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Matter Dependence of the Three-Loop Soft Anomalous Dimension Matrix

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The resummation of soft gluon exchange for QCD hard scattering requires a matrix of anomalous dimensions, which has been computed through two loops. The two-loop matrix is proportional to the one-loop matrix. Recently there have been proposals that this proportionality extends to higher loops. One can test such proposals by computing the dependence of this matrix on the matter content in a generic gauge theory. It is shown that for the matter-dependent part the proportionality extends to three loops for arbitrary massless processes.

High-energy colliders, particularly hadron colliders such as the Tevatron and the imminent Large Hadron Collider (LHC), copiously produce complex events containing multiple jets of hadrons. The production rate for such events can be understood through perturbative quantum chromodynamics (QCD). However, in many cases large logarithms of ratios of kinematic quantities appear, and it is necessary to resum these logarithms to obtain an accurate prediction. For processes in which only two partons appear at the leading order in the perturbation expansion in α_s (Born level), such as quark-antiquark annihilation to an electroweak vector boson, or gluon-gluon fusion into the Higgs boson, such resummations have been carried out to relatively high order [1], thanks in part to a detailed understanding of properties of the Sudakov form factor in QCD [2–4]. In these cases, only one color structure is permitted — the colors of the two partons must be identical. For three partons as well, a unique color structure appears at Born level (an $SU(3)$ generator matrix or structure constant).

However, for four or more partons, such as appear in di-jet production at hadron colliders, the Born level process contains multiple color structures. The emission or virtual exchange of soft gluons can mix these structures. To organize the logarithms from virtual exchange it is convenient to introduce a *soft anomalous dimension matrix* [5–8]. This matrix can be computed from the renormalization properties of vacuum matrix elements of products of Wilson lines, also known as eikonal lines. Each line corresponds to an external parton in the amplitude [6]. At one loop, the soft matrix can only change the colors of two hard partons at a time, because the virtual gluon propagator has only two ends. Such action can be written as a color operator of the form $\mathbf{T}_i^a \mathbf{T}_j^a \equiv \mathbf{T}_i \cdot \mathbf{T}_j$ [9, 10] where $\mathbf{T}_i$ indicates the action on the color of line i, and the adjoint index a of the virtual gluon is summed over.

At higher loops, it might seem that arbitrarily many external partons (eikonal lines) could eventually be connected by soft gluons, resulting in a very complicated soft anomalous dimension matrix. For example, at two loops the structure $f^{abc}\mathbf{T}_i^a \mathbf{T}_j^b \mathbf{T}_k^c$ might have appeared, connecting eikonal lines i, j and k. Rather remarkably, however, such a structure does not appear, for any two-loop amplitude in a massless gauge theory [11, 12]. Also, the two-loop soft anomalous dimension matrix $\mathbf{\Gamma}_S^{(2)}$ is proportional to the one-loop matrix,

$$\mathbf{\Gamma}_S^{(2)} = \left[\gamma_K^{(2)}/\gamma_K^{(1)}\right] \mathbf{\Gamma}_S^{(1)} , \tag{1}$$

where $\gamma_K^{(L)}$ is the L-loop coefficient of the cusp anomalous dimension [13], $\gamma_K(\alpha_s) = \sum_{L=1}^{\infty} \gamma_K^{(L)} (\alpha_s/\pi)^L$, with $\gamma_K^{(1)} = 2C_i$ (C_i is the quadratic Casimir for line i). This result agrees with explicit infrared singularities of various two-loop QCD amplitudes [14–16]. The soft anomalous dimension matrix also appears in the study [17] and resummation [18] of electroweak Sudakov logarithms, relevant for very high-transverse-momentum processes at the LHC. (An analogous "cross" anomalous dimension matrix governs color exchange in forward $2 \to 2$ scattering [19]. As this is not in the fixed-angle regime, the cross and soft anomalous dimensions are not directly comparable.) Recently it has been conjectured that the proportionality (1) might persist to higher loops [20–22]. Ref. [22] also showed that such an ansatz is consistent with a required invariance under rescaling of Wilson-line velocities, but the solution to the consistency conditions is not unique for amplitudes with four or more partons.

In this letter the proportionality (1) is shown to extend to three loops, for the part of $\mathbf{\Gamma}_S^{(3)}$ that depends on the matter content, in any massless gauge theory. The result is established using the same transformation of loop-integration variables employed at two loops in refs. [11, 12]. There is also a term in $\mathbf{\Gamma}_S^{(2)}$ arising from pure gluon exchange. Although the result here shows that certain pure-glue contributions connecting three eikonal lines also vanish, other contributions connecting three and four lines remain to be understood. On the other hand, these contributions are the same in any theory, so it is possible, for example, to extract them from an amplitude in $\mathcal{N} = 4$ super-Yang-Mills theory (SYM), and then apply the result to QCD. In fact, the full color dependence of the three-loop four-point amplitude in $\mathcal{N} = 4$ SYM is known in terms of a handful of loop integrals [20]. Evaluating the integrals (a nontrivial task) would allow a test of the proportionality of the pure-gauge part of the soft anomalous dimension matrix at three loops. The observed uniform *transcendentality* of $\mathcal{N} = 4$ SYM amplitudes [23] already implies that any non-proportional term in $\mathbf{\Gamma}_S^{(3)}$ should be a weight-5

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transcendentality function of kinematic invariants, where $\zeta(n)$, $\ln^n x$ and $\mathrm{Li}_n(x)$ are examples of weight n functions — in addition to the constraints of ref. [22].

Consider the scattering amplitude for n massless partons f_i carrying (all-outgoing) momenta $\{p_i\}$ and color $\{r_i\}$. To analyze the amplitude at fixed angles and large overall momentum scale Q, represent the momenta as $p_i = Qv_i, \quad v_i^2 = 0$, where the v_i are four-velocities. Divergences are regulated using dimensional regularization with $D = 4 - 2\epsilon$. The amplitude's color-dependence can be represented as a vector $|\mathcal{M}\rangle$ in a C-dimensional vector space spanned by the color tensors $\{(c_I)_{\{r_i\}}\}$ [6, 10, 24],

$$|\,\mathcal{M}\rangle \equiv \sum_{I=1}^{C} \mathcal{M}_I\ (c_I)_{\{r_i\}}\ . \tag{2}$$

For example, for n-gluon amplitudes, one choice of basis is the set of multiple traces of generators, $\mathrm{tr}(T^{a_1}\ldots T^{a_k})\,\mathrm{tr}(T^{a_{k+1}}\ldots T^{a_l})\ \ldots$. For amplitudes with two quarks (1 and 3) and two anti-quarks (2 and 4), the basis is much simpler; it is spanned by $\delta_{i_1}^{\bar{\imath}_2}\delta_{i_3}^{\bar{\imath}_4}$ and $\delta_{i_1}^{\bar{\imath}_4}\delta_{i_3}^{\bar{\imath}_2}$.

Infrared divergences of on-shell amplitudes may be factorized into jet functions describing virtual partons collinear with the external lines, and soft functions characterizing the exchange of soft gluons between the hard lines. The general form of the factorized amplitude, for equal factorization and renormalization scales μ, is [24]

$$\begin{aligned}\big|\,\mathcal{M}\left(v_j, Q^2/\mu^2, \alpha_s(\mu), \epsilon\right)\big\rangle \ &=\ \prod_{i=1}^{n} J^{[i]}\left(\alpha_s(\mu), \epsilon\right) \\ \times\, \mathbf{S}\left(v_j, Q^2/\mu^2, \alpha_s(\mu), \epsilon\right) &\big|\,H\left(v_j, Q^2/\mu^2, \alpha_s(\mu)\right)\big\rangle\ , \end{aligned} \tag{3}$$

where $J^{[i]}$ is the jet function for external state i, $\mathbf{S}$ is the soft function, and H is the hard (short-distance) function, which is finite in the infrared.

The jet function for parton i can be expressed to all orders in terms of three anomalous dimensions, $\mathcal{K}^{[i]}$, $\mathcal{G}^{[i]}$ and $\gamma_K^{[i]}$; and $\mathcal{K}^{[i]}$ is completely determined by $\gamma_K^{[i]}$ [3, 4]. This letter concerns the soft function $\mathbf{S}$. It is governed by the soft anomalous dimension matrix $\mathbf{\Gamma}_S$,

$$\begin{aligned}&\mathbf{S}\left(v_i\cdot v_j\, Q^2/\mu^2, \alpha_s(\mu), \epsilon\right) \\ &= \ \mathrm{P}\ \exp\left[-\int_0^{\mu}\frac{d\tilde{\mu}}{\tilde{\mu}}\mathbf{\Gamma}_S\left(\frac{v_i\cdot v_j\, Q^2}{\mu^2}, \bar{\alpha_s}\left(\tilde{\mu},\epsilon\right)\right)\right] \\ &= \ 1 + \frac{1}{2\epsilon}\left(\frac{\alpha_s}{\pi}\right)\mathbf{\Gamma}_S^{(1)} + \frac{1}{8\epsilon^2}\left(\frac{\alpha_s}{\pi}\right)^2\left(\mathbf{\Gamma}_S^{(1)}\right)^2 \\ &\quad - \frac{\beta_0}{16\epsilon^2}\left(\frac{\alpha_s}{\pi}\right)^2\mathbf{\Gamma}_S^{(1)} + \frac{1}{4\epsilon}\left(\frac{\alpha_s}{\pi}\right)^2\mathbf{\Gamma}_S^{(2)} + \ldots\ . \end{aligned} \tag{4}$$

and also involves the D-dimensional running coupling. At one loop, $\bar{\alpha_s}$ is given by

$$\bar{\alpha_s}\left(\tilde{\mu},\epsilon\right) = \alpha_s(\mu)\left(\frac{\mu^2}{\tilde{\mu}^2}\right)^{\epsilon}\sum_{l=0}^{\infty}\left[\frac{\beta_0}{4\pi\epsilon}\left(\left(\frac{\mu^2}{\tilde{\mu}^2}\right)^{\epsilon} - 1\right)\alpha_s(\mu)\right]^l$$

with $\beta_0 = (11C_A - 4T_R n_f)/3$ in QCD.

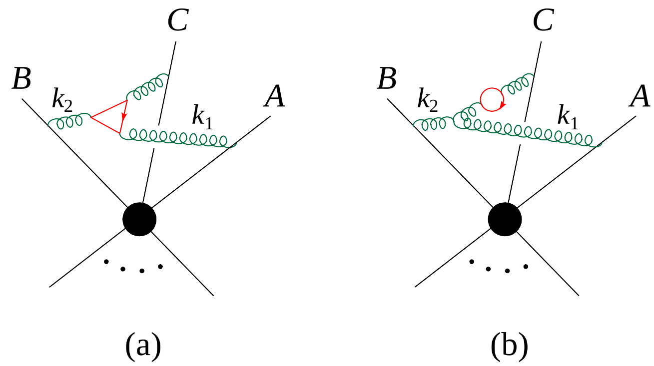

FIG. 1: Three-loop diagrams containing a fermion or scalar loop and three eikonal lines, corresponding to a three-gluon vertex at two loops.

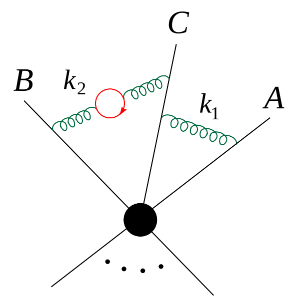

FIG. 2: Three-loop three-eikonal diagram containing a fermion or scalar bubble, and not originating from a three-gluon vertex at two loops.

The one-loop soft matrix is [6, 10]

$$\mathbf{\Gamma}_S^{(1)} \ = \ \frac{1}{2}\sum_{i=1}^{n}\sum_{j\neq i}\mathbf{T}_i\cdot\mathbf{T}_j\ \ln\left(\frac{\mu^2}{-s_{ij}}\right)\ , \tag{5}$$

where $s_{ij} = (p_i + p_j)^2$. Resummed cross sections are determined by the eigenvalues and eigenvectors of $\mathbf{\Gamma}_S$ [7, 8, 25]. Proportionality of $\mathbf{\Gamma}_S^{(2)}$ to $\mathbf{\Gamma}_S^{(1)}$ implies that for fixed kinematics, the same eigenvectors that diagonalize the one-loop matrix also diagonalize the two-loop matrix, simplifying the integration over scale. [11, 12]

The soft anomalous dimension matrix is found from the single-ultraviolet-pole terms in ϵ, for suitable combinations of eikonal lines, as described in detail at one [6] and two loops [12]. At L loops, the maximum number of eikonal lines that can be connected to some gluon is $2L$. However, this maximal number corresponds to a disconnected product of L one-loop single gluon exchanges, which appears in the expansion of the exponential (4). Thus the divergence is removed by one-loop renormalization. The maximum number of eikonal lines that does not correspond to a disconnected product of lower-loop configurations is $L{+}1$. Requiring dependence on the matter content of the theory means that there is a fermion or scalar loop in the diagram; this cuts the maximum number of nontrivially connected legs to L. Such diagrams can be generated from the maximal nontrivial $(L-1)$-loop configurations by inserting bubbles with fermions or scalars, or triangles, or higher-point one-loop diagrams.

At two loops, the key eikonal diagrams for demonstrating proportionality involved three eikonal lines [11, 12], labelled by A, B and C, with velocities v_A, v_B and v_C. The diagrams can be divided into two cases: (*i*) three eikonal lines connected uniquely by a three-gluon vertex, and (*ii*) three eikonal lines connected by two single-gluon exchanges, so that two gluons attach to one of the eikonal lines. Diagram (*i*) was found to vanish. The sum of diagrams of type (*ii*) was found to have only a symmetric color configuration, proportional to $\{\mathbf{T}_A\cdot\mathbf{T}_C\,,\,\mathbf{T}_C\cdot\mathbf{T}_B\}$; the coefficient factorized into an appropriate form to be removed by one-loop renormalization. (It corresponds to second-order terms in the expansion of the exponential (4) in which one leg is common to both factors.)

Inserting a matter triangle or bubble into diagram (*i*) gives the three-loop diagrams in fig. 1(a) and (b). Inserting a bubble into a type (*ii*) diagram gives diagrams of the form shown in fig. 2. I shall show that this insertion does not affect the properties established for the pure-gluon diagrams at two loops. The proof uses exactly the same change of integration variables used at two loops, plus, for diagram 1(a), a handy representation of the one-loop three-gluon vertex $f^{a_1a_2a_3}\Gamma_{\mu_1\mu_2\mu_3}(k_1,k_2,k_3)$, evaluated in an arbitrary gauge [26]. Inserting this representation into diagram 1(a) yields the loop-momentum integral,

$$\begin{aligned}F_{1(\mathrm{a})}(v_A,v_B,v_C)\ &\propto\ f^{abc}\mathbf{T}_A^a\mathbf{T}_B^b\mathbf{T}_C^c\\ \times\int d^Dk_1d^Dk_2\,&\frac{\Gamma_{v_Av_Bv_C}}{k_1^2\,k_2^2\,k_3^2\,v_A\cdot k_1\,v_B\cdot k_2\,v_C\cdot k_3}\,,\end{aligned}\tag{6}$$

omitting the $+i\varepsilon$ prescription, with $k_3=-k_1-k_2$ and

$$\begin{aligned}\Gamma_{v_Av_Bv_C}\ =\ & v_A^{\mu_1}v_B^{\mu_2}v_C^{\mu_3}\Gamma_{\mu_1\mu_2\mu_3}(k_1,k_2,k_3)=A(k_1^2,k_2^2;k_3^2)\,v_A\cdot v_B\,v_C\cdot(k_1-k_2)+B(k_1^2,k_2^2;k_3^2)\,v_A\cdot v_B\,v_C\cdot(k_1+k_2)\\ &-C(k_1^2,k_2^2;k_3^2)(k_1\cdot k_2\,v_A\cdot v_B-v_A\cdot k_2\,v_B\cdot k_1)\,v_C\cdot(k_1-k_2)\\ &+\tfrac{1}{3}S(k_1^2,k_2^2,k_3^2)\,(v_A\cdot k_2\,v_B\cdot k_3\,v_C\cdot k_1+v_A\cdot k_3\,v_B\cdot k_1\,v_C\cdot k_2)\\ &+F(k_1^2,k_2^2;k_3^2)\,(k_1\cdot k_2\,v_A\cdot v_B-v_A\cdot k_2\,v_B\cdot k_1)\,(k_2\cdot k_3\,v_C\cdot k_1-k_1\cdot k_3\,v_C\cdot k_2)\\ &-H(k_1^2,k_2^2,k_3^2)[v_A\cdot v_B(k_2\cdot k_3\,v_C\cdot k_1-k_1\cdot k_3\,v_C\cdot k_2)-\tfrac{1}{3}(v_A\cdot k_2\,v_B\cdot k_3\,v_C\cdot k_1-v_A\cdot k_3\,v_B\cdot k_1\,v_C\cdot k_2)]\\ &+\text{cyclic permutations of }(k_1,A),\,(k_2,B),\,(k_3,C).\end{aligned}\tag{7}$$

The precise forms of the coefficient functions A through H are given in ref. [26], for gluon and massless fermion loops. Here only their symmetry properties are needed: A, C and F are symmetric under exchange of the first two arguments; B is antisymmetric in the first two arguments; S is totally antisymmetric; and H is totally symmetric. Ref. [26] does not explicitly consider the case of a massless scalar, but the vanishing of three-point functions in $\mathcal{N}=4$ SYM implies that the scalar loop must have the same tensor decomposition and symmetry properties.

The change of loop-momentum variables used in refs. [11, 12] was expressed in terms of light-cone variables using the vectors v_A and v_B to define + and − directions. Here it suffices to give the action of the transformation on the various Lorentz products that appear,

$$\begin{aligned}v_A\cdot k_i\ &\to\ \frac{v_A\cdot v_C}{v_B\cdot v_C}\,v_B\cdot k_{\hat{\imath}}\,, &\quad v_C\cdot k_i&\to v_C\cdot k_{\hat{\imath}}\,,\\ v_B\cdot k_i\ &\to\ \frac{v_B\cdot v_C}{v_A\cdot v_C}\,v_A\cdot k_{\hat{\imath}}\,, &\quad k_i\cdot k_j&\to k_{\hat{\imath}}\cdot k_{\hat{\jmath}}\,,\end{aligned}\tag{8}$$

where $\hat{1}=2$, $\hat{2}=1$, and $\hat{3}=3$. The Jacobian for this transformation is unity. It is simple to check that every contracted tensor appearing in eq. (7) has the proper symmetry under (8) so that, combined with the symmetry of the appropriate coefficient function, and the remaining (symmetric) product of propagators in eq. (6), the integrand is antisymmetric. Hence the integral (6) vanishes.[1] For example, the two terms multiplying S in eq. (7) exchange with each other with a positive sign, whereas the corresponding terms multiplying H do so with a negative sign. Note that the transformation (8) is only to be used for the term shown explicitly in eq. (7). For the two images of this term under cyclic permutation, one should use the correspondingly permuted transformation, which respects the symmetry properties of the permuted functions A through H.

For the diagrams in fig. 1(b) and fig. 2, write the bubble diagram as

$$\Pi^{a_1a_2}_{\mu_1\mu_2}(k^2)=\delta^{a_1a_2}(\eta_{\mu_1\mu_2}-\xi k_{\mu_1}k_{\mu_2}/k^2)\Pi(k^2)\,,\tag{9}$$

where $\xi=1$ for the transverse scalar and fermion bubbles, but ξ may differ from 1, depending on the gauge, for a gluon bubble. The $\eta_{\mu_1\mu_2}$ term in eq. (9), inserted into diagrams 1(b) and 2, clearly gives the same tensor structure as at two loops. Thus [11] its contribution to diagram 1(b) vanishes. It is easy to see that the ξ-dependent

[1] Another argument for the vanishing uses total antisymmetry of the integrated result under exchanges of $\{v_A,v_B,v_C\}$, and the impossibility of building such functions [27].

terms are also odd under the transformation (8), so diagram 1(b) vanishes for any ξ.

There are four diagrams of the type shown in fig. 2, related by changing the order of connection on line C, and by moving the matter bubble to the other gluon line. The sum of the four diagrams splits into a color antisymmetric piece, proportional to $[\mathbf{T}_A \cdot \mathbf{T}_C\,,\,\mathbf{T}_C \cdot \mathbf{T}_B]$, whose coefficient vanishes exactly as in ref. [12]; and a color symmetric piece, proportional to $\{\mathbf{T}_A \cdot \mathbf{T}_C\,,\,\mathbf{T}_C \cdot \mathbf{T}_B\}$, containing a factor in the integrand of

$$\begin{aligned} &\frac{1}{v_C\cdot k_1}\Bigg[v_A\cdot v_C\bigg(v_B\cdot v_C-\xi\,\frac{v_B\cdot k_1 v_C\cdot k_1}{k_1^2}\bigg)\Pi(k_1^2) \\ &+(v_A\leftrightarrow v_B,\quad k_1\leftrightarrow k_2)\Bigg]+(v_A\leftrightarrow v_B,\quad k_1\leftrightarrow k_2) \\ =\ &\frac{v_C\cdot(k_1+k_2)}{v_C\cdot k_1\,v_C\cdot k_2}\Bigg[v_A\cdot v_C\bigg(v_B\cdot v_C-\xi\,\frac{v_B\cdot k_1 v_C\cdot k_1}{k_1^2}\bigg) \\ &\times\Pi(k_1^2)\ +\ (v_A\leftrightarrow v_B,\quad k_1\leftrightarrow k_2)\Bigg]. \end{aligned} \tag{10}$$

The latter form implies that this diagram factorizes into the product of a one-loop diagram with the matter-dependent part of the two-loop diagram. Hence it also does not contribute to $\mathbf{\Gamma}_S^{(3)}$. The above arguments also show that, independent of the gauge, the soft matrix does not receive contributions from those three-eikonal pure-glue diagrams containing a gluon loop.

Finally one must address the matter-dependent contributions to $\mathbf{\Gamma}_S^{(3)}$ involving only two eikonal lines. As was the case for the full $\mathbf{\Gamma}_S^{(2)}$, only two of these diagrams (ladder and crossed ladder) do not obviously reduce group theoretically to the form $\mathbf{T}_i \cdot \mathbf{T}_j$. Inserting a matter bubble of the form (9) does not change the group theory. The magnitudes of these two contributions are linked [19, 28], however, by the general non-abelian exponentiation theorem for eikonal webs [29], in such a way that their sum is of the form $\mathbf{T}_i \cdot \mathbf{T}_j$ (after removing the square of the one-loop term). Because this result holds for the case of two partons (i and j of equal colors, so that $\mathbf{T}_i \cdot \mathbf{T}_j \to \mathbf{T}_i^2 = C_i$), the constant of proportionality must be the (matter dependent part of) the cusp anomalous dimension,

$$\mathbf{\Gamma}_S^{(3),\,\mathrm{matter}} = \Big[\gamma_K^{(3),\,\mathrm{matter}}/\gamma_K^{(1)}\Big]\,\mathbf{\Gamma}_S^{(1)}\,. \tag{11}$$

An alternative, very general argument for the constant of proportionality, given that the matrix is proportional, is based on an anomaly under rescalings of the eikonal-line velocities [22]. As noted there, the situation may become more complex at four loops, due to the fact that $\gamma_K^{(4)}$ might contain quartic Casimir terms $\sim (d_A^{abcd})^2$ [28–30].

In summary, the results of this letter show that the three-loop soft anomalous dimension matrix is essentially the same in all massless gauge theories, *i.e.* up to the matter dependence of $\gamma_K^{(3)}$. If proportionality fails for the pure gauge terms, then at least the three-loop properties are constrained by $\mathcal{N}=4$ super-Yang-Mills theory, as well as the velocity rescalings studied in ref. [22].

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