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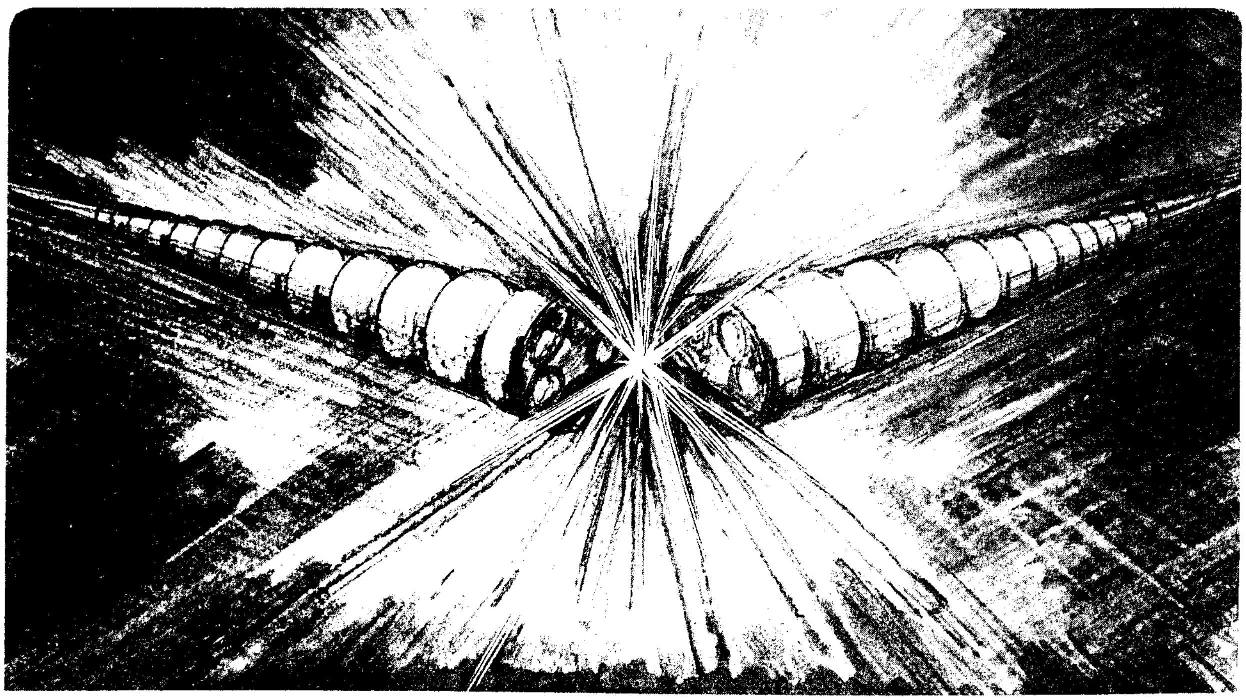

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The stability of empty resonators (or cold cavities) has been widely studied, and is well understood. Here we consider the stability of symmetric resonator systems when there is a free-electron laser (FEL) interaction present within the cavity. We first construct a linear thick-lens model of the FEL and analytically study the dependence of resonator stability on its geometry. Next, we employ a nonlinear, three-dimensional FEL oscillator code to study the dependence of FEL performance on the cavity configuration. The analytic and numerical approaches are compared and it is shown that they agree quite well. It is found that the region of stability is shifted toward longer cavities, and beyond the concentric configuration. Between the confocal and the concentric configurations, where the empty-resonator analysis predicts stability, there now appear regions of instability. We find that operation near the concentric configuration is preferable, and operation very near the confocal should be avoided.

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I. INTRODUCTION

The stability of modes in an empty optical cavity has been well studied and can be found in standard texts[1]. It is found that changing the geometry of the resonator affects its stability, and that near-concentric and near-confocal resonators are the most stable. Based on these studies, in applications to conventional lasers, the preferred configuration is near-confocal. In applications to free-electron lasers (FELs) the choice is less unanimous, though more often near-concentric; however these choices seem to be motivated more by considerations of the amount of power incident on the mirrors, than by those of stability. (Recently, a study of resonators in the confocal configuration has been made by Pantell and Ozcan[2], based on considerations different from ours.)

In the case of FELs, the presence of a wiggler within the resonator introduces a strong, nonlinear, interaction, that must necessarily affect the stability of the system. In the paper paper we study this effect, both analytically (in the linear approximation) and with a computer simulation (that uses the full, nonlinear, FEL equations of motion).

In Section 2 we sketch, briefly, the usual empty-resonator analysis – mainly to define parameters and notation. We also introduce the formalism of Kogelnik [3] for studying resonators with internal optical systems. In Section 3 we construct a simple linear model of the FEL interaction, and apply Kogelnik's formalism to derive the stability criterion. In Section 4 we present details of a three-dimensional, nonlinear, FEL oscillator simulation program that we have developed[4], based on an amplifier program written by Tran and Wurtele[5]. We employed this program to investigate cavity stability in the presence of FEL interactions; results are presented in Section 5. Finally, in Section 6 we present some discussion and conclusions.

II. STABILITY OF RESONATOR SYSTEMS

In the first sub-section, we give a brief over-view of the usual empty-cavity analysis, in order to define notation and to provide a basis of comparison. Next we sketch the elements of Kogelnik's analysis of resonators with internal optical systems, in order to provide the basis for the linear analysis of the next section.

A. Empty Resonators

Consider a resonator system comprising two concave mirrors with radii of curvature R_1 and R_2 respectively, placed a distance D apart. Within the framework of geometrical optics, we can write down a transfer matrix that takes a light ray through one pass within the resonator. Starting out at the right surface of the mirror on the left, this one-turn matrix M_1 can be written as,

$$M_1 = \begin{pmatrix} 1 & 0 \\ -2/R_1 & 1 \end{pmatrix} \begin{pmatrix} 1 & D \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -2/R_2 & 1 \end{pmatrix} \begin{pmatrix} 1 & D \\ 0 & 1 \end{pmatrix}. \tag{1}$$

Since each of the individual matrices has unit determinant, so does the matrix M_1, and hence the whole transformation is symplectic. The condition for stability is $|Tr(M_1)| \leq 2$. Defining the *stability parameters* $g_{1,2}$,

$$g_{1,2} = 1 - \frac{D}{R_{1,2}}, \tag{2}$$

the stability criterion can be written as,

$$0 \leq g_1 g_2 \leq 1. \tag{3}$$

The stability diagram in $g_1 \leftrightarrow g_2$ space is shown in Figure 1. As can be seen, the region of stability is small, and largely confined to the near-concentric and near-confocal configurations. Note that the symmetric resonator is represented by the line $g_1 = g_2$ $(= g$, say). The stability criterion then reduces to $-1 \leq g \leq 1$.

B. Resonators with an internal optical system

A resonator that has an optical system (a set of lenses or lens-like media) within it, can be reduced to an *equivalent* empty resonator, by suitably redefining the stability parameters g_1 and g_2. The stability criterion of Eq. 3, then applies to the modified parameters, and determines the stability of the system.

Kogelnik[3] has studied the problem of deriving the equivalent empty resonator for an arbitrary internal optical system. He does this by first considering a simple thin lens within the resonator, and deriving the modified stability parameters for this case, using imaging rules. He then generalizes to an arbitrary optical system by noting that such a system can be completely described by its effective focal-length f, and the positions of its principal planes, h_1 and h_2.

Now, since the optical parameters, f, h_1 and h_2, can always be written in terms of the elements of the ray matrix for the optical system, it must be possible to write the modified stability parameters g_1 and g_2 (of the equivalent empty resonator) in terms of the matrix elements A, B, C, D (of the internal optical system). Kogelnik finds that[3]

$$\begin{aligned} g_1 &= \left(A - \frac{B}{R_1}\right), \\ g_2 &= \left(D - \frac{B}{R_2}\right). \end{aligned} \tag{4}$$

The stability of the resonator with the internal optical system is then given by Eq.(3), i.e. $0 \leq g_1 g_2 \leq 1$.

III. LINEAR FEL MODEL

In this Section we apply the above formalism, particularly Eqs. 3 and 4, to the case where there is a FEL interaction within the resonator. The wiggler does not completely occupy the space within the resonator; the relevant geometry is shown in

Figure 2.

In order to describe the FEL we choose a linear model wherein the FEL is described by a lens-like medium that has a transversly dependent refractive index n given by

$$n = n_0 \left(1 - \tfrac{b^2}{2} x^2\right). \tag{5}$$

We let the length of the FEL be l, and let it be placed symmetrically within the resonator, a distance w from each mirror (Fig. 2).

The optical system under consideration then consists of a drift space, followed by an FEL interaction, followed by a drift space. The ray transfer matrix for this system can be written as,

$$M_t = \begin{pmatrix} 1 & w \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \cos bl & (1/n_0 b) \sin bl \\ -n_0 b \sin bl & \cos bl \end{pmatrix} \begin{pmatrix} 1 & w \\ 0 & 1 \end{pmatrix}. \tag{6}$$

Multiplying the matrices we can write down the individual elements of the ray matrix:

$$A = \cos bl - w n_0 b \sin bl \tag{7a}$$

$$B = 2w \cos bl + \frac{1}{n_0 b} \sin bl - w^2 n_0 b \sin bl \tag{7b}$$

$$C = -n_0 b \sin bl \tag{7c}$$

$$D = \cos bl - w n_0 b \sin bl. \tag{7d}$$

Noting that $A = D$ and assuming, for simplicity, that the mirrors are symmetric so that $R_1 = R_2$, we find, from Eqs. 4, that for the symmetric case $g_1 = g_2$. Further, the stability condition (Eq. 3) becomes, as before,

$$-1 \leq g \leq 1, \tag{8}$$

where,

$$g = \cos bl \left[1 - \frac{2w}{R}\right] - \sin bl \left[\frac{1}{n_0 bR} + w n_0 b \left(1 - \frac{w}{R}\right)\right]. \qquad (9)$$

Note that if there is no FEL, then $l = 0$, $w = D/2$, and the equation reduces to $g = (1 - D/R)$ — which is just the equation for the symmetric, empty resonator case.

Equations 8 and 9 completely describe the stability of the FEL oscillator system. There are five independent parameters that describe the system – three for the FEL and two for the empty resonator. Since we are primarily interested in how the stability of the system changes with the length D $(= 2w + l)$ of the resonator, we now assign numerical values to the other parameters, and plot g as a function of $x = D/R$. Note that $x = 1$ corresponds to the confocal configuration, and $x = 2$ to the concentric. For an empty resonator, the regime of stability is $0 \leq x \leq 2$.

The results are shown in Figure 3 where b has been varied and the other parameters have been held fixed. For small b the stability region is not very different from that in the empty resonator case. As b increases, however, it becomes possible to have regions of stability even for $x > 2$, i.e. for configurations that are longer than concentric. This is really not surprising, because the extra focusing provided by the FEL keeps the optical beam from diverging. Note also that for sufficiently large b regions of instability appear even for $x \leq 2$ ('over-focusing'). This is potentially an important observation, because this is precisely the region where most FEL oscillators operate.

IV. NONLINEAR FEL OSCILLATOR SIMULATION

The FEL model we chose in the last section was linear and therefore rather simple. The real FEL interaction is nonlinear, and therefore rather complex. One is consequently forced to take recourse to computer simulations. For this purpose we use a three-dimensional FEL oscillator simulation program 'TDAOSC' that we have developed[4]. It is based on the program 'TDA' developed Tran and Wurtele[5], which is a time-independent, three-dimensional, axisymmetric FEL code. It uses the nonlinear KMR[6] equations of motion for the electrons:

$$\frac{d\gamma}{dz} = -k_s\left(1-\frac{\omega_w}{ck_s}\right)\frac{a_w a_s f_B \sin(\theta+\phi_s)}{\gamma}, \tag{10a}$$

$$\frac{d\theta}{dz} = k_w + \frac{\omega_w}{c} - k_s\left(1-\frac{\omega_w}{ck_s}\right)\frac{1+a_w^2+p_x^2+p_y^2+2a_w a_s f_B\cos(\theta+\phi_s)}{2\gamma^2}, \tag{10b}$$

$$\frac{dp_x}{dz} = -\frac{1}{2\gamma}\frac{\partial}{\partial x}a_w^2, \tag{10c}$$

$$\frac{dp_y}{dz} = -\frac{1}{2\gamma}\frac{\partial}{\partial y}a_w^2, \tag{10d}$$

$$\frac{dx}{dz} = \frac{p_x}{\gamma}, \tag{10e}$$

$$\frac{dy}{dz} = \frac{p_y}{\gamma}. \tag{10f}$$

Here k_s, ω_s, a_s refer to the wavenumber, frequency and the rms value of the dimensionless vector potential $a_s (= eA_s/mc)$ of the radiation or signal field. Similarly, the wiggler is characterized by its wavenumber, frequency and dimensionless vector potential, k_w, ω_w and $a_w (= eB_w/mck_w)$. The longitudinal dynamical variables of the electron are γ, the Lorentz factor, and the electron phase $\theta = (k_s+k_w)z - (\omega_s - \omega_w)t$. The transverse dynamical variables are the positions x, y and the momenta p_x, p_y that have been normalized to mc. Finally, f_B is the usual Bessel function factor that is unity for helical wigglers.

The wave equation assumes a single frequency and the paraxial approximation:

$$\left[\frac{\partial}{\partial z} + \frac{1}{2\imath k_s}\nabla_\perp^2\right] a_s e^{\imath\phi_s} = \imath\frac{eZ_0}{mc^2}\frac{f_B}{2k_s}\frac{1}{N}\sum_{j=1}^{N}\delta(x-x_j)\delta(y-y_j)a_s(x,y)\frac{e^{-\imath\theta_j}}{\gamma_j}, \tag{10g}$$

where Z_0 is the vacuum impedance ($= 377\Omega$), and the sum is over all N simulation particles, each carrying a partial current I/N.

In order to model an FEL oscillator, one has to transport the electromagnetic radiation from the end of the wiggler back to its entrance. To implement this we added three segments of a Fresnel-Kirchoff integral[7] that propagate the radiation

(a) from the end of the FEL to the right-hand mirror, (b) from this mirror backwards to the left-hand mirror, and (c) from the left-hand mirror onward to the entrance to the FEL. This completes one pass. In the second transport it is assumed that the radiation does not interact with the electron beam and does not see the wiggler aperture. For the next pass a fresh bunch of electrons is loaded, while the electric field at the entrance to the wiggler is used as input to the amplifier code. This process is iterated for as many passes as necessary; usually until the mode stabilizes and the optical beam power saturates.

Parameters used in the simulations described below are shown in Table 1. In order to consider more realistic configurations, the parameters chosen were based on the then existing design parameters for the CRDL infra-red FEL proposed at the Lawrence Berkeley Laboratory[8]. In this scheme, hole-coupling is used for extracting the optical beam, so in the simulations the right-hand mirror had a hole in the centre. In the design the configuration of the cavity was near-concentric. In order to explore the consequences of changing the configuration, we held all other parameters fixed, and increased only the length of the cavity. The cavity length D was varied between $6.3m$ and $13.3m$. (The confocal configuration corresponds to $D = 6.3m$ and the concentric to $D = 12.6m$.)

V. RESULTS OF THE SIMULATIONS

For a given optical configuration to be acceptable, we require two things: first, the mode profile at the right-hand mirror must have a maximum at the centre, so that power can be effectively coupled out of the beam. Second, the mode profile within the wiggler must have the same characteristic, so that the overlap between the electron beam and the radiation field may be substantial. Note that the cold-cavity modes satisfy these criteria.

We found that for $6.8m < D < 12.6m$ (a region of stability for an empty resonator), the mode profile does, indeed, have a maximum at the centre (though there may be structure at larger radii). At $D = 6.8m$, however, even before the confocal configuration is reached, there is an abrupt change in the profile; the mode now has a minimum at the centre, and the FEL cannot be operated in this configuration (Figure 4).

Significantly, we find that at the concentric end, this situation does not develop until $D = 13.3m$, i.e. for a configuration longer than the concentric. This is in contrast to the empty-resonator case. Thus, the focusing effect of the FEL wiggler results in a shift in the region of stability toward and beyond the concentric configuration.

A closer examination of the region $6.8m < D < 13.3m$ reveals further dynamical structure (Figure 5). We find that near the concentric configuration ($D = 12.6m$), the pass-by-pass evolution of the optical power is smooth, and it saturates in about 10-20 passes. Between $8.3 - 10.3m$, however, we find that there are fairly large pass-to-pass oscillations in the optical power, and that it takes close to 40 passes before some level of stability is achieved. The mode profile too, has substantial structure at larger radii, and hence this regime is not conducive to stable operation of the FEL.

A look at the amount of power coupled out of the hole is also instructive (Figure 6). We notice that at the two extremes of the stability region, the power P_h that is coupled out through the hole is small, because the mode has a minimum at the hole. Starting just beyond the confocal end, at $D = 6.3m$, as the separation D increases, P_h also starts to increase. However, between $8.3m < D < 10.3m$, where there is an instability that causes the saturated power to oscillate, P_h falls substantially. Once beyond this region it increases until beyond the concentric limit, before finally getting into the unstable regime. Note that even at $D = 12.6m$, for the exactly concentric configuration, the mode is smooth and the out-coupled power high.

Thus, our simulations indicate that, for this example, the best operating configuration would be very near the concentric. This allows for the best mode profile as well as the largest amount of out-coupled power.

VI. DISCUSSION AND CONCLUSIONS

In Section 3 we constructed a simple, linear model for the FEL interaction, and were able to investigate, analytically, the stability of a symmetric resonator with an internal FEL wiggler. We found that the regime of stability changes relative to the empty resonator. In particular, that regime can extend beyond the concentric configuration. Additionally, areas of instability can occur even for configurations that were stable for the empty resonator.

In Section 5 we used a nonlinear FEL oscillator program, 'TDAOSC', to extend the investigation into the more realistic regime where the FEL interaction is nonlinear. In addition, the three-dimensional nature of the simulation program allowed us look at mode profiles.

The introduction of nonlinearities can change the dynamics in a fundamental manner. Linear superposition no longer holds, and nonlinear stabilization can occur. This means that for given parameters the mode geometry can be preserved over many passes. However, as the cavity length is changed, there can still be a sudden change in the mode profile. For practical applications the precise nature of the profile is extremely important, because a stable configuration that yields an unacceptable mode profile is still unusable. In this paper we have therefore taken the practical approach and used the nature of the mode profile as an indicator of stability in the system.

We observe good agreement between the results of the simulations and the predictions of the analytic linear model. In particular, the region of stability starts before the confocal configuration, and extends beyond the concentric. In addition, between the confocal and the concentric configurations there exists a region of instability wherein the FEL oscillator should not be operated.

In conclusion, we find that for FEL oscillator applications the near-concentric resonator configuration is preferable, and that operation very near the confocal is not advisable.

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FIGURES

FIG. 1. Stability diagram for the empty asymmetric resonator.

FIG. 2. Schematic of a symmetric resonator that contains a FEL wiggler within it. The length of the wiggler is l, and the length of the drift spaces on either side is w. The FEL is modeled as a lens-like medium.

FIG. 3. Stability parameter g as a function of the normalized cavity length $x(= D/R)$, for the linear resonator system of Figure 2; for different b. The stability condition is $-1 < g < 1$. For the empty resonator this is satisfied for $0 \leq x \leq 2$.

FIG. 4. From the simulation, transverse optical mode profile at the out-coupling mirror, for (a) $D = 11.3m$, and (b) $D = 6.8m$.

FIG. 5. Total optical power as a function of the number of passes, for (a) $D = 12.3m$, and (b) $D = 10.3m$. Data from the simulations.

FIG. 6. The power coupled out through the hole, P_h, as a function of the cavity length D. Data from the simulations.

TABLES

TABLE I. Parameters used in the simulation

Parameter	Value
Wiggler parameter (a_w)	0.637
Wiggler length (L)	2 m
Wiggler period (λ_w)	5 cm
Wiggler bore radius	12.7 mm
Normalized beam emittance (ϵ_n)	2×10^{-5} m-rad
Beam radius (r_b)	0.63 mm
Beam energy (γ)	37.95
Beam current (I)	100 A
Optical wavelength (λ)	25 μm
Initial optical power	1 MW
Radius of curvature of mirrors	6.3 m
Radius of cross-section of mirrors	23 mm
Separation between the mirrors	varied
Number of test particles	1024
Number of radial grid points	128
Number of longitudinal integration steps	40