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ABSTRACT

Models are developed and solved to describe the flow of and deposition from low and high concentration turbidity currents. The shallow water equations are amended to include particle transport to describe the low concentration turbidity currents. The suspension balance model is used to describe the high concentration turbidity currents. Numerical simulations are developed to solve the highly non-linear, free boundary problems associated with these models. Simpler, algebraic scaling relationships are also developed for these models.

The models are successfully validated against field observations of turbidites. With these models, one can take seismic information on the shape of the turbiditic deposit and estimate the particle size, which can be used to determine the porosity and permeability.
EXECUTIVE SUMMARY

Turbidites are formed by the deposition of sand and clay particles from a turbidity current, which is a subsurface, suspension-laden flow driven by the density difference between the current and ambient fluid. The fluid mechanics of this buoyancy-driven, multiphase flow are now fairly well-understood. We developed a computational simulation of the process of deposition as function of the initial volume of the suspension or its flow rate, its initial sediment load, and the local topography (all which will be inferred from cores, logs and seismic data). From the predicted pattern of deposition and the distribution of particle sizes in the deposit, the spatial distribution of porosity and permeability at a centimeter length-scale can be determined. This information is invaluable for running other simulations of oil recovery to determine optimal methods for extraction of oil from the reservoir. Our proposed method is an excellent complement to geostatistical methods for spatial characterization of a reservoir because it includes additional information based on the physics of the formation of the reservoir.

Specifically, we have

- Developed first principle model equations for turbidity flows.
- Developed numerical methods to solve equations for the turbidity flows.
- Developed simulation of flow of and deposition from turbidity flows over arbitrary and dynamical changing topography for multiple particle sizes and multiple events.
- Developed method to convert depositional data into petrophysical data (e.g., porosity, permeability)
1.0 INTRODUCTION

The porous rock and sand of many reservoirs, such as those in the deep waters of the Gulf of Mexico and off the coast of west Africa, are turbidites derived from the deposition of sediment from turbidity or debris flows. These flows are driven by the bulk density difference or buoyancy due to the suspension of dense particles in the less dense ambient fluid. The suspensions are created by submarine landslides, which suspend sediment, and when the dense suspension reaches the basal plain, it spreads like a turbidity current. As this turbidity current propagates, it deposits its sediment on the basal plain, and this deposit may become reservoir sand or rock.

We developed first-principles, process-based models for the formation of turbiditic oil reservoirs to apply these models with core and seismic data using inverse algorithms to characterize the entire porosity and permeability distribution in these oil reservoirs. This paradigm for reservoir characterization will complement geostatistics by taking advantage of our knowledge and understanding of the processes that create turbidites.

We created simulations based on mathematical models of the deterministic transport processes for the dynamics of dilute turbidity flows and concentrated debris flows. With these simulations one can predict the stratigraphy and facies of a reservoir composed of one or many depositional events and, by various means, convert these results to porosity and permeability distributions. Of course initial and boundary conditions are necessary in order to run the simulations. These can be determined by parameter estimation techniques and methods of inversion using the available seismic and core data, which ensures that the results are consistent with known properties of the reservoir. (Well data are honored.)

This method of characterization using deterministic models is valuable and advantageous for several reasons:

• Fundamental physics is the foundation of this method for determining the stratigraphy, facies porosity and permeability of a reservoir, unlike geostatistics, which is a purely a statistical extrapolation based on available data. Thus, additional constraints in the form of physical laws are incorporated into the proposed characterization method, which complements geostatistics.

• Knowledge of the stratigraphy, facies, porosity and permeability throughout the reservoir is particularly critical for the evaluation of a new reservoir in order to decide whether or not to make the large capital investment to develop the field. The ability to accurately characterize a reservoir and thus more accurately evaluate its profitability is clearly a key economic advantage.

The need for improved characterization of turbiditic oil reservoirs is timely. The valuable reservoirs currently being explored and developed in the deep Gulf of Mexico are turbidites. Such deposits, constituting the most active hydrocarbon play in the U.S., are light oil or gas reservoirs found at great depth and in relatively unexplored regions. Economic viability of these reservoirs depends strongly on the production rate; therefore, the ability to estimate the high-quality locations in the reservoir is vital.
We developed three types of forward, process models for the emplacement of sediment by turbidity currents. They are the following:

1) Fluid flow and particle transport equations based on the shallow water equations. This model applies to turbidity currents with low concentrations of suspended sediment. It was developed to simulate the flow and resulting depositional pattern of the sediment over an erodible topography in three dimensions. The model was applied to simulate the formation of a known turbiditic reservoir in the deep Gulf of Mexico.

2) Fluid flow and particle transport equations based on the suspension balance model. This model applies to very concentrated turbidity currents or debris flows. It was developed to simulate the flow and deposition for two-dimensional flows.

3) Scaling relationships among the important physical parameters characterizing the flow of and deposition from turbidity currents. With these algebraic scaling relationships, estimates of the size and shape of depositional patterns can be made from initial conditions for a wide variety of two- and three-dimensional turbidity flows.

More information on these models are discussed below. Detailed information on the models can be found in the Ph.D thesis of L. Srivatsan (2004) and the published paper Srivatsan, Lake and Bonnecaze (2004).

2.0 EXPERIMENTAL – MODEL FOR DILUTE TURBIDITY CURRENTS

The basis for the model of the flow and deposition of a dilute turbidity current is the shallow water equations (see Bonnecaze et al. 1993, 1995). To these we amend conservation equations for the particles in the current and the erodible bed. These resulting equations are:

\[
\frac{\partial h}{\partial t} + \nabla \cdot (uh) = E |u|,
\]

\[
\frac{\partial u h}{\partial t} + \nabla \cdot (uuh) + \frac{1}{2} g_s \nabla \left( \phi h^2 \right) = g_s \sin \theta \phi h \nabla \eta - \frac{1}{2} C_f |u| u,
\]

\[
\frac{\partial \phi_i}{\partial t} + \nabla \cdot (uh \phi_i) = -v_i \phi + E_r v_i,
\]

where \( h \) is the height of the current, \( \eta \) is the topography relative to a constant datum, \( u \) is the velocity vector parallel to the surface, \( \phi \) is the total volume fraction of suspended particles, \( \phi_i \) is the volume fraction of the \( i \)th species. The initial reduced gravity \( g_0 = \left( \rho_p - \rho_s \right) g \phi_0 / \rho_s \), where \( \phi_0 \) is the initial concentration of the turbidity current. \( E_r \) is the erosion rate parameter. The erosion rate is defined as, \( E_r = a \left( Z + Z \right) / \left( 1 + a Z / 0.3 \right) \), where \( a = 1.3 \times 10^{-7} \) is a constant and \( Z = \alpha_z (u/v_s) \). The particle Reynolds number is defined in terms of the particle diameter \( d \) as \( R_p = \sqrt{Rg d^3} / v_s \), \( v \) being the kinematic viscosity of the fluid. The parameters \( (\alpha_1, \alpha_2) \) are assigned the value \( (1, 0.6) \) for \( R_p > 2.36 \) and \( (0.586, 1.23) \) for \( R_p \leq 2.36 \).
The rate of increase of the bed thickness is given in terms of the bed load transport $q_s$ and net deposition of sediment. The evolution of a bed height $\eta$ with an average porosity $\phi$ is expressed as,

$$\left(1 - \phi \right) \frac{\partial \eta}{\partial t} + \frac{\partial q_s}{\partial x} = D,$$

where

$$q_s = \alpha \sqrt{Rgd} \left( \tau^* - \tau_c^* \right)^n,$$

$$\tau^* = \frac{f u^2}{Rgd},$$

and $\tau_c^*$ is the critical Shields stress, a measure of the threshold stress required for sediment motion. A useful correlation for estimating $\tau_c^*$ is given by Brownlie (1981) in terms of the particle Reynolds number as,

$$\tau_c^* = 0.22 R_p^{-0.6} + 0.06 \exp\left(-17.77 R_p^{-0.6}\right)$$

The value of $\alpha_s$ was chosen to be 10 as its value is usually greater than unity. Changing its value from 1 to 100 was found not to affect the result significantly. The value of the exponent $n$ in equation (9) varies from 1.5 to 2.5 and a value of 1.6 was chosen for all the runs.

2.1 RESULTS AND DISCUSSION 1 – MODEL FOR DILUTE TURBIDITY CURRENTS

The recreation of the GoM deposit in the Gulf of Mexico (data supplied by Mobil), is shown below in Figure 1. The base topography, which can be determined from seismic is approximated as a tilted plane. The recreated topography is reconstructed from 325 sedimentation events, and it agrees well with the true seismic image.
Given the particle size distribution and location throughout the deposit, the porosity and permeability can be determined. An example of this is shown in Figure 2,
3.0 EXPERIMENTAL – MODEL FOR CONCENTRATED TURBIDITY CURRENTS

The two-dimensional flow of turbidity currents resulting from the release of fixed volume of suspension through a channel is considered. The initial volume of suspension per unit channel width is $V$. The suspension consists of monodisperse particles of density $\rho_p$. The concentration of particles in the initial suspension is $\phi_{in}$. The suspending fluid as well as ambient fluid is water whose density and viscosity are $\rho_f$ and $\eta_f$. The flow occurs over a rigid, horizontal, non-erodible surface.

The flow is governed by the two-phase SBM equations. The suspension phase is characterized by velocity $\langle u \rangle$ and pressure $P$ (which is expressed in terms of fluid pressure $p_f$ and particle pressure $\Pi_p$). The particle phase is characterized in terms of velocity $\langle u_p \rangle$ and pressure $\Pi$. The scale factors used to non-dimensionalize the model equations are presented next. All length variables are scaled using $x_0 = V^{1/2}$. All velocity variables are scaled using $u_0 = (\rho_p - \rho_f) g \phi_{in} x_0^2 / \eta_f$. This scale factor is obtained by equating the magnitudes of buoyancy and pressure gradient terms of suspension phase momentum equations. The time scale is taken to be $t_0 = x_0 / u_0$. The pressure is scaled using $p_0 = \eta_f u_0 / x_0$. The concentration of particles $\phi$ is scaled using inlet concentration $\phi_{in}$. The non-dimensionalized differential equations governing the flow of viscous turbidity currents are given below:

$$\nabla \cdot \langle u \rangle = 0,$$
\[-\phi e_y - \nabla P + \nabla \cdot \langle \Sigma \rangle^R = 0,\]

\[\begin{align*}
P &= p_f + \frac{(1 + \lambda_2)}{2} \Pi, \\
\langle \Sigma \rangle^R &= \left( \frac{-(1 - \lambda_2)}{2} \Pi + 2\eta_s \frac{\partial \langle u \rangle}{\partial x} \right) + \eta_s \left( \frac{\partial \langle u \rangle}{\partial y} + \frac{\partial \langle v \rangle}{\partial x} \right) + \frac{(1 - \lambda_2)}{2} \Pi + 2\eta_s \frac{\partial \langle v \rangle}{\partial y} \\
\frac{\partial \phi}{\partial t} + \nabla \cdot (\phi \langle \mathbf{u} \rangle_p) &= 0 \\
-N_B \phi e_y + \frac{\phi}{f} (\langle u \rangle - \langle u \rangle_p) + N_B \nabla \cdot \langle \Sigma \rangle_p &= 0.
\end{align*}\]

\[\begin{align*}
\langle \Sigma \rangle_p &= \left( -\Pi + 2\eta_p \frac{\partial \langle u \rangle}{\partial x} \right) + \eta_p \left( \frac{\partial \langle u \rangle}{\partial y} + \frac{\partial \langle v \rangle}{\partial x} \right) \\
&\quad + \eta_p \left( \frac{\partial \langle u \rangle}{\partial y} + \frac{\partial \langle v \rangle}{\partial x} \right) -\lambda_2 \Pi + 2\eta_p \frac{\partial \langle v \rangle}{\partial y}
\end{align*}\]

Here \(\langle \Sigma \rangle^R\) represents reduced suspension stress. The particle pressure (\(\Pi\)), particle and suspension phase viscosities (\(\eta_p\) and \(\eta_s\)) and hindered settling function (\(f\)) depend on particle concentration.

The initial conditions for the simulation of turbidity currents resulting from the release of fixed-volume of suspension are given by

\[\langle \mathbf{u} \rangle = \langle \mathbf{u} \rangle_p = 0,\]

\[\phi = 1, \quad 0 \leq x \leq x_{in}, 0 \leq y \leq h_{in},\]

\[= 0, \quad \text{elsewhere}.\]

Here \(h_{in}\) and \(x_{in} = 1/h_{in}\) are the dimensions of initial area of the suspension. The boundary conditions are derived from well-established assumptions of fluid-mechanics, namely, no-slip and traction-free conditions (Gresho, 1991). They are given below:

\[\langle \mathbf{u} \rangle = \langle \mathbf{u} \rangle_p = 0, \quad \text{along bottom surface},\]

\[\phi = 0, \quad x = x_M,\]

\[= 0, \quad y = y_M.\]
\[ \langle \Sigma \rangle \cdot n = 0, \quad \text{along } \Gamma_N \]

In the boundary condition for \( \phi \) (11), \( x = x_M \) and \( y = y_M \) represent far right and top boundaries of the simulation domain. The Dirichlet boundary \( \Gamma_D \) consists of the bottom surface. The Neumann boundary \( \Gamma_N \) consists of the boundary not included in \( \Gamma_D \).

The dimensionless buoyancy number \( N_B \), which appears in the particle phase momentum equation (6), is given by the ratio of buoyancy to viscous velocity scales, i.e.

\[
N_B \equiv \frac{v_s}{u_0} = \frac{v_f u_s}{g' V}, \quad \text{where} \quad g' = \frac{\left( \rho_p - \rho_f \right) \phi_{in}}{\rho_f} g.
\]

Increase in buoyancy number signifies heavier or bigger particles. The mathematical description of an arbitrary turbidity current in the viscous regime is defined by two dimensionless parameters – (1) buoyancy number \( N_B \) and (2) concentration of particles at inlet \( \phi_{in} \).

### 3.1 RESULTS AND DISCUSSION 2 – MODEL FOR DILUTE TURBIDITY CURRENTS

The model differential equations are solved numerically using a combination of variable-grid finite element method (FEM), an improved projection algorithm and an FEM-based revised flux-corrected transport algorithm. A typical result is shown below in figure 2, which is a plot of the particle concentration throughout the flow. Note that the concentration of near 63% by volume correspond to deposits.

![Figure 3](image.png)

\( N_B = 1.0 \)

Figure 3. Particle concentration following the flow of a hyperpycnal, concentrated turbidity current. Red indicates the highest concentrations. The initial concentration is 10% particles and the buoyancy number is 1.0.
The concentrated flow model, which naturally includes erosion, shows a dearth of particles at the point of release (the origin) due to the high shear and a buildup downstream into a mound.

4.0 EXPERIMENTAL – SCALING RELATIONSHIPS FOR TURBIDITY CURRENTS

In addition to detailed simulations, often simpler algebraic estimates can be useful to describe a deposit. We developed a suite of estimations for a variety of depositional flows depending on the slope, particle concentration, entrainment and other parameters. These predictions were published in Srivatsan, Lake & Bonnecaze (2004). The predictions are based on an analysis of the equations presented in Section 2.0 of this report. Described below is one example of results and application of scaling analysis.

4.1 RESULTS AND DISCUSSION 3 – SCALING RELATIONSHIPS FOR TURBIDITY CURRENTS

Consider the flow of a constant volume turbidity current of volume V down a planar slope oriented at an angle $\theta$. Estimates of the downstream and cross-stream extents of the deposit, $x_0$ and $y_0$, respectively, and the thickness of the deposit $d_0$ are given by the expressions,

$$x_0 \sim \left( \frac{g_0 V^\frac{1}{3}}{v_e \cos^8 \theta} \right)^\frac{1}{9},$$

$$y_0 \sim \left( \frac{V \cos \theta}{\sin \theta} \right)^\frac{1}{3},$$

$$d_0 \sim \left( \frac{V^2 \phi_0 v_e^6 \cos^5 \theta}{(1-\varepsilon)^9 g_0^6 \sin^2 \theta} \right)^\frac{1}{9},$$

where $\varepsilon$ is the porosity of the deposited sediment. These expressions are valid in the limit of negligible entrainment and bottom friction, which are satisfied when

$$N_E = \frac{E \phi_0 x_0}{d_0} \ll 1,$$

$$N_F = \frac{C_f x_0 (1-\varepsilon)}{2 \phi_0 d_0} \ll 1,$$

where $N_E$ and $N_F$ are the entrainment and friction numbers for this system.

Hamlin (1999) measured the thickness and extents of the Ozona turbidites. From these measurements, the O(1) coefficients for the scaling relationships above were determined and it was found that
Here the slope angle is three degrees, the ε = 30% and the particle diameter is 100 μm, as verified experimentally. Predictions of the cross-stream extent and deposit thickness as a function of the downstream extent are plotted in Figure 3 along with the experimental observations. The agreement is very good. Similar comparisons between the theoretical predictions and the field observations are also good, as shown in Srivatsan et al. (2004).

CONCLUSIONS

We have developed simulations and scaling relationships that predict the extent and depth of deposits from turbidity currents flowing over a variety of topographies for a range of input parameters, including particle size, particle size distribution and flow rate history. With these relationships, one can convert seismic measurements of shape and size into information on the particle size and size distribution, which can be expressed in terms of porosity and permeability. This information provides a complementary means to geostatistics for characterizing turbiditic oil reservoirs.
REFERENCES


