} [12, 13], consistent with the linear theory which predicts a gain of 7 GeV in this case. By using test particles in the PIC simulations, we find that because the wake is slightly non-linear, 97 MeV and 1120 MeV can be achieved respectively at 10^{19} and 10^{18} cm^{-3}. We can then predict that we will obtain 10 GeV with $n_0 = 10^{17}$ cm^{-3} over $L_{\mathrm{acc}} = 1$ m. We next perform simulations with the PIC algorithm at $n_0 = 10^{19}$ cm^{-3} where the beam is accelerated to 100 MeV in 1 mm to determine beam loading performance and electron beam quality. Using the scaling laws given here we can deduce the properties of the bunch in a 10^{17} cm^{-3} stage.

BEAM LOADING

A charged electron beam, when propagating into the plasma, creates its own wake which damps the wakefield created by the leading laser pulse. The maximum amount of charge that can be loaded is that for which the wakes created by the electron beam and the laser pulse are equal. For a short bunch ($k_p\sigma_L < 1$) in the linear regime ($E_x/E_0 < 1$) it is given by [14]:

$$N_{\max} = \frac{1}{2k_p r_e}\frac{E_x}{E_0}\frac{k_p^2\sigma_r^2}{H_R} \approx 9.3\times 10^9 \frac{E_x/E_0}{\sqrt{n_0/10^{16}\mathrm{cm}^{-3}}}\frac{k_p^2\sigma_r^2}{H_R} \tag{1}$$

with

$$H_R = \frac{k_p^2\sigma_r^2}{2}\exp\left(k_p^2\sigma_r^2/2\right)\Gamma\left(0,\frac{k_p^2\sigma_r^2}{2}\right) \qquad \text{(3D)} \tag{2}$$

$$H_R = \sqrt{\frac{\pi}{2}}k_p\sigma_r\exp\left(k_p^2\sigma_r^2/2\right)\mathrm{Erfc}\left(\frac{k_p\sigma_r}{\sqrt{2}}\right) \qquad \text{(2D)} \tag{3}$$

where $E_0 = c^2 m k_p/e$ is the cold non-relativistic wavebreaking limit, r_e is the classical electron radius, and $\Gamma(a,x) = \int_x^\infty e^{-t}t^{a-1}dt$.

Simulations verify that Eq. (1) is accurate for a mildly nonlinear laser-driven wakefield ($a_0 \sim 1$). Figure 1 shows the percentage of beam loading in terms of peak ac-

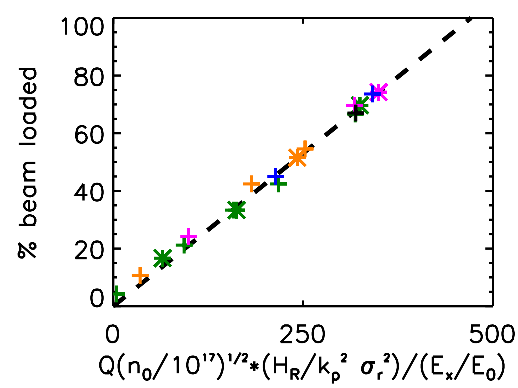

FIGURE 1. Beam loading, in terms of peak accelerating electric field, as a function of charge, normalized by $\sqrt{n_0(\mathrm{cm}^{-3})/10^{17}} \times \left(H_R/k_p^2\sigma_r^2\right)/(E_x/E_0)$. The dashed line is theory, the points are for 2D (+) and 3D (∗) simulations and for the following cases from the lighter gray to the darker: $k_pL = 2$, $a_0 = 1$, $n_0 = 10^{19}$ cm^{-3}, $k_p\sigma_L = 0.05$ and beam radii $k_p\sigma_r = 0.3$ (orange), $k_p\sigma_r = 1$ (magenta), $k_p\sigma_r = 1.8$, (green); scaling with a_0 is shown at $k_pL = 1$, $a_0 = \sqrt{2}$, $n_0 = 10^{19}$ cm^{-3}, $k_p\sigma_r = 0.3$ (blue); scaling with density is shown at $k_pL = 2$, $a_0 = 1$, $n_0 = 10^{18}$ cm^{-3}, $k_p\sigma_r = 1.8$ (black).

celerating electric field as a function of the electron bunch charge, Q, normalized by $\sqrt{n_0(\mathrm{cm}^{-3})/10^{17}} \times \left(H_R/k_p^2\sigma_r^2\right)/(E_x/E_0)$. The dashed line is the coefficient given by Eq. (1) and the points represent different simulated cases. The scaling is verified for different values of σ_r, n_0, and E_x/E_0, with $k_p\sigma_L = 0.05$, for both 2D and 3D simulations. Simulations in 2D at $n_0 = 10^{17}$ cm^{-3} also agree with the scaling. This data implies that we can load 60 pC for $k_p\sigma_r = 0.3$ and 200 pC for $k_p\sigma_r = 1$ with $n_0 = 10^{17}$ cm^{-3} and $a_0 = 1$, $k_pL = 2$ and $k_pw_0 = 5.3$.

ACCELERATING STAGE

We now consider the design of an accelerating stage with beam loading. Since the electron bunch creates its own wake, strong electric field gradients can be introduced inside the bunch which induce energy spread as the electrons are accelerated. It is possible to shape the bunch such that E_x stays longitudinally constant inside the bunch [14]. This can be achieved by using a triangular bunch, or by increasing the bunch length σ_L while keeping the charge constant. Ideally, minimizing the energy spread by this method requires that the bunch stays at the same phase over the acceleration length. This can be achieved by axially tapering the plasma density profile. In principle, some optimum tapering can be found such that the electron bunch does not dephase [15].

Here we simply consider a linear plasma taper and a gaussian electron bunch loaded in the second bucket after the laser pulse. Because the beam is loaded further behind the driver a milder taper is needed. Although the tapering is not optimum and the electron bunch still dephases, energy gain is increased and momentum spread is low. Figure 2(a) shows the electron momentum distribution (solid line) with and without plasma density

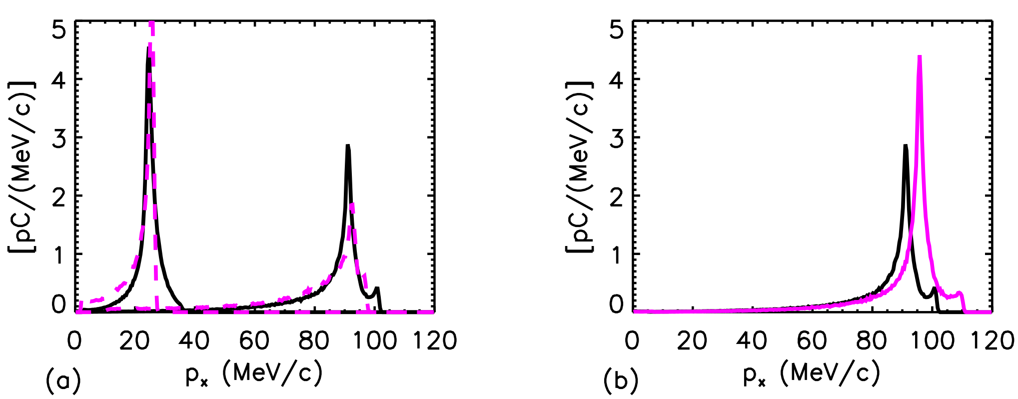

FIGURE 2. (a) Electron (solid line) and positron (dashed line) longitudinal momentum distribution without (peak centered around $p_x \cong 25$ MeV/c) and with (peak centered around $p_x \cong 90$ MeV/c) plasma taper for $a_0 = 1$, $k_pL = 2$, $k_pw_0 = 5.3$, $n_0 = 10^{19}$ cm^{-3}, $k_p\sigma_L = 0.5$, $k_p\sigma_r = 1$, $Q = 22.8$ pC (corresponding to $Q = 228$ pC at $n_0 = 10^{17}$ cm^{-3}). (b) Electron spectra with plasma taper for stages at $n_0 = 10^{19}$ cm^{-3}, $k_pw_0 = 5.3$, $k_p\sigma_L = 0.5$, $k_p\sigma_r = 1$ and with approximately equivalent beam loading at $a_0 = 1$, $k_pL = 2$, $Q = 22.8$ pC (black) and $a_0 = \sqrt{2}$, $k_pL = 1$, $Q = 31.4$ pC [magenta (gray)].

taper with $k_p\sigma_L = 0.5$ (so the accelerating field is constant longitudinally inside the bunch at the loading phase) and $k_p\sigma_r = 1$, and for the laser parameters $a_0 = 1$, $k_pL = 2$ and $k_pw_0 = 5.3$. The charge in the bunch is 22.8 pC, the wakefield being approximately 50% beam loaded. The bunch is loaded with an initial momentum of 10 MeV/c and an initial momentum spread of 0.9 MeV/c (9%) full width at half maximum (FWHM). The plasma taper is of the form $n(x) = n_0(13.2\ x[\mathrm{cm}]+1)$ where $n_0 = 10^{19}$ cm^{-3} on axis, i.e., the plasma density varies by 40% in 0.3 mm. Without plasma taper the electron bunch is accelerated to $\cong$ 24.5 MeV/c (i.e., 14.5 MeV/c energy gain) over 0.3 mm with $\cong$ 12% momentum spread FWHM. With plasma taper the electron bunch is accelerated to $\cong$ 90 MeV/c (i.e., 80 MeV/c energy gain) over $\cong$ 0.7 mm with $\cong$ 3% momentum spread FWHM. The plasma taper has allowed the electron bunch to gain four times the energy and reduced the energy spread by allowing phase locking. Scaling these quantities to a stage at $n_0 = 10^{17}$ cm^{-3}, we can achieve 8 GeV energy gain for a 228 pC charge electron bunch in 0.7 m, and with few percent energy spread.

Because the stage is in the quasi-linear regime ($a_0 \sim 1$) the wake is nearly symmetric and a positron bunch can be accelerated in the same manner. Figure 2(a) shows the positron momentum distribution (dashed line) for the same parameters as for the electron bunch described above. The axial plasma taper is of the form $n(x) = n_0(6.8\ x[\mathrm{cm}]+1)$ with $n_0 = 10^{19}$ cm^{-3} on axis. Without taper, the positron bunch is accelerated to $\cong$ 25.5 MeV/c with 7% momentum spread FWHM in 0.2 mm; with taper, the final energy is $\cong$ 92 MeV/c with 4% momentum spread FWHM in 0.7 mm.

In the previous cases the laser pulse energy is about 30% depleted at the end of the acceleration stage. For a given laser energy, it is possible to increase the efficiency of the stage by reducing the laser pulse length and hence increasing the laser strength a_0. Simulations show that for $k_pL = 1$ and $a_0 = \sqrt{2}$, the laser depletion length is of the order of the dephasing length, i.e. more of the laser energy is transferred into the plasma wake. The higher a_0 in this case results in a 40% higher accelerating field E_x/E_0 and hence more charge can be loaded into the wake. Figure 2(b) shows the electron momentum distribution in the previous case ($k_pL = 2$ and $a_0 = 1$) and in the case $k_pL = 1$ and $a_0 = \sqrt{2}$, corresponding to the same laser energy, and with the bunch parameters $k_p\sigma_L = 0.5$, $k_p\sigma_r = 1$ and $Q = 31.4$ pC. The wake is about 50% beam loaded, as in the previous case. The same axial plasma taper is used. With about 40% more charge than in the previous case, the electron bunch is accelerated to 96 MeV/c (i.e., $\cong$ 86 MeV/c energy gain) with $\cong$ 4% momentum spread FWHM. Scaling these quantities to a stage at $n_0 = 10^{17}$ cm^{-3}, we can achieve almost 9 GeV energy gain for a 300 pC charge electron bunch in 0.7 m, and with few percent energy spread.

CONCLUSION

PIC simulations have been used to model the evolution of an electron beam in a laser driven plasma accelerating structure. By using and extending the scalings of the physical constants with the plasma density, we are able to deduce the properties of the electron beam for longer stages at lower density and with higher energy gain. We have verified the scaling for the guiding, the wake structure and for the beam loading. Using the scaled simulations, we have shown that it is possible to accelerate a few hundreds of pC of charge to about 10 GeV at a density of 10^{17} cm^{-3} using a 110 J, 60–120 fs laser and an axial plasma taper which allows increase of the energy gain and reduction of the momentum spread. In the quasi-linear regime, it is also possible to accelerate positrons quasi-symmetrically. Further work will be done to study the evolution of the emittance of the electron bunch in the accelerating stage and to optimize the laser driver parameters to increase the stage efficiency. Furthermore, the effect of the electron beam betatron oscillation, which is not constant compared to λ_p, on emittance and energy spread will also be studied with scaled simulation at different densities and with reduced models.

ACKNOWLEDGMENTS

The authors thank B.A. Shadwick for insightful discussions. We also acknowledge the assistance of the VORPAL development team. This work was supported by the Director, Office of Science, Office of High Energy Physics, of the U.S. Department of Energy under Contract No. DE-AC02-05CH11231 and DE-FG02-04ER84097 and by SciDAC grant No. DE-FC02-07ER41499. The simulations were performed at the National Energy Research Scientific Computing Center (NERSC).

REFERENCES

1. W. P. Leemans, B. Nagler, A. J. Gonsalves, C. Tóth, K. Nakamura, C. G. R. Geddes, E. Esarey, C. B. Schroeder, and S. M. Hooker, *Nature Phys.* **2**, 696–699 (2006).
2. J. Faure, C. Rechatin, A. Norlin, A. Lifschitz, Y. Glinec, and V. Malka, *Nature* **444**, 737–739 (2006).
3. C. G. R. Geddes, K. Nakamura, G. R. Plateau, C. Tóth, E. Cormier-Michel, E. Esarey, C. B. Schroeder, J. R. Cary, and W. P. Leemans, *Phys. Rev. Let.* **100**, 215004 (2008).
4. A. J. Gonsalves, D. Panasenko, K. Nakamura, C. Lin, E. Monaghan, C. Toth, C. G. R. Geddes, C. B. Schroeder, E. Esarey, and W. P. Leemans, "Injection in a capillary-discharge waveguide using an embedded gas jet," in *this proceedings*, AIP, 2008.
5. E. Esarey, P. Sprangle, J. Krall, and A. Ting, *IEEE Trans. on Plasma Sci.* **24**, 252–288 (1996).
6. E. Esarey, B. A. Shadwick, C. B. Schroeder, and W. P. Leemans, "Nonlinear Pump Depletion and Electron Dephasing in Laser Wakefield Accelerators," in *Eleventh Advanced Accelerator Concepts*, edited by V. Yakimenko, AIP, New York, 2004, vol. 737, pp. 578–584.
7. C. Huang, V. Decyk, C. Ren, M. Zhou, W. Lu, W. Mori, J. Cooley, T. A. Jr., and T. Katsouleas, *J. Comp. Phys.* **217**, 658–679 (2006).
8. J.-L. Vay, *Phys. Rev. Let.* **98**, 130405 (2007).
9. B. Cowan, D. L. Bruhwiler, P. Messmer, K. Paul, C. G. R. Geddes, E. Esarey, and E. Cormier-Michel, "Laser wakefield simulation using a speed-of-light frame envelope model," in *this proceedings*, AIP, 2008.
10. C. Nieter, and J. R. Cary, *J. Comp. Phys.* **196**, 448–473 (2004).
11. L. Gorbunov, and V. Kirsanov, *Zh. Eksp. Teor. Fiz.* **93**, 509–518 (1987).
12. D. L. Bruhwiler, J. R. Cary, B. Cowan, K. Paul, C. G. R. Geddes, P. J. Mullowney, P. Messmer, E. Esarey, E. Cormier-Michel, W. P. Leemans, and J. L. Vay, "New Developments in the Simulation of Advanced Accelerator Concepts," in *this proceedings*, AIP, 2008.
13. C. G. R. Geddes *et al.*, *Journal of Physics: Conference Series* **125**, 012002 (2008).
14. T. Katsouleas, S. Wilks, P. Chen, J. Dawson, and J. Su, *Part. Acc.* **22**, 81–99 (1987).
15. P. Sprangle, B. Hafizi, J. R. Peñano, R. F. Hubbard, A. Ting, C. I. Moore, D. F. Gordon, A. Zigler, D. Kaganovich, and T. M. Antonsen, *Phys. Rev. E* **63**, 056405 (2001).