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Predictions and Observations of Low-shear Beta-induced Shear Alfvén - Acoustic Eigenmodes in Toroidal Plasmas*

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Abstract

New global MHD eigenmode solutions arising in gaps in the low frequency Alfvén -acoustic continuum below the geodesic acoustic mode (GAM) frequency have been found numerically and have been used to explain relatively low frequency experimental signals seen in NSTX and JET tokamaks. These global eigenmodes, referred to here as Beta-induced Alfvén - Acoustic Eigenmodes (BAAE), exist in the low magnetic safety factor region near the extrema of the Alfvén - acoustic continuum. In accordance to the linear dispersion relations, the frequency of these modes shifts as the safety factor, q, decreases. We show that BAAEs can be responsible for observations in JET plasmas at relatively low beta $< 2\%$ as well as in NSTX plasmas at relatively high beta $> 20\%$. In contrast to the mostly electrostatic character of GAMs the new global modes also contain an electromagnetic (magnetic field line bending) component due to the Alfvén coupling, leading to wave phase velocities along the field line that are large compared to the sonic speed. Qualitative agreement between theoretical predictions and observations are found.

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I. INTRODUCTION

An area where up to now there has been sporadic investigation [? ? ? ? ? ?] but where pertinent wave particle interactions in burning plasma regimes need to be investigated, is the theoretical and experimental study of the linear interaction of between shear Alfvén and acoustic waves, two of the fundamental excitations of plasma in MHD theory in toroidal geometry. The interaction is mediated by finite pressure, plasma compressibility and the geodesic curvature. As a result of such interaction additional gaps in the coupled Alfvén - acoustic continuum emerge [? ?] and here we present an analytic model for the structure of these gaps. In addition their Alfvén - acoustic continuum has extrema with respect to the radial position, which arise in the gaps or in the low shear regions. We find with the use of the NOVA code [?] that global modes may occur adjacent to these extrema points, but were not reported before.

We call these new global modes associated with Alfvén - acoustic continuum gaps as the Beta-induced Alfvén - Acoustic Eigenmodes (BAAE) and we find that they can appear in both relatively low and relatively high beta plasmas. We will present data of hitherto unexplained oscillations in JET (relatively low beta) and NSTX (relatively high beta) in order to test how well MHD theory might explain these observations. Numerical simulations indicate that there are qualitative tendencies of the data to follow MHD theory, but our study also indicates there is a need of a comprehensive kinetic theory to be developed to accurately explain the data. Present kinetic theories seem to be insufficiently complete for this problem as we discuss in the summary section.

Numerical calculations for monotonic safety factor, q- profile and observations on JET show that the BAAE modes evolve from zero frequency, when q_0 or q_{min} cross rational values, and is bounded by a certain maximum value, which is determined by the frequency of the Alfvén - acoustic gap. By understanding the range of BAAE frequency excitation and observing or exciting these frequencies in experiments it seems potentially possible to extend the use of MHD to determine q_0 or q_{min}. Such observations could be a very important diagnostic tool for ITER and other burning plasma experiments. These observations would also help to infer the central plasma beta and the ion and electron temperatures.

In JET observations near the lowest possible frequency of the mode, the wave properties are quite similar to that of a shear Alfvén wave which is characterized by a magnetic field line bending perturbation, but with a dispersion that is modified from the conventional shear Alfvén wave dispersion. We have found qualitative agreement between theoretical predictions and observations, but there is a disparity that remains to be resolved in the quantitative matching of the predicted frequency with experimental observations.

In NSTX with the plasma beta around 20%, the BAAE modes are found numerically using the ideal MHD NOVA code. They are located within the Alfvén - acoustic continuum gaps and these BAAEs match hitherto unexplained oscillations in NSTX. They are localized near the region of low shear in plasmas with a reversed shear profile. In this case the frequency is about half the TAE frequency and is close to the geodesic acoustic mode (GAM) frequency. In NSTX, there is typically a strong drive from beam ions and it appears that the BAAEs contribute to a rather strong fast ion radial redistribution and loss. The frequency of the BAAE modes is in good agreement with that measurements when corrected for the Doppler shift due to the rotation.

The paper is organized as follows. In section II we derive expression for the Alfvén - acoustic continuum, its extremum points and the frequency gap. We apply our theory to JET and NSTX plasmas in Sections III and IV, respectively, where NOVA simulation results are also presented. A summary and a discussion of the relation of our work to previous publications are given in Sec. **??**.

II. LOW FREQUENCY ALFVÉN - ACOUSTIC CONTINUUM

We start from the ideal MHD equations for the continuum, which incorporates both the shear Alfvén and the acoustic branches (neglecting drift effects) [**? ?**]

$$\begin{aligned}
\omega^2\rho\frac{|\nabla\psi|^2}{B^2}\xi_s + (\mathbf{B}\cdot\nabla)\frac{|\nabla\psi|^2}{B^2}(\mathbf{B}\cdot\nabla)\,\xi_s \; + \gamma p k_s \nabla\cdot\vec{\xi} \; &= 0, \\
\left(\frac{\gamma p}{B^2}+1\right)\nabla\cdot\vec{\xi} + \frac{\gamma p}{\omega^2\rho}(\mathbf{B}\cdot\nabla)\frac{\mathbf{B}\cdot\nabla}{B^2}\nabla\cdot\vec{\xi} \quad + k_s\xi_s \quad &= 0,
\end{aligned} \tag{1}$$

where ψ is the poloidal magnetic flux, p and ρ are the plasma equilibrium pressure and density, γ is the specific heat ratio, $\xi_s \equiv \vec{\xi}\cdot[\mathbf{B}\times\nabla\psi]\,/\,|\nabla\psi|^2$, $\vec{\xi}$ is the plasma displacement, $k_s \equiv 2\mathbf{k}\cdot[\mathbf{B}\times\nabla\psi]\,/\,|\nabla\psi|^2$ and $\mathbf{k}$ and $\mathbf{B}$ are vectors of the magnetic curvature and field. The baseline physics of the Alfvén - acoustic mode coupling and the formation of the continuum and global modes in toroidal geometry can be understood in the limit of a low beta, high aspect ratio plasma. Making use of this tokamak ordering we find that the expression for the geodesic curvature is, $k_s = 2\varepsilon \sin\theta/q$, where $\varepsilon = r/R \ll 1$ and we neglected corrections of order $O(\varepsilon^2)$ and higher. This is justified for the low mode frequency, such that $\Omega^2 = O(1)$, where we defined $\Omega^2 \equiv (\omega R_0/v_A)^2/\delta$ and $\delta \equiv \gamma\beta/2 = O(\varepsilon^2)$. Thus, keeping only leading order terms we reduce the system of equations Eq.(1) to (for details see Ref.[**?**])

$$\Omega^2 y + \delta^{-1}\partial_{\|}^2 y \quad + 2\sin\theta\, z \quad = 0, \tag{2}$$

$$\Omega^2 z + \partial_{\|}^2 z \ + 2\Omega^2 \sin\theta\, y \ = 0, \tag{3}$$

where $y \equiv \xi_s \varepsilon/q$, $z \equiv \nabla\cdot\vec{\xi}$, $\partial_{\|} \equiv R_0 d/dl$ and l is the distance along the field line. For sufficiently small values of ε and β, numerical solutions of Eqs.(1,3) are found to be almost identical to numerical solutions of the NOVA code [**?**], which solves Eqs.(1). For this section, we can then consider Eqs.(1) and (2,3) to be interchangeable.

Equation (2) is essentially the shear Alfvén wave equation if z term is neglected. Equation (3) for z alone, would be the conventional sound equation. The two equations couple due to the finite compressibility of the plasma in toroidal geometry to give the form shown in Eqs.(2, 3). The coupling of each branch with the dominant poloidal mode number m, is via the $m \pm 1$ sideband harmonics of the other branch. To treat the side band harmonics we employ the following form for the perturbed quantities

$$\begin{pmatrix} y \\ z \end{pmatrix} = \sum_j \begin{pmatrix} y_{j-m}(r) \\ z_{j-m}(r) \end{pmatrix} e^{-i\omega t + ij\theta - in\zeta}, \tag{4}$$

where ζ is the ignorable toroidal angle along the torus, the direction of symmetry.

By substituting Eq.(4) into Eqs.(2,3) to eliminate $z_{j\pm1}$ in terms of y_j and $y_{j\pm2}$ we find a three term recursion relation

$$\left[\Omega^2 - \frac{k_j^2}{\delta} - \Omega^2\left(\frac{1}{\Omega^2 - k_{j-1}^2} + \frac{1}{\Omega^2 - k_{j+1}^2}\right)\right] y_j + \Omega^2\left[\frac{y_{j-2}}{\Omega^2 - k_{j-1}^2} + \frac{y_{j+2}}{\Omega^2 - k_{j+1}^2}\right] = 0.$$

One can readily justify that for $j = 0$ we can ignore the side band terms in the limit $\delta \ll 1$. Then the coefficient of y_0 must vanish and we obtain the following dispersion relation (similar to Ref.[**?**])

$$\left(\Omega^2 - k_0^2/\delta\right)\left(\Omega^2 - k_{+1}^2\right)\left(\Omega^2 - k_{-1}^2\right) \ = \ \Omega^2\left(2\Omega^2 - k_{+1}^2 - k_{-1}^2\right). \tag{5}$$

The left hand side set to zero, represents oscillations in an infinite medium, the two acoustic modes $\Omega^2 = k_{\pm1}^2$ and the shear Alfvén wave $\Omega^2 = k_0^2/\delta$, in the absence of toroidal effects and this is valid for $k_0^2/\delta \gg 1$. However the character of the solutions drastically changes for $k_0^2/\delta \ll 1$.

Then as $|k_{\pm 1}| \gg |k_0|$ in the vicinity of $q = m/n$ we can approximate $k_j^2 = (j/q + k_0)^2 \simeq j^2/q^2 + 2k_0 j/q$. Note, that in the high- m, n limit the radial dependence can be kept in the k_0 term only and $q = q_r = m/n$ can be fixed at the rational surface value, that is $k_j^2 \simeq j^2/q_r^2 + 2k_0 j/q_r$. With the coupling effect from the geodesic curvature, we find in the limit $k_0^2/\delta \ll 1$ two low frequency roots. One is

$$\Omega^2 = 1/q^2 \tag{6}$$

(comparable in frequency to the two acoustic side band frequencies), while the second solution takes the form of a modified shear Alfvén wave

$$\Omega^2 = k_0^2/\delta \left(1 + 2q^2\right). \tag{7}$$

This dispersion also follows from the kinetic theory developed in Ref.[**?**]. As k_0^2/δ gets larger the modified Alfvén wave frequency, Eq.(7), approaches the frequency of the first solution, Eq.(6), but do not cross. The first solution's frequency, Eq.(6), increases slowly, while the character of the Alfvénic branch solution, Eq.(7), changes dramatically and begins to decrease. As one increases k_0^2/δ further, so that it becomes larger than unity, the larger of the two roots approaches the acoustic side band solution

$$\Omega^2 = k_{\pm 1}^2, \tag{8}$$

taken with plus sign (assuming $k_0 \gg \sqrt{\delta}$ or minus sign if $-k_0 \gg \sqrt{\delta}$) while the smaller of the two solutions (i.e. the modified Alfvén wave, Eq.(7) approaches the other acoustic sideband, Eq.(8), with minus sign (or plus sign if $-k_0 \gg \sqrt{\delta}$). The third root of the dispersion relation given by Eq.(5) in the limit $k_0^2/\delta \ll 1$ is found to be the geodesic acoustic mode (GAM) with a frequency which is larger than the other two modes, i.e. $\Omega^2 = 2\left(1 + 1/2q^2\right)$ [**? ? ?**]. As k_0^2/δ increases, this mode changes character and becomes the usual shear Alfvén wave $\Omega^2 = k_0^2/\delta$ when $k_0^2/\delta \gg 1$. The same branch was found in Refs. [**?**] and [**?**], but was called BAE branch.

As we indicated, when $k_0^2/\delta \sim 1/q^2$ Alfvén and acoustic branches approach each other, but their corresponding roots do not cross. Instead, they form a gap (in frequency) structure that needs to be resolved. Now, we can rewrite Eq.(5) in the following form $4k_0^4/\delta q_r^2 -$

$A^2k_0^2/\delta + \Omega^2 (A-2) A = 0$, where $A = \Omega^2 - q_r^{-2}$. We can solve this equation for $k_0^2 (\Omega^2)$ dependence:

$$k_0^2 = \left[A^2 \pm \sqrt{A^4 - 16\delta\Omega^2 A (A-2) / q_r^2} \right] q_r^2/8. \tag{9}$$

This dispersion contains the branches discussed above. We can make use of the properties of Eq.(9) to find the expression for the continuum frequency gap at which $\partial k_0^2/\partial\Omega^2 \to \infty$, that is where k_0^2 has double root. It is easy to see that one of such cases is when $A = 0$, which is found to be a root that corresponds to the upper boundary of the gap, $\Omega = \Omega_+$, where

$$\Omega_+^2 = 1/q_r^2. \tag{10}$$

To find the lower boundary of the gap we consider Eq.(9) together with the double root condition, and find that a cubic equation for A needs to be satisfied: $A^3 - 16\delta (A + q_r^{-2}) (A-2) / q_r^2 = 0$ or

$$\hat{A}^3 - \left(\hat{A} + b\right)\left(\hat{A} - d\right) = 0, \tag{11}$$

where $\hat{A} = q_r^2 A/16\delta$, $b = 1/16\delta$, and $d = 2q_r^2/16\delta$. An exact solution of Eq.(11) can be obtained in general form, but it can be accurately approximated assuming $2q_r^2 > 1$. We find then $\hat{A} = C^{1/3} - d/3C$, where $C = d\sqrt{4d + 27b^2}/6\sqrt{3} - bd/2 > 0$, and the lower boundary gap frequency is expressed by

$$\frac{\Omega_-^2}{\Omega_+^2} = 1 + 16\delta \left(C^{1/3} - \frac{d}{3C^{1/3}} \right). \tag{12}$$

This can be further reduced in the limit of $\delta \ll 1$ to

$$\Omega_-^2 = \Omega_+^2 \left[1 - \left(32q^2\delta \right)^{1/3} \right]. \tag{13}$$

However, this last form is accurate only for very low plasma beta, $\delta < 0.2\%$.

Now we compare different continuum solutions obtained here from Eqs.(2,3), with NOVA solutions obtained from Eqs.(1) for a specific case based on the JET plasma discussed later. In our numerical work the adiabaticity index is taken as $\gamma = 11/8$, which corresponds to equal ion and electron temperatures $T_e = T_i$ in $q \gg 1$ limit [**?**]. For now we take circular magnetic surfaces with the following plasma parameters: the major radius $R_0 = 2.90m$, the minor radius $a = 0.0945m$ taken ten times smaller then actual JET plasma radius, plasma

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