An improved model for the strain dependence of the superconducting properties of Nb_3Sn

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Abstract. We propose an improved model for the strain dependence of the superconducting properties of Nb_3Sn. The model is based on the three dimensional strain tensor and derived in terms of the first, second and third invariants, and improves an existing model that only includes the second invariant. The axial form of the new model accurately accounts for the experimentally observed dependence of the effective upper critical magnetic field (H^*_{c2}) on axial strain, i.e. a quasi-parabolic strain dependence, asymmetry, and an upturn at large compressive axial strain. An accurate model that accounts for the three dimensional nature of strain is important for scaling relations for the critical current that are used to model magnet performance based on wire measurements.

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1. Introduction

The development of accurate scaling relations for the critical current density (J_c) as a function of magnetic field (H), temperature (T) and strain (ε) in Nb_3Sn wires is of key importance for the analysis of magnet performance based on wire results, and for a minimization of the number of measurements that are required to fully parameterize an unknown wire. A prime user of such scaling relations is the International Thermonuclear Experimental Reactor (ITER) community, who use scaling to analyze magnet performance and in developing and implementing the acceptance tests of conductor batches from various manufactures.

The ITER community recently adopted a modern scaling relation that was derived elsewhere for the description of axial strain experiments on Nb_3Sn wires [1], to improve on an earlier form [2] of such a relation [3]. It can be shown [4] that the relation proposed in [1] can be written as:

$$
\begin{aligned}
&J_c\left(H,T,\varepsilon\right) = \sqrt{2}C\mu_0 H_c^*\left(t\right) h^{p-1}\left(1-h\right)^q \qquad (1)\\
&\text{with}\\
&H_c^*\left(t\right) \cong H_c^*\left(0\right)\left(1-t^2\right),\\
&t \equiv \frac{T}{T_c^*\left(0,\varepsilon\right)} \qquad \text{and} \qquad h \equiv \frac{H}{H_{c2}^*\left(T,\varepsilon\right)},\\
&T_c^*\left(0,\varepsilon\right) = T_{cm}^*\left(0\right) s\left(\varepsilon\right)^{\frac{1}{3}},\\
&H_{c2}^*\left(T,\varepsilon\right) = H_{c2m}^*\left(0\right) MDG\left(t\right) s\left(\varepsilon\right),\\
&MDG\left(t\right) \equiv \frac{H_{c2}^*\left(t\right)_{\mathrm{MDG}}}{H_{c2}^*\left(0\right)_{\mathrm{MDG}}} \cong \left(1-t^{1.52}\right),
\end{aligned}
$$

In which C, p, and q are constants, H_c^* is the effective thermodynamic critical magnetic field, T_c^* is the effective critical temperature, and T_{cm}^* and H_{c2m}^* are maximum occurring values. $H_{c2}^*(t)_{\mathrm{MDG}}$ and $H_{c2}^*(0)_{\mathrm{MDG}}$ are solutions to the Maki–De Gennes relation:

$$
\ln\left(t\right) = \psi\left(\frac{1}{2}\right) - \psi\left(\frac{1}{2} + \frac{\hbar D^*\left(\varepsilon\right)\mu_0 H_{c2}^*\left(T,\varepsilon\right)}{2\phi_0 k_B T}\right), \qquad (2)
$$

in which ψ represents the digamma function, and D^* represents the effective diffusion constant for the normal conducting electrons. D^* is related to the slope of $H_{c2}^*(T,\varepsilon)$ at $T_c^*(0,\varepsilon)$ through [5]:

$$
\left(\frac{\partial \mu_0 H_{c2}^*\left(T,\varepsilon\right)}{\partial T}\right)_{T=T_c^*(0,\varepsilon)} = -\frac{4\phi_0 k_B}{\pi^2 \hbar D^*\left(\varepsilon\right)}, \qquad (3)
$$

indicating that D^* has to depend on strain to account for the experimentally observable change in the slope at $T_c^*(0,\varepsilon)$, which originates from the stronger strain dependence of $H_{c2}^*(0,\varepsilon)$ compared to $T_c^*(0,\varepsilon)$.

$MDG(t)$ can be approximated by $\left(1-t^{1.52}\right)$ to arrive at an explicit relation, but at the cost of errors in $H_{c2}^*(T,\varepsilon)$ of up to 0.4 T. In [1] $p = 0.5$ and $q = 2$ to retain generality, thereby limiting the validity range to below about 80% of $H_{c2}^*(T,\varepsilon)$, above which compositional distributions become important. In the ITER form [3], p and q are allowed to vary at the cost of an increased number of fit parameters, to account for variable grain sizes in wires that modify pinning behavior (through a variable p), and to describe the property variation close to $H_{c2}^*(T,\varepsilon)$ that originates from compositional distributions in wires (through a variable q). All strain dependence in (1) is contained

in the function $s(\varepsilon)$, which is defined as the normalized strain dependence of H^*_{c2}, and is the focus of this paper.

2. History of strain models

Various forms for $s(\varepsilon)$ exist in the literature. One dimensional, axial models for $s(\varepsilon)$ are Ekin's original power-law model [6], the extended power-law model [7], and a polynomial form proposed by the Durham University group [8]. Early work by Testardi [9] and Welch [10], in combination with experiments on quasi-two dimensional Nb_3Sn tape conductors, led to the proposal of the original deviatoric strain model by Ten Haken in 1994 [11], and its introduction in an $s(\varepsilon)$ form for scaling relations in 1999 [12, 13]. Although the original deviatoric strain model accounts for the three dimensional nature of strain and improves the accuracy with respect to the original power-law model in the high compressive axial strain region, its derivation is still largely empirical and it uses only the second strain invariant.

Significant progress was made in the three dimensional descriptions of the strain dependence by Markiewicz in 2004, who calculated the change in the phonon spectrum as a function of strain and coupled that through the Eliashberg relations to a change in T_c [14, 15]. Markiewicz was able to show that in axial strain experiments the first invariant provides an initial, almost constant, reduction of T_c, the second invariant provides the main parabolic-like dependence, and the third invariant introduces asymmetry and an upturn for large compressive strain [16]. The calculations involved were, unfortunately, too extensive to be appropriate for scaling relations. This led in 2006 to a proposed simplification in the form of a polynomial fit [17] that is very similar to the Durham University polynomial model. Based on the important conclusions from Markiewicz his work, the original deviatoric strain model was modified in a somewhat artificial way by introducing asymmetry through a rotation of Ten Haken's original deviatoric strain model. This led to the modified deviatoric strain model as proposed in [1], which accounts for the parabolic-like strain dependence including asymmetry, but does not account for the upturn at large compressive axial strains.

The first scaling attempts performed for ITER [3] using the relations in [1] with variable p and q and the rotational modified deviatoric strain model on 20 data sets highlighted three important conclusions with respect to the strain descriptions: 1) As to be expected the polynomial forms provide the highest accuracy within the bounds of the fit; 2) polynomial models have coefficients whose values depend on the fit range and are not suitable for extrapolation outside of the measured data range; 3) the largest deviations in the present scaling relation occur at the large axial compressive strains since the rotational modified deviatoric strain model does not account for the observable upturn at these strain values. In addition to this it can be noted that three dimensional models are highly relevant for the extensive finite element modeling that is presently being performed on Nb_3Sn wires, cables and magnets. Based on these conclusions it is desirable to derive a three dimensional form for $s(\varepsilon)$ which in axial form accounts for the quasi-parabolic dependence, the change in curvature (upturn) at large compressive axial strains, and asymmetry. The model has to account for the underlying physics, but be simple enough to be applicable in scaling relations for $J_c(H, T, \varepsilon)$. The derivation and test of such a model is the purpose of this article.

3. Modified strain model

3.1. Invariants of the strain tensor

The strain tensor, $\boldsymbol{\epsilon}$, which has components ε_{ij}, can be uniquely decomposed into a spherical part and traceless deviatoric part as $\boldsymbol{\epsilon} = (\varepsilon_{\mathrm{hyd}}/3)\mathbf{I} + \boldsymbol{\epsilon}_{\mathrm{dev}}$, where $\boldsymbol{\epsilon}_{\mathrm{dev}}$ is the deviatoric part of the strain, $\mathbf{I}$ is the second order identity tensor, and $\varepsilon_{\mathrm{hyd}}$ is the spherical part of the strain which coincides with the hydrostatic strain. In component form, this is given by $\varepsilon_{ij} = (\varepsilon_{\mathrm{hyd}}/3)\delta_{ij} + \chi_{ij}$, where δ_{ij} is the Kronecker delta and χ_{ij} are the components of the deviatoric strain tensor. In the remainder of this article the symbol $\boldsymbol{\epsilon}$ will be used to represent tensorial strain quantities, while the symbol ε will be used for scalar valued strain quantities. Since the deviatoric strain is traceless, the spherical strain must be given by

$$\varepsilon_{\mathrm{hyd}} = \varepsilon_{11} + \varepsilon_{22} + \varepsilon_{33}; \tag{4}$$

therefore, the components of the deviatoric strain tensor are

$$\chi_{ij} = \varepsilon_{ij} - \frac{1}{3}(\varepsilon_{11} + \varepsilon_{22} + \varepsilon_{33})\delta_{ij}. \tag{5}$$

For an isotropic elastic material, the strain energy density function may be expressed in terms of the invariants of the strain tensor. Under the full rotation group, the strain tensor has three independent invariants. These may be chosen as the first invariant of the strain tensor, I_1 and the second and third invariants of the deviatoric strain tensor, J_2 and J_3. For this choice, the invariants are given by

$$\begin{array}{c} I_1 = \mathrm{Tr}(\boldsymbol{\epsilon}) = \varepsilon_{\mathrm{hyd}} \\ J_2 = \frac{1}{2}\boldsymbol{\epsilon}_{\mathrm{dev}} : \boldsymbol{\epsilon}_{\mathrm{dev}} \\ J_3 = \det(\boldsymbol{\epsilon}_{\mathrm{dev}}) \end{array} . \tag{6}$$

In terms of the principal strains, $(\varepsilon_1, \varepsilon_2, \varepsilon_3)$, the invariants are given by

$$\begin{array}{c} I_1 = \varepsilon_1 + \varepsilon_2 + \varepsilon_3 \\ J_2 = \frac{1}{6}\left((\varepsilon_1 - \varepsilon_2)^2 + (\varepsilon_2 - \varepsilon_3)^2 + (\varepsilon_3 - \varepsilon_1)^2\right) \\ J_3 = (\varepsilon_1 - \frac{1}{3}I_1)(\varepsilon_2 - \frac{1}{3}I_1)(\varepsilon_3 - \frac{1}{3}I_1) \end{array} . \tag{7}$$

3.2. Three dimensional strain model

The goal of this work is to develop a three dimensional strain model that is physically based, allows extrapolation, and is simple enough to be applied in scaling relations. The original Ten Haken deviatoric strain model [11] meets the above criteria, however, it does not capture the observed deviation from linearity at large compressive strains. Markiewicz [16] linked this asymmetry and upturn at large compressive strains to the third invariant of the deviatoric strain tensor which is neglected in the previous deviatoric strain models. H^*_{c2} as a function of strain is here assumed to take the following form,

$$\frac{H^*_{c2}(\boldsymbol{\epsilon})}{H^*_{c2}(\boldsymbol{\epsilon} = \mathbf{0})} = (1 - C_{\mathrm{h}} I_1)\left(1 - C_{\mathrm{d},1}\sqrt{J_2} - C_{\mathrm{d},2} J_3\right), \tag{8}$$

where C_{h}, $C_{\mathrm{d},1}$, and $C_{\mathrm{d},2}$ are constants which must be determined experimentally. This model is partly based on the original deviatoric strain model in that it predicts a linear dependence on $\sqrt{J_2}$, but it also includes the dependence on I_1 and J_3. Similarities can also be drawn with the model proposed by Markiewicz [17], which includes all three invariants. In Markiewicz's model, the strain function is taken to be the product of

a function of I_1 and a separate function of J_2 and J_3, since in the detailed invariant analysis [16] it was observed that the hydrostatic part enters the strain function largely separately from the deviatoric parts of the strain. We have chosen to take the same approach in our strain model, and in fact the form proposed in equation 8 is very similar to the reciprocal of the form proposed by Markiewicz in [17] with the exception that we introduce the functional dependence on J_2 as $\sqrt{J_2}$ while he uses a second order polynomial (note that this is consistent with a series expansion). This is motivated by the experimental validation of the Ten Haken deviatoric strain model for a limited strain range. It is important to note that the model described in equation 8 is not rigorously derived, however, it is based on several important analytical and experimental observations and it meets the requirements that were outlined at the beginning of this section.

3.3. Uniaxial strain model

Since many of the studies on strain dependence are based on uniaxial strain experiments on multifilamentary wire conductors, in this section we derive a uniaxial version of the strain model given in equation 8. For uniaxial tests on wire conductors, an idealization of the strain state in the Nb_3Sn filaments, under the assumption of transverse isotropy and linear elasticity, can be expressed as

$$\begin{array}{l} \varepsilon_1 = \varepsilon_\mathrm{a} + \varepsilon_{1,0} \\ \varepsilon_2 = -\nu\varepsilon_\mathrm{a} + \varepsilon_{2,0} \\ \varepsilon_3 = -\nu\varepsilon_\mathrm{a} + \varepsilon_{3,0} \end{array} , \tag{9}$$

where ε_a is the applied uniaxial strain in the 1-direction, ν is the effective Poisson ratio of the conductor, and $\varepsilon_{1,0}$, $\varepsilon_{2,0}$, and $\varepsilon_{3,0}$ are the residual strains due to thermal mismatch in the 1, 2, and 3 directions respectively. Here it is assumed that the direction of applied strain coincides with the principal 1-direction, consistent with a long cylinder assumption for wire geometries. The residual strain values will clearly vary as a function of position inside of the conductor, however, here we consider average effective values. Also, note that ν is an effective parameter that depends on the properties and the layout of all of the materials inside the wire. In general ν may vary as a function of applied strain due to effects such as plasticity. In this case ν is assumed to be constant, consistent with a full plasticity analysis in which the strain state of Nb_3Sn remains approximately linear with a relatively constant Poisson ratio as reported by Markiewicz [17].

It is useful to define the following quantities

$$\begin{array}{l} \bar{\varepsilon} = \frac{1}{2}(\varepsilon_{2,0} + \varepsilon_{3,0}) \\ \Delta\varepsilon = \frac{1}{2}(\varepsilon_{2,0} - \varepsilon_{3,0}) \end{array} ; \tag{10}$$

these are the average of the transverse residual strain components and the difference in the transverse residual strain components, respectively. Substituting equations 9 and 10 into equation 7, the second invariant of the deviatoric strain is given by

$$J_2 = \frac{1}{3}(1+\nu)^2\left((\varepsilon_\mathrm{a} - \delta)^2 + (\varepsilon_{0,\mathrm{a}})^2\right), \tag{11}$$

where

$$\begin{array}{l} \delta = \frac{1}{1+\nu}(\bar{\varepsilon} - \varepsilon_{1,0}) \\ \varepsilon_{0,\mathrm{a}} = \frac{\sqrt{3}}{1+\nu}|\Delta\varepsilon| \end{array} . \tag{12}$$

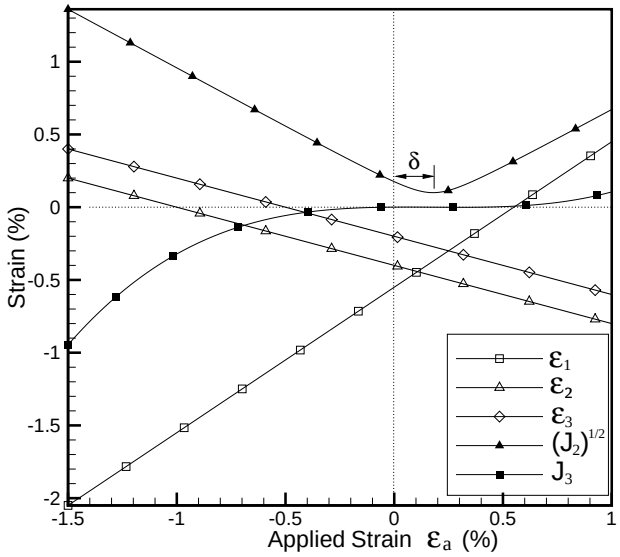

Figure 1. The Strain components (ε_1, ε_2, and ε_3) and the invariants of the deviatoric strain ($\sqrt{J_2}$ and J_3) as a function of applied strain for the linear elastic transversely isotropic model.

The quantity δ represents the value of the applied strain at the point of minimum J_2, while $\varepsilon_{0,\mathrm{a}} = \frac{\sqrt{3}}{1+\nu}\sqrt{J_{2,min}}$ is proportional to the minimum value of $\sqrt{J_2}$ (see figure 1). It is interesting to note that, for this simplified model, δ is dependent on the difference between the average of the transverse residual strain components and the longitudinal strain, while the minimum of $\sqrt{J_2}$ only depends on the difference in the transverse residual strain components. Turning our attention to the third invariant of the deviatoric strain, J_3 can be expressed in terms of the principal strain components by substituting equations 9 and 10 into equation 7, giving

$$J_3 = \frac{2}{27}(1+\nu)^3\left((\varepsilon_{\mathrm{a}}-\delta)^3 - 3(\varepsilon_{0,\mathrm{a}})^2(\varepsilon_{\mathrm{a}}-\delta)\right). \tag{13}$$

Note that when $\varepsilon_{0,a}$ is small ($< 0.3\%$), the cubic term dominates and $J_3 \approx \frac{2}{27}(1+\nu)^3(\varepsilon_{\mathrm{a}}-\delta)^3$. It is interesting to note that the roots of J_3 occur at $\varepsilon_{\mathrm{a}} = \delta - \frac{3\Delta\varepsilon}{1+\nu}$, $\varepsilon_{\mathrm{a}} = \delta$, and $\varepsilon_{\mathrm{a}} = \delta + \frac{3\Delta\varepsilon}{1+\nu}$. Therefore, J_3 is centered around $\varepsilon_{\mathrm{a}} = \delta$ and also vanishes at that value of applied strain, which is the same value at which J_2 is a minimum. Figure 1 shows the strain components along with $\sqrt{J_2}$ and J_3 as a function of the applied strain ε_a for reasonable values of $\varepsilon_{1,0}$, $\varepsilon_{2,0}$, and $\varepsilon_{3,0}$, which are chosen simply to represent how $\sqrt{J_2}$ and J_3 vary as a function of applied strain, but are not rigorously determined.

In our proposed model, the dependence on the first invariant is described by the term $1 - C_{\mathrm{h}}I_1$. Markiewicz shows in [16] that the effect of I_1 on $s(\varepsilon)$ is to produce a relatively substantial offset from the maximum value and a relatively weak linear variation as a function of applied strain. Therefore, as a first approximation, the dependence of $s(\varepsilon)$ on I_1 can be replaced by a constant term $1 - C_{\mathrm{h}}I_1|_{\varepsilon_a=\varepsilon_m}$, where ε_{m} is the thermal pre-strain in the conductor. This term includes the hydrostatic strain reduction of H^*_{c2} due to I_1 but does not include the (weakly) linear strain dependence.

This assumption allows us to eliminate one free parameter at the cost of a slight loss in accuracy, which can be estimated to be approximately 1% for a range of intrinsic axial strain varying from -1.5% to 0.5% [16].

Using this assumption, the maximum of H^*_{c2} occurs very close to $\varepsilon_a = \delta$; therefore, it can be considered that $\varepsilon_m \approx \delta$. For typical model fit parameters (see equation 15 and table 2), ε_m is found to deviate from δ by less than 0.02% strain. Using equations 8, 11, and 13 along with an approximation to the intrinsic axial strain as $\varepsilon_I = \varepsilon_a - \delta \approx \varepsilon_a - \varepsilon_m$, and the assumption that the hydrostatic strain dependence is approximately constant, the maximum of H^*_{c2} can be approximated as

$$H^*_{c2}(\varepsilon_I = 0) = \left(H^*_{c2}(\boldsymbol{\epsilon} = \mathbf{0})\right)\left(1 - C_h I_1|_{\varepsilon_I=0}\right)\left(1 - C_{a,1}\varepsilon_{0,a}\right), \tag{14}$$

where $C_{a,1} = \frac{1+\nu}{\sqrt{3}} C_{d,1}$. With the results from equations 11 and 13, the model can now be written in terms of an applied axial strain for uniaxial experiments. Using the assumption that the hydrostatic strain dependence can approximately be regarded as constant, substituting equations 11 and 13 into equation 8, and normalizing by the maximum H^*_{c2}, the uniaxial applied strain model is given by

$$\begin{aligned} s(\varepsilon_I) &= \frac{H^*_{c2}(\varepsilon_I)}{H^*_{c2}(\varepsilon_I = 0)} \\ &= \frac{1 - C_{a,1}\sqrt{(\varepsilon_I)^2 + (\varepsilon_{0,a})^2} - C_{a,2}\left((\varepsilon_I)^3 - 3(\varepsilon_{0,a})^2\varepsilon_I\right)}{1 - C_{a,1}\varepsilon_{0,a}}, \end{aligned} \tag{15}$$

where $C_{a,2} = \frac{2}{27}(1+\nu)^3 C_{d,2}$. Note that the parameters $C_{a,1}$ and $C_{a,2}$ (equation 15) are related to the parameters $C_{d,1}$ and $C_{d,2}$ (equation 8) through the effective Poisson ratio, while δ and $\varepsilon_{0,a}$ depend on the initial strain state (residual strains) in the conductor. If the effect of the third invariant is neglected ($C_{a,2} = 0$), the axial strain model reduces to Ten Haken's uniaxial strain model.

4. Parameters of the strain function

4.1. Fitting procedures

It was previously suggested by Ten Haken [13] that a similar parameter to $C_{d,1}$ in his model may be an intrinsic property of Nb_3Sn. This was surmised since the value of C_a in his axial strain model, which corresponds to $C_{a,1}$ in equation 15 with $C_{a,2} = 0$, had little variation for different sample wires. It is of interest to determine if we also see this same behaviour for $C_{a,1}$ and $C_{a,2}$ in our model. For this purpose the axial strain model (equation 15) is used to fit experimental data from seven different wires (taken from the excellent data at the Durham University superconductivity group website [18] as summarized in [8]), and three different fitting procedures are performed in which the number of free parameters is varied. If $C_{d,1}$ and $C_{d,2}$ are indeed intrinsic material parameters, then $C_{a,1}$ and $C_{a,2}$ should show some correlation across the different wire layouts except for variations associated with ν. Note that the parameters δ and $\varepsilon_{0,a}$ are closely related to the initial strain state in the material, which depends on the substrate material used for a specific experiment. For the first fitting procedure, all four parameters $C_{a,1}$, $C_{a,2}$, δ, and $\varepsilon_{0,a}$ are treated independently. This should produce the best fit for each individual sample since it has the most free parameters. However, if $C_{d,1}$ and $C_{d,2}$ are indeed related or constant, the functional form (15) should be in accordance with all of the data sets for related or constant values of $C_{a,1}$ and $C_{a,2}$. Perhaps, the simplest relation between $C_{a,1}$ and $C_{a,2}$ is a constant ratio $C_{a,2} = RC_{a,1}$,

where R is a universal constant. This is similar to the suggestion made in [3] that the parameters in the rotationally modified deviatoric strain model [1] are related by a linear equation. For the second fitting procedure, the error over all of the data sets is minimized by varying $C_{\mathrm{a},1}$, δ, and $\varepsilon_{0,\mathrm{a}}$ in each individual data set and by varying the global constant R. Once the optimal value of R is determined, a second step is performed where the error is minimized in each individual data set by varying $C_{\mathrm{a},1}$, δ, and $\varepsilon_{0,\mathrm{a}}$. For the final fitting procedure, the global error is minimized by varying two global constants, $C_{\mathrm{a},1}$ and $C_{\mathrm{a},2}$, and two local constants in each data set, δ, and $\varepsilon_{0,\mathrm{a}}$. Once the optimal values of $C_{\mathrm{a},1}$ and $C_{\mathrm{a},2}$ are determined, a second step is performed where the error is minimized in each individual data set by varying δ and $\varepsilon_{0,a}$.

Jewell [19] showed that in several sample conductors cracks form in the Nb_3Sn for applied strains in the filaments slightly above 0.3% at room temperature. More generally, the onset of irreversibility in axial strain experiments occurs at $\varepsilon_{\mathrm{I}} =$ 0.1 to 0.6%, depending on the sample and experiment, but is mostly located around $\varepsilon_{\mathrm{I}} =$ 0.4 to 0.5%. The model described in this article is only valid for elastic behaviour, and strongly deviating data can reasonably be expected to be caused by plasticity and/or crack formation. Of these, plasticity is not necessarily detectable by irreversibility in an experiment where an elastic substrate determines the strain state of the sample, as evident from the reversibility at large compressive strain. For all three of the fitting procedures, only data points with an intrinsic strain satisfying $\varepsilon_{\mathrm{I}} < 0.3\%$ are used, since for larger ε_{I} deviations from the model are observed in four of the data sets. The error in each individual data set, e_i, is defined as

$$e_i = \left[\frac{1}{N_d^i} \sum_{j=1}^{N_d^i} \left(s(\varepsilon_{\mathrm{I},j}) - s_j \right)^2 \right]^{\frac{1}{2}}, \tag{16}$$

where the normalization value N_d^i is the number of data points in set i, and $\varepsilon_{\mathrm{I},j}$ is the value of the intrinsic strain at point j, which has a corresponding strain function value s_j. The total error over all data sets is defined as

$$e_{tot} = \left[\frac{1}{N_s} \sum_{j=1}^{N_s} (e_i)^2 \right]^{\frac{1}{2}}, \tag{17}$$

where in this case $N_s = 7$ since the fits were performed for seven different wires.

4.2. Results

Using the fitting procedures described above, a value of $R = 1.034 \times 10^3$ is obtained from the second fitting procedure, and values of $C_{\mathrm{a},1} = 40.8$ and $C_{\mathrm{a},2} = 3.25 \times 10^4$ are obtained from the third fitting procedure. Table 1 shows both the total fitting error and the fitting error for each of the data sets in terms of the number of free fitting parameters. As is expected, the error increases as the number of free parameters decreases. However, the increase in the error is relatively small when four free parameters are reduced to three for all of the sample wires. When compared with the error for three free parameters, the increase in error with two free parameters is also relatively small for all except the RRP strand. Figure 2 shows the data fits that are obtained with a constant ratio between $C_{\mathrm{a},1}$ and $C_{\mathrm{a},2}$. This second fitting procedure is chosen to represent the data fits since it provides a good balance between fitting accuracy and the number of free parameters that are needed. Table

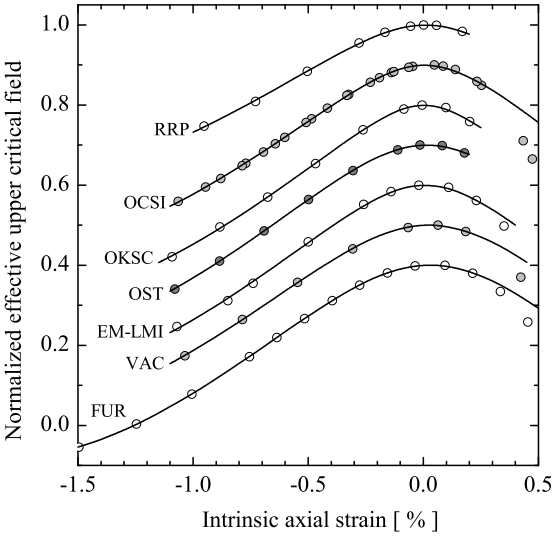

Figure 2. Experimental data [8, 18] for seven different strands along with the uniaxial strain model fits for three free parameters with $C_{\mathrm{a},2} = (1.034 \times 10^3) C_{\mathrm{a},1}$. Each individual data set is offset by a value of 0.1 along the vertical axis for clarity.

Table 1. Total and individual data set fitting error as a function of the number of free parameters for the intrinsic strain range $\varepsilon_I < 0.3$. For three free parameters, $C_{\mathrm{a},2} = (1.034 \times 10^3) C_{\mathrm{a},1}$, and for two free parameters, $C_{\mathrm{a},1} = 40.8$ and $C_{\mathrm{a},2} = 3.25 \times 10^4$. Details on the different strands can be found in [8].

N_p	OCSI	RRP	OKSC	OST	EM-LMI	VAC	FUR	e_{tot}
4	1.6E-03	7.1E-04	1.9E-03	7.2E-04	1.5E-03	5.2E-04	7.8E-04	1.2E-03
3	1.7E-03	1.4E-03	2.2E-03	9.1E-04	3.7E-03	8.6E-04	8.5E-04	1.9E-03
2	2.3E-03	9.2E-03	3.2E-03	3.2E-03	5.3E-03	6.0E-04	2.6E-03	4.6E-03

Table 2. Computed parameters and the error for the fitting procedure with three free parameters ($C_{\mathrm{a},2} = (1.034 \times 10^3) C_{\mathrm{a},1}$).

	OCSI	RRP	OKSC	OST	EM-LMI	VAC	FUR
δ (%)	0.046	0.055	0.085	0.111	0.260	0.306	0.276
$\varepsilon_{0,a}$ (%)	0.254	0.294	0.200	0.360	0.289	0.416	0.451
$C_{a,1}$	41.6	35.7	43.6	43.7	43.6	41.9	42.6
$C_{a,2} \times 10^{-4}$	4.30	3.69	4.50	4.52	4.51	4.33	4.41

2 shows the parameters that are obtained when only three free parameters are used and $C_{\mathrm{a},2} = (1.034 \times 10^3) C_{\mathrm{a},1}$. Note that the values of $C_{\mathrm{a},1}$ and $C_{\mathrm{a},2}$ have only small variations for all except the RRP strand.

5. Summary

(1) In this work a three dimensional strain model that is physically based, allows extrapolation, and is simple enough to be applied in scaling relations is developed. The model is based on the first invariant of the strain tensor and the second and third invariants of the deviatoric strain tensor. A one dimensional strain model is derived from the general three dimensional form and verified with data from uniaxial strain experiments. Specifically, this model reproduces the observed upturn in the strain function at large compressive strains.

(2) The model is used to fit the experimental data with varying numbers of free parameters. As is expected, the fitting error decreases with increasing number of fit parameters. However, as is evident from figure 2 the model produces an exceptional fit for 3 free parameters with a constant ratio between $C_{\mathrm{a},2}$ and $C_{\mathrm{a},1}$.

(3) When a constant ratio between $C_{\mathrm{a},2}$ and $C_{\mathrm{a},1}$ is used, the values of $C_{\mathrm{a},1}$ and $C_{\mathrm{a},2}$ have only small variations for all except the RRP strand.

(4) The performance of this model is assessed by determining how well the model fits data from uniaxial strain experiments. However, it remains to be seen how well the model performs with the direct use of the three dimensional form (equation 8) in a detailed mechanical analysis.

(5) The model is found to be applicable for $-1.5\% < \varepsilon_I < 0.3\%$ for the data used in this study. Fits to experimental data should generally be restricted to the reversible region. The behaviour at large tensile axial strains may be attributed to inelastic behaviours such as cracking, which are not taken into account in the elastic model.

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References

[1] A. Godeke, B. ten Haken, H. H. J. ten Kate, and D. C. Larbalestier. A general scaling relation for the critical current density in Nb_3Sn wires. *Supercond. Sci. and Techn.*, 19:R100, 2006.

[2] L. T. Summers, M. W. Guinan, J. R. Miller, and P. A. Hahn. A model for the prediction of Nb_3Sn critical current as a function of field, temperature, strain, and radiation damage. *IEEE Trans. Magn.*, 27(2):2041, 1991.

[3] L. Bottura and B. Bordini. $J_C(B, T, \varepsilon)$ parameterization for the ITER Nb_3Sn production. Paper 5LY06, presented at the 2008 Appl. Supercond. Conf., Chicago, IL, USA.

[4] M. G. T. Mentink. Critical surface parameterization of high J_c RRP Nb_3Sn strand. Internship report, Lawrence Berkeley National Laboratory, Berkeley, CA, USA, 2008.

[5] A. Godeke, M. C. Jewell, C. M. Fischer, A. A. Squitieri, P. J. Lee, and D. C. Larbalestier. The upper critical field of filamentary Nb_3Sn conductors. *J. Appl. Phys.*, 97:093909, 2005.

[6] J. W. Ekin. Strain scaling law for the flux pinning in practical superconductors. part 1: Basic relationship and application to Nb_3Sn conductors. *Cryogenics*, 20:611, 1980.

[7] J. W. Ekin. *Experimental techniques for low-temperature measurements*. Oxford University Press, New York, 2006.

[8] X. F. Lu, D. M. J. Taylor, and D. P. Hampshire. Critical current scaling laws for advanced Nb_3Sn superconducting strands for fusion applications with six free parameters. *Supercond. Sci. and Techn.*, 21:105016, 2008.

[9] L. R. Testardi. Unusual strain dependence of T_c and related effects for high-temperature (A15 structure) superconductors: Sound velocity at the superconducting phase transition. *Phys. Rev. B*, 3:95, 1971.

[10] D. O. Welch. Alternation of the superconducting properties of A15 compounds and elementary composite superconductors by nonhydrostatic elastic strain. *Adv. Cryo. Eng. (Materials)*, 26:48, 1980.

[11] B. ten Haken. *Strain effects on the critical properties of high-field superconductors*. PhD thesis, Univ. of Twente, Enschede, The Netherlands, 1994.

[12] A. Godeke, B. ten Haken, and H. H. J. ten Kate. Scaling of the critical current in ITER type niobium-tin superconductors in relation to the applied field, temperature and uni-axial applied strain. *IEEE Trans. Appl. Supercond.*, 9(2):161, 1999.

[13] B. ten Haken, A. Godeke, and H. H. J. ten Kate. The strain dependence of the critical properties of Nb_3Sn conductors. *J. Appl. Phys.*, 85:3247, 1999.

[14] W. D. Markiewicz. Elastic stiffness model for the critical temperature T_c of Nb_3Sn including strain dependence. *Cryogenics*, 44:767, 2004.

[15] W. D. Markiewicz. Invariant formulation of the strain dependence of the critical temperature t_c of Nb_3Sn in a three term approximation. *Cryogenics*, 44:895, 2004.

[16] W. D. Markiewicz. Invariant strain analysis of the critical temperature T_c of Nb_3Sn. *IEEE Trans. Appl. Supercond.*, 15(2):3368, 2005.

[17] W. D. Markiewicz. Invariant temperature and field strain functions for Nb_3Sn composite superconductors. *Cryogenics*, 46:846, 2006.

[18] Durham-Superconductivity-Group, 2008. http://www.dur.ac.uk/superconductivity.durham/.

[19] M. C. Jewell, P. J. Lee, and D. C. Larbalestier. The influence of Nb_3Sn strand geometry on filament breakage under bend strain as revealed by metallography. *Supercond. Sci. and Techn.*, 16:1005, 2003.