Using nanoscale transistors to measure single donor spins in semiconductors

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Abstract. We propose a technique for measuring the state of a single donor electron spin using a field-effect transistor induced two-dimensional electron gas and electrically detected magnetic resonance techniques. The scheme is facilitated by hyperfine coupling to the donor nucleus. We analyze the potential sensitivity and outline experimental requirements. Our measurement provides a single-shot, projective, and quantum non-demolition measurement of an electron-encoded qubit state.

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The spin of electrons bound to donors in silicon is considered a promising candidate qubit for quantum information processing [1]. An integral part of any quantum computation architecture is the capacity for high-fidelity qubit readout; however, the detection of spin states of single donor electrons and nuclei in silicon has remained elusive. In this paper we analyze spin dependent scattering between conduction electrons and neutral donors as a spin-to-charge-transport conversion technique, and show that quantum non-demolition (QND) measurements of single electron spin-encoded qubit states are realistically achievable when mediated via nuclear spin states. Such a measurement will also be of value to the developing field of spintronics where the electrical detection of spin states is valuable. Our readout takes advantage of two features: i) the ability to perform electron spin resonance spectroscopy using a two-dimensional electron gas (2DEG), and ii) the hyperfine shift induced on dopant electron Zeeman energies by the dopant nuclear spin state. In the following, we first describe the experimental apparatus and the techniques of 2DEG mediated spin resonance spectroscopy, and then we present our proposal for spin state measurement, analyze the sensitivity of the measurement scheme and establish the key factors that determine signal-to-noise.

The physical setting. Figure 1(a) shows a cross section of a 2DEG spin readout device with a single implanted donor. Prior studies have used similar devices with bulk-doping [2, 3] or a large number of implanted donors (10^6) [4] in the 2DEG channel. The 2DEG is operated in accumulation mode and thus scattering occurs between conduction electrons and electron(s) bound to the shallow donor(s). The basic principle exploited in these studies is the role of the exchange interaction in electron-electron scattering. At a scattering event between a conduction electron and a loosely bound donor impurity electron, the Pauli principle demands that the combined wave function of the two electrons be antisymmetric with respect to coordinate exchange. This constraint, together with the

FIGURE 1. (a) A cross section of the field-effect transistor (FET) used to create the 2DEG. In order to reduce qubit decoherence, it is beneficial to implant into isotopically purified silicon. (b) Four-level system of electron-nuclear spin degrees of freedom in the secular approximation. The energy eigenstates are the eigenstates of σ_z^e and σ_z^n .

fact that the combined spin state can be symmetric (triplet) or antisymmetric (singlet), imposes a correlation between the spatial and spin parts of the wave function and results in an effective spin dependence of the scattering matrix, leading to a spin dependent conductance. Application of a static magnetic field will partially polarize conduction and impurity electrons leading to excess triplet scattering. A microwave drive will alter these equilibrium polarizations when on resonance with impurity (or conduction) electron Zeeman energies and hence alter the ratio of singlet versus triplet scattering events, registering as a change in the 2DEG current. Thus, the spin dependent 2DEG current can be used as a detector of spin resonance and accordingly this technique is commonly known as *electrically detected magnetic resonance* (EDMR).

From hereon we will consider the experimental setup described above in the particular situation where there is a single donor P nucleus present, the electron spin of which encodes the quantum information that we wish to measure. A crucial question in the context of quantum computing is whether the spin-dependent 2DEG current can be used to measure the state of an electron-spin qubit, as spin-dependent tunneling processes have been employed [5]. The fundamental concern here is whether the spin exchange scattering interaction at the core of the spin-dependent 2DEG current allows for a quantum state measurement of a single donor impurity electron spin. One might expect that because the exchange interaction is destructive (in the sense that it will change the state of the target (donor electron) spin with some probability), it will only produce a faithful measurement if the time over which it acts is extremely short. Indeed, one can show using a minimal model of the exchange scattering that the relaxation of a general electron spin state is rapid across virtually all reasonable scattering parameter ranges [6]. The estimated measurement induced population mixing time of $T_{mix} \sim 1 - 10 ns$, is drastically smaller than the required measurement integration time: $\tau_m \sim 10^{-3} s$ (see below for a calculation of τ_m). This makes it impossible to faithfully map the electron spin state onto the meter variable, and hence impossible to perform a single electron spin state measurement using the 2DEG current directly. However, as we will now show, it is possible to make use of the nuclear spin degree of freedom in order to utilize EDMR for a projective and QND measurement of single spin states. The key is that the state of the nuclear spin affects the Zeeman splitting of the electron spin (and thus its resonant frequency) via the mutual hyperfine coupling.

Nuclear spin mediated spin measurement. The low-energy, low-temperature Hamiltonian describing the electron and nuclear spins of a phosphorous dopant in a static magnetic field, $\mathbf{B} = B\hat{z}$ is $H = \frac{1}{2} [g_e\mu_B B\sigma_z^e - g_n\mu_n B\sigma_z^n] + A\sigma^e \cdot \sigma^n$ where μ_B and μ_n are the Bohr and nuclear magnetons, g_e (g_n) is the electron (nuclear) g -factor, and A characterizes the strength of the hyperfine interaction between the two spins (we set $\hbar = 1$ throughout the paper). For moderate and large values of B , the σ_z terms dominate and we can make the *secular approximation*, to arrive at: $H \approx 1/2 [g_e\mu_B B\sigma_z^e - g_n\mu_n B\sigma_z^n] + A\sigma_z^e \sigma_z^n$. The energy levels and eigenstates of this Hamiltonian are shown in Fig. 1(b). Note that we have ignored the coupling of both spins to uncontrolled degrees of freedom such as paramagnetic defects and phonons (coupling to lattice spins can be mitigated by the use of a ^{28}Si substrate). These environmental couplings will contribute to decoherence of the nuclear and electron spin states (e.g. [7]), and we will simply assume that this results in some effective relaxation and dephasing of the electron and nuclear spins. We see that the resonance frequency (Zeeman energy) of the electron is a function of the nuclear spin state. Therefore, our strategy will be to transfer the qubit state from the electron to the nucleus and then use EDMR to measure the nuclear spin. This is in effect a spin-to-resonance-to-charge conversion measurement. The state transfer can be performed by applying resonant microwave and radio frequency pulses to implement the qubit SWAP gate, or a variant of it (for more details, see Ref. [6]). Recently, coherent information transfer between electron and nuclear spins of P atoms in silicon was demonstrated in Ref. [8].

Measurement sensitivity and measurement-induced decoherence. Here we address the issue of the sensitivity of the differential EDMR current in the limit of single donor scattering. The on-resonant (I) and off-resonant (I_0) current differential (normalized) is [6]: $\frac{\Delta I}{I_0} \equiv \frac{I - I_0}{I_0} \approx -\alpha s \langle \sigma_z \rangle_i^{ss} P_c^0 \frac{1/\tau_n}{1/\tau_t}$. Here $\alpha \equiv \langle \Sigma_s - \Sigma_t \rangle|_{z=z_i} / \langle \Sigma_s + 3\Sigma_t \rangle|_{z=z_i}$, Σ_s and Σ_t are singlet and triplet scattering cross sections, respectively, and $\langle \cdot \rangle|_{z=z_i}$ denotes an average over the scattering region with the donor location in z (see Fig. 1(a)) held fixed [2, 9]. $\langle \sigma_z \rangle_i^{ss}$ is the equilibrium “polarization” of the single donor, an equilibrium spin projection value that is achieved in $\sim 10\text{ns}$, much faster than the observable times scales of the measurement [6]. s is a saturation parameters that characterizes how much microwave power is absorbed by the impurity and conduction electrons. The final term in the expression for the current differential represents the ratio between impurity scattering ($1/\tau_n$) and total scattering ($1/\tau_t$) rates. We assume $1/\tau_t = 1/\tau_0 + 1/\tau_n$, where $1/\tau_0$ is the scattering rate due to all other processes (such as surface roughness scattering and Coulomb scattering by charged defects).

To estimate the expected magnitude of this current differential, we begin by considering present state of the art 2DEG mediated EDMR experiments where this current differential is $\sim 10^{-7}$ (with $T \sim 5\text{K}$, $B \sim 0.3\text{T}$, a 2DEG channel area of $160 \times 20\mu\text{m}^2$, probe current $1\mu\text{A}$, and a donor density of $2 \times 10^{11}\text{donors/cm}^2$) [4]. We assume that α' will be similar for the single donor device as in current experiments. Then in scaling down to a single donor, the first aspect to consider is the scattering rate ratio: $\rho \equiv \frac{1/\tau_n}{1/\tau_t}$. To first order this ratio can be kept constant if we scale the 2DEG area concomitantly with the donor number. From the channel area and density of current experiments, we extrapolate that a 2DEG area of $\sim 30 \times 30\text{nm}^2$ – well within the realm of current technology [10] – would keep ρ unchanged. Optimization of donor depth might relax this

size requirement [9]. A higher order analysis would require detailed investigation of the device specific interface and intrinsic contributions to the other scattering processes, and hence to the channel mobility and τ_0 . Related to this concern, the mobility of the 2DEG channel can be improved – e.g., by using hydrogen passivation to mitigate surface roughness at the oxide interface [11] – to increase ρ . We conservatively estimate a factor of 10 increase in $\Delta I/I_0$ from such improvements. The saturation parameter $s \sim 1$ for large enough microwave powers in the recent measurements [4] and so does not present an area for improvement. Finally, an avenue for significant improvement in signal is to increase the conduction electron polarization, P_c^0 , which is currently $\sim 0.1 - 1\%$. This polarization is roughly proportional to the applied static magnetic field, and therefore a factor of 10 improvement is possible by operating at $B = 3T$. Additionally, spin injection techniques can be employed to achieve $P_c^0 > 10\%$ (e.g. [12, 13]), resulting in a 100-fold improvement in $\Delta I/I_0$. Hence, by improvements in device scaling and channel mobility, and by incorporating spin injection, we estimate a realistic, improved current differential of $\Delta I/I_0 \sim 10^{-4}$. Given this $\Delta I/I_0$ and a probe current of $I_0 \sim 1\mu A$, to achieve an SNR of 10 through shot-noise limited detection we require a τ_m satisfying $\left(\frac{\Delta I}{I_0}\right) \frac{I_0 \tau_m}{e} > 10\sqrt{\frac{I_0 \tau_m}{e}}$, where the left hand side is the signal, the right hand side is the accumulated shot-noise multiplied by the SNR, and e is the fundamental unit of electric charge. Solving this yields a measurement integration time of $\tau_m \sim 10^{-3} s$.

The final step in completing the measurement analysis is to confirm that the state of the nucleus does not flip within the measurement time, especially in the presence of the EDMR electron driving. We have shown that in moderate to large magnetic fields this is indeed the case, and the nuclear T_1 in the presence of electron driving will be comfortably larger than τ_m [6]. Also, we note that this has very recently been confirmed by explicit measurements of nuclear T_1 under the conditions of electron driving [14]. This analysis implies that once the measurement collapses onto a nuclear basis state, the nuclear spin state effectively remains there, and therefore the EDMR measurement satisfies the QND requirement on the qubit state. This QND aspect also makes the measurement technique an effective method for initializing the state of the nuclear spin.

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