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Distributed Generation Investment by a Microgrid under Uncertainty

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ABSTRACT. This paper examines a California-based microgrid's decision to invest in a distributed generation (DG) unit fuelled by natural gas. While the long-term natural gas generation cost is stochastic, we initially assume that the microgrid may purchase electricity at a fixed retail rate from its utility. Using the real options approach, we find a natural gas generation cost threshold that triggers DG investment. Furthermore, the consideration of operational flexibility by the microgrid increases DG investment, while the option to disconnect from the utility is not attractive. By allowing the electricity price to be stochastic, we next determine an investment threshold boundary and find that high electricity price volatility relative to that of natural gas generation cost delays investment while simultaneously increasing the value of the investment. We conclude by using this result to find the implicit option value of the DG unit when two sources of uncertainty exist.

KEYWORDS. Distributed Generation; Real Options; Optimal Investment; Energy Markets.

1. INTRODUCTION

The objective of deregulated electricity sectors is to improve economic efficiency by providing market signals to participants (see [1]). In competitive electricity markets, both producers and

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consumers should make decisions better suited to their circumstances than those in the regulated paradigm. Traditionally, the viewpoint was that the electricity sector exhibits characteristics of a "natural monopoly," i.e., costs that decline with output. This necessitated a vertically integrated structure with franchise monopolies operating in distinct regions. Such an industrial structure, however, turned the potentially competitive generation function of the sector into a *de facto* monopoly. Indeed, little evidence exists that large generation companies are necessary to achieve economies of scale (see [2]). Due to the observed inefficiencies of this structure, such as low capital and labour productivity, a gap between retail and wholesale prices, and poor energy efficiency because of heat and distribution losses (see [3]), many jurisdictions have deregulated their electricity sectors over the past twenty years. In a broad sense, these measures have kept functions with "natural monopoly" characteristics, such as distribution and transmission, under the control of regulatory agencies, while opening up the generation and retailing functions to competition. As with other sectors of the economy, greater efficiency may then be achieved by matching demand and supply in a decentralised fashion such that consumers and producers make decisions based on their own utility- and profit-maximising objectives.

By potentially providing high-resolution price signals to market participants, electricity market restructuring may also enable the emergence of microgrids, which are energy sources and sinks that normally operate in parallel with the grid, but which can function as islands. To do so, a microgrid must apply local on-site control and will contain one of (or a combination of) the following: fossil- or biomass-fired distributed generation (DG) units possibly with combined heat and power (CHP) applications matched to non-electrical energy requirements such as building heating and/or cooling, demand control, or local renewable harvesting (see [4]). Facilitated by such exposure to price signals and a favourable regulatory regime, a greater proportion of electricity generation may then take place closer to loads than in the familiar centralised, regulated paradigm (see [5]). Although microgrids were historically a response to reliability issues and transmission constraints, they now also have an economic imperative due to the possibility of financial incentives, e.g., avoiding energy purchases during peak periods and/or creation of carbon credits. Thus, microgrids utilising small-scale, on-site DG offer tangible benefits stemming from the possibility of lower-cost electricity and greater system energy efficiency along with a lower carbon footprint derived from CHP applications. Additionally, microgrids may offer "qualitative" improvements, such as power quality and reliability (PQR) more customised to end-use requirements and improved prospects for the adoption of small-scale renewable energy technologies. PQR benefits are characterised as "qualitative" here not because they are without genuine economic value, but rather because methods for incorporating their contribution to microgrid value streams are currently rudimentary. Nevertheless, considerable regulatory barriers currently inhibit more widespread adoption of DG, ranging from poorly defined and enforced interconnection standards to tariff components such as standby charges and exit fees. Ample research and folklore exists on the struggles that self-generators face when trying to coexist in the power system with enormous entrenched electricity utilities (see [6]). However, we focus here on solely the economics of DG investment by a microgrid facing risk from uncertain electricity and fuel prices.

From a microgrid's perspective, making investment and operational decisions concerning DG units should involve an assessment of the uncertainty in both electricity and fuel prices. While the former are largely fixed in the case of utility-provided time-of-use (TOU) tariffs, the latter may exhibit considerable volatility, being typically subject to monthly procurement cost revision. A microgrid should, therefore, account for this uncertainty when making investment and operational decisions. Whereas in previous studies we have strived to model the economics and thermodynamics of a microgrid in a detailed, but purely deterministic, setting (see, for example, [7], [8], and [9]), here, we incorporate randomness into our analysis, thereby necessitating abstraction from some real-world considerations.[1] Specifically, in this paper, we examine the problem of a California-based[2] microgrid via the real options approach to determine fuel cost thresholds below which it is optimal for the microgrid to install DG. We next extend the basic model to allow for operational flexibility and also evaluate the option to disconnect entirely from the utility. Finally, we permit the electricity price to be stochastic in order to examine its impact on the microgrid's investment decision. Where possible, we contrast the results yielded by the real options approach with those implied by a traditional, deterministic discounted cash flow (DCF) analysis.

2. PROBLEM FORMULATION

We assume that a microgrid has a constant electric load,[3] $\frac{Q}{8760}$ (kW_e), that it must serve via either utility purchases or a DG unit in which it has the discretion to invest. The turnkey cost of the DG unit, I (US$), is deterministic and includes all purchase, shipping, and installation costs. For now, we assume a deterministic utility electricity price, P (US$/$kWh_e$), whereas the long-term natural gas generation cost, C (US$/$kWh_e$),[4] evolves according to a geometric Brownian motion (GBM) process as follows (see [11]):

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[1] A related paper investigates sequential DG and CHP investment strategies under uncertainty (see [10]).

[2] The California case is compelling not only because of the state's efforts with deregulation and cost-reflective retail tariff rates for electricity, but also because of its large agricultural, commercial, and industrial energy base. Furthermore, the state has several DG subsidy programmes funded both by the California Public Utilities Commission, e.g., the California Self-Generation Incentive Program, and the California Energy Commission, e.g., the Rebate Program for Wind & Fuel Cell Renewable Energy Electric-Generating Systems.

[3] The constant-load assumption may be relaxed without complicating the analysis as long as the load remains deterministic. Indeed, as long as the conditional expected cash flows may be calculated, e.g., by integration, then including time-varying loads poses no technical difficulty. However, for clarity of exposition and in order to explain the intuition behind the microgrid's decision-making under uncertainty, we avoid this real-world feature.

[4] This is calculated by multiplying the natural gas fuel cost (US$/kWh) by the heat rate (kWh/kWh_e) of the DG unit.

$$dC = \alpha C dt + \sigma C dz \tag{1}$$

This implies that successive percentage changes in C are independent of each other, which is a reasonable model for the long-term evolution of commodity prices (see [12]). Consequently, the natural gas generation cost at time t given the initial cost, C_0, is lognormally distributed with mean $C_0 e^{\alpha t}$. Here, α is the annual growth rate of C, σ is the annual volatility of C, and dz is the increment to a standard Wiener process. We additionally define the annual real risk-free interest rate as r and let δ be the convenience yield associated with natural gas generation, i.e., the risk-adjusted rate of return on natural gas generation minus its growth rate. Intuitively, this is the opportunity cost of retaining the option to invest. For a financial call option, δ is the forgone dividend. Finally, we assume that once the DG unit is installed, its effective lifetime is infinite due to the possibility of maintenance upgrades. This simplification is further justified by the fact that the discrepancy between the present value (PV) of a perpetuity and the PV of an annuity decreases with the length of the time horizon. For example, if the actual lifetime of a DG unit is thirty years, then the ratio of the latter to the former per dollar for $\delta = 0.04$ is $1 - \frac{1}{(1+\delta)^{30}} = 0.6917$.

Given price uncertainty and managerial flexibility, we use the real options approach (see [13]) to model the microgrid's decision-making. This approach is appropriate because it trades off in continuous time the PV of benefits from immediate investment with its associated costs. Specifically, the real options approach includes not only the tangible investment costs such as the turnkey cost, but also the opportunity cost of exercising the option to invest, which is the loss of the discretion to wait for more information. Indeed, at times, it may be better to retain the

For example, the 500 kW_e DG unit considered in this paper has a heat rate of 3.01. If the price of natural gas is option to invest even for a project that is "in the money" from the DCF perspective. Analogous to the pricing of financial call options (see [14]), the real options approach constructs a risk-free portfolio using a short position on the underlying asset and then equates its expected appreciation (net of any dividend payments) to the instantaneous risk-free rate that could have been earned by investing in the portfolio. For a perpetual option, the resulting partial differential equation (PDE) from this "no arbitrage" condition becomes an ordinary differential equation (ODE), which is solved analytically using boundary conditions. As part of the solution, an investment threshold price for the underlying asset is obtained, at which investment is triggered. If an investment opportunity also offers embedded options, such as the discretion to abandon or to suspend and re-start production, then the approach is still valid, but may not yield closed-form analytical solutions. Nevertheless, the threshold prices and resulting option values may be obtained numerically.

Of course, as with any quantitative approach, the real options one has its limitations. In particular, the assumption that a risk-free portfolio may be constructed does not apply in some commodities markets. Furthermore, the price process may not be exogenous, nor may the parameters governing asset price dynamics be constant. The first issue is usually avoided by trading a surrogate underlying commodity that is closely related to the one in question, whereas the second may be addressed by modelling the industry equilibrium (see [15]). As natural gas, the fuel for the DG unit we consider, is widely traded, the first issue is not relevant. And, since we take the perspective of a microgrid, i.e., a market participant small enough not to have any impact on the equilibrium price, the second issue is also not of concern here. The third issue (pertaining to asset price parameters) is important and should be accounted for using a stochastic volatility model (see [16]). However, since the purpose of this paper is to illustrate the

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currently US$0.0324/kWh, then the natural gas generation cost is US$0.097524/kWh$_e$.

microgrid's behaviour under uncertainty, the additional insight provided by stochastic volatility is approximated by doing sensitivity analysis on the volatility parameter. In Section 3.3, we also consider a case with stochastic electricity prices, but we first turn to the base case with a fixed electricity price.

3. NUMERICAL EXAMPLES

3.1. Case 1: Option to Invest

We first consider a simple case in which the microgrid has the option to invest in a 500 kW_e DG unit without any operational flexibility. Specifically, assume that there are two states of the world: one in which the microgrid holds the option to invest in DG and the other in which it has exercised the option to meet its load solely from on-site generation. In the former situation, the microgrid receives no incremental electricity cost savings, and in the latter, its present value of cost savings is $\frac{P}{r}-\frac{C}{\delta}+\frac{X}{rQ}$ per kWh_e, which is simply the difference between two perpetuities plus the PV of the savings from avoiding the utility customer fee. Note that the natural gas generation cost is discounted using the risk-adjusted convenience yield to account for the uncertainty in C. Here, X (in US$) is the annual customer charge paid to the utility. For the San Diego Gas and Electric (SDG&E) utility, this was approximately US$50 per month in the year 2000, which implies $X = 600$.

Letting $V_0(C)$ be the value per kWh_e of the option to invest and $V_1(C)=\frac{P}{r}-\frac{C}{\delta}+\frac{X}{rQ}$ be the expected PV per kWh_e of the microgrid's cost savings with DG installation, we now construct a risk-free portfolio, Φ, consisting of one unit of $V_0(C)$ and short $V_0'(C)$ units of natural gas (kWh_e equivalent). By equating the instantaneous risk-free return on an investment of amount Φ to the expected appreciation of Φ less any dividend payments, we obtain:

$$r\Phi dt = E[d\Phi] - \delta C V_0'(C)dt \tag{2}$$

In order to simplify the right-hand side of Eq. (2), we first note that:

$$\begin{aligned} &\Phi = V_0(C) - C V_0'(C) \\ &\Rightarrow d\Phi = dV_0 - V_0'(C)dC \end{aligned} \tag{3}$$

Next, we apply Itô's Lemma to obtain:

$$dV_0 = V_0'(C)dC + \frac{1}{2}V_0''(C)(dC)^2 \tag{4}$$

By substituting Eq. (4) into Eq. (3) and taking expectations, we obtain:

$$\begin{aligned} &d\Phi = \frac{1}{2}V_0''(C)(dC)^2 \\ &\Rightarrow E[d\Phi] = E\left[\frac{1}{2}V_0''(C)(dC)^2\right] \\ &\Rightarrow E[d\Phi] = \frac{1}{2}V_0''(C)\sigma^2 C^2 dt \end{aligned} \tag{5}$$

Finally, we substitute Eq. (5) into the right-hand side of Eq. (2) and re-arrange to obtain the following ODE:

$$\frac{1}{2}\sigma^2 C^2 V_0''(C) + (r-\delta)C V_0'(C) - rV_0(C) = 0 \tag{6}$$

Applying the boundary condition[5] $\lim_{C\to\infty} V_0(C) = 0$, the solution to Eq. (6) is:

$$V_0(C) = A_2 C^{\beta_2} \tag{7}$$

Here, $A_2 > 0$ and $\beta_2 = \frac{1}{2} - \frac{(r-\delta)}{\sigma^2} - \sqrt{\left(\frac{(r-\delta)}{\sigma^2} - \frac{1}{2}\right)^2 + \frac{2r}{\sigma^2}} < 0$ are constants, with A_2 to be determined endogenously.[6] Note that Eq. (7) implies that the value of the option to invest is

[5] This boundary condition implies that the option to invest becomes worthless as the natural gas generation cost increases without bound.

high when the natural gas generation cost is low. Using the following value-matching and smooth-pasting conditions, we solve for A_2 and the investment threshold cost, C_I:

$$V_0(C_I) = V_1(C_I) - \frac{I}{Q} \tag{8}$$

$$V_0'(C_I) = V_1'(C_I) \tag{9}$$

Equation (8) states that upon exercise, the microgrid receives a cash flow equal to the PV from an installed DG unit minus the investment cost. As for Eq. (9), it is a first-order condition that equates the marginal benefit of delaying investment (stemming from more information about the natural gas price) with the marginal cost of delaying investment (due to a lower PV) at the point of exercise; indeed, if they do not equate, then the holder of the option would be better off exercising it either earlier or later.

The parameters for this and subsequent cases (unless otherwise indicated) are given in Table 1 and correspond roughly to the situation for a microgrid in the service territory of the SDG&E utility during the year 2000 (see [9]). Using these data, we find that $\beta_2 = -4.2405$, while the closed-form solutions to Eqs. (8) and (9) reveal that $C_I = \frac{\beta_2 \delta}{\beta_2 - 1}\left(\frac{P}{r} + \frac{X}{rQ} - \frac{I}{Q}\right) = 0.077334$ and $A_2 = -\frac{1}{\delta \beta_2 C_I^{\beta_2 - 1}} = 8.8112 \times 10^{-6}$. In other words, the microgrid should install a 500 kW_e DG unit only if the natural gas generation cost decreases to US\$0.077334/kWh_e. As illustrated in Fig. 1, the value to retain the option to invest in DG is greater than the net present value (NPV) of the cost savings from installed DG as long as the natural gas cost is greater than US\$0.077334/kWh_e. At C_I, the value of the option is exactly equal to the NPV of cost savings,

[6] β_2 is simply the negative root of the characteristic quadratic equation $\frac{1}{2}\sigma^2 \beta(\beta - 1) + (r - \delta)\beta - r = 0$.

thereby triggering investment. Note that $V_0(C)$ is defined only over $C \geq C_I$ since the option to invest is exercised for any generation cost below the threshold value. Thus, the difference between the two curves is the value of waiting for more information by delaying investment, which is also the opportunity cost of investment. Performing sensitivity analysis as in Fig. 2 by varying the value of σ, *ceteris paribus*,[7] illustrates that the investment threshold decreases with increasing σ, thereby signifying an increase in the value of the option to invest stemming from a greater value of waiting when there is more uncertainty.

For comparison, a completely deterministic evaluation of this investment opportunity recommends waiting until the natural gas generation cost drops to $C_I^{\det} = P + \frac{X}{Q} - \frac{rI}{Q} = 0.0956$ before installing DG.[8] This is because the deterministic DCF decision rule is to invest as long as the NPV of the active DG unit is non-negative:

$$\begin{aligned} &\frac{P}{r} - \frac{C}{r} + \frac{X}{rQ} - \frac{I}{Q} \geq 0 \\ &\Rightarrow \frac{C}{r} \leq \frac{P}{r} + \frac{X}{rQ} - \frac{I}{Q} \\ &\Rightarrow C_I^{\det} = P + \frac{X}{Q} - \frac{rI}{Q} \end{aligned} \tag{10}$$

Intuitively, Eq. (10) states that investment under certainty occurs if the cost of on-site generation plus the amortised investment cost per kWh_e of the DG unit is less than the electricity price and the customer charge per kWh_e. Since this approach neglects the opportunity cost of waiting[9]

[7] Technically, since δ depends on σ, it should also be varied with σ. However, it is kept constant here simply to illustrate the sensitivity of C_I on σ.

[8] This is the same answer if one uses $\sigma = 0.000001$ in the expression for C_I.

[9] As mentioned earlier, there is a value from waiting that is included in the real options approach. Once the investment opportunity is exercised, this value is no longer available. Hence, it is an opportunity cost to investing since it is an implicit cost that is added on to the direct investment cost, I, which then postpones investment

before investing, it suggests investing sooner than the real options approach. Consequently, the value of the investment opportunity according to the DCF approach at $C_0 = P = 0.10$ is $\frac{P}{r} - \frac{C_0}{r} + \frac{X}{rQ} - \frac{I}{Q} = -0.1107$, whereas with the real options approach it is $V_0(C_0) = A_2 C_0^{\beta_2} = 0.1533$. Thus, a microgrid would not be willing to invest now in DG via the deterministic DCF approach, but may do so in the future if it uses the real options one. In the next section, we examine how may be facilitated by consideration of operational flexibility.

3.2. Case 2: Operational Flexibility and the Option to Disconnect

Typically, DG units have the capability to be turned on and off at a cost in response to the natural gas cost. Furthermore, if the natural gas generation cost decreases sufficiently, then the microgrid may consider disconnecting from the utility altogether in order to save on customer charges. We now investigate the options to shut down and re-start the installed DG unit at costs S and R (both in US$), respectively, as well as the option to disconnect permanently from the utility. We, thus, have four states in this system as opposed to two in Section 3.1:

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exercise.

0. DG is not installed

1. DG is installed and operational with utility connection available

2. DG is installed and switched off with utility connection available

3. DG is installed and operational without utility connection

In state 0, the PV of the microgrid's cost savings is simply the following:[10]

$$V_0(C) = A_2^F C^{\beta_2} \tag{11}$$

In state 1, the microgrid may exercise the option to suspend DG operation if the cost of natural gas generation becomes sufficiently high relative to the price of utility-provided electricity. Alternatively, if the cost of natural gas generation decreases sufficiently relative to P, then the microgrid may exercise the option to disconnect permanently from the utility. Otherwise, if it operates the DG unit, then it obtains the PV of cost savings relative to utility electricity purchases. Therefore, the PV of the microgrid's cost savings in the second state is:

$$V_1(C) = B_1 C^{\beta_1} + B_2 C^{\beta_2} + \frac{P}{r} - \frac{C}{\delta} \tag{12}$$

Here, $B_1 > 0$, $B_2 > 0$, $\beta_1 = \frac{1}{2} - \frac{(r-\delta)}{\sigma^2} + \sqrt{\left(\frac{(r-\delta)}{\sigma^2} - \frac{1}{2}\right)^2 + \frac{2r}{\sigma^2}} = 5.2405 > 1$, and $\beta_2 = -4.2405$ (as in Section 3.1) are constants, with B_1 and B_2 endogenous to the system. Note that the first term of Eq. (12) is the option to shut down, which becomes more valuable as the natural gas generation cost increases, the second term is the option to disconnect, which becomes more valuable as the natural gas generation cost decreases, and the remaining terms comprise the PV of cost savings per kWh_e from using DG rather than utility purchases.

[10] The superscript refers to a case with flexibility in order to distinguish the coefficient from that in Section 3.1.

In a suspended state, the microgrid's PV of cost savings reflects the option to re-start:

$$V_2(C) = D_2 C^{\beta_2} \tag{13}$$

Again, $D_2 > 0$ is an endogenous constant, and Eq. (13) indicates that when DG operation is suspended, the microgrid does not accrue any cost savings and re-starts the DG unit when the cost of natural gas generation decreases sufficiently. Finally, from the operational state, the microgrid may also choose to disconnect permanently from the utility should the cost of natural gas generation drop significantly. If it exercises this option, then the PV of its costs savings is similar to that in state 1 from the previous numerical example:

$$V_3(C) = \frac{P}{r} - \frac{C}{\delta} + \frac{X}{rQ} \tag{14}$$

Given the value functions in Eqs. (11), (12), (13), and (14), we also require the following value-matching and smooth-pasting conditions (as well as the ones in Eqs. (8) and (9)) describing transitions among the states[11]:

$$V_1(C_S) = V_2(C_S) - \frac{S}{Q} \tag{15}$$

$$V_1'(C_S) = V_2'(C_S) \tag{16}$$

$$V_2(C_R) = V_1(C_R) - \frac{R}{Q} \tag{17}$$

$$V_2'(C_R) = V_1'(C_R) \tag{18}$$

$$V_1(C_X) = V_3(C_X) \tag{19}$$

$$V_1'(C_X) = V_3'(C_X) \tag{20}$$

[11] The intuition behind these is similar to that for Eqs. (8) and (9).

These eight equations may be solved numerically for the following eight unknowns: the four endogenous constants A_2^F, B_1, B_2, and D_2, along with the investment threshold cost with operational flexibility, C_I^F, the shutdown and re-start threshold costs, C_S and C_R, respectively, and the disconnection threshold cost, C_X.

Since the resulting system of equations is highly non-linear, there is no analytical solution to it. Nevertheless, for $S = R = 5000$ and the parameters in Table 1, we numerically obtain $A_2^F = 1.0721 \times 10^{-5}$, $B_1 = 45085$, $B_2 = 1.712 \times 10^{-9}$, $D_2 = 1.4896 \times 10^{-5}$, $C_I^F = 0.085$, $C_S = 0.104$, $C_R = 0.096$, and $C_X = 0.038$.[12] Note that $C_I^F > C_I$, i.e., investment is accelerated due to operational flexibility. These results imply that operational flexibility makes it easier for the microgrid to invest by increasing the NPV of an installed DG unit since the microgrid could always revert to utility purchases in case of sustained increases in the natural gas generation cost. In addition, the microgrid waits until the natural gas generation cost is well above (below) P before turning off (on) the DG unit. This reflects not only the explicit cost S (R) of turning off (on) an active (inactive) DG unit, but also the implicit cost R (S) that the microgrid incurs in the future if it ever re-starts (shuts down) an inactive (active) DG unit. In effect, the microgrid wants to avoid a situation in which it turns off (on) a marginally unprofitable (profitable) unit only to have to turn it back on (off) again shortly. Although the option to disconnect is

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[12] It should be noted that the shutdown and re-start costs are estimated using the annual variable operating and maintenance (O&M) cost for this DG unit as follows: $\frac{US\$0.012}{kWh_e} \cdot 8760h \cdot 500kW_e = US\$52{,}560$. In other words, the approximate annual incremental O&M costs of this DG unit assuming constant output at rated capacity is US$50,000. Since the optimal operating strategy of the microgrid is not known in advance, it is not clear how this additional amount should be allocated to each shutdown and re-start decision. As a convention, we assume that, on average, there will be ten such decisions during a year, which yields $S = 5000$ and $R = 5000$ as additional costs associated with operational flexibility.

available, it does not seem likely to be exercised in the short term because the PV of its benefit, i.e., the cost savings from not having to pay the utility's customer charge, is outweighed by the option to buy electricity from the utility in the event of natural gas generation cost increases. Therefore, the disconnection cost threshold is set much lower than P; effectively, the microgrid must expect sustained positive cash flows from DG operation in the future before exercising the disconnection option.

Fig. 3 illustrates the value of the re-start option along with $V_0(C)$ and $V_1(C)$. Again, we draw the curves only over the regions in which they are defined. Specifically, $V_0(C)$, $V_1(C)$, $V_2(C)$, and $V_3(C)$ exist over $C \geq C_I^F$, $C_S \geq C \geq C_X$, $C \geq C_R$, and $C \geq C_X$, respectively. Since C_X is very low, neither it nor $V_3(C)$ is indicated on the graph. We observe from Eqs. (12) and (14) that $V_1(C)$ is almost linear, i.e., similar to $V_3(C)$, for low C, and then resembles the option to re-start, $V_2(C)$, for intermediate C. Furthermore, at each threshold price, the discontinuous jump between the appropriate curves is equal to the fixed cost per kWh_e of switching states. For example, at C_I^F, it is optimal to invest in the DG unit, i.e., jump from curve $V_0(C)$ to $V_1(C)$, at which point the difference between the two curves reflects the investment cost per kWh_e.

As indicated in Section 3.1, greater uncertainty also makes the microgrid more hesitant to act, whether to invest, suspend, re-start, or disconnect. Fig. 4 illustrates the effect of natural gas generation cost volatility on the relevant thresholds, where we again treat δ and σ as independent parameters. In particular, greater volatility at a relatively high natural gas cost not only increases the option value to invest, but also delays investment as the microgrid waits for the natural gas cost to fall sufficiently in order to ensure that any cost decrease is sustainable. In fact, the microgrid becomes more likely to face extremely high and unbounded costs while simultaneously experiencing extremely low, but bounded (by zero), costs. Since the effect of the former is stronger than that of the latter, the microgrid is more reluctant to invest. Therefore, higher natural gas cost volatility makes investment in DG both more valuable and less likely.[13] This also has the effect of delaying any decision to suspend DG operation because the implicit reward of shutting down, i.e., the option to re-start DG, increases with natural gas volatility. Similarly, disconnecting completely from the utility becomes highly improbable for even a moderate level of volatility. Hence, there is a wider zone of inaction as volatility increases.

3.3. Case 3: Stochastic Electricity Price

Although end-use consumers in California do not currently face real-time (volatile) electricity prices, they did for a few months in the years 1999 and 2000 (see [17]). We model this perturbation by allowing the electricity price to evolve according to a GBM process as follows:

$$dP = \alpha_P P dt + \sigma_P P dz_P \tag{21}$$

In addition, we assume that δ_P is the convenience yield on electricity and that the electricity price has instantaneous correlation ρdt with the cost of natural gas generation, which may be very high if natural gas figures prominently in the overall generation fuel mix. Indeed, in California, almost 38% of the electricity generated uses natural gas as its fuel (see [18]). Proceeding analogously to Section 3.1, we construct a risk-free portfolio by using the option to invest in DG, $F(P,C)$, along with short positions on both P and C. Letting $p \equiv \frac{P}{C}$, $F(P,C) = Cf(p)$, and assuming the option value is homogeneous[14] in (P,C), we obtain the following ODE (see the Appendix for details):

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[13] The DG investment opportunity is similar to a put option on natural gas generation, which increases in value with the volatility of the underlying asset since this makes extremely low prices more probable.

[14] This is permissible as long as the NPV of the active investment depends only on P and C. In order to ensure

$$\frac{1}{2}\left(\sigma_P{}^2 - 2\rho\sigma_P\sigma + \sigma^2\right)p^2 f''(p) + \left(\delta - \delta_P\right)pf'(p) - \delta f(p) = 0 \tag{22}$$

The closed-form solution to this after applying the value-matching and smooth-pasting conditions $f(p_I) = \frac{p_I}{\delta_P} - \frac{1}{\delta}$ and $f'(p_I) = \frac{1}{\delta_P}$, respectively, is $f(p) = a_1 p^{\gamma_1}$, where $p_I = \frac{\gamma_1 \delta_P}{(\gamma_1 - 1)\delta}$ is the investment threshold ratio, $a_1 = \frac{1}{\gamma_1 p_I^{\gamma_1 - 1}\delta_P}$, and $\gamma_1 = \frac{-(t-s) + \sqrt{(t-s)^2 - 4su}}{2s} > 1$, using the constants $s \equiv \frac{\sigma_P{}^2 - 2\rho\sigma_P\sigma + \sigma^2}{2}$, $t \equiv \delta - \delta_P$, and $u \equiv -\delta$. This is analogous to the solution to the ODE in Eq. (6) of Section 3.1 except that the boundary condition implies that the option value is increasing in the underlying variable, p.

Using a deterministic electricity price as in Section 3.1 indicates that for $P = 0.10$, the investment cost threshold is $C_I = 0.0809$ and $A_2 = 1.1172 \times 10^{-5}$, whereas the model with a stochastic electricity price reveals $C_I = 0.0867$ for $\sigma_P = 0.03$ and $\rho = 0.80$ (see Figs. 5 and 6). For the latter, we obtain $\gamma_1 = 7.545$, $a_1 = 1.3066$, and $p_I = 1.1528$, and the corresponding option values to invest in the two cases given an initial natural gas generation cost equal to P are US\$0.85M and US\$0.57M, respectively. Intuitively, relatively low volatility in P along with high ρ reduces the scope for large cost savings from DG operation, thereby making it more attractive to invest sooner when both prices are stochastic. Relative to the case with a deterministic electricity price, when C decreases here, P also decreases, but relatively slightly, which cuts the cost savings from on-site generation. Conversely, when $C > P$, the losses from on-site generation are reduced. However, since only savings from on-site generation are bounded, they are more affected; hence, this reduces the option value of waiting.

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this, we set both the turnkey cost of the unit and the utility customer charge equal to zero.

In contrast, when P is relatively volatile, there is greater chance of both exceptionally high and low cost savings from DG operation. In this case, it is beneficial for the microgrid to wait longer. Here, if C decreases, then P also decreases, but to a greater extent (reflecting its larger relative volatility), which results in economic losses from distributed generation investment that are bounded as P cannot become negative. Alternatively, if C increases, then P also increases by a relatively greater amount, which results in savings from on-site generation that are unbounded as P can conceivably increase without limit. Consequently, this greater relative uncertainty increases the option value of the DG investment opportunity. For example, if $\sigma_P = 0.12$, then $C_I = 0.0753$, and the option value to invest is US$1.14M (see Fig. 7). Finally, using the value for p_I, we create an investment threshold boundary to indicate the highest possible natural gas generation cost that permits DG investment for each level of electricity price (see Fig. 8). As σ_P increases, *ceteris paribus*, the slope of the boundary increases, thereby reducing the size of the region in which investment is optimal.

4. SUMMARY

The ongoing deregulation of electricity industries worldwide provides scope for more decentralised decision-making as a means towards improving economic efficiency in electricity provision. To the extent that policymakers seek to create incentives for market participants to act in a socially beneficial manner, they should aim to achieve outcomes in which energy resources are utilised consistent with their relative marginal values, inclusive of environmental footprint. In a competitive regime, however, market participants should also account for price risk when making decisions, a dimension that was largely absent in the era of price-regulated vertical integration in both electricity and natural gas supply. As we indicate in Section 3.1, there is a loss in investment value from neglecting the managerial flexibility in decision-making under uncertainty. Therefore, the standard DCF approach for analysing investment and operations should be either modified or replaced by techniques that explicitly factor in the roles of price volatility and managerial discretion.

This paper takes a real options approach to analyse the investment and operational decisions of a California microgrid. Threshold fuel costs are derived for triggering investment in DG, suspending and re-starting DG operations, and disconnecting from the electricity utility altogether. First, in the case without operational flexibility, the DG investment opportunity is worth more with the real options approach than with the deterministic DCF one. Indeed, even if it is not worthwhile to proceed with DG installation immediately, the right to make such an investment is more attractive once managerial discretion under uncertainty is considered. As a sensitivity analysis, increasing the natural gas generation cost volatility decreases the investment threshold subsequently since the opportunity cost of killing the option to wait is also greater when there is more uncertainty.

Next, it is observed that greater operational flexibility makes DG investment more attractive for the microgrid, while the disconnection option is exercised only in the rare case of sustained natural gas cost decreases. Due to the fixed shutdown and re-start costs, it is optimal for the microgrid to suspend (re-start) the DG unit only when the natural gas generation cost rises above (falls below) the electricity price. Again, sensitivity analysis with the natural gas generation cost volatility provides some insight: more uncertainty in the system causes the microgrid to wait longer before making operational changes as it would not want to turn off (on) a marginally unprofitable unit only to have to turn it back on (off) shortly thereafter. More profoundly, as the natural gas generation cost volatility increases, it increases the value of the shutdown (re-start) option; however, this also increases the opportunity cost of shutting down (re-starting), thereby making it optimal to delay the action. An extension to the basic model to permit stochastic electricity prices (positively correlated with natural gas generation costs) indicates that relatively low volatility in electricity prices increases immediate investment in DG as the microgrid has less chance of sustained cost saving increases from waiting longer. Conversely, highly volatile electricity prices decrease immediate investment as the microgrid is able to increase cost savings by waiting.

Although it is not possible to verify or refute our findings quantitatively, their qualitative insights have practical bearing for commercial and industrial consumers in a large, deregulated jurisdiction such as California. For example, a case study performed for Joseph Gallo Dairy Farms of Atwater, CA indicates that the 700 kW_e of on-site generation currently in operation would not have been economical without subsidies, a methane digester for producing biogas (used as fuel instead of propane), or the opportunity to proceed with the investment in stages (see [19]). This outcome highlights the importance of incorporating managerial flexibility in the planning stage as well as the risk from facing volatile fuel prices. Since both are features of the real options approach, we feel that it may provide more meaningful insights into the behaviour of microgrids than the standard DCF one.

While the analysis conducted here is purely economic, it should be noted that the diffusion of emerging microgrid technologies will likely be determined as much by the regulatory environment as by economic fundamentals. Indeed, analysing, designing, and permitting actual microgrids will be a lengthy and costly process, which also adds a great deal of uncertainty to project costs. Thus, we recognise that a project financial analysis is no more than an early step along the long road to project commissioning. Actual systems will require considerable engineering as well as legal and regulatory work before real-world viability could be fully assessed. Nevertheless, in this paper, we have endeavoured to identify the loss in value when risk and uncertainty are not accounted for as well as to explicate the changes in decision-making that should be considered relative to the deterministic DCF approach. Given economic uncertainty in other energy sectors, we believe that the approach outlined here would be appropriate in those settings as well since it enables decision-makers to consider the benefits of timing and operational flexibility in mitigating price or cost risk. For future work, we intend to examine incremental investment under uncertainty in a portfolio of alternatively sized DG technologies by a microgrid (see, for example, [20] , [21], and [22]) as well as the options to upgrade to CHP-enabled DG units and to sell electricity back to the grid or to nearby consumers.

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APPENDIX

Here, we derive the value of the option to invest in a DG unit without operational flexibility or turnkey costs when there is uncertainty in both the electricity price and the marginal cost of natural gas generation. As in Section 3.1, we construct a risk-free portfolio, Φ, by using the option to invest in DG, $F(P,C)$, along with short positions on both P and C:

$$\Phi = F - F_P P - F_C C \qquad \text{(A-1)}$$

Totally differentiating Eq. (A-1) implies:

$$d\Phi = dF - F_P dP - F_C dC \qquad \text{(A-2)}$$

Applying Itô's Lemma, we obtain:

$$dF = F_P dP + F_C dC + \frac{1}{2} F_{PP} (dP)^2 + \frac{1}{2} F_{CC} (dC)^2 + F_{PC} dPdC \qquad \text{(A-3)}$$

We next substitute Eq. (A-3) into Eq. (A-2) and take expectations:

$$\begin{aligned} & d\Phi = \frac{1}{2} F_{PP} (dP)^2 + \frac{1}{2} F_{CC} (dC)^2 + F_{PC} dPdC \\ & \Rightarrow E[d\Phi] = \frac{1}{2} F_{PP} \sigma_P^2 P^2 dt + \frac{1}{2} F_{CC} \sigma^2 C^2 dt + F_{PC} \sigma_P \sigma PC \rho dt \end{aligned} \qquad \text{(A-4)}$$

From the no-arbitrage condition, the instantaneous risk-free return on an investment of amount Φ must equal the expected appreciation of Φ less any dividend payments:

$$r\Phi dt = E[d\Phi] - \delta_P P F_P dt - \delta C F_C dt \qquad \text{(A-5)}$$

Substituting Eqs. (A-1) and (A-4) into Eq. (A-5), we obtain:

$$\begin{aligned} & rF - rF_P P - rF_C C = \frac{1}{2} F_{PP} \sigma_P^2 P^2 + \frac{1}{2} F_{CC} \sigma^2 C^2 + F_{PC} \sigma_P \sigma \rho PC - \delta_P P F_P - \delta C F_C \\ & \Rightarrow \frac{1}{2} F_{PP} \sigma_P^2 P^2 + \frac{1}{2} F_{CC} \sigma^2 C^2 + F_{PC} \sigma_P \sigma \rho PC + (r - \delta_P) P F_P + (r - \delta) C F_C - rF = 0 \end{aligned} \qquad \text{(A-6)}$$

By using the change of variables $p \equiv \frac{P}{C}$, $F(P,C) = Cf(p)$, and assuming the option value is homogeneous in (P,C), we convert the PDE in Eq. (A-6) into the following ODE (see [13]):

$$\frac{1}{2}\left(\sigma_P{}^2 - 2\rho\sigma_P\sigma + \sigma^2\right)p^2 f''(p) + (\delta - \delta_P)pf'(p) - \delta f(p) = 0 \tag{A-7}$$

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FIGURE CAPTIONS

Figure 1. Value of Investment Opportunity for $\sigma = 0.06$ *(Case 1).*

Figure 2. Investment Threshold Cost (Case 1).

Figure 3. Value of Investment Opportunity for $\sigma = 0.06$ *(Including Options to Shutdown, Re-start, and Disconnect).*

Figure 4. Investment, Shutdown, Re-start, and Disconnection Cost Thresholds.

Figure 5. Value of Investment Opportunity (Deterministic Electricity Price and Zero Turnkey Cost).

Figure 6. Value of Investment Opportunity (Low-Volatility Stochastic Electricity Price and Zero Turnkey Cost).

Figure 7. Value of Investment Opportunity (High-Volatility Stochastic Electricity Price and Zero Turnkey Cost).

Figure 8. Investment Threshold Boundary for High-Volatility Stochastic Electricity Price.

TABLES

Table 1. Base Case Parameter Values.

Parameter	Value
P	US\$0.10/kWh$_e$
I	US\$0.50M
$\frac{Q}{8760}$	500 kW$_e$
X	US\$600
σ	0.06
δ	0.04
r	0.04

FIGURES

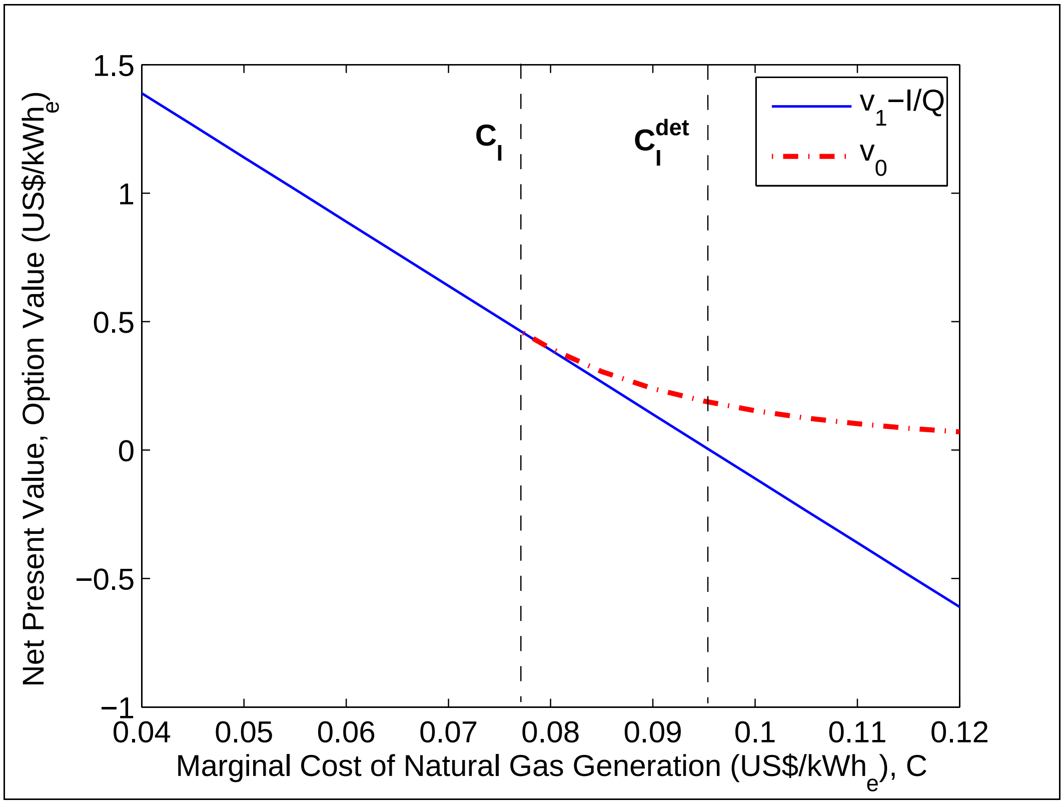
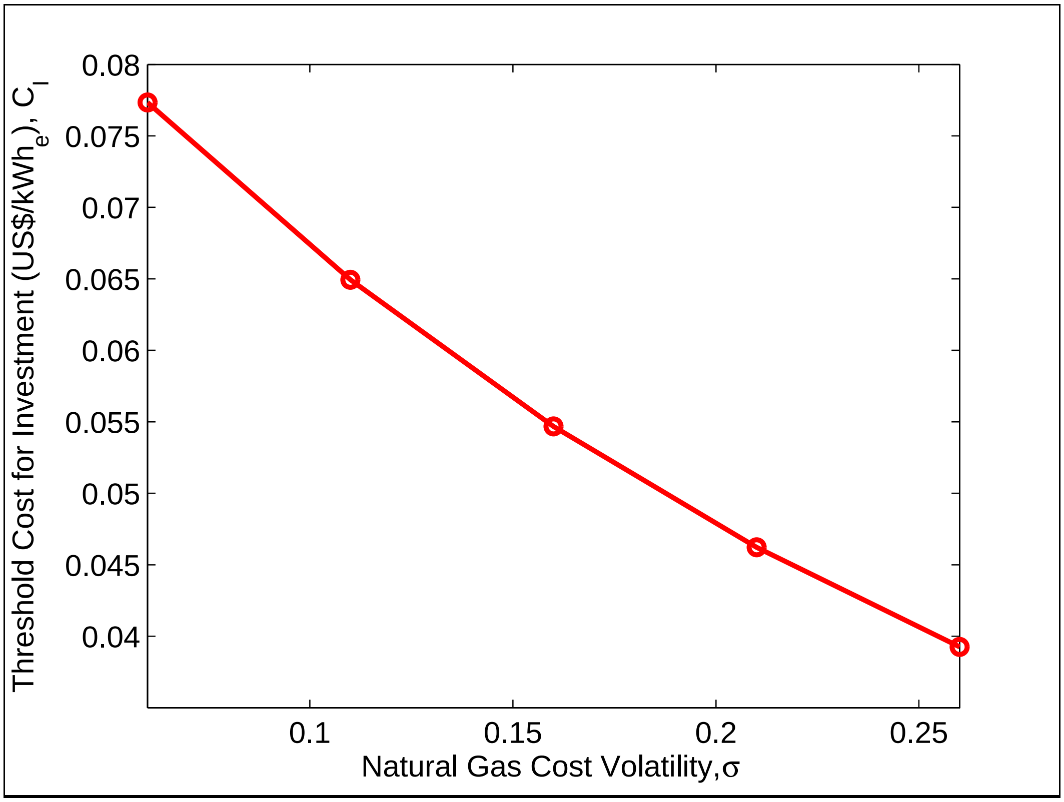
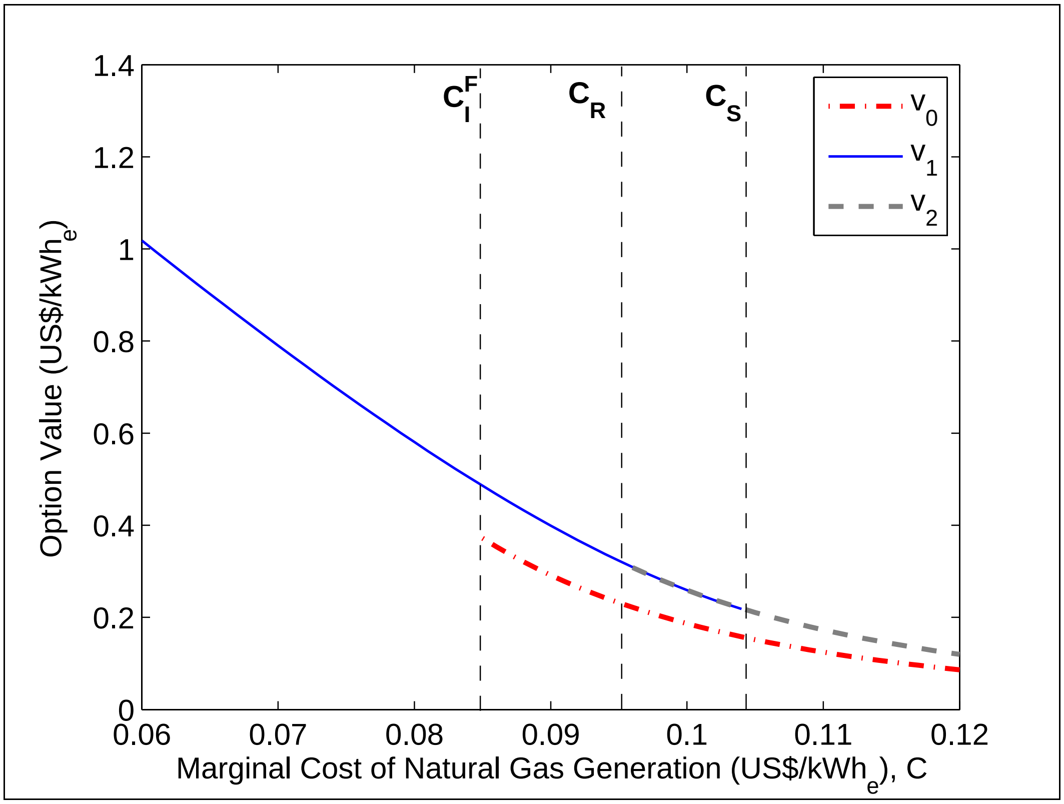


Figure 2.



0.08
0.075
0.07
0.065
0.06
0.055
0.05
0.045
0.04
Threshold Cost for Investment (US$/kWh$_e$), C_I
0.1
0.15
0.2
0.25
Natural Gas Cost Volatility,σ

Figure 2.



Option Value (US$/kWh$_e$)
Marginal Cost of Natural Gas Generation (US$/kWh$_e$), C
C_I^F
C_R
C_S
v_0
v_1
v_2
1.4
1.2
1
0.8
0.6
0.4
0.2
0
0.06
0.07
0.08
0.09
0.1
0.11
0.12

Figure 3.

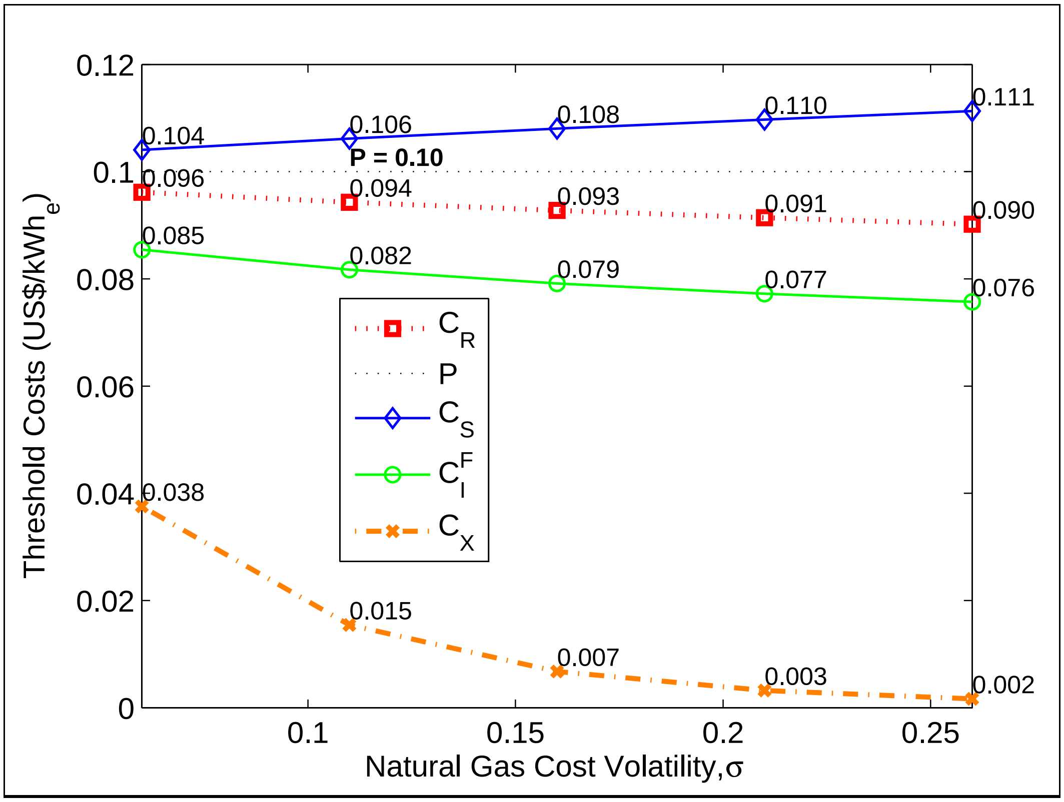

0.12
0.1
0.08
0.06
0.04
0.02
0
0.1
0.15
0.2
0.25
Threshold Costs (US$/kWh_e)
Natural Gas Cost Volatility,σ
P = 0.10
0.104
0.106
0.108
0.110
0.111
0.096
0.094
0.093
0.091
0.090
0.085
0.082
0.079
0.077
0.076
0.038
0.015
0.007
0.003
0.002
C_R
P
C_S
C_I^F
C_X

Figure 4.

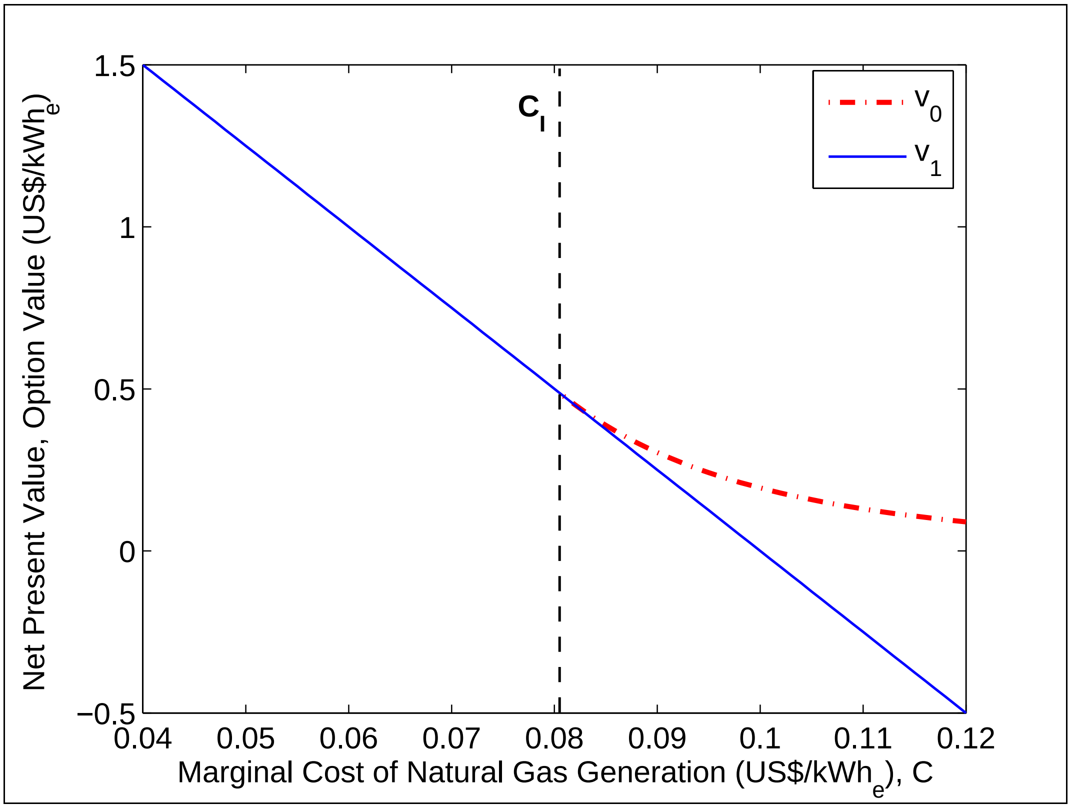
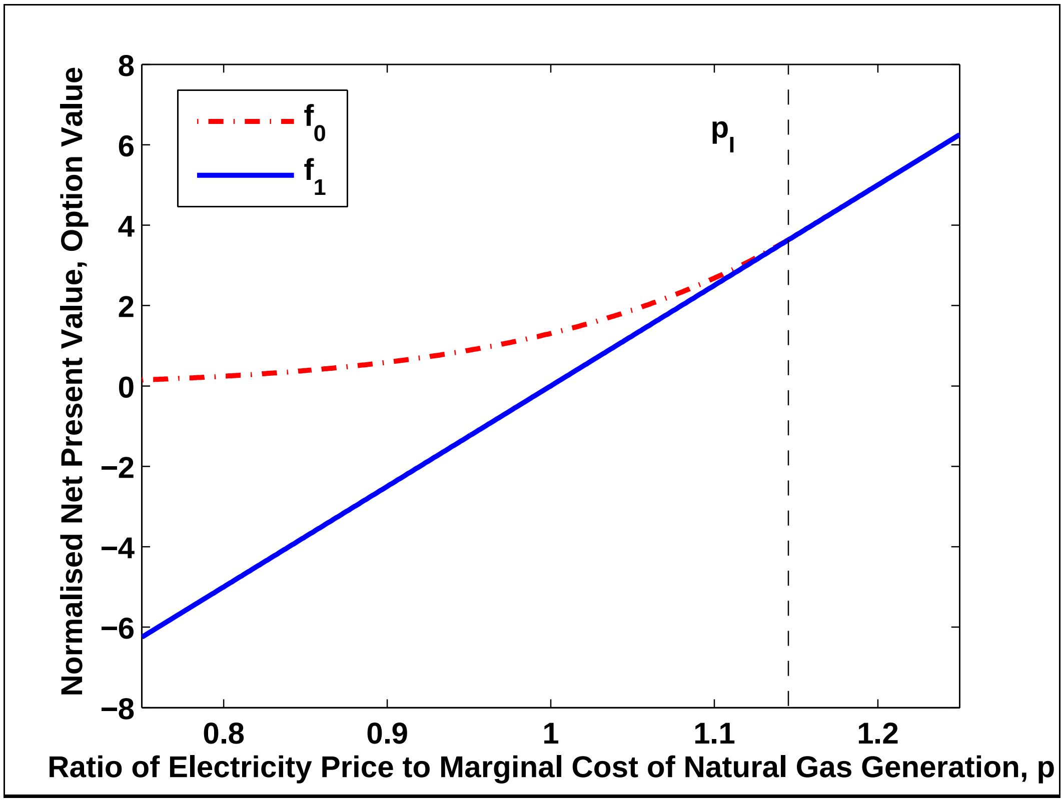
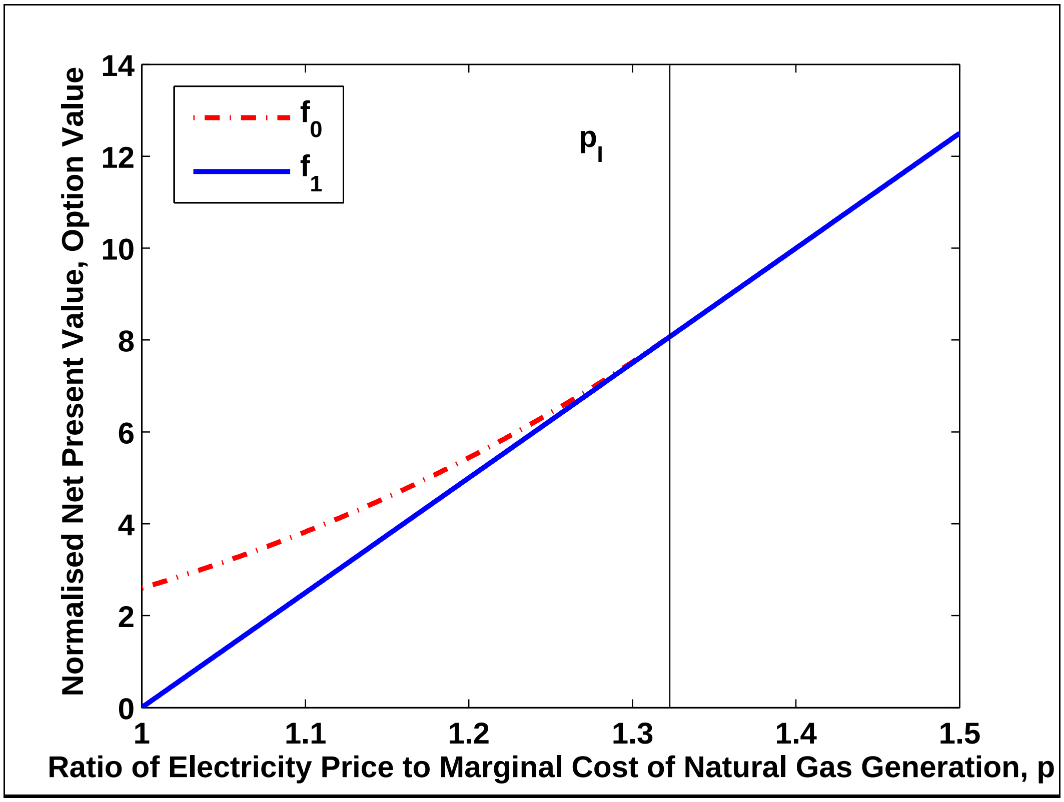

Net Present Value, Option Value (US$/kWh_e)
Marginal Cost of Natural Gas Generation (US$/kWh_e), C
C_I
V_0
V_1
1.5
1
0.5
0
−0.5
0.04
0.05
0.06
0.07
0.08
0.09
0.1
0.11
0.12

Figure 5.



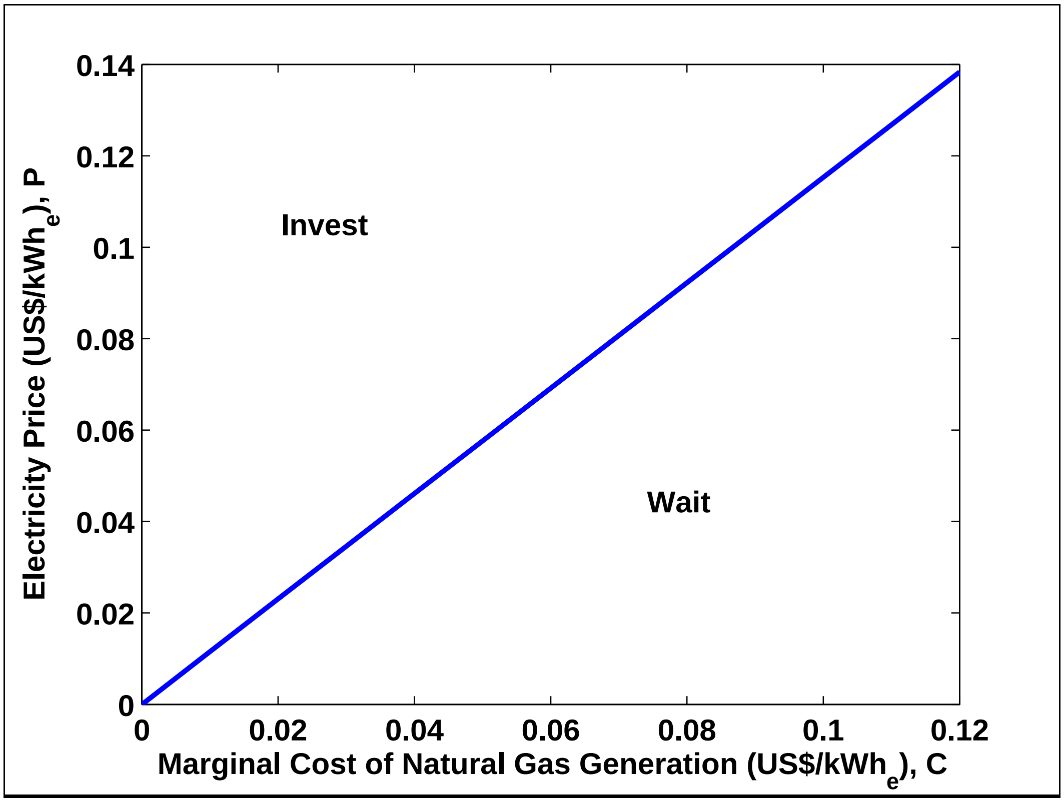
8
6
4
2
0
−2
−4
−6
−8
Normalised Net Present Value, Option Value
f0
f1
pI
0.8
0.9
1
1.1
1.2
Ratio of Electricity Price to Marginal Cost of Natural Gas Generation, p

Figure 6.



14
12
10
8
6
4
2
0
1
1.1
1.2
1.3
1.4
1.5
Normalised Net Present Value, Option Value
Ratio of Electricity Price to Marginal Cost of Natural Gas Generation, p
f_0
f_1
p_I

Figure 7.



0.14
0.12
0.1
0.08
0.06
0.04
0.02
0
Electricity Price (US$/kWh$_e$), P
Invest
Wait
0
0.02
0.04
0.06
0.08
0.1
0.12
Marginal Cost of Natural Gas Generation (US$/kWh$_e$), C

Figure 8.