Evaluation of a Semi-Implicit Numerical Algorithm for a Rate-Dependent Ductile Failure Model

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Outline

- History of the TEPLA model
- Current Version of the model
- Results
A survey conducted in the mid-80’s revealed that the mathematical descriptions of ductile fracture tended to apply to either tensile tests or spall tests. The objective behind the development of the TEPLA was then a unification of these disparate phenomena into a single model.
TEPLA History - 2
J. of Appl. Phys., Vol. 64, No. 12, 1988, pp. 6699-6712

Gurson Flow Surface

\[ \tau = \sqrt{\frac{3}{8} s_{ij} s_{ij}} \]
\[ \delta = -\frac{3}{2} \frac{p}{Y} \]
\[ F(s_{ij}, p, Y, \phi) = 4\tau^2 - Y^2 \left[ 1 + \phi^2 - 2\phi \cosh \delta \right] \]

Coupled Shear/Porosity Failure Criterion
\[ \gamma_f = \gamma_0 + \Delta \gamma \exp \left[ a \left( \frac{p}{2\tau} \right) \right] \left( \frac{\phi}{\phi_f} \right)^2 + \left( \frac{\gamma}{\gamma_f} \right)^2 = 1 \]

Coupled Evolution Rules for:
Deviator \[ d\tau = \frac{3\mu}{4\tau} \left( s_{ij} d\epsilon_{ij} - s_{ij} d\epsilon_{ij}^p \right) \]
Porosity \[ d\phi = (1 - \phi) d\epsilon_{kk}^p \]
Pressure \[ dp = -(1 - \phi) B_s d\ln v + \rho \Gamma_s T ds + [(1 + \Gamma_s)p - (1 - \phi) B_s] d\ln(1 - \phi) \]
Gurson Flow Surface

\[ \tau = \frac{1}{2} \sqrt{\frac{3}{2} s_{ij} s_{ij}} \quad \delta = -\frac{3}{2} \frac{P}{Y} \]

\[ \dot{Y} = H \sqrt{\frac{2}{3} \dot{e}_{ij}^p \dot{e}_{ij}^p} \]

\[ Y = [Y_0 + H (\dot{e}_p)^{an}] [1 + b_n \ln (\dot{e}_p)] \]

\[ F(s_{ij}, p, Y, \phi) = 4\tau^2 - Y^2 [1 + (q\phi)^2 - 2q\phi \cosh \delta] \]

Coupled Evolution Rules for:

Deviator \[ \dot{s}_{ij} = 2G \left( \dot{e}_{ij} - \dot{e}_{ij}^p \right) \]

Porosity \[ \dot{\phi} = (1 - \phi) \dot{e}_{kk}^p \]

Pressure \[ \dot{P} = [B - (1 + \Gamma_s)P] \dot{e}_{kk}^p - B\dot{e}_{kk}^p + \Gamma_s s \]

Updating the plastic strain rate Three methods investigated (associative flow, return, hybrid)
Strain Softening

**Problem:** Softening leads to a change in the set of governing equations for the dynamic IBVP from hyperbolic to elliptic and the problem becomes ill-posed.

**Manifestation (Simo 1989)**
- The strains localize to a narrow band (set of measure zero)
- Classical local dissipation becomes meaningless since no dissipation can take place in a localized set of zero Borel measure
- Numerical simulation of softening materials exhibit a totally spurious mesh dependency
- For elastic and rate independent materials, the governing equations exhibit a local loss of ellipticity which precludes wave propagation

**Possible Fixes (Simo 1989)**
- Mesh dependent modulus $H^h$
- Nonlocal methods (higher-order spatial derivatives)
- Viscoplasticity (Higher order temporal derivatives)
Gurson Flow Surface

\[ \tau = \sqrt{\frac{3}{2} s_{ij} s_{ij}} \]

\[ \delta = -\frac{3}{2} \frac{p}{Y} \]

\[ F(s_{ij}, p, Y, \phi) = \tau^2 - Y^2 \left[ 1 + (q\phi)^2 - 2q\phi \cosh \delta \right] \]

Coupled Shear/Porosity Failure Criterion

\[ \gamma_f = \gamma_0 + \Delta \gamma \exp \left[ a \left( \frac{p}{2\tau} \right)_f \right] \left( \frac{\phi}{\phi_f} \right)^2 + \left( \frac{\gamma}{\gamma_f} \right)^2 = 1 \]

Coupled Evolution Rules for:

Deviator \[ \tilde{s}_{ij} = 2G \left( \tilde{e}_{ij} - \tilde{\varepsilon}_{ij}^p \right) \]

Porosity \[ \dot{\phi} = (1 - \phi)\dot{\varepsilon}_{kk}^p \]

Pressure \[ \dot{P} = \Gamma_s \tilde{s}_{ij}\tilde{e}_{ij} - B\dot{\varepsilon}_{kk}^p + \alpha\dot{\varepsilon}_{kk}^p \]
Viscoplasticity

\[ \dot{\sigma}_{ij} = f' \dot{e} + \tau_r \dot{e} \]

Length scale: \( l = k \frac{\tau r c}{E} \)

Rate parameter: \( \tau_r = \eta \left( \frac{1 - \phi_0}{\phi_0} \right)^{2/3} + \left( \frac{1 - \phi}{\phi} \right)^{1/3} \)
Current TEPLA Model

Gurson Flow Surface

\[ \tau = \sqrt{\frac{3}{2} s_{ij} s_{ij}} \]

\[ \delta = -q_2 \frac{3}{2} \frac{p}{Y} \]

\[ F(s_{ij}, p, Y, \phi) = \left( \frac{\tau}{Y} \right)^2 - \left[ 1 + q_3 \phi^2 - 2q_1 \phi \cosh \delta \right] \]

Coupled Shear/Porosity Failure Criterion

\[ \epsilon_f = \sqrt{1 - \left( \frac{\phi}{\phi_f} \right)^2 \left[ D_1 + D_2 \exp \left( D_3 \frac{P}{Y} \right) \right]} \]

Coupled Evolution Rules for:

Deviator \[ \dot{s}_{ij} = 2G \left( \dot{e}_{ij} - \dot{e}_{ij}^p \right) - s_{ik} W_{kj} + W_{ik} s_{kj} \]

Porosity \[ \dot{\phi} = (1 - \phi) \dot{e}_{kk}^p \]

Pressure \[ \dot{P} = \Gamma s s_{ij} \dot{e}_{ij} - B \dot{e}_{kk}^p + \alpha \dot{e}_{kk}^p \]
Implicit Algorithm - 1

1. Solve for the trial state

\[ s_{i,j}^t = s_{i,j}^n + 2G \dot{e}_{ij} \Delta t + \dot{r}_{ij} \Delta t \quad \quad P^t = P^n - B \dot{e}_{kk} \Delta t \quad \quad \phi^t = \phi^n \]

2. Solve for the equilibrium state

Implicit time integration leads to four coupled nonlinear equations which must be solved simultaneously:

\[ C = \left( 1 + \frac{6G}{Y} \right) \frac{\bar{\tau}}{Y} - \frac{\tau^t}{Y} = 0 \]

\[ D = \bar{\delta} + \frac{3}{2} q_2 \frac{\lambda}{Y_f} \left[ 3 \frac{q_1 q_2}{Y_f} \alpha \ddot{\phi} \sinh \bar{\delta} + 2 \Gamma_s \left( \frac{\bar{\tau}}{Y} \right)^2 \right] - \delta^t = 0 \]

\[ E = \bar{\phi} - \phi^t - 3(1 - \bar{\phi}) \frac{\lambda}{Y_f} \frac{\lambda}{Y_f} \ddot{\phi} \sinh \bar{\delta} = 0 \]

\[ F = \bar{F} = \left( \frac{\bar{\tau}}{Y} \right)^2 - \left( 1 + q_3 \bar{\phi}^2 - 2q_1 \ddot{\phi} \cosh \bar{\delta} \right) = 0 \]

Which must be solved simultaneously for independent variables: \( \frac{\bar{\tau}}{Y}, \bar{\delta}, \bar{\phi}, \frac{\lambda}{Y} \).
3. Solve for the final state

\[ s_{ij}^{n+1} = \frac{\xi}{\Delta t} + \frac{\tau t}{\Delta t + 1} s_{ij}^t \]

\[ p^{n+1} = \frac{p^t + \frac{\alpha \Delta t}{\tau_r} \bar{P} + \frac{\tau_s}{3G} (\tau^t - \tau^{n+1}) \tau^{n+1}}{1 + \frac{\alpha \Delta t}{\tau_r}} \]

\[ \phi^{n+1} = \frac{\phi^n + \dot{\epsilon}_{kk}^p \Delta t}{1 + \dot{\epsilon}_{kk}^p \Delta t} \]
Flyer Plate Experiment

Target free sfc.

Impact sfc.

Flyer free sfc.

U_p

or

p

t

p

Visar

Flyer

Manganin Gage

PMMA

Target

U_p vs TIME, (Ms)

TIME, (Ms)
Copper Results

158.5 m/s

128.6 m/s
Tantalum Results – 1

200 m/s

Visar

300 μm

TIME (μs)

U_s (m/s)
Tantalum Results – 2

220 m/s

Visar

TIME (μs)

U_{ls} (m/s)

Tantalum

Data

Code

300 μm
Tantalum Results – 3

- 254 m/s

![Visar Image]

![Image of Tantalum Surface]

![Graph of Time vs. Velocity (m/s) with markers for Tantalum, Data, and Code]

TIME (µs)
Conclusions

• Few results shown
  Much more needed for validation

• Time step problem overcome – yes
  Quantification needed

• 1d, 2d, 3d nuances

• Parameter set for variety of materials