Measurement of $D^0 - \overline{D^0}$ mixing from a time-dependent amplitude analysis of $D^0 \to K^{+}\pi^-\pi^0$ decays


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We present evidence of $D^0 \rightarrow \bar{D}^0$ mixing using a time-dependent amplitude analysis of the decay $D^0 \rightarrow K^+ \pi^- \pi^0$ in a data sample of 384 fb$^{-1}$ collected with the BABAR detector at the PEP-II $e^+e^-$ collider at SLAC. Assuming $CP$ conservation, we measure the mixing parameters $\delta_{K^{*+0}} = [2.61 \pm 0.08 \text{ (stat.)} \pm 0.39 \text{ (syst.)}]^\circ$, $y_{K^{*+0}} = [0.06 \pm 0.04 \text{ (stat.)} \pm 0.34 \text{ (syst.)}]^\circ$. The confidence
level for the data to be consistent with the no-mixing hypothesis is 0.1%, including systematic uncertainties. This result is inconsistent with the no-mixing hypothesis with a significance of 3.2 standard deviations. We find no evidence of CP violation in mixing.

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The mixing between the flavor eigenstates $|D^0\rangle$ and $|\bar{D}^0\rangle$ of the neutral $D$ meson depends upon the mass and width differences of the mass eigenstates. In the standard model (SM), the $D^0, \bar{D}^0$ mixing contribution from short-distance effects is negligible [1]. This is due to Glashow-Iliopoulos-Maiani suppression of the first two quark generations and Cabibbo-Kobayashi-Maskawa suppression of the third. Long-distance effects from intermediate states that couple to both $D^0$ and $\bar{D}^0$ dominate. Their contributions to the mixing parameters are difficult to predict, but are estimated to be of the order $10^{-3}$–$10^{-2}$ [1]. Several recent studies report evidence for mixing parameters at the 1% level [2]. This is consistent with some SM expectations and provides constraints on new physics models [3]. If mixing occurs, the physical eigenstates $|D_{1,2}\rangle = |pD^0\rangle \pm q|\bar{D}^0\rangle$ must have different masses $M_{1,2}$ or widths $\Gamma_{1,2}$. Here $|p|^2 + |q|^2 = 1$. Mixing is parametrized using $x \equiv 2(M_1 - M_2)/(\Gamma_1 + \Gamma_2)$ and $y \equiv (\Gamma_1 - \Gamma_2)/(\Gamma_1 + \Gamma_2)$, where (2) refers to the almost CP-even (oddly) eigenstate. If CP is conserved, then $|p/q| = 1$ and $\arg(q/p \cdot A_f/A_{\bar{f}}) = 0$. Here $A_f$ ($A_{\bar{f}}$) is the amplitude of the transition of the $D^0$ ($\bar{D}^0$) to the final state $f$.

In this letter, we analyze events in which the flavor of the $D^0$ is measured at production. We present the first time-dependent amplitude analysis of the $D^0 \to K^+\pi^-\pi^0$ Dalitz plot to extract the mixing parameters. Previously, we studied the time dependence of $D^0 \to K^+\pi^-\pi^0$ decays integrated over large regions of the Dalitz plot. We found no evidence for mixing [3]. However, certain regions of the phase space are more sensitive to mixing than others (see below). Therefore the amplitude analysis is potentially more sensitive than our previous work. The wrong-sign (WS) decays $D^0 \to K^+\pi^-\pi^0$ and the more copious right-sign (RS) decays $D^0 \to K^-\pi^+\pi^0$ are reconstructed. The RS decays proceed through a Cabibbo-favored (CF) amplitude. The WS decays proceed through a coherent sum of a doubly Cabibbo-suppressed (DCS) amplitude and a CF amplitude produced by mixing. We identify RS and WS decays by reconstructing the $D^+ \to D^0\pi^+, D^0 \to K\pi\pi^0$ decay chain. The flavor of the $D^0$ candidate is determined from the charge of the low-momentum pion ($\pi^+$). The DCS and the CF amplitudes are described with isobar models [3] as outlined below.

The time-dependent decay rate is a function of the Dalitz variables $s_{12} = m_K^2 + m_\pi^2$ and $s_{13} = m_{\pi^0}^2$. It depends on the DCS amplitude $A_f(s_{12}, s_{13}) = \langle f | H | D^0 \rangle$ and the CF amplitude $A_{\bar{f}}(s_{12}, s_{13}) = \langle \bar{f} | \bar{H} | \bar{D}^0 \rangle$, where

$$\bar{f} = K^+\pi^-\pi^0.$$ In the limit $|x|, |y| \ll 1$, it is given by:

$$\frac{dN_f(s_{12}, s_{13}, t)}{ds_{12}ds_{13}dt} = e^{-\Gamma t} \left[ |A_f|^2 + |A_{\bar{f}}|^2 \right] y \cos \delta_f - x \sin \delta_f (\Gamma t) + \frac{x^2 + y^2}{4} |\bar{A}_f|^2 (\Gamma t)^2 \right]$$

where $\delta_f(s_{12}, s_{13}) = \arg[A_f(s_{12}, s_{13})/A_{\bar{f}}(s_{12}, s_{13})]$. The first term in Eq. (1) is the DCS contribution to the WS rate; the second term is a pure mixing contribution; the second term arises from the interference between DCS and mixing CF amplitudes. We determine the CF amplitude $A_{\bar{f}}$ in a time-independent Dalitz plot analysis of the RS decay sample, and use it in the analysis of the WS sample. The DCS amplitude $A_f$ is extracted along with the mixing parameters. In the isobar approach, $A_f$ and $A_{\bar{f}}$ are described as a coherent sum of amplitudes, where each amplitude accounts for a resonance contribution. From inspection of the Dalitz plots (Fig. 10-b), WS decays proceed primarily through the resonance $D^0 \to K^+\pi^-$, while RS decays proceed primarily through $D^0 \to K^-\rho^+$. For both $A_f$ and $A_{\bar{f}}$, one complex amplitude must be fixed arbitrarily.

The advantage of an amplitude analysis is that the interference term in Eq. (1) produces a variation in average decay time as a function of position in the Dalitz plot that is sensitive to the complex amplitudes of the resonant isobars as well as the mixing parameters. In this study, the change in the average decay time and the interference between the $D^0 \to K^+\pi^-$ and $D^0 \to \rho^-K^+$ amplitudes are the origin of our sensitivity to mixing. Our analysis is sensitive to $x$ and $y$ in the form $y'_{K\pi\pi^0} \equiv x \cos \delta_{K\pi\pi^0} - y \sin \delta_{K\pi\pi^0}$ and $x'_{K\pi\pi^0} \equiv x \cos \delta_{K\pi\pi^0} + y \sin \delta_{K\pi\pi^0}$ where $\delta_{K\pi\pi^0}$ is the strong-interaction phase difference between the DCS $D^0 \to \rho^-K^+$ and the CF $\bar{D}^0 \to K^+\rho^-$ amplitudes, and cannot be determined in this analysis (note that $\delta_{K\pi\pi^0}$ is in general different from the analogous phase in other decays). Mixing is implied through a non-zero value of $x'_{K\pi\pi^0}$ or $y'_{K\pi\pi^0}$. We define $A_f(s_{12}, s_{13}) = r_0 A_f^{DCS}(s_{12}, s_{13})$ and $A_{\bar{f}}(s_{12}, s_{13}) = A_{\bar{f}}^{CF}(s_{12}, s_{13})$. Here

$$r_0^2 = N_{WS}/ \left( N_{RS} \cdot \int N_f(s_{12}, s_{13}, t)ds_{12}ds_{13}dt \right)$$

where $N_{WS}$ ($N_{RS}$) is the number of WS (RS) events in the sample. The variation of average decay time in the Dalitz plot due to mixing depends on the ratios $x'_{K\pi\pi^0}/r_0$ and $y'_{K\pi\pi^0}/r_0$; these are the parameters that we extract.
directly from the data and use to determine the significance of the mixing result.

The amplitudes entering the WS analysis are described as a sum of isobar components \( A_j \) that are parametrized with Breit-Wigner functions, \( A_j^{CF/DCS} = \sum_{j=1}^{n_{CC/DCS}} a_{j,CF} \alpha_j \langle m^2_{K+\pi-}, m^2_{\pi-K-} \rangle_0 \), where \( a_j \) and \( \alpha_j \) are the strong interaction amplitudes and phases of the \( j^{th} \) resonant amplitude \( \alpha_j \). For the \( K-\pi \) S-wave component we use a parametrization derived from \( K-\pi \) scattering data \[2\], which consists of a \( K_0(1430) \) resonance together with an effective non-resonant component.

We analyze a data sample of 384 fb\(^{-1}\) collected with the BaBar detector \[3\] at the PEP-II \( e^+e^- \) collider at SLAC near a center-of-mass energy of 10.58 GeV. Charged tracks are reconstructed with a silicon-strip detector (SVT) and a drift chamber (DCH), both in a 1.5 T magnetic field. Particle identification is based on measurements of ionization energy loss \( dE/\text{d}x \) in the SVT and DCH together with measurements from a Cherenkov ring-imaging device. Photon energies are measured with a CsI(Tl) calorimeter. All selection criteria, the fit procedure and the systematic error analysis are discussed in more detail before we search for evidence of mixing in the data.

Selection criteria are based partly on those of Ref.\[3\] and are identical for the RS and WS samples. We require the \( \pi^+ \) candidates to have a transverse momentum \( p_t^{LAB} > 0.12 \text{ GeV}/c \), where \( LAB \) indicates the laboratory frame. We reject electrons that mimic \( \pi^+ \) using \( dE/\text{d}x \) measurements. We use kinematic selection criteria to eliminate electrons from pair conversions. The energies of photon candidates used to form \( \pi^0 \) candidates are required to be greater than 0.1 GeV; the invariant mass of photon pairs forming a \( \pi^0 \) must be in the range \( 0.09 < m_{\pi^+\pi^-} < 0.16 \text{ GeV}/c^2 \). We require the \( \pi^0 \) momentum \( p_t^{LAB} \) to be greater than 0.35 GeV/c. The reconstructed invariant mass for the \( D^0 \) candidates must have \( 1.74 < m_{K\pi\pi^0} < 1.98 \text{ GeV}/c^2 \). The \( \pi^0 \) and \( D^0 \) masses are then set equal to their nominal values \[3\] and the \( D^+ \) is refitted \[10\] with the constraint that its production point lies within the beam spot region. The \( D^{+} \) invariant mass and \( D^0 \) measured decay time \( t_{K\pi\pi^0} \) are derived from this fit. We require \( 0.139 < \Delta m < 0.155 \text{ GeV}/c^2 \) from \( \Delta m \equiv m_{K\pi^0\pi^-} - m_{K\pi^0} \). To reject \( D^* \) candidates from \( B \) decays, we require the \( D^0 \) center-of-mass momentum to be greater than 2.4 GeV/c. For events that contain multiple \( D^* \) candidates with shared tracks, the candidate that yields the largest fit probability for the decay chain is retained. The three-dimensional flight path is used to determine \( t_{K\pi^0\pi^-} \) and its uncertainty \( \sigma_t \). For signal events, the typical value of \( \sigma_t \) is 0.23 ps; we accept \( D^* \) candidates with \( \sigma_t < 0.50 \) ps.

We extract the signal and background yields from a binned extended maximum likelihood fit to the \( m_{K\pi^0\pi^-} \) and \( \Delta m \) distributions (Fig.\[3\]-d). For subsequent analysis, we retain \( D^* \) candidates in the signal region defined as \( 0.1449 < \Delta m < 0.1459 \text{ GeV}/c^2 \) and \( 1.8495 < m_{K\pi^0} < 1.8795 \text{ GeV}/c^2 \). Our final RS (WS) sample is composed of 658,986 (3009) events with a purity of 99\% (50\%). The efficiency of the signal region selection is 64.6\%.

![Dalitz Plots](image)

**FIG. 1:** Dalitz plots for the (a) RS and (b) WS \( D^0 \) samples. The reconstructed (c) \( D^0 \) mass and (d) \( \Delta m \) distributions for the WS sample requiring respectively (c) \( 0.1449 < \Delta m < 0.1459 \text{ GeV}/c^2 \) and (d) \( 1.8495 < m_{K\pi^0} < 1.8795 \text{ GeV}/c^2 \). The fit results used to extract the yields are shown by the superimposed curves. The light histogram represents the mistag background, while the dark histogram shows the combinatoric background.

The RS sample is used to determine the CF isobar model parameters \( a_{j,CF}^{\pi^0} \) and \( \alpha_{j,CF}^{\pi^0} \), as well as the decay time resolution function, which is parametrized as a sum of three Gaussian functions with a common mean, with widths given by the per-event \( \sigma_t \) times a different scale factor for each Gaussian. The reconstructed RS \( D^0 \) signal decay time distribution (Fig.\[3\]a) is described by a probability density function (PDF) consisting of an exponential function convolved with the resolution function. The resolution function parameters and \( D^0 \) lifetime are determined in an unbinned maximum likelihood fit. The mean value of the resolution function is found to be \( 4.2 \pm 0.7 \) fs. This value is consistent with the magnitude expected from instrumental effects, and the associated systematic uncertainty is determined by setting the value to zero. As a cross-check we determine the \( D^0 \) mean lifetime to be \( 409.9 \pm 0.8 \) (stat. only) fs, in agreement with the world average \( 410.1 \pm 1.5 \) (stat. + syst.) fs \[9\].

The \( D^0 \) candidates in the WS signal region can be divided into three categories: signal events, combinatorial background, and incorrectly tagged RS events (mistag), each one described by its own PDF, whose parameters are determined in an unbinned maximum likelihood fit.
During the fit procedure, the number of events in each category is fixed to the value obtained from the fit to the $m_{K\pi\nu}$ and $\Delta m$ distributions.

The PDF describing the WS time-dependent Dalitz plot is given by Eq. (1) convolved with the $t_{K\pi\nu}$ resolution function. The $\sigma_1$ PDFs for signal and background are taken from the RS data. The DCS amplitudes and phases for each resonance and the mixing parameters are determined in the fit. The CF Dalitz plot amplitudes arising from mixing are taken from the fit to the RS sample described in the previous paragraph. The mistag events are parametrized using an empirical PDF obtained from the RS data, since mistag events contain correctly reconstructed RS $D^0$ decays. The PDF describing the combinatorial background is constructed by averaging the $(s_{12}, s_{13}, t_{K\pi\nu})$ distributions obtained from the WS $m_{K\pi\nu}$ sidebands: this accounts for correlations between those three variables that might be present in the data.

The results of the time-dependent fit of the WS data, the $a_{j}^{DCS}$, $\delta_{j}^{DCS}$ and fit fractions $f_j$ (2), are given in Table I. The fit fraction of the non-resonant contribution to the $K^-\pi^+$ S-wave is absorbed into the $K_0^{*-}(1430)$ fit function. Projections of the fit results are shown in Fig. 2b-d. The change in log likelihood ($-2\Delta \ln L$) between the fit with mixing and with no mixing ($x_{K\pi\nu}/r_0 = y_{K\pi\nu}/r_0 = 0$) is 13.5 units, including systematic uncertainties. For two degrees of freedom, the confidence level that the result is due to no-mixing is 0.1%. The significance of the mixing result is equivalent to 3.2 standard deviations, and thus constitutes evidence for $D^0$-$\bar{D}^0$ mixing.

<table>
<thead>
<tr>
<th>Resonance</th>
<th>$a_j^{DCS}$</th>
<th>$\delta_j^{DCS}$ (degrees)</th>
<th>$f_j$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho^{0}(770)$</td>
<td>1 (fixed)</td>
<td>0 (fixed)</td>
<td>39.8 $\pm$ 6.5</td>
</tr>
<tr>
<td>$K_2^{*-}(1430)$</td>
<td>0.088 $\pm$ 0.017</td>
<td>$-17.2 \pm 12.9$</td>
<td>2.0 $\pm$ 0.7</td>
</tr>
<tr>
<td>$K_0^{*-}(1430)$</td>
<td>6.78 $\pm$ 1.00</td>
<td>69.1 $\pm$ 10.9</td>
<td>13.1 $\pm$ 3.3</td>
</tr>
<tr>
<td>$K_0^{*}(892)$</td>
<td>0.899 $\pm$ 0.005</td>
<td>$-171.0 \pm 5.9$</td>
<td>35.6 $\pm$ 5.5</td>
</tr>
<tr>
<td>$K_0^{0}(1430)$</td>
<td>1.65 $\pm$ 0.59</td>
<td>$-44.4 \pm 18.5$</td>
<td>2.8 $\pm$ 1.5</td>
</tr>
<tr>
<td>$K_0^{*0}(892)$</td>
<td>0.398 $\pm$ 0.038</td>
<td>$-24.1 \pm 9.8$</td>
<td>6.5 $\pm$ 1.4</td>
</tr>
<tr>
<td>$\rho^{0}(1700)$</td>
<td>5.4 $\pm$ 1.6</td>
<td>157.4 $\pm$ 20.3</td>
<td>2.0 $\pm$ 1.1</td>
</tr>
</tbody>
</table>

Using a Bayesian approach, by integrating the likelihood function with respect to $x_{K\pi\nu}$ and $y_{K\pi\nu}$, assuming a flat prior distribution, we obtain $x_{K\pi\nu} = (2.61^{+0.57}_{-0.67}$ (stat.) $\pm 0.39$ (syst.))% and $y_{K\pi\nu} = (0.06^{+0.65}_{-0.64}$ (stat.) $\pm 0.34$ (syst.))% with a correlation of $-0.75$.

![Figure 2](image-url)

**FIG. 2:** (a) Proper time distribution for RS events with the fit result superimposed. The distribution of background events is shown by the shaded histogram. (b) Proper time distribution for WS events. (c, d) $m_{K\pi\nu}$ and $m_{K\pi\nu}$ projections with superimposed fit results (line). The light histogram represents the mistag background, while the dark histogram shows the combinatoric background.

Extensive validation of this fitting procedure is performed using Monte Carlo (MC) experiments based on the PDF shapes and DCS amplitudes extracted from data. The validation studies are performed over a wide range of mixing parameters. These studies demonstrate that the fit correctly determines the mixing parameters to within a small offset of 0.243.σ, where σ is the statistical uncertainty. These small biases are a consequence of the relatively small size of our data sample and become negligible if MC samples with higher statistics are used. We correct the final result for this offset.

Sources of systematic uncertainty for $x_{K\pi\nu}/r_0 = y_{K\pi\nu}/r_0$, related to the choice of the isobar model and the experimental assumptions, are considered. For each effect we refit the data with an alternative assumption and extract the overall correlated uncertainty for the fitted parameters. We estimate the Dalitz model uncertainties [0.38σ (0.35σ)], where σ is the statistical uncertainty, by varying the mass and the width of each resonance within their error and by using alternative parametrizations for the isobar components $A_j$ in the fit: the largest error arises from uncertainties in the $K^*$ and $\rho$ parameters and from
uncertainties in the parametrization of the $K - \pi$ S-wave. Systematic uncertainties related to the number of signal and background events [0.15σ (0.22σ)] are evaluated by varying them according to their statistical uncertainties. Similarly, the definition of the signal region, the $\sigma_i$ requirement, and the selection of the best $D^*$ candidate are varied. The effect on the mixing parameters is $0.50\sigma$ (0.37σ). Variations in efficiency across the Dalitz plot contribute systematic uncertainties of $0.09\sigma$ (0.10σ).

The $I_{K\pi\pi0}$ resolution function parameters are varied within their errors. The offset is also set to zero. The systematic effect is $0.11\sigma$ (0.09σ). The total systematic error on $x'_{K\pi\pi0}/r_0$, $y'_{K\pi\pi0}/r_0$, is $0.57\sigma$ (0.66σ).

The same procedure is applied separately to the WS $D^0$-tagged (+) and $\bar{D}^0$-tagged (−) events to search for $CP$ violation in mixing. We find $x^+_{K\pi\pi0} = (2.33^{+0.64}_{-0.90} \pm 0.39)\%$, $y^+_{K\pi\pi0}$ = ($-0.05^{+0.63}_{-0.67} \pm 0.50$)%, $x^-_{K\pi\pi0} = (3.55^{+0.83}_{-0.83} \pm 0.65)$% and $y^-_{K\pi\pi0} = (-0.54^{+0.10}_{-0.10} \pm 0.41)$%, respectively, and thus observe no evidence for $CP$ violation. The correlation between $x^+_{K\pi\pi0}$ ($x^-_{K\pi\pi0}$) and $y^+_{K\pi\pi0}$ ($y^-_{K\pi\pi0}$) is $-0.69 (-0.66)$.

In conclusion, our data are inconsistent with the no-mixing hypothesis with a significance of 3.2 standard deviations. Our results thus constitute evidence for mixing. For the mixing parameters we find $x^+_{K\pi\pi0} = (2.61^{+0.57}_{-0.68} \pm 0.39)$% and $y^+_{K\pi\pi0} = (-0.06^{+0.55}_{-0.64} \pm 0.34)$% with a correlation of $-0.75$. These values are consistent with our previous result [3] and with some SM estimates for mixing. No evidence for $CP$ violation is found.

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