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LOWER BOUNDS ON SELF-FOCUSING SO AS TO MAINTAIN RING INTEGRITY NEAR THE INITIATION OF ACCELERATION IN AN ELECTRON RING ACCELERATOR*

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ABSTRACT

Relationships necessary for ring stability are derived between the self-focusing forces of an electron ring and the magnetic field gradient defocusing forces present near and just subsequent to the start of ring acceleration.

1. INTRODUCTION

It is well known that if an electron ring is accelerated too quickly it will leave behind ions, since they are too massive to keep up with the electrons[1]. If the ions are supplying the ring self-focusing[2] the ring will consequently lose integrity. Thus there are upper limits on the magnitudes of the axial electric field, E_z, or the radial magnetic field, B_r, which accelerate the ring. Below the limits, ring stability is maintained and also ion acceleration is accomplished.

Often ring self-focusing is predominantly supplied by images[3]. The above-mentioned restriction on E_z or B_r is then no longer necessary for maintaining ring integrity (although still vital for ion acceleration). There are even in this case, however, restrictions on B_r or E_z that must be satisfied in order to have ring axial integrity. These restrictions must be satisfied no matter what the source of the self-focusing.

The limits on the accelerating forces acting on the ring during the transition from the magnetic potential well, where the ring is formed and loaded with ions, to the region where the ring is subject to the main accelerating force, requires particular attention. This transition is obtained, at least in all the schemes considered up to now, by decreasing the depth of the potential well and at the same time introducing an axially varying radial magnetic field B_r. Prior to, and right up to, the start of ring axial acceleration with time-independent external fields (spillout) the ring is subject to the field, B_r, which creates nonelastic forces on electrons. These forces, unless counteracted by adequately large self-focusing forces, will pull the ring apart in the axial (z) direction.

Electrons in the ring have a spread in energy, and hence in equilibrium radii. Thus, because of the radial variation of B_r, there is a force tending to tear the ring apart.

In summary, for given ring parameters, there is an upper bound (most stringent at the spillout point) on $(\partial^2 B_r/\partial z^2)$ and on $(\partial B_r/\partial r)$ for maintaining ring integrity up to, and at, spillout.

Subsequent to spill also, energy spread in the ring combines with B_r and $\partial B_r/\partial r$ to tend to pull the ring apart axially. At the same time, the unfavorable sign of $\partial B_r/\partial z$ (just subsequent to spill) also has a defocusing effect. Once again there are limits that must be observed, for given ring parameters, in order to maintain ring integrity.

In this paper we examine a very simple model and obtain rough estimates relating the ring self-focusing[3)], Q_s^2, to ring parameters, to B_r, and to the B_r derivatives. We obtain a critical lower limit, Q^2_{crit}, on Q_s^2.

For parameters[4)] characteristic of the Lawrence Radiation Laboratory Compressor III we find that Q^2_{crit} is sufficiently small that Q_s^2 can be larger than Q^2_{crit}, but still small enough that--with the aid of the image cylinder--operation is possible with the incoherent tune, Q_R, less than unity. This conclusion is valid for a ring of small minor radius (of the order of 0.5 cm or less). On the other hand, Q^2_{crit} varies with the ring minor radius, so that if the minor radius is 2.0 cm (perhaps the situation if there is a resonance crossing during compression) then Q^2_{crit} is excessively large, and ring integrity will be lost during spillout.

The general analysis is presented in Sections 2, 3, and 4 of this paper, with the Appendix supplying details of the postspill analysis. Section 5 is devoted to a numerical example employing the parameters of the LRL Compressor III. The final section (Section 6) contains three general remarks.

2. ANALYSIS FOR A MONOCHROMATIC RING IN THE PRESPILL PHASE

Typical curves showing B_r vs z (at a fixed radius) in the neighborhood of the spill point are shown in Fig. 1[4]. We approximate B_r by the form

$$B_r(z) = \frac{\partial B_r}{\partial z}(z_e)\,(z - z_e) + \frac{1}{2}\,\frac{\partial^2 B_r}{\partial z^2}(z_e)\,(z - z_e)^2 \ . \qquad (1)$$

The z motion (with azimuthal angle θ as an independent variable) is governed by the potential function

$$V = \frac{1}{2}\,Q_s^{\,2}\,\xi^2 - \frac{eR^2}{m_o\gamma c}\left[\frac{\partial B_r}{\partial z}(z_e)\,\frac{\xi^2}{2} + \frac{\partial^2 B_r}{\partial z^2}(z_e)\,\frac{\xi^3}{6}\right] \ , \qquad (2)$$

where $\xi = z - z_e$ is the amplitude of an electron in its motion about the equilibrium position z_e, R is the equilibrium radius of the beam, which is related to $B_z(z_e)$ by

$$R = \frac{m_o\gamma c}{eB_z(z_e)} \ , \qquad (3)$$

and γ is the ratio of an electron energy to its rest mass $m_o c^2$. The quantity $Q_s^{\,2}$ is the ring self-focusing, which will have contributions (negative) from curvature effects, from image terms (positive, one hopes), and from ions (positive).

The potential of Eq. (2) may be written in the form

$$V = \frac{1}{2} Q^2 \xi^2 - \frac{R}{6}\left[\frac{\frac{\partial^2 B_r}{\partial z^2}(z_e)}{B_z(z_e)}\right] \xi^3 , \tag{4}$$

which is plotted in Fig. 2. From the figure it is clear that the ring minor radius a must be less than ξ_{max} for stability. Thus we have the stability criterion

$$a < \xi_{max} = \frac{2Q^2}{R}\left[\frac{B_z(z_e)}{\frac{\partial^2 B_r}{\partial z^2}(z_e)}\right] . \tag{5}$$

Actually the requirement is that there be adequate stable phase volume to contain the ring. This requirement is (roughly) a condition on $Q_s a^2$; we assume, in this analysis, that a has been chosen so as to satisfy the phase-volume condition. Thus Eq. (5) is to be considered as a condition on ξ_{max}, for given a.

At the spill point $(\partial B_r/\partial z)$ is zero, and Q^2 takes its smallest value of the prespill phase, namely, Q_s^2. Thus Eq. (5) is most stringent when evaluated at spill, i.e., when $z_e = z_{sp}$:

$$Q_s^2 > \frac{Ra}{2} \frac{\frac{\partial^2 B_r}{\partial z^2}(z_{sp})}{B_z(z_{sp})} . \tag{6}$$

3. EFFECT OF ENERGY SPREAD IN THE PRESPILL PHASE

Because of energy spread in the ring, particles have a spread in equilibrium radii. Since B_r varies with r, particles of different energy feel different forces, which effect also tends to cause axial spreading of the ring. It may be taken into account by augmenting Eq. (2) with a term

$$-\frac{R^2}{B_z(z_e)}\left[\frac{\partial B_r}{\partial r}(z_e)\right]\left(\frac{\Delta E}{E}\right)\xi\ , \tag{7}$$

where $(\Delta E/E)$ is the energy spread in the ring.

The criterion of Eq. (5) now becomes

$$a < \frac{\xi_{max}}{2} + \left\{\left(\frac{\xi_{max}}{2}\right)^2 - \frac{2R\left[\frac{\partial B_r}{\partial r}(z_e)\right]}{\left[\frac{\partial^2 B_r}{\partial z^2}(z_e)\right]}\left(\frac{\Delta E}{E}\right)\right\}^{\frac{1}{2}}\ , \tag{8}$$

where ξ_{max} is given by Eq. (5) [and is clearly the maximum of the potential when $(\Delta E/E) = 0$]. The condition $\xi_{max} > a$ is now replaced by

$$\xi_{max} > a + 2\left(\frac{R}{a}\right)\left(\frac{\Delta E}{E}\right)\left|\frac{\frac{\partial B_r}{\partial r}(z_e)}{\frac{\partial^2 B_r}{\partial z^2}(z_e)}\right|\ , \tag{9}$$

which may be transformed into the form [corresponding to Eq. (6)]

$$Q_s^{\,2} > \frac{Ra}{2}\left[\frac{\dfrac{\partial^2 B_r}{\partial z^2}(z_{sp})}{B_z(z_{sp})}\right] + \frac{R^2}{a}\left(\frac{\Delta E}{E}\right)\left|\frac{\dfrac{\partial B_r}{\partial r}(z_{sp})}{B_z(z_{sp})}\right| \quad . \tag{10}$$

4. POSTSPILL ANALYSIS

Dynamics of independent electrons is described by the principle of least action:

$$\delta \int (\underset{\sim}{p}_{mech} - \underset{\sim}{A}) \cdot d\underset{\sim}{s} = 0 \quad , \tag{11}$$

with the mechanical momentum measured in units of "magnetic rigidity." From Eq. (11) follow the equations of motion,

$$\frac{d}{d\theta}\left[\frac{pr'}{D}\right] - \frac{pr}{D} + r\,B_z = 0 \; ,$$

$$\frac{d}{d\theta}\left[\frac{pz'}{D}\right] - r\,B_r = 0 \; . \tag{12}$$

where p is the magnitude of the mechanical momentum, and

$$D = [r^2 + r'^2 + z'^2]^{\frac{1}{2}} \; , \tag{13}$$

and primes denote derivatives with respect to θ.

We wish to study motion of electrons in the neighborhood of a central--or reference--electron. For the reference particle we write

$$r = r_o(t) \; ,$$

$$z = z_o(t) \; . \tag{14}$$

For an arbitrary electron we write

$$r = r_o(t) + \eta(t) ,$$

$$z = z_o(t) + \xi(t) ,$$

$$p = p_o + \Delta p . \tag{15}$$

Inserting Eq. (15) into Eqs. (12), and keeping only first-order terms, we obtain (by steps detailed in the Appendix)

$$\frac{p_o r_o''}{r_o} - p_o + r_o \, B_z(r_o, z_o) = 0 , \tag{16}$$

$$\frac{p_o z_o''}{r_o} - r_o \, B_r(r_o, z_o) = 0 , \tag{17}$$

$$\eta'' + \eta = r_o \left(\frac{\Delta p}{p_o}\right) , \tag{18}$$

$$0 = \xi'' - \left[\frac{r_o^{\,2}}{p_o} \frac{\partial B_r}{\partial z}(r_o, z_o)\right] \xi - \left[\frac{2 r_o}{p_o} B_r(r_o, z_o) + \frac{r_o^{\,2}}{p_o} \frac{\partial B_r}{\partial r}(r_o, z_o)\right] \left(\frac{\Delta p}{p_o}\right) r_o . \tag{19}$$

Equations (16) and (17) determine the reference trajectory, whereas Eqs. (18) and (19) describe electron motion relative to the reference particle.

It suffices, for evaluation of the coefficients in the equations for ξ, to use the approximate solution of Eq. (18), namely,

$$p_o = R\ B_z(R, z_o) \quad , \tag{20}$$

where we have identified r_o as the ring radius R. Furthermore, we must augment Eq. (19) with the self-focusing terms $Q_s^{\,2}\xi$.

The coefficients in Eq. (19) are, of course, functions of θ. However, they are slowly varying functions of θ under the assumption that B_r and B_z vary slowly in space and β_z is small. Thus we approximately solve Eq. (19) by taking the coefficients as constants. The general solution is of the form

$$\xi = A\ e^{i\omega t} + B \ , \tag{21}$$

where B is proportional to $(\Delta p/p_o)$.

The eigenfrequency is, to first order, given by

$$\omega^2 = -\frac{R}{B_z}\frac{\partial B_r}{\partial z} + Q_s^{\,2} \ . \tag{22}$$

The nonoscillatory term is, to first order,

$$B = \frac{R\left(\dfrac{\Delta p}{p_o}\right)}{\omega^2}\left[\frac{2B_r}{B_z} + \frac{R}{B_z}\frac{\partial B_r}{\partial r}\right] \ . \tag{23}$$

Ring integrity in the z direction (there is no problem in the r direction) requires

$$\omega^2 > 0 \;,$$

$$B < a \;. \tag{24}$$

The condition on ω^2 is necessary to prevent ring explosion, whereas the condition on B is a self-consistency requirement. In summary, and expressing Eq. (24) as a condition on the self-focusing term $Q_s^{\,2}$, we have the conditions

$$Q_s^{\,2} > \frac{R}{B_z}\frac{\partial B_r}{\partial z} \tag{25}$$

and

$$Q_s^{\,2} > \frac{R}{a}\left(\frac{\Delta E}{E}\right)\left|\frac{2B_r}{B_z} + \frac{R}{B_z}\frac{\partial B_r}{\partial r}\right| + \frac{R}{B_z}\frac{\partial B_r}{\partial z} \;. \tag{26}$$

Clearly satisfying Eq. (26) is sufficient, since Eq. (25) is a less strong condition than Eq. (26).

5. A NUMERICAL EXAMPLE: THE LRL COMPRESSOR III

We adopt, for the purpose of demonstrating the significance of the requirements of Eqs. (10) and (26), the values characteristic of the LRL Compressor III[5]:

$$R = 3.2 \text{ cm}, \qquad \frac{\partial^2 B_r}{\partial z^2}(z_{sp}) \approx 3 \frac{G}{cm^2},$$

$$\frac{\Delta E}{E} = 2.0\%, \qquad \frac{\partial B_r}{\partial r}(z_{sp}) \approx 5 \frac{G}{cm},$$

$$B_z = 17 \text{ kG}, \qquad \frac{\partial B_r}{\partial B}(z_{sp}) \approx 2 \frac{G}{cm},$$

$$a = 0.5 \text{ cm}, \qquad B_r \approx 50 \text{ G}. \tag{27}$$

The radial field corresponds to a rather "poor" adjustment of operating conditions, such as might have been the case in the first experiments. One obtains

$$Q_s^2 > [1.4 + 1.2] \times 10^{-4} \qquad \text{[prespill condition of Eq. (10)]},$$

$$Q_s^2 > 3.9 \times 10^{-4} \qquad \text{[postspill stability of Eq. (25)]},$$

$$Q_s^2 > 1.3 \times 10^{-3} \qquad \text{[postspill self-consistency of Eq. (26)]}. \tag{28}$$

Self-focusing of this magnitude is available from the image cylinder and ion focusing. There is an ion loading percentage low enough to keep ν_R well below unity and large enough to satisfy Eq. (28), but it might be hard to achieve in practice. For a "good" adjustment of operating conditions the field derivatives are much smaller than the values used above (for example; $\partial^2 B_r/\partial z^2$ is only 1/25 as large, in one computational example, than the value in the "poor" case) and there exists a wider range of ion loading satisfying Eq. (28) and $\nu_R < 1$.

If, however, a is larger than 0.5 cm (such as might be the result of a blowup caused by excessive ion loading in poor vacuum conditions, causing a crossing of the incoherent $\nu_R = 1$ resonance), then neither images nor ions could supply the required values of Q_s^2. In this circumstance one would observe a diffuse spill ("peel-off") rather than a fast spill, as was, in fact, the case in the first experiments with Compressor III[4].

6. THREE REMARKS

Remark #1

It is interesting to inquire whether the postspill condition for focusing is necessary: Perhaps; even in $Q_s^2 \approx 0$, the rate of blowup is sufficiently small that the increase in ring size is tolerable for the short (≈ 50 cm) acceleration length of a typical model. A very good acceleration column has

$$\frac{\partial B_r}{\partial z} \approx 0.4 \text{ G/cm} , \tag{29}$$

with the ring covering (say) 24 cm in 90 nsec. In this case the uncompensated blowup e-folds by

$$\left[\frac{R}{B_z}\frac{\partial B_r}{\partial z}\right]^{\frac{1}{2}} \frac{ct}{R} \approx 7.3 , \tag{30}$$

which is clearly unacceptable; Condition ([illegible]) must be observed.

Remark #2

For a ring of rather good quality, ion self-focusing is very powerful, and adequate--by itself--to overcome curvature terms in Q_s^2. In this case one can contemplate operation in which no image cylinder is used (and hence $\nu_R = 1$ is crossed, but--perhaps--rapidly enough to be innocuous). Assuming the ion self-focusing to be much larger than the curvature effects, we may ignore the latter and write

$$Q_s^{\,2} \approx \frac{N_e R r_e f}{\pi \gamma a^2} , \tag{31}$$

where N_e is the number of electrons in the ring, r_e is the classical electron radius, γ is the ratio of the electron energy to its rest energy, and f is the fraction of electrical neutralization of the ring.

Inserting Eq. (31) into Eqs. (10) and (26), we obtain lower bounds on f:

$$f > \frac{\pi\gamma}{N_e r_e}\left[\frac{a^3}{2B_z}\,\frac{\partial^2 B_r}{\partial z^2} + \frac{Ra}{B_z}\left(\frac{\Delta E}{E}\right)\left|\frac{\partial B_r}{\partial r}\right|\right] , \tag{32}$$

$$f > \frac{\pi\gamma}{N_e r_e}\left\{\left(\frac{\Delta E}{E}\right) a \left|\frac{2B_r}{B_z} + \frac{R}{B_z}\,\frac{\partial B_r}{\partial r}\right| + \frac{a^2}{B_z}\,\frac{\partial B_r}{\partial z}\right\} . \tag{33}$$

It must be remembered that a necessary requirement for the validity of Eqs. (32) and (34) is that ion self-focusing dominates curvature effects. These last formulas are of interest in that the dependence upon ring parameters is explicit, in particular, the important dependence upon N_e and a.

Remark #3

It is amusing to relate the postspill condition of Eq. (33) in its dependence upon B_r to the condition for ring acceleration without the loss of ions. This last-mentioned condition is, for ions of mass M and ionization Ze,

$$B_r < \frac{N_e Ze}{\pi Ra}\left(\frac{m\gamma}{M}\right) . \qquad (34)$$

The B_r term of Eq. (33) (which actually is the numerically most significant term in the case of Compressor III) yields

$$B_r < \frac{N_e\ e\ f}{2\pi\ Ra\left(\frac{\Delta E}{E}\right)} . \qquad (35)$$

The condition of Eq. (35) will automatically be satisfied, provided the ion-acceleration condition of Eq. (34) is satisfied, if

$$\frac{f}{2\left(\frac{\Delta E}{E}\right)} > \frac{Zm\gamma}{M} . \qquad (36)$$

Since, for usually contemplated parameters, $f > 2\%$, $\Delta E/E \approx 2\%$, and $Zm\gamma/M \approx 1/100$, we see that Eq. (36) is satisfied: the left-hand side is at least 50 times as large as the right-hand side.

However, all this holds only for a strongly ion-self-focused ring. When it does not, then satisfying the ion acceleration condition of Eq. (34) does not guarantee satisfying the ring integrity conditions of Eq. (26).

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APPENDIX. DERIVATION OF SIMPLE EQUATIONS FOR POSTSPILL MOTION

In this appendix we derive Eqs. (16) through (19), from Eqs. (12), (13), (14), and (15). We employ the fact that $\frac{r'}{r_o}$, $\frac{z'}{r_o}$, $\frac{r''}{r_o}$, $\frac{z''}{r_o}$, $\frac{\xi}{r_o}$, and $\frac{\Delta p}{p_o}$ are small quantities.

Thus we expand Eq. (12), keeping only terms through second order. It is necessary to keep second-order terms because the relative motion in the z direction (described by ξ) is only weakly defocusing and is described (to lowest order) by second-order terms. In more detail, it can be seen in the answers [Eqs. (16)-(19)] that in zero order (B_z constant, $B_r = 0$) $r_o'' = z_o'' = 0$. The particles oscillate (strongly) in the r direction about a uniformly moving ring of constant radius. In first order (B_z slowly changing, $B_r/B_z \ll 1$) the reference particle accelerates slowly, and particles oscillate in the r direction but $\xi'' = 0$. Only in second order does the ξ equation describe ξ oscillations.

To second order, Eqs. (12) become:

$$\frac{pr''}{r} - \frac{1}{2}\, p\, \frac{r'^2}{r^2} + \frac{1}{2}\, p\, \frac{z'^2}{r^2} + r\, B_z = p \;,$$

$$\frac{pz''}{r} - p\, \frac{r'z'}{r^2} - r\, B_r = 0 \;. \tag{37}$$

Introducing Eqs. (14) and (15), and then isolating the reference particle, we obtain for it

$$\frac{p_o r''_o}{r_o} - \frac{1}{2} p_o \frac{r'^2_o}{r_o^2} + \frac{1}{2} p \frac{z'^2_o}{r_o^2} + r_o B_{zo} = p_o ,$$

$$p_o \frac{z''_o}{r_o} - p_o \frac{r'_o z'_o}{r_o^2} - r_o B_{ro} = 0 . \qquad (38)$$

Neglecting terms of second order in these first-order equation yields Eqs. (16) and (17) of the text.

From Eqs. (37) we obtain linear equations in η and ξ, namely:

$$\frac{p_o \eta''}{r_o} + B_{zo}\eta + \left[r_o \frac{\partial B_r}{\partial r} - p_o \frac{r''_o}{r_o^2}\right]\eta - \frac{p_o}{r_o^2}\left\{r'_o \eta' - z'_o \xi'\right\} + \xi r_o \frac{\partial B_z}{\partial z} = \Delta p ,$$

$$\frac{p_o \xi''}{r_o} - r_o \frac{\partial B_r}{\partial z}\xi - \left[2B_{ro} + r_o \frac{\partial B_r}{\partial r}\right]\eta - \frac{p_o}{r_o^2}\left[r'_o \xi' + z'_o \eta'\right] = 0 .$$

(39)

In the equation for η there is a first-order focusing term, so we may neglect second-order terms. In the ξ equation we may neglect fast oscillating η terms and replace η with $(r_o \Delta p/p_o)$. We obtain

$$\eta'' + \eta - \left[\frac{r_o'\eta'}{r_o} - \frac{z_o'\xi'}{r_o}\right] = r_o \frac{\Delta p}{p_o} ,$$

$$\xi'' - \frac{r_o^2}{p_o}\frac{\partial B_r}{\partial z}\xi - \left[2\frac{r_o}{p_o}B_{ro} + \frac{r_o^2}{p_o}\frac{\partial B_r}{\partial r}\right]\left(\frac{\Delta p}{p_o}\right)r_o$$

$$- \left[\frac{r_o'\xi'}{r_o} + \frac{z_o'\eta'}{r_o}\right] = 0 . \tag{40}$$

We have carefully retained second-order terms involving η' and ξ', since they produce antidamping. However, they are negligible; they simply describe the well-known increase in beam major and minor radii during expansion acceleration--a small effect in the early expansion phase. Dropping these terms, we obtain Eqs. (18) and (19) of the text.

REFERENCES

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1. V. I. Veksler et al., in Proceedings of the Sixth International Conference on Accelerators, Cambridge, Massachusetts, September 1967, Cambridge Electron Accelerator Laboratory Report CEAL-2000, p. 289.

2. See, for example, A. M. Sessler, in Symposium on Electron Ring Accelerators, Berkeley, California, February 1968, Lawrence Radiation Laboratory Report UCRL-18103, Feb. 1968, p. 25.

3. L. J. Laslett, On the Focussing Effects Arising From the Self-Fields of a Toroidal Beam, Lawrence Radiation Laboratory Internal Report ERAN-30, 1969; W. A. Perkins, Ion Focusing and Image Focusing During Rollout and Spillout, Lawrence Radiation Laboratory Internal Report ERAN-32, June 1969.

4. D. Keefe, ERA Development at Berkeley (Lawrence Radiation Laboratory Report UCRL-18896, Sept. 1969), submitted for publication in the Proceedings of the Seventh International Conference on High Energy Accelerators.

5. See Reference 4; also private communication from W. Perkins and A. Garren (Lawrence Radiation Laboratory, Berkeley, California).

FIGURE CAPTIONS

Fig. 1. Radial field, B_r, as a function of z, for times near the spill time t_3. The curve corresponding to t_2 is used to define z_e--the point where $B_r = 0$ and $\partial B_r/\partial z < 0$. Spillout is close to z_{sp}.

Fig. 2. Potential V as a function of amplitude ξ.

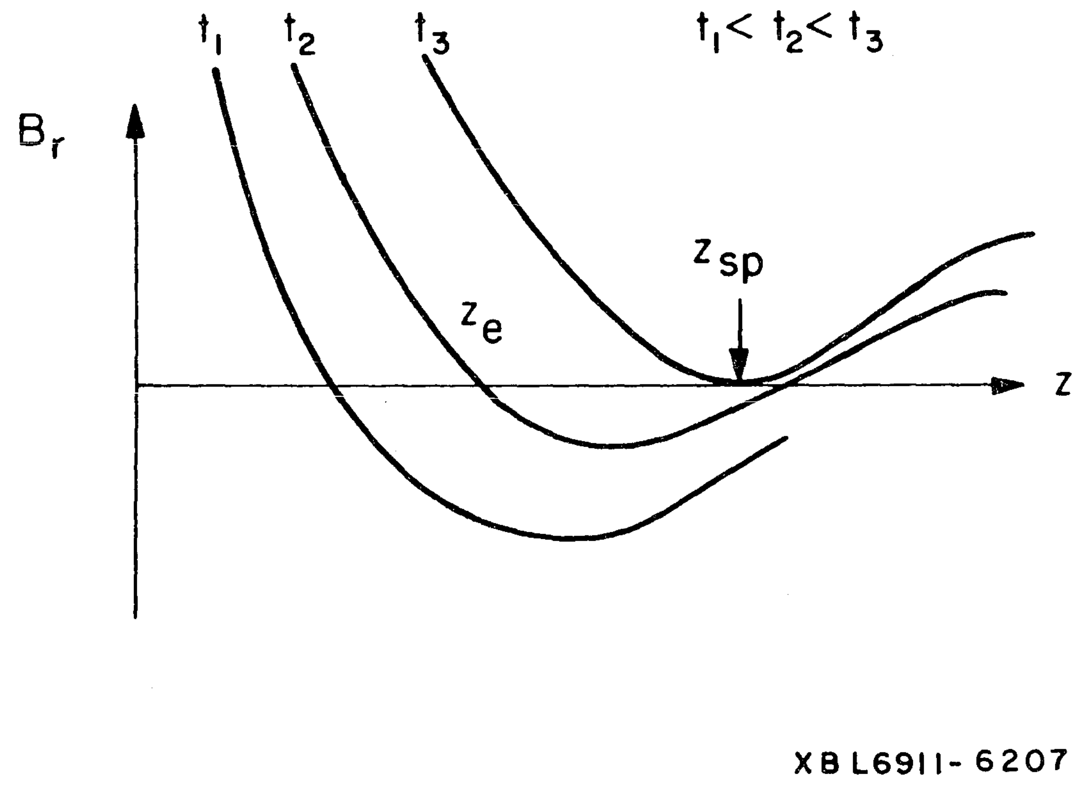

Fig. 1

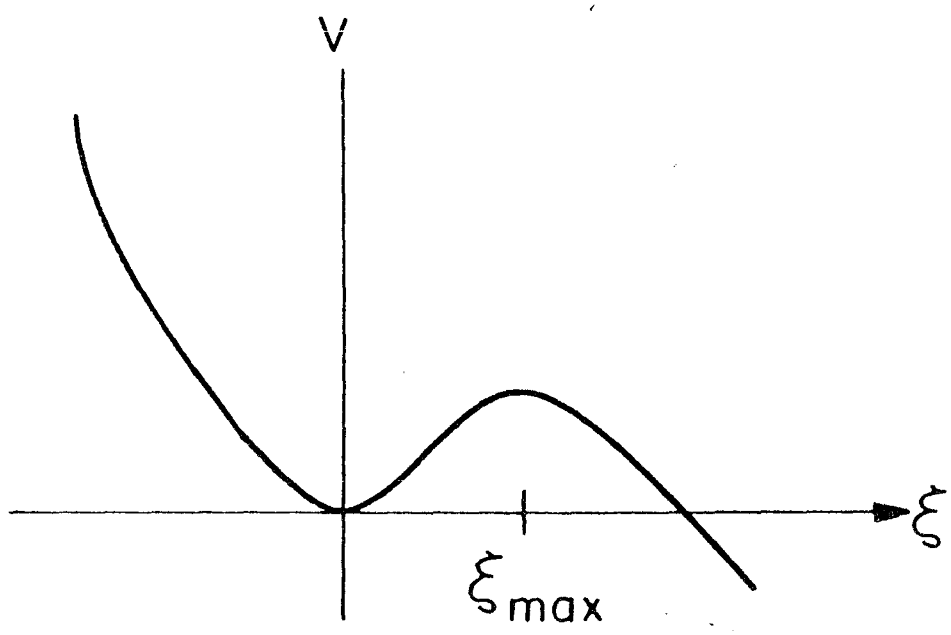

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Fig. 2