EVIDENCE FOR MERGING OR DISRUPTION OF RED GALAXIES FROM THE EVOLUTION OF THEIR CLUSTERING*

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ABSTRACT

The formation and evolution of massive red galaxies form a crucial test of theories of galaxy formation based on hierarchical assembly. In this letter we use observations of the clustering of luminous red galaxies from the Boötes field and N-body simulations to argue that about 1/3 of the most luminous satellite galaxies appear to undergo merging or disruption within massive halos between $z \simeq 0.9$ and $z \simeq 0.5$.

Subject headings: galaxies: evolution – galaxies: halos

1. INTRODUCTION

The assembly of the most massive galaxies is a key test of cold dark matter (CDM) models of galaxy formation, as the ongoing growth of massive galaxies via mergers is a generic feature of hierarchical CDM models. Observationally, the most massive galaxies have little ongoing star formation, and the bulk of their stellar mass was formed at z > 2 (e.g. Bower, Lucey & Ellis 1992; Trager et al. 2000; Cool et al. 2006, and references therein). If there is appreciable growth of these galaxies at z < 1, this must be due to galaxy mergers, as predicted by the hierarchical CDM models.

Evidence for the ongoing assembly of massive galaxies is inconclusive. While the stellar mass within the red galaxy population has doubled since z = 1 (Bell et al. 2004; Willmer et al. 2006; Brown et al. 2006a), this appears to be due to the truncation of star formation in blue galaxies, and the role of mergers is poorly known. van Dokkum (2005) and Bell et al. (2006a,b), using close galaxy pairs, conclude that L_{\star} red galaxies grow rapidly via mergers since z = 1, while Masjedi et al. (2006), using similar techniques, find that the merger rate of $4L_{\star}$ red galaxies is only ~ 1% Gyr⁻¹. Using the galaxy space density, Brown et al. (2006a) find that the stellar masses of $4L_{\star}$ red galaxies grow by $\simeq 25\%$ since $z \sim 0.7$, while others find no significant growth over similar redshift ranges (e.g., Bundy et al. 2006; Cimatti, Daddi & Renzini 2006; Caputi et al. 2006; Wake et al. 2006).

There is an additional route to constraining the evolution of galaxies, which is to use their clustering properties. Building upon the theoretically understood evolution of the dark matter halo population, we can obtain complementary constraints which bypass the model dependence of stellar evolution or merger times as a function of projected distance. We illustrate this approach in this *Letter*, presenting evidence from the evolution of their clustering that luminous red galaxies undergo merging or disruption between $z \sim 0.9$ and $z \sim 0.5$.

2. THE OBSERVATIONAL SAMPLE

We use galaxies in the 9 deg² Boötes field, which has been imaged in the optical and infrared by the NOAO Deep Wide-Field (NDWFS; Jannuzi & Dey 1999) and *Spitzer* IRAC Shallow Surveys (Eisenhardt et al. 2004). We use a luminous (>

 $1.6L_{\star}$) subset of the Boötes red galaxy sample which was selected from the imaging using empirical photometric redshifts and an evolving restframe U-V color criterion (Brown et al. 2006a). This subset includes galaxy samples in three redshift slices: 0.4 < z < 0.6, 0.6 < z < 0.8 and 0.8 < z < 1.0 with comoving volumes of 2.4, 3.5 and $5.2 \times 10^6 (h^{-1} \text{Mpc})^3$ respectively, and at each redshift, galaxies are selected to be above a luminosity threshold such that the sample has a constant comoving number density ($\bar{n} = 10^{-3} h^3 \text{Mpc}^{-3}$). Our results are based on the observed evolution of the angular clustering of these samples, containing a few thousand galaxies each. We transform from models of the spatial clustering to angular clustering using a redshift distribution model which accounts for the small measured uncertainties of the photometric redshifts ($\sigma_z \lesssim 0.05$). We describe the clustering measurements and modeling in detail in Brown et al. (2006b).

3. MODELING GALAXY CLUSTERING

Our galaxy samples have been selected to have constant We find that their clustering evolves very little, with $\xi(6h^{-1}\text{Mpc},z) \simeq 1$. We begin with an analytic argument to show that these galaxies cannot be undergoing a common luminosity evolution history, such as pure passive evolution, without any mergers¹. In such a scenario each galaxy preserves its identity and no galaxy leaves or enters the sample. If we assume that galaxies and mass follow the same velocity field, and retain their identities, then the continuity equation in the linear regime requires that $\dot{\delta}_{gal} = \dot{\delta}_{m}$ (Peebles 1980). If we define $\delta_{gal}(z) = b(z)\delta_m(z)$ and the growth function $D(z) \equiv \delta_{\rm m}(z)/\delta_{\rm m}(0)$ then $b(z) = 1 + D^{-1}(z)[b(0) - 1]$ (Fry 1996). As shown in Figure 1, this prediction is in good agreement with our numerical simulations with passive evolution (see § 3.3). Assuming scale-independent, deterministic biasing therefore predicts evolution in ξ which is not in agreement with the observations. In fact, we find that passive evolution cannot fit the trend of the central values of the clustering strength for any plausible cosmology.

To go further we need a way of connecting the galaxies we observe with the host dark matter halos whose evolution the-

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FIG. 1.— (Top) The correlation length, r_0 , from a power-law fit for the sample as a function of z from Brown et al. (2006b). (Bottom) The evolution of the large-scale bias, assuming a dark matter power spectrum with $\sigma_8 = 0.8$. The solid line is $b(z) = 1 + D^{-1} [b(0) - 1]$ (see text), the open circles are measured from our "passive mocks" and the solid circles are from the mocks which best fit $w(\theta)$ at each redshift.

ory predicts. The halo model (see e.g. Cooray & Sheth 2002 for a review) has provided us with such a physically informative and flexible means of describing galaxy bias. The key insight is that an accurate prediction of galaxy clustering requires a knowledge of the occupation distribution of objects in halos (the HOD) and their spatial distribution. In combination with ingredients from N-body simulations a specified HOD makes strong predictions about a wide array of galaxy clustering statistics. The formalism thus allows us to use observations of galaxy clustering to constrain the connection between galaxies and their host dark matter halos at each z, and in particular to show that the luminous galaxies in the NDWFS have undergone significant merging or disruption between $z \simeq 0.9$ and $z \simeq 0.5$ by comparing the HOD inferred from the $z \simeq 0.5$ clustering data to one which is passively evolved from the inferred HOD at $z \simeq 0.9$.

3.1. Simulations and mock catalogs

Our modeling of galaxy clustering is based on mock catalogs constructed within the HOD framework by populating halos in a cosmological N-body simulation. We use a high resolution simulation of a Λ CDM cosmology ($\Omega_M = 0.25 =$ $1 - \Omega_\Lambda$, $\Omega_B = 0.043$, h = 0.72, $n_s = 0.97$ and $\sigma_8 = 0.8$). The linear theory power spectrum was computed by evolution of the coupled Einstein, fluid and Boltzmann equations using the code described in White & Scott (1995). This code agrees well with *CMBfast* (Seljak & Zaldarriaga 1996), see e.g. Seljak et al. (2003). The simulation employed 1024³ particles of mass $8 \times 10^9 h^{-1} M_{\odot}$ in a periodic cube of side $500 h^{-1}$ Mpc using a *TreePM* code (White 2002). The Plummer equivalent softening was $18 h^{-1}$ kpc (comoving).

For each output we generate a catalog of halos using the Friends-of-Friends (FoF) algorithm (Davis et al. 1985) with a linking length of $0.168 \times$ the mean inter-particle spacing. This procedure partitions the particles into equivalence classes, by linking together all particle pairs separated by less than a distance *b*. The halos correspond roughly to particles with $\rho > 3/(2\pi b^3) \simeq 100$ times the background density. Our mass definition uses the sum of the particle masses in the halo, however to obtain better correspondence between our defi-

nition of halo mass and that implicitly defined by the mass functions of Sheth & Tormen (1999) and Jenkins et al. (2000) we rescaled the masses by $M/M_{\rm fof} = 1+0.01 (\ln M_{\rm fof} - 23.5)$ where $M_{\rm fof}$ is the FoF mass in units of $h^{-1}M_{\odot}$. With this redefinition the mass function in the simulation lies between those of Sheth & Tormen (1999) and Jenkins et al. (2000), differing from them by less than 10% in the mass range of interest.

To make mock catalogs we use a halo model which distinguishes between central and satellite galaxies. We choose a mean occupancy of halos: $N(M) \equiv \langle N_{gal}(M_{halo}) \rangle$. Each halo either hosts a central galaxy or does not, while the number of satellites is Poisson distributed about a mean N_{sat} . With the luminosity-threshold samples, we parameterize N(M) = $N_{cen} + N_{sat}$ with 4 parameters (e.g. Zheng et al. 2005)

$$N_{\rm cen}(M) = \frac{1}{2} \, {\rm erfc} \left[\frac{\ln(M_{\rm cut}/M)}{\sqrt{2}\sigma} \right] \tag{1}$$

and

$$N_{\rm sat}(M) = \left(\frac{M - M_{\rm cut}}{M_1}\right)^{\alpha} \tag{2}$$

for $M > M_{\text{cut}}$ and zero otherwise. Different functional forms have been proposed in the literature, but the current form is flexible enough for our purposes. Including a different low mass roll-off in the satellite term, following Tinker et al. (2005) and Conroy, Wechsler & Kravtsov (2006), does not alter our basic conclusions.

Given an HOD and the halo catalogs we can produce a mock catalog in one of two ways. Central galaxies always live at the minimum of the halo potential. We either place satellite galaxies assuming an NFW profile (Navarro, Frenk & White 1996) with a concentration-mass relation fit to the halos in the simulation or anoint n_{sat} dark matter particles, chosen at random, as galaxies. The two methods produce very similar, though not identical, clustering, with the biggest differences on Mpc scales. An analytic model (described in Zheng 2004; Tinker et al. 2005) also produces very similar results. The differences between the methods are smaller than the observational errors, so we shall neglect them henceforth.

3.2. Comparing with data

From the model galaxy positions we compute $\xi(r)$ in real space by direct pair counting in the periodic box for separations $< 20 h^{-1}$ Mpc. Beyond $20 h^{-1}$ Mpc we extrapolate assuming a constant bias. The redshift distribution is used to convert $\xi(r)$ into $w(\theta)$ using Eq. (50) of Simon (2006), giving the predicted clustering for any set of HOD parameters. We fit to the data assuming Gaussian errors with the covariance matrices of Brown et al. (2006b), and assume a 5% error on the number density of galaxies. Figure 2 compares the best fitting HOD model predictions to the data at $z \simeq 0.9$ and 0.5.

In order to propagate the observational errors into uncertainties in the HOD (Eqs. 1,2) we used a Markov chain Monte-Carlo method (e.g. Gilks, Richardson & Spiegelhalter 1996) as detailed in Brown et al. (2006b). We found that the data were unable to rule out models with $\sigma \gg 1$ and $\alpha \ll 1$, which we regard as unlikely for large red galaxies, so we impose a prior which penalizes $\sigma > 1$ and $\alpha \simeq 0$. Tests indicate that surveys twice as large would not need this prior, though with this prior even our chains converged well. Because the mock catalog generation using NFW profiles is very fast and requires little memory we use this to generate the chains.



FIG. 2.— The angular correlation function, $w(\theta)$, for the 0.4 < z < 0.6 and 0.8 < z < 1.0 slices. Open circles with error bars represent the Boötes data, the solid line is the best fitting HOD model prediction and (in the lower panel) the dashed line is the prediction of the best fitting $z \simeq 0.9$ model passively evolved to $z \simeq 0.5$.

3.3. Passive evolution

For a subset of 250 of the models at $z \simeq 0.9$ we use the particle-based method to produce mock catalogs which we passively evolve to $z \simeq 0.5$ simply by tracking the particles based on their ID. The positions and halo memberships of these tagged particles are followed. We expect the HOD to be more robust than the small-scale clustering for these particles², but we show the latter in Figure 2 for completeness. Looking at the difference in the HODs is also more informative, and gives us a clue as to what physics may be missing from pure passive evolution.

4. DISCUSSION

A comparison of the HOD of the passively evolved samples with the HODs which best fit the $z \simeq 0.5$ data indicates that evolution produces too many galaxies in high mass halos, as shown in Figure 3. A similar conclusion can be reached by comparing the clustering of the passively evolved models to the data in Figure 2 — the excess clustering on small scales from passive models clearly indicates that the models



FIG. 3.— HODs for the $z \simeq 0.5$ sample. The shaded area indicates the mean and standard deviation in the HOD from the Markov chains, fit to the $w(\theta)$ data at $z \simeq 0.5$. The mass scale has been increased by 7% as described in the text. Open triangles indicate the HOD from the models that fit the $z \simeq 0.9$ data evolved to $z \simeq 0.5$ by tracking particles. Squares assume 33% of the satellites in the passively evolved mocks disappear between $z \simeq 0.9$ and $z \simeq 0.5$.

overpredict galaxy pairs within the same halo, i.e., they predict too many satellite galaxies in high mass halos. Another indication of the excess is that the satellite fraction in the passively evolved models is 0.24 ± 0.02 while that in the best fitting models is 0.18 ± 0.02 .

There must be some physical process which reduces the number of galaxies in massive halos, and the most natural candidates are dynamical friction and tidal stripping which act to merge massive satellites with the central galaxy or disrupt them. Based on Figure 3, at high halo masses $\sim 1/3$ of the satellites in the passively evolved catalogs must have disappeared by $z \simeq 0.5$. We caution that our calculation ignores sources and sinks. If galaxies can enter or leave the sample due to rapid evolution of their star formation rates, interpretting the evolution of galaxy clustering is non-trivial. However, z < 1 blue galaxies are rare, and (at least at $z \simeq 0$) red galaxies have little cold gas to fuel renewed bursts of star formation.

There is a subtlety to bear in mind with these statistics. Merging results in a small fraction of $z \simeq 0.9$ galaxies disappearing by $z \simeq 0.5$. So we should really compare the passively evolved catalog (with $\bar{n} = 10^{-3} h^3 \text{Mpc}^{-3}$) to a fitted one with a slightly lower number density – the "true" $z \simeq 0.5$ descendants of the $z \simeq 0.9$ galaxies. The cut-off mass scale for this catalog would be slightly larger than our fits and we should shift the mass definition in the HOD accordingly. We estimate the size of this shift, by considering the evolution of the central galaxies, to be $\approx 7\%$ and in Figure 3 we increase the mass scale of the HOD inferred from the $z \simeq 0.5$ data accordingly (though this shift does not change our conclusions).

To estimate the impact of the disappeared satellites on the growth of the central galaxy or the boost of the intracluster light (ICL) in the halo, we make the following simple model. By matching the number density of halos above mass M with that of galaxies above luminosity L (we use the *B*-band LFs of Brown et al. 2006a), we can relate the central galaxy luminosity to halo mass (the luminous end of the LF is dominated by central galaxies as shown in Zheng et al. 2005). For our sample, at $z \simeq 0.5$ a halo of $5 \times 10^{13} h^{-1} M_{\odot}$ on average hosts a

² The small-scale clustering depends on the evolution of the subhalos inside of the host halo, and due to finite force and mass resolution these may not be correctly modeled in massive halos.

central galaxy of $\approx 5 L_{\star}$, with $L_{\rm cen} \propto M^{0.36}$ for massive halos. If we also assume that the satellite LF has the same shape as the global LF, we can use the fitted HODs to find the number of satellites and integrate to measure their total light. We find that in such halos satellites on average have a total lumnosity of 2.6 L_{\star} , so if this is what is left after 1/3 of the satellites disappeared, the satellites would have contributed $\approx 25\%$ of the current stellar mass to the central galaxy or a similar mass to the ICL. For halos of $10^{14} h^{-1} M_{\odot}$ the contribution is $\approx 40\%$. These numbers are upper bounds, since satellites may have lost just enough mass to leave our sample. Note that the stellar material resides in the inner regions of the halo, where the potential well is the deepest, and is the last material to be disrupted. An "average" satellite would need to lose 40% of its stellar mass, or decrease its surface brightness, to drop out of the sample. Analyzing samples with lower luminosities would help us constrain this.

We are unable, without further modeling, to differentiate between satellites fueling the growth of the central galaxy or the ICL, which at lower redshift comprises $\approx 5-10\%$ of the stellar mass in groups and clusters (Gonzalez et al. 2000; Zibetti et al. 2005; Aguerri et al. 2006). We can, however, form an $L_{cen} - M$ relation at $z \simeq 0.9$ as above. By matching progenitor halos to their descendants at $z \simeq 0.5$ and assuming 0.48 *B*-band magnitudes of fading we find $\approx 10\%$ growth in the stellar mass of the central galaxy between $z \simeq 0.9$ and 0.5. This would suggest the satellites also build an ICL component (the total stellar mass in the disappeared satellites, if it all ended up as ICL, constitute $\sim 15\%$ of the total halo luminosity above the threshold). However this result relies on our assumption of a uniform LF and on stellar population evolution predictions, and needs to be constrained by observations of the ICL. If we argue that there is little extended light at high z, either the satellites may have lost just enough mass to leave the sample or the "extra" mass would accrete onto the central galaxy. We regard this dichotomy as an open question requiring further investigation.

Our conclusions are necessarily tentative due to the limited volume of the NDWFS Boötes survey, which does not probe the mass function above $10^{14} h^{-1} M_{\odot}$ well. There are several

areas where more or different data would be beneficial. Tests using models of larger surveys³ indicate that doubling the survey volume removes the islands of parameter space which we have excluded with priors and shrinks the errors on the HOD parameters by $\approx \sqrt{2}$. A measurement of the space density of groups richer than several members would shrink the errors on the high mass end of the HOD dramatically, but would require more volume than we have at present to contain a representative sample of rich groups. We also investigated the dependence of our results upon cosmology using similar simulations with different parameters. Our results remain robust within the currently allowed range of models.

This preliminary investigation shows the power of clustering measures to inform questions of the formation and evolution of galaxies. We find evidence for evolution in the red galaxy HOD very different than that predicted by pure passive evolution models. Our result is largely independent of models of red galaxy stellar populations and estimates of dynamical friction scales. With more data from the NDWFS and future surveys we hope to be able to trace in detail the formation history of the most massive galaxies in the Universe.

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³ The wider area survey need not be as deep as NDWFS.

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