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A POSSIBLE CONNECTION BETWEEN DARK ENERGY AND THE HIERARCHY*

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Recently it was suggested that the dark energy maybe related to the well-known hierarchy between the Planck scale ($\sim 10^{19}$GeV) and the TeV scale. The same brane-world setup to address this hierarchy problem may also in principle address the smallness problem of dark energy. Specifically, the Planck-SM hierarchy ratio was viewed as a quantum gravity-related, dimensionless fine structure constant where various physical energy scales in the system are associated with the Planck mass through different powers of the 'gravity fine structure constant'. In this paper we provide a toy model based on the Randall-Sundrum geometry where SUSY-breaking is induced by the coupling between a SUSY-breaking Higgs field on the brane and the KK gravitinos. We show that the associated Casimir energy density indeed conforms with the dark energy scale.

Keywords: Dark energy; brane-world; Casimir energy.

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1. Introduction

Recent observations[1,2,3] imply that dark energy is likely to be a cosmological constant with vacuum energy density $\rho_{\rm DE} \sim (10^{-3}{\rm eV})^4$. If dark energy is indeed a cosmological constant ($w = -1$) which never changes in space and time, then it must be a fundamental property of the spacetime. One cannot but note that the energy scale of this fundamental property of the vacuum is so much smaller than that of the standard model of particle physics, which is $\sim$TeV, by a factor $\sim 10^{-15}$. Why is this energy gap so huge?

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There has been another long-standing hierarchy problem in physics, i.e., the existence of a huge gap between the Planck scale of quantum gravity at 10^{19}GeV and that of the standard model gauge interactions at TeV, by a factor $\sim 10^{16}$. As is well-known, there have been two interesting solutions to this hierarchy problem proposed in recent years: the Arkani-Hamed-Dimopoulos-Dvali (ADD) model[4] and the Randall-Sundrum (RS) model.[5] In both models the brane-world scenario is invoked where the 3-brane is imbedded in the extra dimensions, and the SM fields are confined to the brane while gravity fields reside in the bulk. In the case of ADD, the extra spatial dimensions are flat. The gravity is weak (or the Planck scale is huge) because it is diluted by the largeness of the extra dimensions in which it resides. In the alternative solution proposed by RS, the gravity is weak on the TeV brane because its strength is exponentially suppressed by the warp factor descended from the Planck brane.

The surprising numerical coincidence between these two energy gaps prompts to the wonder: Are these two hierarchy problems related? Recently one of us[6] suggested that these two hierarchies are indeed related. If one equates the two energy gaps as

$$\frac{\rho_{\rm DE}^{1/4}}{M_{\rm SM}} \simeq \frac{M_{\rm SM}}{M_{\rm Pl}} \equiv \alpha_G, \tag{1}$$

then it suggests that the underlying dynamics that gives rise to the dark energy is related to the Planck scale through the intermediary of the Standard Model (SM) scale, at $\sim$ TeV. This may not be too much of a stretch. After all, the cosmological constant as a manifestation of vacuum energy is necessarily connected with the structure of spacetime and quantum fields. But Eq. (1) actually implies more. It suggests that their possible connection must be mediated and inverted by the TeV physics. We should like to caution, however, that the dark energy scale is actually not on the same footing as the other two energy scales. Whereas TeV scale represents the interaction strengths of the Standard Model and possibly its supersymmetric extension, and the Planck scale that for the gravitational interaction, the dark energy scale is not associated with any new interaction strength per se. After all, there are only four fundamental interactions in this world. It is therefore clear that the dark energy scale must not be a primary fundamental scale in physics, but rather a deduced, secondary quantity. In this regard, Eq. (1) serves to explicate the relationship of the dark energy scale with that of the four fundamental interactions. That is, the underlying mechanism that induces the dark energy must be resulted from a double suppression by the same hierarchy factor descended from the Planck scale:

$$\rho_{\rm DE}^{1/4} \simeq \frac{M_{\rm SM}}{M_{\rm pl}} M_{\rm SM} = \left(\frac{M_{\rm SM}}{M_{\rm Pl}}\right)^2 M_{\rm Pl} = \alpha_G^2 M_{\rm Pl}. \tag{2}$$

We note that such a situation is not unique in physics. For example, in atomic physics the hydrogen ground state energy is suppressed from the electron rest mass by two powers of the fine structure constant, α. Analogous to that, the Planck-SM hierarchy ratio can be viewed as a 'gravity fine structure constant', α_G, through different powers of which are various physical energy scales in the system associated with the Planck mass.

In our original attempt,[7,8] we investigated the Casimir energy on a supersymmetry-breaking brane as dark energy based on the ADD-like geometry. But instead of invoking Eq. (2) as our guidance, we looked for the general constraint on the various fundamental energy scales if the Casimir energy so induced was to be interpreted as the dark energy. More recently we invoke Eq. (2) as our guidance in search of the underlying dynamics for the dark energy.[6] There we followed the Gherghetta-Pommarol (GP) mechanism[9,10] of twisted and untwisted boundary conditions for gravitino and graviton, respectively, to break supersymmetry and to induce their mass shifts. Unfortunately while the gravitino massless mode did develop a highly suppressed mass shift, higher mode KK masses nevertheless acquire mass shifts at the TeV level. As a result the final 4D Casimir energy on the TeV brane was too large for our desire. In this paper we follow the same philosophy to rely on the above Ansatz as our guidance in constructing our model for dark energy. However, instead of the GP twisted-untwisted boundary conditions, we invoke a Higgs coupling on the TeV brane for SUSY-breaking. We show that the smallness of the dark energy arises naturally in this new approach.

In Sec. 2 we introduce a toy model for dark energy guided by the above relation. The basic building blocks of our model remain unchanged: The gravity sector lives in the bulk while the standard model as well as the breaking of supersymmetry occur on the TeV brane in the RS geometry. For simplicity, we invoke a Dirac fermion in a hypermultiplet to represent the gravitino. We then calculate the KK gravitino mass shift induced by its coupling with a Higgs field on the TeV brane. We show that the resultant graviton-gravitino mass difference for non-zero KK modes can be as small as $\alpha_G^3 M_{\rm Pl}$. In Sec. 3 we derive the associated Casimir energy on the TeV brane. Summing over all the KK modes, we demonstrate that the resultant Casimir energy scales exactly as what we expect for the dark energy.

2. SUSY-Breaking Induced Graviton-Gravitino Mass Difference

In this section we demonstrate how a bulk field can develop a tiny mass shift between its super-partners at the tree level due to the breaking of supersymmetry on the TeV brane. It suffices our purpose to study the coupling of the brane Higgs with the bulk spin 1/2 Dirac fermion field as a simple-minded representation of the gravitino field. Bona fide Higgs coupling with the spin 3/2 gravitino field will be presented in a separate paper. The same Higgs field would only couple with the graviton at the loop level, and we shall ignore it here.

The Randall-Sundrum model invokes the following metric:

$$ds^2 = e^{-2\sigma}\eta_{\mu\nu}dx^\mu dx^\nu + a^2 dy^2 \,, \tag{3}$$

where $\sigma = \sigma(y) = ka|y|, \mu, \nu = 0, 1, 2, 3, -\pi \le y \le \pi$, and a is the radius and k the curvature of the orbifold $\mathcal{S}^1/\mathcal{Z}_2$ in the compactified 5th dimension y. The hidden, or Planck, brane locates at $y = 0$ while the visible, or TeV, brane locates at $y = \pi$. As is well-known, the Planck-SM hierarchy is bridged if $ka \sim \mathcal{O}(10)$ so that the mass scale at $y = \pi$ is suppressed by the warp factor α_G. It is customary to take $k \sim M_{\rm Pl}$. So in the RS model the extra dimension size a is only about 10 times the Planck length. We follow the original RS construct where only the gravity sector lives in the bulk while all other fields in the standard model are confined on the TeV brane.

Supersymmetry in a slice of AdS space-time has been investigated by various authors.[17,9,10] Since for simplicity we invoke a Dirac fermion in the bulk to represent the gravitino, we will consider the hypermultiplet which consists of $H = (\phi^i, \Psi)$, where ϕ^i are two complex scalars and Ψ is a Dirac fermion. The 5D action has the form:[9]

$$S_5 = \int d^4x \int dy \sqrt{-g}\Big[|\partial_M \phi^i|^2 + i\bar{\Psi}\gamma^M D_M \Psi + m^2_{\phi^i}|\phi^i|^2 - i m_\Psi \bar{\Psi}\Psi\Big] . \tag{4}$$

Note that under our metric convention, $\sqrt{-g} = a \exp -4\sigma$. The equation of motion for the hypermultiplet fields, written in terms of the RS metric, is

$$\Big[e^{2\sigma}\eta^{\mu\nu}\partial_\mu\partial_{nu} + e^{s\sigma}\partial_5(e^{-s\sigma}\partial_5) - M^2_\Phi\Big]\mathrm{H}(x^\mu, y) = 0 , \tag{5}$$

where $\mathrm{H} = [\phi, e^{-2\sigma}\Psi_{L,R}]$ with $s = [4, 1]$. The L or R component stands for even or odd under the Z_2 symmetry.

The Kaluza-Klein (KK) decomposition and the associated eigen-modes for bosons and fermions in the RS geometry have been well studied in recent years.[19,20,9,10] Goldberger and Wise[19] first studied the behavior of bulk scalar field in the RS model. Flachi *et al.*[20] investigated that for the bulk fermion field. Gherghetta and Pomarol (GP1)[9] extended the study to different supermultiplets in the bulk. The bulk gravitino field in the RS AdS geometry was studied in details in a second paper by Gherghetta and Pomarol (GP2).[10] Here we briefly summarize those results relevant to our discussion. Decomposing the 5D fields as

$$\mathrm{H}(x^\mu, y) = \frac{1}{\sqrt{2\pi a}} \sum_n H^{(n)}(x^\mu) f^{(n)}(y) , \tag{6}$$

where the Kaluza-Klein modes $f^{(n)}(y)$ obey the orthonormal condition

$$\frac{1}{2\pi}\int_{-\pi}^{\pi} dy e^{(2-s)\sigma} f^{(n)}(y) f^{(m)}(y) = \delta_{nm} , \tag{7}$$

GP1 solved the equation of motion and found the y-dependent KK eigenfunction as

$$f^{(n)}_L = \frac{1}{N_n} e^{s\sigma/2}\Big[J_\nu\Big(\frac{m_n}{k}e^\sigma\Big) + b_\nu Y_\nu\Big(\frac{m_n}{k}e^\sigma\Big)\Big], \tag{8}$$

$$f^{(n)}_R = \frac{\sigma'}{kN_n} e^{s\sigma/2}\Big[J_{\nu-1}\Big(\frac{m_n}{k}e^\sigma\Big) + b_{\nu-1} Y_{\nu-1}\Big(\frac{m_n}{k}e^\sigma\Big)\Big], \tag{9}$$

where m_n is the 4D mass for the nth mode and b_ν is a constant which satisfies the boundary condition. For the hypermultiplet, $\nu = |c + 1/2|$ for ϕ^1 and Ψ_L, and $\nu = |c - 1/2|$ for ϕ^2 and Ψ_R, where c is related to the 5D fermion mass by $m_\Psi = c\sigma'$ and is a free parameter in this case. As was shown in GP1, the Dirac fermion zero mode wavefunction scales as $f_L^{(0)}(y) \sim \exp[(2 - c)\sigma]$. Since our purpose is to mimic the gravitino with the Dirac field, where the gravitino massless zero mode wavefunction scales as[10] $f_L^{(0)}(y) \sim \exp(-\sigma/2)$, we fix c at 5/2 so that the Dirac fermion massless mode has the same y-dependence in its wavefunction. Under this construction, the massless mode is localized at the Planck brane ($y = 0$) whereas the higher KK modes localize at the TeV brane ($y = \pi$), which are not sensitive to the choice of c. This is because the asymptotic behavior of Bessel functions are insensitive to its index. The boundary condition imposed on b_ν dictates that

$$\frac{J_\nu(m_n/k)}{Y_\nu(m_n/k)} = \frac{J_\nu(\alpha_G^{-1} m_n/k)}{Y_\nu(\alpha_G^{-1} m_n/k)}\,. \tag{10}$$

Solving this equation, the 4D KK masses, which are identical for both the even and odd modes, are found to be

$$m_n \simeq \alpha_G \Big(n + \frac{3}{4}\Big)\pi k \qquad (n \gg 0)\,. \tag{11}$$

Note that in the asymptotic limit the KK mass spectrum energy gap between adjacent modes is independent of n, and is $\sim \pi\alpha_G M_{\rm Pl} \sim$ TeV. This has important impact on our Casimir energy. We will return to this point in the next section.

Now we introduce the following action as a perturbation to break SUSY:

$$S_{\Phi\Psi} = \int d^4x \int \frac{dy}{a}\delta(y-\pi)\sqrt{-g}g_5\Phi(x)\bar\Psi(x,y)\Psi(x,y) \tag{12}$$

$$= \sum_{n=0}^{\infty}\int \frac{dy}{a}\delta(y-\pi)\sqrt{-g}\frac{1}{2\pi a}[e^{2\sigma}f^{(n)}]^2 \int d^4x g_5\Phi(x)\bar\Psi^{(n)}(x)\Psi^{(n)}(x) \tag{13}$$

$$\equiv \sum_{n=0}^{\infty}\int d^4x \delta m_n \bar\Psi^{(n)}(x)\Psi^{(n)}(x)\,, \tag{14}$$

where Φ is the Higgs field on the brane, $f^{(n)} = f_L^{(n)} + f_R^{(n)}$ and g_5 the 5D Higgs-gravitino Yukawa coupling. The KK gravitino mass-shift for the nth mode is thus

$$\delta m_n \equiv g_5\langle\Phi\rangle \int \frac{dy}{a}\delta(y-\pi)\sqrt{-g}\frac{1}{2\pi a}[e^{2\sigma}f^{(n)}]^2\,, \tag{15}$$

where $\langle\Phi\rangle$ is the vacuum expectation value (VEV) of the Higgs field, which we assume to be $\sim$ TeV. Since g_5 is the strength of the coupling between the Higgs and the gravity sector, it should be proportional to the mass of the Higgs. On the other hand, g_5 has the dimensionality of $\sim$1/mass. But the only fundamental mass scale in the RS system is the Planck mass. Thus $g_5 \sim {\rm TeV}/M_{\rm Pl}^2 \sim \alpha_G/M_{\rm Pl}$. Physically, the y-integral represents the probability of finding the nth mode KK gravitino on the TeV brane.

On the TeV brane where $y = \pi$, the argument of the Bessel functions is $\alpha_G^{-1} m_n/k \sim n\pi \gg 1$. In this limit, and inserting the asymptotic mass spectrum, we find

$$\lim_{z\to\infty} J_\nu(z) = \sqrt{\frac{2}{\pi z}}\cos(z - \frac{\nu\pi}{2} - \frac{\pi}{4}) \simeq \sqrt{\frac{2}{n\pi}}\cos((n-1)\pi) = \pm\sqrt{\frac{2}{n\pi}}\,, \quad (16)$$

$$\lim_{z\to\infty} Y_\nu(z) = \sqrt{\frac{2}{\pi z}}\sin(z - \frac{\nu\pi}{2} - \frac{\pi}{4}) \simeq \sqrt{\frac{2}{n\pi}}\sin((n-1)\pi) = 0\,. \quad (17)$$

On the other hand, the normalization constant for the nth mode can be determined from Eq. (7), and it can be shown that N_n is independent of s:

$$N_n \simeq \frac{\alpha_G^{-1}}{\sqrt{2\pi ka}} J_\nu(\alpha_G^{-1} m_n/k) \simeq \frac{\alpha_G^{-1}}{\sqrt{n\pi^2 ka}}\,. \quad (18)$$

It is interesting to note that in the asymptotic limit the KK gravitino wavefunction on the TeV brane is independent of both ν and n:

$$f^{(n)}(y=\pi) \simeq \sqrt{2\pi ka}\,\alpha_G^{1/2}\,. \quad (19)$$

Collecting all the σ-dependence, the y-integral of the SUSY-breaking action scales as

$$\frac{1}{2\pi a}\int \frac{dy}{a}\delta(y-\pi)\sqrt{-g}[e^{2\sigma} f^{(n)}]^2 \simeq k\alpha_G\,. \quad (20)$$

Putting these together, we obtain the SUSY-breaking induced KK gravitino mass shift:

$$\delta m_n \sim \alpha_G^3 M_{\rm Pl}\,. \quad (21)$$

3. Casimir Energy in the Brane World

Casimir effect has been considered as a possible origin for the dark energy by many authors.[11,12,13,14,15,7,8] It is known that the conventional Casimir energy in the ordinary 3+1 dimensional spacetime cannot provide repulsive gravity necessary for dark energy. Conversely, Casimir energy on a 3-brane imbedded in a higher-dimensional world with suitable boundary conditions can in principle give rise to a positive cosmological constant. Typically for flat extra-dimensions, the resulting Casimir energy density on the 3-brane scales as

$$\rho^{(4)}_{\rm Casimir} \sim a^{-4}\,, \quad (22)$$

where a is the extra dimension size. As summarized by Milton,[11] the required extra dimension sizes for it to conform with the supposed dark energy would have to be very large. In the case of $n = 2$, the Casimir energy is roughly consistent with that required for the ADD solution to the Planck-SM hierarchy. The scaling of the Casimir energy is modified if the system is supersymmetric but broken on the brane. Let such a SUSY-breaking induce KK mass shift be δm_n. Then the 4D Casimir energy on the brane becomes[6]

$$\rho^{(4)}_{\rm Casimir} \sim a^{-2}\delta m_n^2\,. \quad (23)$$

The dependence on a^{-2} results from the summation over all KK modes, where the energy gap in the KK mass spectrum is commensurate with the periodicity of the compact extra-dimension, i.e., $m_n - m_{n-1} \sim 1/a$.

Casimir energy in the RS geometry has been investigated by several authors.[22,23,24] The bottom line is that it retains the same generic scaling as that in the flat space. However, care must be taken in identifying the KK mass spectrum energy gap. In the RS geometry the KK mass spectrum gap does not scale trivially as $1/a \sim M_{\rm Pl}$. As we have seen from the previous section, the gap in the KK mass spectrum scales as $m_n - m_{n-1} \simeq \alpha_G \pi k \simeq$ TeV instead. Thus the 4D Casimir energy on the TeV brane in the RS geometry scales as

$$\rho^{(4)}_{\rm Casimir} \sim [\alpha_G M_{\rm Pl}]^2 \delta m_n^2 \,. \tag{24}$$

Inserting the δm_n derived in the previous section into the above expression, we arrive at

$$\rho^{(4)}_{\rm Casimir} \sim \alpha_G^8 M_{\rm Pl}^4 \sim \left[\left(\frac{M_{\rm SM}}{M_{\rm Pl}}\right)^2 M_{\rm Pl}\right]^4 \sim \rho_{\rm DE} \,. \tag{25}$$

It is remarkable that, guided by our Ansatz of Eq. (2), we manage to construct a model for dark energy where the vacuum energy is indeed suppressed from the Planck scale by two powers of α_G.

4. Summary

Recent observational evidence indicates that the dark energy may actually be the cosmological constant. We argue that the numerical coincidence between the SM-Planck hierarchy and the inverted SM-DE hierarchy implies a deeper connection between the two. Relying on this connection as our guidance, we investigate the possibility of interpreting the Casimir energy density induced in a SUSY-breaking brane world as the dark energy. Invoking the RS warp geometry and Higgs-gravitino coupling on the TeV brane for SUSY-breaking, we demonstrate that the 4D Casimir energy on the brane indeed scales as $\alpha_G^2 M_{\rm Pl}$, just right for the dark energy. This is quite remarkable.

For the sake of simplicity, in this paper we have invoked Dirac fermion to represent the gravitino field. It is natural to wonder whether our result remains unchanged if a bona fide spin 3/2 gravitino field is invoked. We will report on this in a separate paper.

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