Measurement of Ratios of Branching Fractions and CP-Violating Asymmetries of $B^\pm \to D^*K^\pm$ Decays


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The measurements are sensitive to the unitarity triangle angle $\gamma$ and the CP asymmetries ($A_{CP}^\pm$). We report a study of $B^\pm \to D^* K^\pm$ decays with $D^*$ decaying to $D\pi^0$ or $D\gamma$, using 383 $\times 10^6 B\bar{B}$ pairs collected at the $\Upsilon(4S)$ resonance with the BABAR detector at the SLAC PEP-II B-Factor. The $D$ meson decays under study include a non-CP mode ($K^\pm \pi^\mp$), CP-even modes ($K^\pm K^\mp$, $\pi^\pm \pi^\mp$) and CP-odd modes ($K^0_s \pi^0$, $K^0\phi$, $K^0\omega$). We measure ratios ($R_{CP}^\pm$) of branching fractions of decays to CP eigenmode states and to flavor-specific states as well as CP asymmetries ($A_{CP}^\pm$). These measurements are sensitive to the unitarity triangle angle $\gamma$. We obtain $A_{CP}^+ = -0.11 \pm 0.09 \pm 0.01,$
\[ R_{CP}^* = 1.31 \pm 0.13 \pm 0.04, \text{ and } A_{CP}^* = 0.06 \pm 0.10 \pm 0.02, \quad R_{CP}^{\pm} = 1.10 \pm 0.12 \pm 0.04, \] where the first error is statistical and the second error is systematic. Translating our results into an alternative parametrization, widely used for related measurements, we obtain \( x_1^* = 0.11 \pm 0.06 \pm 0.02 \) and \( x_2^* = 0.00 \pm 0.06 \pm 0.02 \). No significant CP-violating charge asymmetry is found in either the flavor-specific mode \( D \to K^{\pm} \pi^\mp \) or in \( B^{\pm} \to D^{*\pi^\pm} \) decays.


I. INTRODUCTION

In the Standard Model (SM), CP-violating phenomena are a consequence of a single complex phase in the Cabibbo-Kobayashi-Maskawa (CKM) quark-mixing matrix [1]. The \( B^{\pm} \to D^{(*)} K^{(*)\pm} \) decay modes provide a theoretically clean determination of the unitarity triangle angle \( \gamma \), since the latter is equal to the relative phase between the CKM- and color-favored \( b \to c \) and the CKM- and color-suppressed \( b \to u \) decay amplitudes that are dominant in the considered decays. The method proposed by Gronau, London and Wyler (GLW) makes use of the direct CP violation in the interference between the amplitudes for \( B^{\pm} \to D^{0} K^{\pm} \) and \( B^{\pm} \to \overline{D}^{0} K^{\pm} \) decays when the \( D^0 \) and \( \overline{D}^0 \) mesons decay to the same CP eigenstate [2, 3]. The same approach is equally applicable when the \( D \) and/or the \( K \) meson is replaced with its excited state. In this paper we use the \( B^{\pm} \to D^{*} K^{\pm} \) decay.

In the case of \( B^{\pm} \to D^{*} K^{\pm} \) decays, one defines the CP-violating charge asymmetry

\[ A_{CP}^* \equiv \frac{\mathcal{B}(B^- \to D_{CP}^+ K^-) - \mathcal{B}(B^+ \to D_{CP}^+ K^+)}{\mathcal{B}(B^- \to D_{CP}^- K^-) + \mathcal{B}(B^+ \to D_{CP}^- K^+)} \] and the ratio of branching fractions for the decays to CP eigenmodes and flavor-specific states

\[ R_{CP}^* \equiv \frac{\mathcal{B}(B^- \to D_{CP}^+ K^-) + \mathcal{B}(B^+ \to D_{CP}^+ K^+)}{[\mathcal{B}(B^- \to D^{0} K^-) + \mathcal{B}(B^+ \to \overline{D}^{0} K^+)]/2}. \]

We refer to the companion of the charmed meson in the final state as the prompt track. Experimentally, it is convenient to normalize the branching fractions of the decays with a prompt kaon in the final state to those of the similar decays with a prompt pion that have a larger branching fraction. The ratio \( R_{CP}^* \) can then be expressed as

\[ R_{CP}^* \approx \frac{R^*_+}{R^*_-}, \] where \( R^*_+ \) and \( R^*_- \) are the \( K/\pi \) ratios

\[ R^*_+ \equiv \frac{\mathcal{B}(B^- \to D_{CP}^+ K^-) + \mathcal{B}(B^+ \to D_{CP}^+ K^+)}{\mathcal{B}(B^- \to D_{CP}^- K^-) + \mathcal{B}(B^+ \to D_{CP}^- K^+)} \] and

\[ R^*_- \equiv \frac{\mathcal{B}(B^- \to D^{0} K^-) + \mathcal{B}(B^+ \to \overline{D}^{0} K^+)}{\mathcal{B}(B^- \to D^{0} K^-) + \mathcal{B}(B^+ \to \overline{D}^{0} K^+)} \].

The ratio \( R^*_+ \) is predicted to be of the order of \( (f_K/f_{\pi}) \times \left| V_{us}/V_{ud} \right|^2 = 0.080 \pm 0.002 \) [5], where \( f_K \) and \( f_{\pi} \) are the form factors of the mesons. Equation (3) would be an equality if CP violation was completely absent in \( B \to D^{*\pi} \) decays. Defining the charge asymmetry

\[ A^*_K \equiv \frac{\mathcal{B}(B^- \to D^{*h} h^-) - \mathcal{B}(B^+ \to D^{*h} h^+)}{\mathcal{B}(B^- \to D^{*h} h^-) + \mathcal{B}(B^+ \to D^{*h} h^+)} \] (noted \( A^*_K \) when referring to \( h = \pi \) and \( h = K \) respectively), this approximation implies that the pion charge asymmetry \( A^*_K \) should be compatible with zero, as should be the kaon charge asymmetry \( A^*_K \) for the flavor-specific modes \( D \to K^{\pm} \pi^{\mp} \). The possible bias induced by this approximation is expected to be small since the ratio of the amplitudes of \( B^- \to \overline{D}^{0} \pi^- \) and \( B^- \to D^{0} \pi^- \) processes is predicted to be of the order of 1% [6] in the SM, and will be accounted for in the systematic uncertainties.

Most experimental systematic uncertainties, such as those related to the reconstruction of the \( D^* \), and the uncertainties on the \( D \) decay branching fractions, cancel in the \( K/\pi \) ratios \( R^*_+ \) and \( R^*_- \). By neglecting the small [7, 8] \( D^{0} - \overline{D}^{0} \) mixing [9] and CP violation in \( D^{0} \) decays, \( R_{CP}^* \) and \( A_{CP}^* \) are related to \( \gamma \) through

\[ R_{CP}^* = 1 + r_B^2 \pm 2r_B^2 \cos \delta_B \cos \gamma, \] and

\[ A_{CP}^* = \frac{\pm r_B \sin \delta_B \sin \gamma}{1 + r_B^2 \pm 2r_B \cos \delta_B \cos \gamma}, \]

\( \delta_B \) is the phase of the \( D^{0} \) mixing.

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TABLE I: Past measurements of parameters related to the measurement of $\gamma$ in $B^{\pm} \to D^{\mp}K^{\pm}$ decays by the GLW method.

<table>
<thead>
<tr>
<th></th>
<th>$A_{CP^+}$</th>
<th>$A_{CP^-}$</th>
<th>$R_{CP^+}$</th>
<th>$R_{CP^-}$</th>
<th>$R^*$</th>
</tr>
</thead>
<tbody>
<tr>
<td>BABAR [12]</td>
<td>$-0.10 \pm 0.23^{+0.04}_{-0.04}$</td>
<td>$-$</td>
<td>$1.06 \pm 0.26^{+0.10}_{-0.09}$</td>
<td>$-$</td>
<td>$0.0813 \pm 0.0040^{+0.0042}_{-0.0031}$</td>
</tr>
<tr>
<td>Belle [13, 14]</td>
<td>$-0.20 \pm 0.22 \pm 0.04$</td>
<td>$0.13 \pm 0.30 \pm 0.08$</td>
<td>$1.41 \pm 0.25 \pm 0.06$</td>
<td>$1.15 \pm 0.31 \pm 0.12$</td>
<td>$0.078 \pm 0.019 \pm 0.009$</td>
</tr>
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</table>

where $r^*_B$ is the magnitude of the ratio of the amplitudes for the processes $B^- \to \bar{D}^0K^-$ and $B^- \to D^0K^-$, and $\delta_B^*$ is the relative strong phase between these two amplitudes. The ratio $r^*_B$ involves a CKM factor $|V_{ub}V_{cs}/V_{ub}V_{us}| \approx 0.44 \pm 0.05$ [5] and a color suppression factor that has been estimated to lie between $0.26 \pm 0.07 \pm 0.05$ [10] and 0.44 [6], so that $r^*_B$ is predicted to be in the range $0.1 - 0.2$. More recent calculations that take into account final state interactions [11] yield predictions of $r^*_B = 0.09 \pm 0.02$.

The latest results by BABAR and Belle are reported in references [12] and [13, 14], respectively. BABAR used $123 \times 10^3$ $B\bar{B}$ pairs with $D^\pm \to \pi^\pm \nu \bar{\nu}$ and $D$ reconstructed in the $C P$-even modes $K^+K^-$ and $\pi^+\pi^-$, and non-$C P$ modes $K^0\pi^0 \bar{\pi} \bar{\nu}$ and $K^0\pi^+\pi^- \pi^0$. BABAR used $275 \times 10^3$ $B\bar{B}$ pairs with $D^+ \to \pi^+ \nu \bar{\nu}$ and $D$ reconstructed in the $C P$-even modes $K^+K^-$ and $\pi^+\pi^-$, and non-$C P$ modes $K^0\pi^0 \bar{\pi} \bar{\nu}$ and $K^0\pi^+\pi^- \pi^0$. The results are summarized in Table I. Similar studies have been performed on the channels $B^\pm \to D^\mp K^\pm$ [13] and $D^\pm \to K^\mp$ [17].

The BABAR [18] and Belle [19] experiments have recently obtained estimates of $r^*_B$ and $\delta_B^*$ parameters from the overlap of the $D^0$ and $\bar{D}^0$ decays in the Dalitz planes of some three-body $D$ decays. BABAR obtains $r^*_B = 0.135 \pm 0.051$ and $\delta_B^* = (-63^{+36}_{-26})^\circ$, while Belle obtains $r^*_B = 0.21 \pm 0.08 \pm 0.02 \pm 0.05$ and $\delta_B^* = (342^{+23}_{-23} \pm 4 \pm 23)^\circ$ (where the first error is statistical, the second is the experimental systematic uncertainty and the third reflects the uncertainty on the $D$ decay Dalitz models).

In this paper, by using $(383 \pm 4) \times 10^6$ $B\bar{B}$ pairs, we update the results of our previous study of $B^\pm \to D^*K^\mp$ decays [12] for decays to the $C P$-even modes $K^+K^-$, $\pi^+\pi^-$ and to the flavor-specific modes $K^\mp\pi^\pm$, and we extend it to the $C P$-odd modes $K^0\pi^0$, $K^0\omega$ and $K^0\phi$, and to $D^+ \to D\gamma$. Due to parity and angular-momentum conservation, the $C P$ eigenvalue of the $D^*$ is inferred from that of the $D$, when the $C P$ eigenvalue of the neutral charm companion ($\gamma$ or $\pi^0$) is taken into account [20]: $C P(D^*) = C P(D)$ when $D^* \to D\gamma$, and $C P(D^*) = -C P(D)$ when $D^* \to D\gamma$.

Experimental results can also be presented using the “cartesian coordinates”

$$ (x^*_\pm, y^*_\pm) \equiv (r^*_B \cos(\delta_B^* \pm \gamma), r^*_B \sin(\delta_B^* \pm \gamma)), $$

which have the advantage of having Gaussian uncertainties, and of being uncorrelated and unbiased ($r^*_B$, being positive, is biased towards larger values in low precision measurements, whereas $x^*_\pm$ and $y^*_\pm$ show no such bias) [21]. The parameters $x^*_\pm$ can be obtained from $R_{CP^\pm}$ and $A_{CP^\pm}$,

$$ x^*_\pm = \frac{R_{CP^+} (1 + A_{CP^+}) - R_{CP^-} (1 + A_{CP^-})}{4}. $$

The measurements presented in this paper have no direct sensitivity to $y^*_1$, in contrast to Dalitz analyses. However an indirect constraint can be obtained using

$$ (r^*_B)^2 = x^*_2 + y^*_2 = \frac{R_{CP^+}^2 + R_{CP^-}^2 - 2}{2}. $$

Note that there are four observables in these parameterizations, either $(A_{CP^+}, R_{CP^+}, A_{CP^-}, R_{CP^-})$ or $(x^*_1, y^*_1, x^*_2$ and $y^*_2$), while there are only three independent fundamental parameters ($\gamma$, $r^*_B$ and $\delta^*_B$). The set of parameters must therefore fulfill one constraint, which can be $\kappa = 0$, where

$$ \kappa \equiv R_{CP^+} A_{CP^+} + A_{CP^-} R_{CP^-}. $$

II. THE DATASET AND BABAR DETECTOR

The results presented in this paper are based on data collected with the BABAR detector at the PEP-II asymmetric-energy $e^+e^-$ storage ring of the Stanford Linear Accelerator Center. At PEP-II, 9.0 GeV electrons and 3.1 GeV positrons collide at a center-of-mass energy of 10.58 GeV, which corresponds to the mass of the $T(4S)$ resonance. The asymmetric beam energies result in a boost from the laboratory to the center-of-mass frame of $\beta\gamma \approx 0.56$. The dataset analyzed in this paper corresponds to an integrated luminosity of 347 fb$^{-1}$ at the $T(4S)$ resonance.

The BABAR detector is described in detail elsewhere [22]. Surrounding the interaction point is a five-layer double-sided silicon vertex tracker (SVT), which measures the trajectories of charged particles. A 40-layer drift chamber (DCH) provides measurements of the momenta of charged particles. Both the SVT and DCH are located inside a 1.5 T magnetic field provided by a solenoid magnet. Charged hadron identification is achieved through measurements of particle energy-loss in the tracking system and the Cherenkov angle obtained from a detector of internally reflected Cherenkov light (DIRC). A CsI(Tl) electromagnetic calorimeter (EMC) provides photon detection, electron identification, and $\pi^0$ reconstruction. Finally, the instrumented flux return
(IFR) of the magnet enables discrimination of muons from pions. For the most recent 134 fb−1 of data, a fraction of the resistive plate chambers constituting the muon system has been replaced by limited streamer tubes [23].

We use Monte Carlo (MC) simulation to study the detector acceptance and backgrounds. The MC uses the EvtGen generator [24] and GEANT4 [25] to simulate the passage of particles through matter.

III. RECONSTRUCTION OF B CANDIDATES

We perform an exclusive reconstruction of the full B meson decay chain, in the modes described in the introduction, starting from the final stable products (charged-particle tracks and neutral electromagnetic deposits in the EMC).

The π0 candidates used to form an ω, a D or a D* candidate are reconstructed from pairs of photons with energies larger than 30 MeV, and shower shapes consistent with electromagnetic showers, with invariant mass in the range 115 < m_γγ < 150 MeV/c^2. In addition, the π0 candidates used to form a D* candidate are required to have center-of-mass frame momenta p_γγ < 450 MeV/c. The ω candidates are reconstructed in the three-body decay ω → π^+π^−π^0, with an invariant mass within 50 MeV/c^2 of the world average [5]. We reconstruct K^0_s → π^+π^− candidates from pairs of oppositely charged tracks that are consistent with having originated from a common vertex position and with an invariant mass within 25 MeV/c^2 of the world average [5]. We reconstruct φ → K^+K^− candidates from pairs of oppositely charged tracks with particle identification inconsistent with a pion hypothesis, that are consistent with having originated from a common vertex position, and that have invariant mass within 30 MeV/c^2 of the world average [5].

Only two-body D decays are considered in this study. The D candidates are reconstructed from their two daughters that are required to be consistent with having originated from a common vertex position. In the case of D → K^0_sπ^0, in which vertexing of the K^0_sπ^0 system would yield a poor geometrical constraint, a beam spot constraint is added to the fit in order to force the D daughters to originate from the interaction region.

The D* candidates are formed from D and π^0 or γ candidates. These photon candidates are required to have energies larger than 100 MeV and shower shapes consistent with electromagnetic showers. The D* candidates are required to fulfill 130 < ∆m < 170 MeV/c^2 and 80 < ∆m < 180 MeV/c^2, respectively, where ∆m is the invariant mass difference between the D* and the D candidate.

The π^0, K^0_s, D and D* candidates are refitted with mass constraints before their four-momenta are used to reconstruct the B decay chain. We form B candidates from D* candidates and charged tracks, fitted with a beam spot constraint. We characterize B candidates by two kinematic variables: the difference between the reconstructed energy of the B candidate and the beam energy in the center-of-mass frame ΔE_B = E^γ − √s/2, and the beam-energy substituted mass m_{ES} = √(s/2 + p_0 · p_B)^2/E_0^2 − p_B^2, where (E_0, p_0) and (E_B, p_B) are the four-momenta of the Y(4S) and B meson candidate, respectively, the asterisk denotes the Y(4S) rest frame, and √s is the total energy in the T(4S) rest frame. The subscript K in ΔE_B indicates that the kaon hypothesis has been assumed for the prompt track in the computation of ΔE.

For a correctly reconstructed B meson having decayed to a D^*K final state, ΔE_B is expected to peak near zero, with an R.M.S. of about 16 MeV, and m_{ES} is expected to peak near the B-meson mass 5.279 GeV/c^2, with an R.M.S. that is almost independent of the channel and close to 3 MeV/c^2. For a B → D^*π decay reconstructed as B → D^*K with a correctly identified D^*, the ΔE_B peak is shifted by approximately +50 MeV. At reconstruction level, the loose requirements 5.2 < m_{ES} < 5.3 GeV/c^2 and |ΔE_B| < 0.2 GeV are applied to the B meson candidate.

We form a Fisher discriminant F [26] to distinguish signal events from the significant background due to e^+e^− → q̅q (q = u, d, s, c) continuum events. Six variables are used:

- L_0 and L_2, the zeroth and second angular moments of the energy flow along the B thrust axis. They are defined as \sum_i p_i and \sum_i p_i \cos^2 \theta_i respectively, where p_i is the momentum and \theta_i is the angle with respect to the thrust axis of the B candidate, both in the center-of-mass frame, for all tracks and neutral clusters not used to reconstruct the B meson;
- R_2, the ratio of the second to the zeroth Fox-Wolfram moment [27] of charged tracks and neutral clusters in the center-of-mass frame;
- |cos θ_B|, where θ_B is the angle between the momentum of the B candidate and the boost direction of the e^+e^- center-of-mass frame;
- |cos θ_{thrust}|, where θ_{thrust} is the angle between the B candidate thrust vector and the beam axis in the center-of-mass frame;
- |cos θ_T|, where θ_T is the angle between the B candidate thrust axis and the thrust axis of the rest of the event in the center-of-mass frame (where the rest of the event corresponds to reconstructed particles not associated with the B candidate).

IV. SELECTION OF B CANDIDATES

After the preliminary event reconstruction, a large amount of background remains in the signal candidate sample. In this section we describe the additional selection criteria used to reduce the background.

The selection of each B → D^*K decay mode is optimized separately, by the maximization of the sensitivity...
$S/\sqrt{S+B+1}$, where $\sqrt{S+B+1}$ is a symmetrized approximation of the Poisson uncertainty on the measurement of $S+B$. The numbers $S$ and $B$ of signal and background expected events are estimated from, respectively, high-statistics exclusive MC samples, and a cocktail of generic $B^+B^-$ (with signal events removed), $B^0\bar{B}^0$ and $q\bar{q}$ MC samples.

In the optimization procedure, we include requirements on all variables, including those that will be relaxed during the fit, and including tightening requirements that have been made in the reconstruction stage. Our optimization procedure is similar to that used in Ref. [28], and allows us to determine the optimal set of variables as well as the optimal requirements on those variables, by the examination of the signal-to-background ratio distributions [29]. The final set of variables on which we apply selection optimization is:

- the $B$ candidate-related variables $m_{ES}$ and $\Delta E_K$ introduced above;
- the mass $m_{D^0}$ of the $D$ candidate before the mass constraint is applied and the mass difference $\Delta m$;
- likelihood ratios for the prompt track, that are evaluated making use of the Cherenkov angle information from the DIRC, and of the $dE/dx$ information provided by the tracking system. Explicitly, we compute likelihoods $L_h$ for particle identification (PID) hypotheses $h=K,\pi,p$ and make requirements on the ratios $L_K/(L_K+L_\pi)$ and $L_K/(L_K+L_p)$. We also require that the track is not identified as an electron or muon;
- likelihood ratios for pion and kaon candidates that are daughters of two-body $D$ decays;
- the value of the Fisher variable $F$;
- the invariant masses of the $K^0_S,\phi,\pi^0$ and $\omega$ candidates, when relevant, and before the mass constraints. Furthermore, for decays involving $K^0_S$ candidates, we include the ratio of the flight length of $K^0_S$ candidates in the transverse plane divided by its uncertainty, and require it to be larger than two. For decays involving $\omega$ candidates, we include $|\cos(\theta_\omega)|$, where $\theta_\omega$ is the angle between the normal to the pion decay plane and the $D$ direction in the $\omega$ rest-frame.

The selection requirements applied to these variables are mode-dependent, except for the prompt track PID requirements $L_K/(L_K+L_\pi) > 0.9$ and $L_K/(L_K+L_p) > 0.2$ that are applied to all $B \to D^*K$ channels.

The selection of the $B^{\pm} \to D^*\pi^{\pm}$ modes is identical to that of the $B^{\pm} \to D^*K^{\pm}$ modes, except for the prompt-track PID that is reversed ($L_K/(L_K+L_\pi) < 0.2$).

A fraction of the events have several $B$ candidates selected: the average multiplicity varies from 1.07 to 1.66 for $D^* \to D^*0$ and from 1.00 to 1.25 for $D^* \to D\gamma$, depending on the channel. We select the $B$ candidate that has the $B$ vertex fit with the largest probability. This best-candidate procedure is used during the optimization of the selection that we have described above. The probability of selection of the well-reconstructed signal candidate is mode dependent and is in the range 56–72% for $D^* \to D_0^0$ decays and in the range 68–81% for $D^* \to D\gamma$ decays.

V. MAXIMUM LIKELIHOOD FIT

The dominant contribution to the remaining background after event selection is from $B$ decays, including a significant amount of feed-across from $B^{\pm} \to D^*\pi^{\pm}$ decays. Therefore the measurement is performed with an unbinned likelihood fit [30, 31] based on two variables that best discriminate this background, namely $\Delta E_K$ and a PID variable $T_R$ defined below.

As the prompt track PID likelihood ratio $R \equiv L_K/(L_K + L_\pi)$ is very strongly peaked near zero for pions and near one for kaons, we use a pseudo-logarithmic change of variable

$$T_R = \log_{10} \left( \frac{R + \epsilon}{1 - R + \epsilon} \right).$$

Where we include a small positive number $\epsilon = 10^{-7}$, so that $T_R$ is always defined, with $T_R = +7$ for $R = 1$ (“perfect kaons”) and $T_R = -7$ for $R = 0$ (“perfect pions”).

The measurement is performed with an extended unbinned maximum likelihood function

$$\mathcal{L} = \frac{e^{-N'}(N')^N}{N!} \prod_{i=1}^{N} \mathcal{P}_i,$$

where $N$ is the number of events in the sample to fit, $N'$ is the expected number, and for event $i$

$$\mathcal{P}_i = \frac{1}{N'} \sum_j N_j \mathcal{P}_j^i,$$

where $j = D^*K, D^*\pi, B_K, B_\pi$ is one of four events categories: signal kaon and pion, and background kaon and pion respectively, where the background is a combination of continuum, $B^+B^-$ and $B^{0}\bar{B}^0$ events. The quantity $\mathcal{P}_j^i$ is the probability density function (PDF) for event $i$ and category $j$, and $N_j$ is the number of events in category $j$.

For the signal categories, the distance between the kaon and pion $\Delta E_K$ peaks provides powerful separation between pions and kaons, in addition to PID. For the background categories, we use mutually exclusive likelihood-based pion and kaon selectors, that in particular contain requirements of $R > 0.9$ (kaon) and $R < 0.1$ (pion) respectively. For consistency and symmetry reasons, the whole region $0.1 < R < 0.9$ is removed for all categories, including the signal categories used in the fit. The correlations between $T_R$ and $\Delta E_K$ are found to be small (compatible with zero for the signal $K$ and for
the background categories, and with $-6\%$ for the $\pi$ signal category) therefore a factorized approximation is used

$$P_i(\Delta E_K, T_R) = P_i^* (\Delta E_K) P_i^1 (T_R). \quad (16)$$

We have checked that no bias is introduced by this approximation by simulating a large number of experiments in which the signal is taken from the large statistics exclusive MC samples used for estimating these correlations.

The PDFs used in the fit are determined from MC samples. The signal $\Delta E_K$ PDFs are parameterized with double Gaussian functions. The background PDFs are mode-dependent functions chosen to best represent the MC background distributions: they include Gaussian, exponential, and third-order Chebyshev polynomial functions. The complicated shape of the $\Delta E_K$ distribution of the $B_s$ category arises from the contributions from several distinct components: at low $\Delta E_K$ values, $B^\pm \to D^{\mp} \rho^{\pm}$ decays dominate; in the signal region, the background is mainly composed of $\gamma \to \pi^0$ cross-feed and of $B^\pm \to D^{\mp} \pi^\pm$, the latter of which dominates at high $\Delta E_K$ values. The $T_R$ PDFs are histograms, determined from MC samples, with a binning $\Delta T_R = 0.5$ and, therefore, 28 bins. MC-based studies have shown that the results of such fits do not depend on the number of bins $n_b$ as long as $n_b > 2$.

We correct for a small discrepancy in PID efficiencies between data and MC, using high-statistics high-purity kaon and pion samples from inclusive $D^{\pm} \to D_{\pi^{\pm}}$, $D \to K^{\pm} \pi^\pm$ data. The difference in track momentum spectra between these control samples and the exclusive modes studied in the present analysis is accounted for in the correction procedure. This is achieved by weighting the control sample $T_R$ PDF by the ratio of the MC to control sample prompt track momentum distributions for both cases of the prompt track being a kaon or a pion. An example of the PDFs used for the channel $D^* \to D \pi^0$, $D \to K^{\pm} \pi^\pm$ is shown in Fig. 1.

For signal events with a pion prompt track, for which $\Delta E_\pi$ (the subscript $\pi$ indicates that the pion hypothesis has been assumed for the prompt track in the computation of $\Delta E$) is close to zero, the relation

$$\Delta E_K - \Delta E_\pi \approx \frac{1}{2p} \frac{E_{T(4S)}}{m_{T(4S)}} (m_K^2 - m_\pi^2), \quad (17)$$

introduces a mild dependence of $\Delta E_K$ on the momentum $p$ of the prompt track. The parameters $E_{T(4S)}$ and $m_{T(4S)}$, $m_K$, $m_\pi$ denote the energy of the $e^+ e^-$ system in the laboratory frame and the masses of the mesons, respectively. Fits taking this dependence into account do not show any significant improvement, nor degradation.

Fits performed on the $B^+ B^-$, $B^{0 \bar{B}^{0}}$ and $q\bar{q}$ background MC samples show no significant bias. Similar fits with either pion or kaon signal events removed yield numbers of signal events compatible with zero for the removed category. This indicates that the factorization approximation made for the background PDF does not create any bias on the number of fitted signal events.

Signal efficiencies are estimated from fits on high statistics exclusive MC samples and summarized in Table II. We perform separate fits for each $D^*$ decay mode, and subsequently perform a weighted average to obtain our final results for $R_{CP}^*$ and $A_{CP}^*$. The free parameters of each fit are itemized here:

- the charge-averaged $K/\pi$ ratio (one parameter, $R^*_\pm$ or $R^*$, whenever relevant);
- the number of pion signal events (one parameter);
- the pion and kaon charge asymmetries $A^*_h$ of the signal (two parameters);
TABLE II: Selection efficiencies (in %) for channels used in this analysis for each decay mode of the $D$ (statistical uncertainties only).

<table>
<thead>
<tr>
<th>Channel</th>
<th>Efficiency (in %)</th>
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</thead>
<tbody>
<tr>
<td>$(D\pi^0)K^\pm$</td>
<td>21.0 ± 0.1</td>
</tr>
<tr>
<td>$(D\gamma)K^\pm$</td>
<td>14.6 ± 0.1</td>
</tr>
<tr>
<td>$(D\pi^0)\pi^\pm$</td>
<td>20.4 ± 0.1</td>
</tr>
<tr>
<td>$(D\gamma)\pi^\pm$</td>
<td>8.9 ± 0.1</td>
</tr>
<tr>
<td>$K_0^0\pi^0$</td>
<td>4.4 ± 0.1</td>
</tr>
<tr>
<td>$K_0^0\phi$</td>
<td>10.3 ± 0.1</td>
</tr>
<tr>
<td>$K_0^0\phi$</td>
<td>13.5 ± 0.1</td>
</tr>
<tr>
<td>$K_0^0\phi$</td>
<td>10.4 ± 0.1</td>
</tr>
<tr>
<td>$K_0^0\phi$</td>
<td>13.7 ± 0.1</td>
</tr>
</tbody>
</table>

VI. SYSTEMATIC UNCERTAINTIES

The systematic uncertainties are summarized in Table III.

The contribution of the determination of the $\Delta E_K$ signal PDFs to the systematic uncertainty is estimated by varying the parameters that are fixed in the fit by one standard deviation ($\pm 1\sigma$). The contribution of the $\Delta E_K$ PDFs of the $B_K$ category for the $D^{*0} \rightarrow D\pi^0$ decays is determined similarly.

For the other $\Delta E_K$ background PDFs, this approach cannot be used, as the correlations between parameters are not small. The contribution to the systematics due to the limited MC statistics used to determine the parameters of the PDFs is obtained in the following way. We determine two separate parameterizations of the PDFs on two halves of the MC sample, and perform the fits with them. We take half the difference between the obtained results as an estimate of the systematics.

The contribution of the determination of the $T_{R\gamma}$ signal PDFs to the systematic uncertainty is estimated by performing an additional fit without the correction of the small discrepancy between data and MC described above. The difference between the results of both fits is taken as an estimate of the uncertainty.

The systematic uncertainty introduced by the limited knowledge of $B^\pm \rightarrow D^0\rho^\pm$ and $B^0 \rightarrow D^{*+}\pi^-$ branching fractions is estimated from MC samples by performing a fit on a sample in which the number of these events is varied by $\pm 1\sigma$ [5].

Differences in the interactions of positively and negatively charged kaons with the detector and the possible charge asymmetry of the detector are studied using the exclusive MC samples. Asymmetries of $(-1.0 \pm 0.2\%)$ and $(0.2 \pm 0.2\%)$ are observed for kaon and pion modes, respectively, for the $CP$ modes. A correction of $+1\%$ is applied to the measured values of $A_{CP}^\pm$. The simulation of the detector charge asymmetry has been compared to the actual value in the data in a previous analysis of $B$ decays to $K\pi$ [32]. The possible discrepancy has been found to be smaller than $1\%$.

The $\pi^0 \leftrightarrow \gamma$ cross-feed can reduce the value of $A_{CP}^\pm$, because for a given $D_{CP}$ final state, $D_{CP}$ has the same $CP$ value as $D_{CP}$ if decaying to $D\pi^0$ and the opposite $CP$ value if decaying to $D\gamma$ [20]. This “$CP$ dilution” is estimated from MC samples by performing a fit in which the potential feed-through has been completely removed. The effect is similar among modes and is accounted for by an uncertainty of $0.5\%$ for $D\pi^0$ modes and of $1.0\%$ for $D\gamma$ modes.

In the case of $D$ decays to $K_0^0\phi$ and $K_0^0\omega$, the $CP$-
TABLE IV: Summary of measurements of the charge asymmetries $A_K^*$; the CP eigenmode to flavor specific mode ratios $R_{CP}^*$, and the cartesian parameters $x^*$, for $B^\pm$ decays to CP even and CP odd eigenmodes $D_{CP}K^\pm$. The value of $A_K^*$ for the flavor-specific control mode with $D \rightarrow K^\pm\pi^\mp$ is also given.

<table>
<thead>
<tr>
<th>Flavor specific</th>
<th>$A_K^*$</th>
<th>$R_{CP}^*$</th>
<th>$x^*$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$0.06 \pm 0.04 \pm 0.01$</td>
<td>$-$</td>
<td>$-$</td>
<td>$-$</td>
</tr>
<tr>
<td>$0.11 \pm 0.09 \pm 0.01$</td>
<td>$1.31 \pm 0.13 \pm 0.04$</td>
<td>$0.11 \pm 0.06 \pm 0.02$</td>
<td></td>
</tr>
<tr>
<td>$0.06 \pm 0.10 \pm 0.02$</td>
<td>$1.10 \pm 0.12 \pm 0.04$</td>
<td>$0.00 \pm 0.06 \pm 0.02$</td>
<td></td>
</tr>
</tbody>
</table>

Table VI: Number of events measured in this analysis (statistical uncertainties only).

<table>
<thead>
<tr>
<th>$(D\pi^0)K^\pm$</th>
<th>$(D\gamma)K^\pm$</th>
<th>$(D\pi^\pm)\pi^\mp$</th>
<th>$(D\gamma)\pi^\pm$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K^\pm\pi^\pm$</td>
<td>$874 \pm 44$</td>
<td>$536 \pm 36$</td>
<td>$10729 \pm 133$</td>
</tr>
<tr>
<td>$K\pi^*$</td>
<td>$31 \pm 8$</td>
<td>$15 \pm 6$</td>
<td>$292 \pm 20$</td>
</tr>
<tr>
<td>$K_S^0\pi^0$</td>
<td>$110 \pm 14$</td>
<td>$62 \pm 12$</td>
<td>$987 \pm 43$</td>
</tr>
<tr>
<td>$K_S^0\pi^\pm$</td>
<td>$86 \pm 14$</td>
<td>$62 \pm 11$</td>
<td>$900 \pm 38$</td>
</tr>
<tr>
<td>$K_S^0\omega$</td>
<td>$43 \pm 11$</td>
<td>$29 \pm 9$</td>
<td>$419 \pm 31$</td>
</tr>
<tr>
<td>$K_S^0\phi$</td>
<td>$19 \pm 6$</td>
<td>$21 \pm 6$</td>
<td>$262 \pm 22$</td>
</tr>
</tbody>
</table>

We also obtain $R^* = 0.0802 \pm 0.0031 \pm 0.0018$, compatible with the theoretical prediction given in the introduction, and $(r_B^*)^2 = 0.20 \pm 0.09 \pm 0.03$. We find $\kappa = -0.08 \pm 0.16 \pm 0.02$ (defined in Eq. 12), consistent with zero as expected. We confirm the large value of $(r_B^*)^2$ that can be inferred from the previous measurements [13] based on the GLW method, with a precision improved by a factor two.

Using the value of $\gamma = 67.6 \pm 4.0$ obtained by a SM-based fit of the CKM matrix [33] and the values of $r_B^*$ and $\delta_B^*$ from Ref. [18], we predict $A_{CP^+}^* = -0.18 \pm 0.10$, $R_{CP^+}^* = 1.06 \pm 0.06$, $A_{CP^-}^* = 0.20 \pm 0.10$ and $R_{CP^-}^* = 0.98 \pm 0.05$. Our results are compatible with these predictions.

We also compute the cartesian coordinates with the channel $D \rightarrow K^0\phi$ removed, so as to facilitate the comparison with results obtained with the Dalitz method [18]:

$$x_+^* = 0.09 \pm 0.07 \pm 0.02,$$

$$x_-^* = -0.02 \pm 0.06 \pm 0.02,$$

$$R_{CP}^* = 0.22 \pm 0.09 \pm 0.03.$$
FIG. 2: $\Delta E_K$ distributions, with a cut on the PID variable $T_R > 0$ to enhance the kaon part of the sample. Distributions are shown for each of the decays modes: (left) $D^* \rightarrow D\pi^0$ and (right) $D^* \rightarrow D\gamma$, with the $D$ decay modes indicated on the left of the figure. Dots denote the distribution of the data. Curves denote the PDFs for the various categories: signal $K$ (long-dashed curve); signal $\pi$ (dotted curve); background $K$ (short-dashed curve); background $\pi$ does not appear because the $T_R > 0$ cut completely removes this category. The thick curve denotes the total PDF.
for \( R_{CP} \) and \( A_{CP} \) are at least a factor of two more precise than previous measurements \([12, 13]\). The precision of our results for \( x_{+} \) is comparable to that obtained from Dalitz plot analyses \([18, 19]\). No significant charge asymmetry is observed in the pion modes. These results supersede our previous measurements \([12]\).

**IX. ACKNOWLEDGMENTS**

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