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Investment and Upgrade in Distributed Generation under Uncertainty

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Abstract

The ongoing deregulation of electricity industries worldwide is providing incentives for microgrids to use small-scale distributed generation (DG) and combined heat and power (CHP) applications via heat exchangers (HXs) to meet local energy loads. Although the electric-only efficiency of DG is lower than that of central-station production, relatively high tariff rates and the potential for CHP applications increase the attraction of on-site generation. Nevertheless, a microgrid contemplating the installation of gas-fired DG has to be aware of the uncertainty in the natural gas price. Treatment of uncertainty via real options increases the value of the investment opportunity, which then delays the adoption decision as the opportunity cost of exercising the investment option increases as well. In this paper, we take the perspective of a microgrid that can proceed in a sequential manner with DG capacity and HX investment in order to reduce its exposure to risk from natural gas price volatility. In particular, with the availability of the HX, the microgrid faces a tradeoff between reducing its exposure to the natural gas price and maximising its cost savings. By varying the volatility parameter, we find that the microgrid prefers a direct investment strategy for low levels of volatility and a sequential one for higher levels of volatility.

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1 Introduction

For the first hundred years of its existence, the electricity industry was largely centrally planned and operated under government regulation. Indeed, due to the natural monopoly attributes of its transmission sector, the electricity industry was organised along vertically integrated lines with incumbent investor-owned utilities (IOUs) providing all related services in a geographical location. Although such an arrangement allowed the internalisation of many operating complementarities, e.g., such as those between transmission and generation, it, nevertheless, turned the potentially competitive generation and retailing sectors into *de facto* monopolies with the associated inefficiencies (see Joskow (1987)). The scale of the deadweight losses from under-investment in generation capacity became apparent in the 1960s as demand for electricity continued to increase in many industrialised countries (see Marnay and Venkataramanan (2006)). In an effort to increase economic efficiency and to meet growing demand in this industry, many governmental authorities worldwide have deregulated their electricity industries over the past twenty years. Regardless of the contours of these reforms (see Wilson (2002) for more detail), they have attempted to transmit price signals to decision-makers (whether consumers or producers) in an effort to incentivise both usage and supply of electricity.

Such deregulation of the electric power industry also provides incentives for the adoption of distributed generation (DG)[1] by microgrids, which are localised networks of DG and combined heat and power (CHP)[2] applications matched to local energy requirements. Although the electric-only efficiency of DG is lower than that of central-station generation, the former becomes economically attractive when CHP applications are utilised to meet heat loads via heat exchangers (HXs). Furthermore, the persistence of relatively high fixed-tariff rates for volumetric electricity consumption (even in this era of deregulation) implies that DG becomes attractive to commercial entities that may be able to organise themselves into microgrids. In previous work, we have performed detailed economic and thermodynamic analyses of DG investment and operation in purely deterministic settings based on a cost-minimising mixed-integer linear programme (see Siddiqui et al. (2005) and Siddiqui et al. (2007)). In almost all of the case studies for California, we find that adoption of gas-fired DG is attractive, with on-site generators typically covering a large fraction of the electric load (as well as a large fraction of overall energy needs). However, if a microgrid does decide to install gas-fired DG on site, then it should not treat the natural gas price as being deterministic (see Figure 1). Indeed, the uncertainty from historically volatile natural

[1]DG refers to small-scale, on-site generators, usually with capacities under 1 MW_e.

[2]CHP refers to the capture of waste heat from on-site generation and its subsequent utilisation to offset heat loads.

gas prices may inhibit investment in DG as microgrids would have to be sure that such a costly project would not lose its value in the forseeable future. For example, historically, since 1967 to 2005, the annual percentage change in the natural gas price for commercial users in California has been 13.44%, with the annual volatility equal to 17.82% (according to data from the US Energy Information Administration (EIA), which are available at `http://tonto.eia.doe.gov/dnav/ng/ng_pri_sum_dcu_SCA_a.htm`). Hence, any analysis of DG must take into account this uncertainty, which may then delay investment.

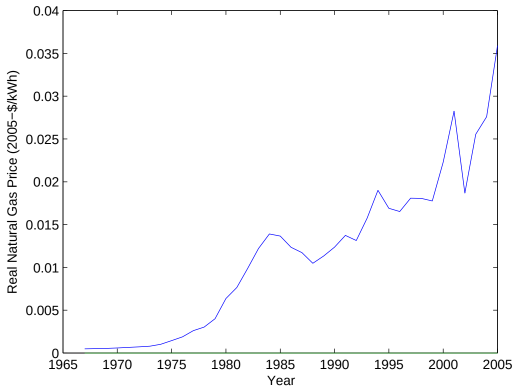

Figure 1: Historical Natural Gas Price Data for California Commercial Users (source: US EIA, `http://tonto.eia.doe.gov/dnav/ng/ng_pri_sum_dcu_SCA_a.htm`)

The risk from such price uncertainty may be managed via investment timing and modularity by using real options (see Brennan and Schwartz (1985) and Dixit and Pindyck (1994)) to model the microgrid's decision making. This approach is appropriate because it trades off in continuous time the benefits from immediate investment with its associated costs. Specifically, the real options approach includes not only tangible investment costs such as the capital cost, but also the opportunity cost of exercising the option to invest, which is the loss of the discretion to wait for more information about the price process. Thus, analogous to the pricing of financial call options (see Black and Scholes (1973)), it may be better to retain the option to invest even for a project that is "in the money" from the deterministic discounted cash flow (DCF) perspective.

Since deregulation provides both opportunities and challenges for the adoption of DG, how should then a typical microgrid proceed with its investment decision? At a fundamental level, the large DG systems recommended by comprehensive, but deterministic, models of customer adoption would be too risky if natural gas prices exhibit the amount of volatility that they have done recently. If a microgrid instead uses the real options approach to analyse its investment decision, then it invariably recommends delaying the installation of DG. What has not been addressed in the literature, however, is if the microgrid is able to modularise its DG and HX adoption by proceeding in a sequential manner when appropriate. In this paper, we take this approach to investigate various investment and upgrade strategies in gas-fired DG. Notably, we focus both on the option to upgrade the capacity and to install a HX after an initial DG unit is purchased that serves the base electric load of the microgrid. We find that for low levels of uncertainty in the natural gas price, direct investment in a DG-HX package that covers the base electric load and the heat load of the microgrid is desirable. This then leaves the microgrid with the option to upgrade its capacity. However, for moderate levels of uncertainty in the natural gas price, a purely sequential investment approach becomes optimal since it enables the microgrid to proceed with investment in each on-site device without bearing all of the risk associated with a large installation and to benefit from swings in the natural gas price by upgrading either to a peak unit or a HX as appropriate.[3] The advantages of a sequential approach have also been illustrated within the context of nuclear power plants (see Gollier et al. (2005)). Of course, we focus purely on the economics of DG adoption while neglecting some of the wider regulatory issues, such as poorly defined and enforced interconnection standards as well as back-up charges and exit fees associated with tariff design. Nevertheless, we hope to provide some insight into the tradeoff between managing the risk exposure to volatile natural gas prices and reducing the costs of meeting on-site energy loads.

The structure of this paper is as follows:

- Section 2 introduces the problem of the microgrid and models it using the real options approach
- Section 3 describes the financial, technological, and energy load data used for our case studies
- Section 4 presents the results of three numerical examples we use to illustrate the intuition behind the investment strategy of the microgrid
- Section 5 summarises the paper and offers directions for future research on this topic

2 Problem Formulation

2.1 Assumptions

We take the perspective of a California-based microgrid that holds the perpetual option to invest in a gas-fired DG unit. If this option is exercised, then the microgrid can cover its constant base electric load,

[3] In Fleten et al. (2007), a model for finding optimal investment timing and capacity choice under uncertainty for DG is presented. The model is developed for renewable energy resources and is based on the assumption that the capacity alternatives are mutually exclusive, i.e., that the capacity of a small-scale windmill or hydropower plant has to be decided prior to investment (see Décamps et al. (2006)). Meanwhile, Wickart and Madlener (2007) considers alternative investments in steam-boiler or cogeneration plants using real options. In this paper that concerns gas-fired DG, capacity can be added to the system later, which will change investment price thresholds due to the increased flexibility.

$Q_{EB}/8760$ (in kW_e), and additionally receive options to upgrade to a peak DG unit as well as a HX. If it exercises the peak DG upgrade option, then the microgrid is able to cover its constant peak electric load, $(Q_{EB} + 2Q_{EP})/8760$ (in kW_e), where Q_{EP} is the annual additional electricity that is consumed during the peak hours of each day, i.e., 0800 to 2000.[4] Similarly, upon exercising the HX upgrade option, the microgrid is able to utilise CHP to meet at least part of its constant heat load, $Q_H/8760$ (in kW).[5] Figure 2 summarises the set of states and transitions possible for the microgrid. Since DG and HX units have short installation lead times, we assume that they are operational instantaneously after being ordered.

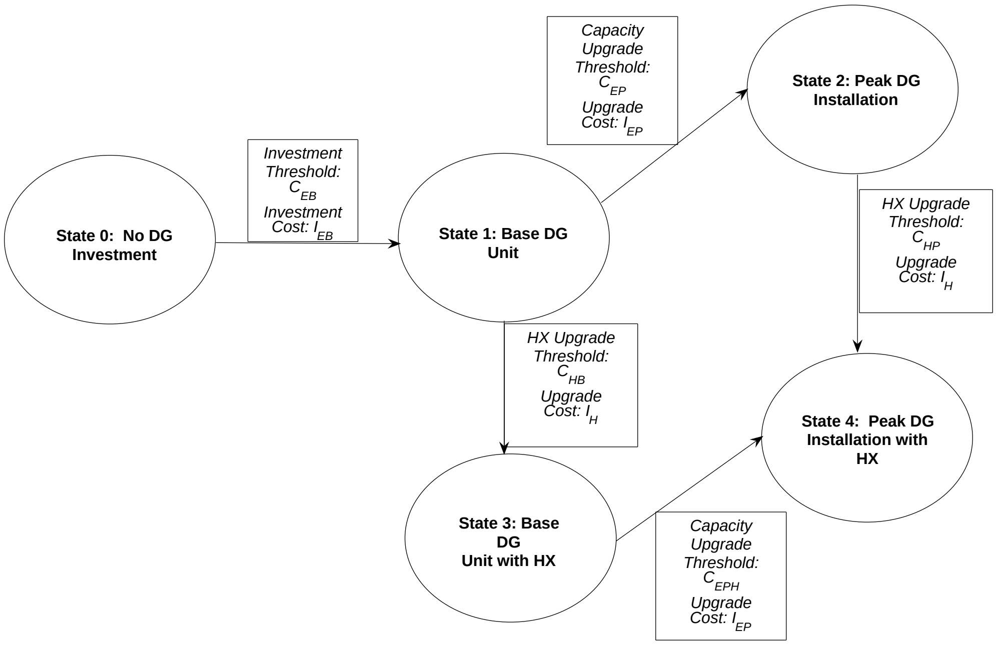

Figure 2: State-Transition Diagram

Prior to the initial investment, the microgrid meets both Q_{EB} and Q_{EP} through utility electricity purchases at a constant price of P (in $/kWh$_e$).[6] At this stage, it must meet Q_H through natural

[4]We assume that the microgrid's peak electric load is one-and-a-half times its base load, but has a duration of only one-half. Therefore, if Q_{EP} is the annual additional electricity used during peak hours, then the peak load is exactly twice what it would be if the additional electric load during peak hours had a duration of one.

[5]We use the units kWh_e and kWh to distinguish electrical and heat energy, respectively. Although natural gas prices are often quoted in $/MMBTU or $/therm, we find kWh a convenient unit since the effective natural gas generation cost (in $/kWh$_e$) may be readily obtained via the heat rate to enable a direct comparison of on- and off-site costs of electricity.

[6]This is assumed constant because under most California utility tariffs, the electricity price may be revised only

gas, the price of which, C_t (in \$/kWh), may exhibit considerable monthly variability and is modelled as evolving exogenously according to a geometric Brownian motion (GBM) process.[7] In particular, $dC_t = \alpha C_t dt + \sigma C_t dz_t$, where $\{z_t, t \geq 0\}$ is a standard Brownian motion process, α is the annual percentage growth rate, and σ is the annual percentage volatility. The exogeneity assumption is justified because a microgrid has a small load relative to that of the system and is, thus, a price taker.

2.2 Simple Investment Analysis with Real Options

If the microgrid wishes to invest in the base DG unit at a capital cost of I_{EB} (in \$), then because the natural gas price is random and there is flexibility over timing, it must trade off in continuous time the present value of cash flows from immediate investment and the time value of information revealed by delaying the investment decision (see Dixit and Pindyck (1994)). The real options approach indicates that there is a threshold natural gas price, C_{EB}, below which investment is optimal, i.e., the natural gas generation cost (the effective cost of producing a kWh$_e$ on-site, or the natural gas price times the heat rate) has to be sufficiently lower than P less the amortised capital cost before investment is triggered.

To illustrate this concept, we summarise an example from Siddiqui and Marnay (2008), in which investment in a base DG unit only is analysed using the real options approach. We denote the value of the option to invest in a DG unit with an infinite lifetime[8] as $V_0(C)$ when the natural gas price is C, while the present value (PV) of an installed unit at the same price is:

$$\begin{aligned} V_1(C) &= \int_0^\infty PQ_{EB}e^{-\rho t}dt - \int_0^\infty \mathcal{E}[C_t|C]\epsilon_B Q_{EB}e^{-\rho t}dt \\ \Rightarrow V_1(C) &= -\frac{PQ_{EB}e^{-\rho t}}{\rho}\Big]_0^\infty - \int_0^\infty Ce^{\alpha t}\epsilon_B Q_{EB}e^{-\rho t}dt \\ \Rightarrow V_1(C) &= \frac{PQ_{EB}}{\rho} + \frac{C\epsilon_B Q_{EB}e^{-(\rho-\alpha)t}}{\rho-\alpha}\Big]_0^\infty \\ \Rightarrow V_1(C) &= \frac{PQ_{EB}}{\rho} - \frac{C\epsilon_B Q_{EB}}{\rho-\alpha} \end{aligned} \quad (1)$$

This is simply the PV of perpetual cost savings from using DG to meet Q_{EB} instead of utility purchases

periodically.

[7]Although geometric mean-reverting (GMR) processes (perhaps with jump diffusion features) are often posited for energy prices, we use GBM for its analytical tractability. Depending on the parameters, the results may be rather different with a GMR assumption. For example, a GMR process will not tend to stray far from its long-term mean if the mean-reversion rate is high. A comprehensive analysis of energy prices using 127 years of data has shown the mean-reversion rate for energy prices to be low (see Pindyck (1999)). Moreover, the same paper estimates the volatility to be stable. These two findings together imply that the GBM assumption may be reasonable for long-term energy prices.

[8]We assume that once the DG unit is installed, its effective lifetime is infinite due to the possibility of maintenance upgrades. This simplification is further justified by the fact that the discrepancy between the PV of a perpetuity and the PV of an annuity due decreases with the length of the time horizon.

of electricity. Here, $\rho > \alpha$ is an arbitrary discount rate, and ϵ_B is the heat rate of the gas-fired DG unit (in kWh/kWh$_e$).

We proceed by using the dynamic programming approach, i.e., via the following Bellman Equation, to find the value of the option to invest, $V_0(C)$:

$$\rho V_0(C)dt = \mathcal{E}[dV_0(C)] \tag{2}$$

This states that the instantaneous rate of return on $V_0(C)$ must equal its expected appreciation. Next, by applying Itô's Lemma to the right-hand side of Equation 2, we obtain the following:

$$\begin{aligned} dV_0(C) &= V_0^{'}(C)dC + \tfrac{1}{2}V_0^{''}(C)(dC)^2 \\ \Rightarrow \quad \mathcal{E}[dV_0(C)] &= \alpha C V_0^{'}(C)dt + \tfrac{1}{2}\sigma^2 C^2 V_0^{''}(C)dt \end{aligned} \tag{3}$$

Then, by substituting Equation 3 into Equation 2, we derive the following ODE:

$$\frac{1}{2}\sigma^2 C^2 V_0^{''}(C) + \alpha C V_0^{'}(C) - \rho V_0(C) = 0 \tag{4}$$

If we apply the boundary condition $\lim_{C\to\infty} V_0(C) = 0$, i.e., the value of the option to invest becomes worthless as the natural gas price increases without bound, then the solution to the ODE in Equation 4 is:

$$V_0(C) = A_2^{'} C^{\beta_2}, \text{ if } C \geq C_I \tag{5}$$

Here, $A_2^{'}$ is a positive endogenous constant, whereas β_2 is the negative root of the characteristic quadratic equation $\frac{1}{2}\sigma^2\beta(\beta-1) + \alpha\beta - \rho = 0$, which has the following roots:

$$\beta_1 = \frac{1}{2} - \alpha/\sigma^2 + \sqrt{\left[\alpha/\sigma^2 - \frac{1}{2}\right]^2 + 2\rho/\sigma^2} > 1 \tag{6}$$

$$\beta_2 = \frac{1}{2} - \alpha/\sigma^2 - \sqrt{\left[\alpha/\sigma^2 - \frac{1}{2}\right]^2 + 2\rho/\sigma^2} < 0 \tag{7}$$

Now, the endogenous constant, $A_2^{'}$, as well as the investment threshold price, C_I, are determined by using the value-matching and smooth-pasting conditions:

$$\begin{aligned} V_0(C_I) &= V_1(C_I) - I_{EB} \\ \Rightarrow A_2^{'} C_I^{\beta_2} &= \tfrac{P}{\rho} Q_{EB} - \tfrac{C_I}{\rho-\alpha}\epsilon_B Q_{EB} - I_{EB} \end{aligned} \tag{8}$$

$$\begin{aligned} V_0'(C_I) &= V_1'(C_I) \\ \Rightarrow \beta_2 A_2^{'} C_I^{\beta_2 - 1} &= -\frac{1}{\rho-\alpha}\epsilon_B Q_{EB} \end{aligned} \tag{9}$$

Equation 8 states that upon exercise of the option, the microgrid receives the net present value (NPV) of an active DG unit, whereas Equation 9 is a first-order necessary condition that equates the marginal benefit of delaying investment (stemming from additional information about the natural gas price process) to its marginal cost (due to the time value of money) at the point of exercise. From these two conditions, closed-form analytical solutions may be found for the two unknowns, C_I and $A_2^{'}$:

$$C_I = \left(\frac{(\rho-\alpha)\beta_2}{\beta_2 - 1}\right)\left(\frac{P}{\rho\epsilon_B} - \frac{I_{EB}}{\epsilon_B Q_{EB}}\right) \tag{10}$$

$$A_2^{'} = -\tfrac{(C_I)^{1-\beta_2}}{(\rho-\alpha)\beta_2}\epsilon_B Q_{EB} \tag{11}$$

According to the deterministic DCF approach, however, investment in the DG unit occurs as long as its NPV is positive, i.e.,

$$\begin{aligned} & NPV(C) \geq 0 \\ & \Rightarrow V_1(C) - I_{EB} \geq 0 \\ & \Rightarrow \tfrac{PQ_{EB}}{\rho} - \tfrac{C\epsilon_B Q_{EB}}{\rho-\alpha} - I_{EB} \geq 0 \\ & \Rightarrow C_I^{det} = \tfrac{(\rho-\alpha)P}{\rho\epsilon_B} - \tfrac{(\rho-\alpha)I_{EB}}{\epsilon_B Q_{EB}} \end{aligned} \tag{12}$$

Since $\frac{\beta_2}{\beta_2-1} < 1$, comparing Equations 10 and 12 indicates that $C_I < C_I^{det}$. Therefore, with the real options approach, investment in DG has greater value (since $V_0(C) > V_1(C) - I_{EB}$ for $C > C_I$), but occurs at a lower natural gas price threshold as the opportunity cost of exercising the option is also greater.

2.3 Compound Investment and Upgrade

We now expand this framework to allow for investment not only in a base DG unit, but also in a DG unit that can cover the microgrid's peak electric load as well as in a HX to cover its heat load. Depending on the amount of uncertainty present, the microgrid may find it more beneficial to modularise its investment rather than installing the base DG unit, peak DG unit, and HX all at once. The sequence of possible transitions among states is outlined in Figure 2. Initially, if no investment has occurred, then the microgrid simply holds the value of the option to invest, which is worth the following:

$$V_0(C) = A_2 C^{\beta_2}, \text{ if } C \geq C_{EB} \tag{13}$$

From this position, known as state 0, the microgrid can install a base DG unit if it proceeds sequentially. Once this initial investment has occurred, the microgrid enters state 1, where it gains the NPV of cash flows from using the DG unit to meet its base electric load[9] as well as the value of the options to

[9]Unlike Siddiqui and Marnay (2008), we do not investigate operational flexibility since our focus here is on alternative investment strategies.

upgrade to a peak DG unit and a HX.[10] If we again assume that the lifetimes of all on-site equipment are infinite, then the PV of base electric load cost savings is simply the difference between two perpetuities, viz., the PV of offset electricity purchases ($\frac{P}{\rho}Q_{EB}$) and the PV of fuel expenses of the base DG unit ($\frac{C}{\rho-\alpha}\epsilon_B Q_{EB}$), plus the amount saved by not having to pay the power demand charge ($\frac{D_E}{8760\rho}Q_{EB}$), where D_E is the power demand rate in \$/kW$_e$ per annum. Thus, the total PV in state 1 is $PV_B(C) = \frac{P}{\rho}Q_{EB} - \frac{C}{\rho-\alpha}\epsilon_B Q_{EB} + \frac{D_E}{8760\rho}Q_{EB}$. The microgrid's total value in this state, inclusive of the capacity and HX upgrade options, is therefore:

$$V_1(C) = PV_B(C) + B_1 C^{\beta_1} + B_2 C^{\beta_2}, \text{ if } C_{EP} \leq C \leq C_{HB} \tag{14}$$

Note that B_1 and B_2 are positive endogenous constants,[11] whereas β_1 and β_2 are defined in Equations 6 and 7. Consequently, Equation 14 indicates that the value of the microgrid in state 1 is the PV of a base DG unit plus the options to upgrade to a HX (captured by the second term, which increases in C because as the natural gas price increases, it becomes more attractive to use CHP applications instead of purchasing natural gas) and to upgrade its capacity (captured by the third term, which decreases in C because as the natural gas price increases, it becomes less attractive to meet the electric load from on-site generation).

From state 1, where only a base DG unit is installed, the microgrid may next install either a peak DG unit or a HX. In case of the former upgrade, the microgrid enters state 2 by optimally waiting for the natural gas price to drop further, i.e., to a threshold C_{EP}. With a peak DG unit installed at a cost of I_{EP} (in \$) in addition to the base DG unit, the microgrid is then able to cover its entire electric load. Here, the PV of the microgrid is $PV_B(C)+PV_P(C)$, where $PV_P(C) = \frac{P}{\rho}Q_{EP} - \frac{C}{\rho-\alpha}\epsilon_P Q_{EP} + \frac{D_E}{8760\rho}2Q_{EP} + X_E/\rho$, while X_E is the annual electricity customer charge from the utility (in \$), which we assume is waived if the microgrid covers its entire electric load as in state 2. Thus, the value of the microgrid in state 2 is:

$$V_2(C) = PV_B(C) + PV_P(C) + D_1 C^{\beta_1}, \text{ if } C \leq C_{HP} \tag{15}$$

The last term in Equation 15 is the value of the option to upgrade to a HX.

Alternatively, from state 1, the HX upgrade option may be exercised once the natural gas price exceeds a threshold, C_{HB}, prior to dropping below the capacity upgrade threshold, C_{EP}. Intuitively, the upgrade will not occur when C is low because the resulting cost saving does not justify the capital cost associated with the upgrade, I_H (in \$). Typically, we would expect $C_{HB} > C_{EB}$, but it may be

[10] Although these options are not mutually exclusive, i.e., if the microgrid exercises one, then the other one is still available in the future, only one may be taken at a time. Therefore, the option values are modelled as in Dixit and Pindyck (1994), p. 232.

[11] Since the microgrid cannot exercise both options simultaneously, the constants B_1 and B_2 do not have the same values as if each option were available separately.

possible for $C_{HB} < C_{EB}$, in which case the microgrid instantaneously upgrades to a HX after having installed a base DG unit. If the microgrid proceeds to upgrade its base DG system with a HX, then it incurs a capital cost of I_H and enters state 3. In turn, it receives not only the PV of base electric cost savings, but also the PV associated with heating cost savings. This latter term is simply the present value of forgone natural gas purchases for the heat load displaced by the application of CHP, i.e., it is equal to $PV_H(C) = \frac{C}{\rho-\alpha}\min\{Q_H, \gamma Q_{EB}\}$, where γ is the kWh of useful heat produced by each kWh$_e$ of on-site generation from the base DG unit. Therefore, its total NPV in this state is $PV_B(C) + PV_H(C)$, and its total value includes also the subsequent option to upgrade its capacity:

$$V_3(C) = PV_B(C) + PV_H(C) + F_2 C^{\beta_2}, \text{ if } C \geq C_{EPH} \tag{16}$$

Finally, from either state 2 or 3, the microgrid can complete its on-site energy system by upgrading either to a HX or a peak DG unit, respectively. In this final state (known as state 4), the value of the microgrid is simply the PV of the installed equipment:

$$V_4(C) = PV_B(C) + PV_P(C) + PV_H(C) \tag{17}$$

In order to solve for the five endogenous constants and five investment thresholds, we use the following five value-matching and five smooth-pasting conditions:

$$\begin{aligned} V_0(C_{EB}) &= V_1(C_{EB}) - I_{EB} \\ \Rightarrow A_2 C_{EB}^{\beta_2} &= B_1 C_{EB}^{\beta_1} + B_2 C_{EB}^{\beta_2} + PV_B(C_{EB}) - I_{EB} \end{aligned} \tag{18}$$

$$\begin{aligned} V_0'(C_{EB}) &= V_1'(C_{EB}) \\ \Rightarrow \beta_2 A_2 C_{EB}^{\beta_2-1} &= \beta_1 B_1 C_{EB}^{\beta_1-1} + \beta_2 B_2 C_{EB}^{\beta_2-1} - \tfrac{1}{\rho-\alpha}\epsilon_B Q_{EB} \end{aligned} \tag{19}$$

$$\begin{aligned} V_1(C_{EP}) &= V_2(C_{EP}) - I_{EP} \\ \Rightarrow B_1 C_{EP}^{\beta_1} + B_2 C_{EP}^{\beta_2} &= D_1 C_{EP}^{\beta_1} + PV_P(C_{EP}) - I_{EP} \end{aligned} \tag{20}$$

$$\begin{aligned} V_1'(C_{EP}) &= V_2'(C_{EP}) \\ \Rightarrow \quad \beta_1 B_1 C_{EP}^{\beta_1-1} + \beta_2 B_2 C_{EP}^{\beta_2-1} &= \beta_1 D_1 C_{EP}^{\beta_1-1} - \tfrac{1}{\rho-\alpha}\epsilon_P Q_{EP} \end{aligned} \tag{21}$$

$$\begin{aligned} V_1(C_{HB}) &= V_3(C_{HB}) - I_H \\ \Rightarrow B_1 C_{HB}^{\beta_1} + B_2 C_{HB}^{\beta_2} &= F_2 C_{HB}^{\beta_2} + PV_H(C_{HB}) - I_H \end{aligned} \tag{22}$$

$$\begin{aligned} V_1'(C_{HB}) &= V_3'(C_{HB}) \\ \Rightarrow \quad \beta_1 B_1 C_{HB}^{\beta_1-1} + \beta_2 B_2 C_{HB}^{\beta_2-1} &= \beta_2 F_2 C_{HB}^{\beta_2-1} + \tfrac{1}{\rho-\alpha}\min\{Q_H, \gamma Q_{EB}\} \end{aligned} \tag{23}$$

$$V_2(C_{HP}) = V_4(C_{HP}) - I_H$$
$$\Rightarrow D_1 C_{HP}^{\beta_1} = PV_H(C_{HP}) - I_H \tag{24}$$

$$V_2'(C_{HP}) = V_4'(C_{HP})$$
$$\Rightarrow \quad \beta_1 D_1 C_{HP}^{\beta_1 - 1} = \tfrac{1}{\rho-\alpha} \min\{Q_H, \gamma Q_{EB}\} \tag{25}$$

$$V_3(C_{EPH}) = V_4(C_{EPH}) - I_{EP}$$
$$\Rightarrow F_2 C_{EPH}^{\beta_2} = PV_P(C_{EPH}) - I_{EP} \tag{26}$$

$$V_3'(C_{EPH}) = V_4'(C_{EPH})$$
$$\Rightarrow \quad \beta_2 F_2 C_{EPH}^{\beta_2 - 1} = -\tfrac{1}{\rho-\alpha} \epsilon_P Q_{EP} \tag{27}$$

Since the resulting system of Equations 18 to 27 is highly non-linear, there are no closed-form analytical solutions to most of the ten unknowns. Nevertheless, this system may be solved numerically for specific parameter values, which is what we do in Section 4.3. It should be noted that Figure 2 does not indicate alternative investment strategies in which the microgrid invests directly in an energy system rather than proceeding sequentially. For example, there are three alternatives to the procedure outlined in Figure 2:

1. Invest directly in a peak DG system with a HX, i.e., go directly from state 0 to state 4
2. Invest directly in a base DG unit coupled with a HX and wait for the opportunity to upgrade capacity, i.e., go directly from state 0 to state 3
3. Invest directly in a peak DG system and wait for the opportunity to upgrade to a HX, i.e., go directly from state 0 to state 2

In Section 4, we shall contrast these direct investment approaches with the sequential one outlined here by using data from a California-based microgrid. However, we first discuss and outline the financial, technological, and energy load data for our model.

3 Data

Since we are interested in analysing a California-based microgrid, we use data from Siddiqui et al. (2007), which performs case studies on numerous test sites in the San Francisco area. For simplicity, we assume that both the base electric and heat loads are constant at 500 kW$_e$ and 100 kW during each hour of the year, respectively. This implies that Q_{EB} = 500 kW$_e$ × 8760 h = 4380 MWh$_e$ and

$Q_H = 100\text{kW} \times 8760 \text{ h} = 876$ MWh. By contrast, the peak load is constant at 250 kW$_e$ for only twelve hours each day and is zero otherwise. Therefore, $Q_{EP} = \frac{1}{2} \times 250 \text{ kW}_e \times 8760 \text{ h} = 1095 \text{ MWh}_e$ (see Figure 3). Since San Francisco is in the service territory of the Pacific Gas & Electric Company (PG&E), we apply its tariff, which is summarised in Table 1.

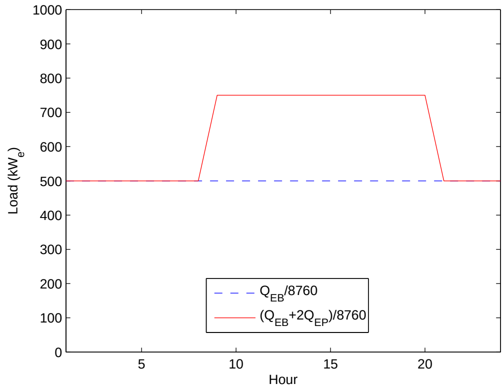

Figure 3: Electric Load Data for Commercial Microgrid

Parameter	Value
P	\$0.10/kWh$_e$
D_E	\$144/kW$_e$
X_E	\$2100

Table 1: PG&E Tariff Information

It should be noted that the electric demand charge, D_E, is \$12 per kW$_e$ per month, which becomes \$144 per kW$_e$ per annum. Similarly, the annual electric customer charge, X_E, is \$175 per month, which is \$2100 per annum. Other financial data are the discount rate, ρ, which we assume to be 6% per annum, i.e., approximately the same as that charged for a secured loan in California during 2006, and the growth rate of the natural gas price, α, which we set equal to zero without loss of generality in order to focus on the stochastic aspect of the natural gas price. Indeed, this can be considered as a numéraire with all other parameters defined relative to it (see Wickart and Madlener (2007)). Finally, the initial natural gas price is $C_0 = \$0.0324$/kWh (as of April 2006), and we allow its volatility, σ, to vary between 0 and 0.45 to reflect the range of the volatility estimate from the historical time series plotted in Figure 1. For example, the volatility was approximately 0.10 during the 1990s and has been close to 0.30 since 2000.

For the technological data, we use the capital cost and heat rate (or, useful heat recovered per kWh$_e$ of electricity generated in case of the HX) from Siddiqui et al. (2007) (see Table 2). For reference, the capital cost of a gas-fired reciprocating engine is \$795/kW$_e$ while that of a gas-fired microturbine is \$1400/kW$_e$. The capital cost of the HX is estimated as the difference in cost between a CHP-enabled reciprocating engine and one without a HX. Thermodynamically, the heat rate of the base and peak DG units are $\epsilon_B = 3.01$ and $\epsilon_P = 3.57$, respectively, i.e., they have fuel-to-electric conversion efficiencies of $\frac{1}{3.01} = 0.33$ and $\frac{1}{3.57} = 0.28$, respectively. If waste heat from this base DG unit is recycled to offset the natural gas purchases used to meet the heat load, then 1.55 kWh of heat are available for each kWh$_e$ of electricity generated, i.e., $\gamma = 1.55$. It may be seen that the peak DG unit not only is less economically attractive than the base DG unit, but also has a lower utilisation rate since it will run for only half the hours in each day if installed. Consequently, the natural gas price would have to decrease noticeably for the peak DG unit to be adopted. We illustrate properties of this investment and upgrade problem in Section 4 using the given data.

Equipment	Capital Cost (\$)	Heat Rate (DG) or Useful Heat (HX) (kWh/kWh$_e$)
500 kW$_e$ DG unit	$I_{EB} = 397500$	$\epsilon_B = 3.01$
250 kW$_e$ DG unit	$I_{EP} = 350000$	$\epsilon_P = 3.57$
HX	$I_H = 135000$	$\gamma = 1.55$

Table 2: Equipment Data

4 Results

Given the data in Section 3, we solve the microgrid's sequential investment and upgrade problem to find natural gas price investment and upgrade thresholds for various levels of σ. Since our objective is to compare the results of a sequential investment strategy with those of direct ones, we need to develop the intuition for both capacity and HX upgrade decisions. Towards this end, we perform two preliminary case studies:

- First, in Section 4.1, we address capacity upgrades by solving the investment problem of a microgrid that has the perpetual option to invest in a base DG unit, which then provides it with the option to upgrade to an additional DG unit capable of covering its peak electric load. By comparing such a sequential investment strategy with a direct one (in which the microgrid may invest only in a DG system that covers its total electric load), we extract the option value of flexibility from the capacity upgrade only.
- Then, in Section 4.2, we focus on the HX upgrade decision by tackling the investment problem of a microgrid that has the perpetual option to invest in a base DG unit, which then entitles it to install

a HX to meet its heat load via CHP applications. Again, if we compare this sequential investment strategy with a direct one (in which the microgrid may invest only in a packaged DG-HX system), then we can calculate the option value of flexibility associated with the HX upgrade.

Finally, in Section 4.3, we solve the complete capacity and HX problem as indicated in Section 2.3. Specifically, we obtain natural gas price investment and upgrade thresholds over a range of σ. By comparing the option value of the sequential investment strategy with those of the three alternative approaches, we obtain the option value of flexibility and are able to determine the range of σ for which each strategy is preferred.

4.1 Numerical Example 1: Base DG Investment with Capacity Upgrade

If the microgrid ignores the heat load and considers only the option to invest in a base DG unit along with the subsequent option to upgrade to a peak DG unit, then there are three states of the world:

- State 0: the DG unit has not yet been installed, and the microgrid meets Q_{EB} and Q_{EP} through grid purchases at P. From the real options analysis, it can be shown that the value of the investment opportunity in this state is $V_0^{cap}(C) = A_2^{cap} C^{\beta_2}$, if $C \geq C_{EB}^{cap}$, where A_2^{cap} is a positive endogenous constant and C_{EB}^{cap} is the base DG unit investment threshold.

- State 1: here, the base DG unit has been installed. The cost savings to the microgrid in this state equal $PV_B(C)$, and the value of the option to upgrade to a peak DG unit is $B_2^{cap} C^{\beta_2}$, where B_2^{cap} is a positive endogenous constant. As in Section 2.3, it can be shown that the value to the microgrid in this state is $V_1^{cap}(C) = PV_B(C) + B_2^{cap} C^{\beta_2}$, if $C \geq C_{EP}^{cap}$, where C_{EP}^{cap} is the peak DG unit upgrade threshold.

- State 2: after the peak DG unit is installed, the microgrid is able to cover its entire electric load. Hence, the microgrid's value in this state is the PV of all electric load cost savings, i.e., $V_2^{cap}(C) = PV_B(C) + PV_P(C)$.

As discussed in Section 2.3, the endogenous constants and relevant thresholds need to be determined. By analytically solving the value-matching and smooth-pasting conditions in Appendix A, we obtain these four unknowns :

$$C_{EP}^{cap} = \left(\frac{(\rho-\alpha)\beta_2}{\beta_2-1}\right)\left(\frac{P}{\rho\epsilon_P} + \frac{2D_E}{8760\rho\epsilon_P} + \frac{X_E}{\rho\epsilon_P Q_{EP}} - \frac{I_{EP}}{\epsilon_P Q_{EP}}\right) \tag{28}$$

$$B_2^{cap} = -\frac{\epsilon_P Q_{EP} (C_{EP}^{cap})^{1-\beta_2}}{(\rho-\alpha)\beta_2} \tag{29}$$

$$C_{EB}^{cap} = \left(\frac{(\rho-\alpha)\beta_2}{\beta_2-1}\right)\left(\frac{P}{\rho\epsilon_B} + \frac{D_E}{8760\rho\epsilon_B} - \frac{I_{EB}}{\epsilon_B Q_{EB}}\right) \tag{30}$$

$$A_2^{cap} = -\frac{\epsilon_B Q_{EB} (C_{EB}^{cap})^{1-\beta_2}}{(\rho-\alpha)\beta_2} - \frac{\epsilon_P Q_{EP} (C_{EP}^{cap})^{1-\beta_2}}{(\rho-\alpha)\beta_2} \tag{31}$$

By comparing Equations 28 and 30 with Equation 10, it can be seen that even when there is the sequential capacity upgrade option, the natural gas price thresholds are the same as if investment were occurring independently in base and peak DG units. In other words, investment in a base DG unit with the subsequent option to upgrade capacity occurs at precisely the same natural gas price threshold as if the upgrade option did not exist.[12] This myopic result holds because the option to upgrade capacity does not reduce the microgrid's net exposure to natural gas prices by allowing it to benefit from high natural gas prices as well; indeed, the profitability of the peak DG unit decreases with the natural gas price, just as that of the base DG unit. Therefore, the microgrid can proceed with this sequential investment opportunity as if it were evaluating independent investment in the two DG units.

In spite of this myopic outcome, there is one advantage to proceeding in a sequential manner: by first installing a base DG unit and then waiting to see what happens to the natural gas price, the microgrid is able to hedge against high natural gas prices in the future. By contrast, a direct investment strategy, in which enough capacity is installed to cover the peak electric load, exposes a larger system with higher capital costs to adverse natural gas price fluctuations. Consequently, the investment threshold price for the full-capacity DG system is lower than C_{EB}^{cap} as the microgrid needs to be more cautious before installing a larger on-site system. Due to the absence of this risk-hedging feature, the option value of the direct investment approach is less than that of the sequential one. We can confirm this by letting $V_0^{cap,D}(C) = A_2^{cap,D} C^{\beta_2}$, if $C \geq C_I^{cap,D}$, and $V_2^{cap,D}(C) = PV_B(C) + PV_P(C)$ be the values to the microgrid in the two states when using the direct investment approach.[13] Again, by setting up relevant value-matching and smooth-pasting conditions (see Appendix B), we can solve for the investment threshold price, $C_I^{cap,D}$, and the endogenous constant, $A_2^{cap,D}$:

$$C_I^{cap,D} = \left(\frac{(\rho-\alpha)\beta_2}{\beta_2 - 1}\right)\left(\frac{\frac{P(Q_{EB}+Q_{EP})}{\rho} + \frac{D_E(Q_{EB}+2Q_{EP})}{8760\rho} + \frac{X_E}{\rho} - I_{EB} - I_{EP}}{\epsilon_B Q_{EB} + \epsilon_P Q_{EP}}\right) \tag{32}$$

$$A_2^{cap,D} = -\frac{(\epsilon_B Q_{EB} + \epsilon_P Q_{EP})(C_I^{cap,D})^{1-\beta_2}}{(\rho-\alpha)\beta_2} \tag{33}$$

The sequential strategy allows the microgrid to proceed with the investment sooner than in the direct case, i.e., it is always the result that $C_{EB}^{cap} > C_I^{cap,D}$ for the data used in this case study. Indeed, the opportunity to install a base DG unit before waiting for more information about the natural gas price is worth more than the opportunity to profit from a large system. This is illustrated in Figures 4 and 5: in the former, the microgrid waits for the natural gas price to drop to a lower level than

[12]With the sequential investment approach, the option to install a peak DG unit does not become available until the base DG unit has been installed already.

[13]Note that with this direct strategy, the microgrid proceeds directly from state 0 to state 2.

C_{EB}^{cap} before investing in the entire system. The difference between the direct and sequential investment value curves at the current natural gas price, $V_0^{cap}(C_0) - V_0^{cap,D}(C_0)$, is the option value of flexibility in making capacity upgrades (see Figure 6). We find that the sequential investment strategy is always more valuable than the direct one because it allows more precision over the timing of the capacity upgrade. For example, it may be more beneficial to delay installation of the peak DG unit to a future time period with lower natural gas prices. However, the value of this advantage erodes as the natural gas price volatility increases since this makes higher natural gas prices more likely; thus, even initial DG investment becomes more risky, an effect that is not offset by the peak DG upgrade option because it also decreases in profitability as the natural gas price increases. Hence, the effectiveness of hedging risk by proceeding sequentially diminishes because higher natural gas prices imply that there is less chance that any DG will ever be installed, thereby making the upgrade timing issue less important.

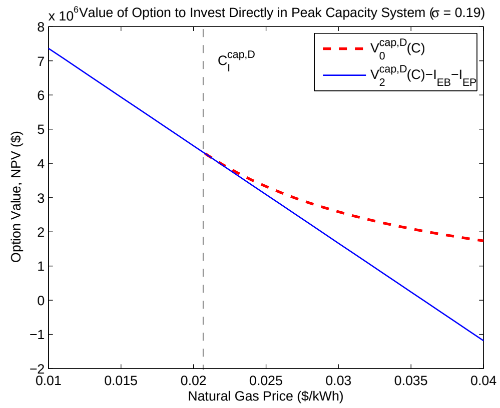

Figure 4: Value of Option to Invest Directly in Peak Capacity System ($\sigma = 0.19$)

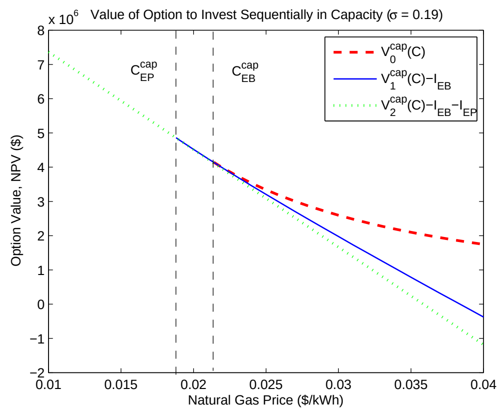

Figure 5: Value of Option to Invest Sequentially in Capacity ($\sigma = 0.19$)

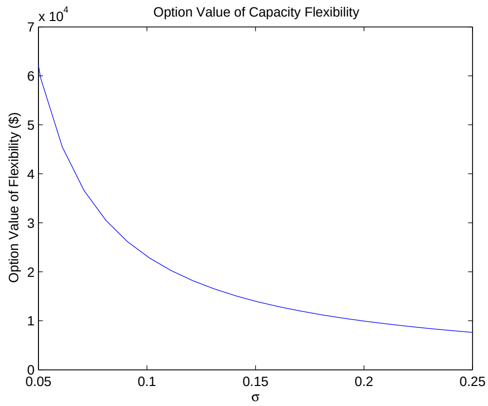

Figure 6: Option Value of Capacity Flexibility

As a sensitivity analysis, we next calculate the investment and upgrade threshold prices from the sequential strategy for a range of σ and compare them to the investment threshold price indicated by the direct investment strategy. In Figure 7, we plot both C_{EB}^{cap} and C_{EP}^{cap} together with $C_I^{cap,D}$ as well as the deterministic investment threshold for the large system, $C_I^{cap,det}$. As expected, the investment and upgrade thresholds under uncertainty decrease with natural gas price volatility, i.e., more uncertainty in the model makes the microgrid more cautious. More subtly, higher volatility increases the opportunity cost of exercising the investment or upgrade option because it is in precisely such a situation that the value of waiting for more information about the natural gas price is greater. In summary, at low levels of natural gas price volatility, the direct investment strategy is both more exposed to the natural gas price and less able to optimise the timing of the peak DG unit's installation. As σ increases, however, the latter deficiency of the direct investment strategy diminishes because it becomes less likely for the peak DG unit to be profitable. In the next section, we similarly examine the tradeoffs inherent in direct and sequential investment strategies involving an HX upgrade option.

4.2 Numerical Example 2: Base DG Investment with HX Upgrade

In this section, we neglect the peak electric load and instead find investment and upgrade threshold prices for the CHP problem using the real options approach and compare them to the results provided by a direct investment approach, i.e., one in which the microgrid has only the option to invest in a CHP-enabled base DG unit for a capital cost of $I_{EB} + I_H$. Again, by doing sensitivity analysis on natural gas price volatility, we would like to determine when it is optimal to install a base DG unit directly with a HX and when it is better to make the investment sequentially.

There are the following three states of the world in this setup:

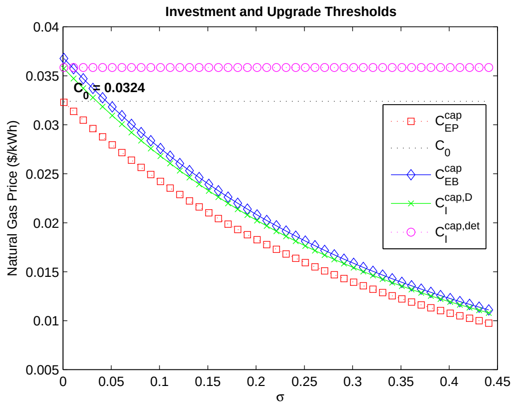

Figure 7: Natural Gas Price Investment and Upgrade Thresholds for Capacity Problem

- State 0: the base DG unit has not yet been installed, and the microgrid meets Q_{EB} and Q_H through grid purchases at P and natural gas purchases at C_t, respectively. From the real options approach, it can be shown that the value of the investment opportunity in this state is $V_0^{hx}(C) = A_2^{hx} C^{\beta_2}$, if $C \geq C_{EB}^{hx}$, where A_2^{hx} is a positive endogenous constant.

- State 1: here, the base DG unit has been installed, but without CHP capability. The cost savings to the microgrid in this state equal $PV_B(C)$, and the value of the option to upgrade is $B_1^{hx} C^{\beta_1}$, where B_1^{hx} is a positive endogenous constant. Therefore, the value of the microgrid in this state is $V_1^{hx}(C) = PV_B(C) + B_1^{hx} C^{\beta_1}$, if $C \leq C_{HB}^{hx}$.

- State 2: after the HX unit is installed, the microgrid is able to recover waste heat from on-site generation to meet its heat load. Hence, the microgrid's value in this state is the PV of baseload electric and heat cost savings, i.e., $V_2^{hx}(C) = PV_B(C) + PV_H(C)$.

We then solve for the two endogenous constants and two investment thresholds by using the two value-matching and two smooth-pasting conditions in Appendix C. Only the upgrade threshold and option value coefficient may be solved analytically:

$$C_{HB}^{hx} = \left(\frac{(\rho-\alpha)\beta_1}{\beta_1 - 1}\right) \frac{I_H}{\min\{Q_H, \gamma Q_{EB}\}} \tag{34}$$

$$B_1^{hx} = \frac{(C_{HB}^{hx})^{1-\beta_1}}{(\rho-\alpha)\beta_1} \min\{Q_H, \gamma Q_{EB}\} \tag{35}$$

Since the remaining two equations are highly non-linear, A_2^{hx} and C_{EB}^{hx} must be obtained numerically for specific data. Furthermore, it is worth noting that the upgrade option does not make economic sense if $C_{EB}^{hx} > C_{HB}^{hx}$, i.e., if the initial investment in the base DG unit is accompanied by the HX. In that case, the problem is one of direct investment in a base DG unit packaged with a HX, which is what we turn to next.

A perpetual option to invest directly in a base DG unit packaged with HX is worth $V_0^{hx,D}(C) = A_2^{hx,D}C^{\beta_2}$ (as long as $C > C_I^{hx,D}$), and the value of an active investment is $V_2^{hx,D}(C) = PV_B(C) + PV_H(C)$. The investment threshold price, $C_I^{hx,D}$, and endogenous constant, $A_2^{hx,D}$, are determined analytically by solving the value-matching and smooth-pasting conditions in Appendix D:

$$C_I^{hx,D} = \left(\frac{(\rho-\alpha)\beta_2}{\beta_2-1}\right)\frac{\left(\frac{PQ_{EB}}{\rho}+\frac{D_EQ_{EB}}{8760\rho}-I_{EB}-I_H\right)}{\epsilon_BQ_{EB}-\min\{Q_H,\gamma Q_{EB}\}} \tag{36}$$

$$A_2^{hx,D} = \frac{(C_I^{hx,D})^{1-\beta_2}}{(\rho-\alpha)\beta_2}\min\{Q_H,\gamma Q_{EB}\} - \frac{(C_I^{hx,D})^{1-\beta_2}}{(\rho-\alpha)\beta_2}\epsilon_BQ_{EB} \tag{37}$$

We again run the model for a range of σ, going from 0 to 0.44. For a low level of natural gas price volatility, it is optimal to invest directly in a CHP-enabled DG unit if the investment threshold price, $C_I^{hx,D}$, is reached because there is not much risk from investing, and therefore, not much additional value to waiting. When the volatility increases to 0.26, it becomes advantageous to proceed sequentially, and thus, the investment threshold price is less than the upgrade threshold price, i.e., $C_{EB}^{hx} < C_{HB}^{hx}$, which implies that due to the greater risk, there is value to waiting after having made the initial investment in DG. Indeed, after the initial investment in DG is triggered, if there is a subsequent natural gas price increase to C_{HB}^{hx}, then it becomes optimal to upgrade to a HX. Therefore, unlike the example in Section 4.1, the investment and upgrade decisions are not myopic because the profitability of the HX increases with the natural gas price, which is contrary to the case with the capacity upgrade option. The corresponding value curves are indicated in Figures 8 and 9.

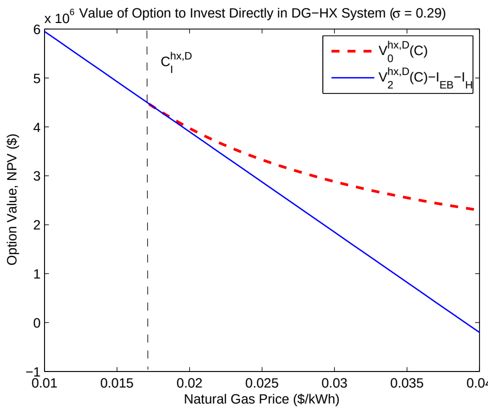

Figure 8: Value of Option to Invest Directly in Base DG-HX System ($\sigma = 0.29$)

At higher levels of volatility, investment is triggered in the sequential system at a slightly lower natural gas price than for the packaged system, i.e., $C_{EB}^{hx} < C_I^{hx,D}$ (see Table 3 and Figure 10, which

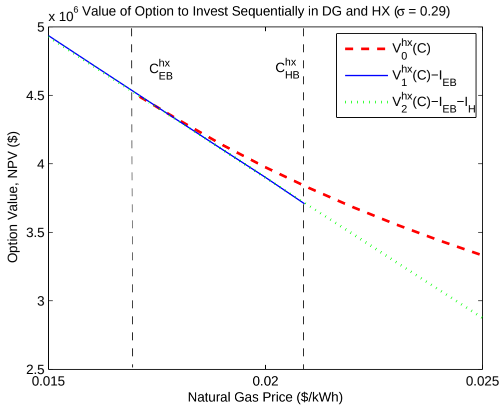

Figure 9: Value of Option to Invest Sequentially in Base DG Unit with HX Upgrade ($\sigma = 0.29$)

also illustrates C^0_{EB}, the threshold at which investment in a base DG unit only occurs). This is because the direct investment strategy is less exposed to the natural gas price, i.e., the microgrid's losses from generation when the natural gas price increases are partially offset by gains from on-site heat production. However, the sequential strategy has greater value overall because it allows the microgrid to optimise the timing of the HX's installation. Due to this added value, the opportunity cost of exercising the option is also greater, thereby providing the seemingly counter-intuitive outcome of $C^{hx}_{EB} < C^{hx,D}_I$.

The additional value from a more flexible investment strategy is illustrated in Figure 11: note that for high natural gas price volatility, the value of the sequential investment option (at the current natural gas price), $V^{hx}_0(C_0)$, is greater than the value of the direct investment option in the packaged DG and HX unit, $V^{hx,D}_0(C_0)$. The option value of this flexibility increases with natural gas price volatility because the HX upgrade option becomes more valuable when higher natural gas prices are more likely, in contrast to the outcome of Section 4.1, where the capacity upgrade with a peak DG unit becomes less valuable with natural gas price increases. Hence, with the HX upgrade option, there is a tradeoff between lower net exposure to the natural gas price (via direct investment) and greater cost savings from optimal timing of the HX adoption (via sequential investment). For low levels of natural gas price volatility, the direct strategy is preferred because it is less likely that the natural gas price will increase in the future and, thus, result in greater cost savings from CHP applications. At high levels of natural gas price volatility, the situation is reversed: it is better to proceed sequentially in order to take advantage of future natural gas price increases.

σ	$C_I^{hx,D}$	C_{EB}^{hx}
0.30	0.0168	0.0167
0.35	0.0147	0.0146
0.40	0.0130	0.0128

Table 3: Base DG Unit Natural Gas Investment Threshold Prices ($/kWh)

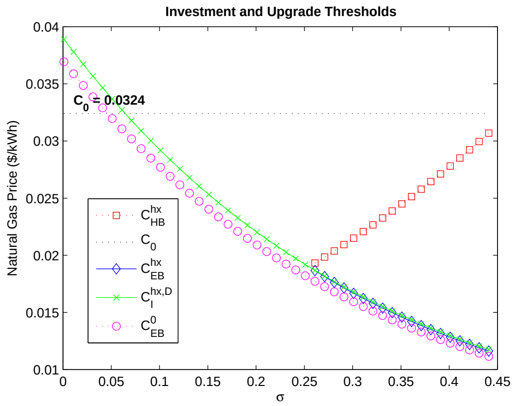

Figure 10: Natural Gas Price Investment and Upgrade Thresholds for CHP Problem

4.3 Numerical Example 3: DG Investment with Both Capacity and HX Upgrade

We now turn to the microgrid's full capacity and HX investment problem with both upgrade options from Section 2.3. To recapitulate, the microgrid may invest in DG units and a HX in a sequential manner as outlined in Figure 2 (henceforth known as strategy 4) or take a more direct approach (strategies 1 through 3 as indicated in Section 2.3). The option values for the former strategy are given in Equations 13 through 17, while the endogenous constants and natural gas price thresholds may be found by solving the value-matching and smooth-pasting conditions in Equations 18 through 27. Only some of the thresholds may be found analytically. For example, solving Equations 26 and 27 simultaneously yields the following:

$$C_{EPH} = \left(\frac{(\rho-\alpha)\beta_2}{\beta_2-1}\right)\left(\frac{P}{\rho\epsilon_P} + \frac{2D_E}{8760\rho\epsilon_P} + \frac{X_E}{\rho\epsilon_P Q_{EP}} - \frac{I_{EP}}{\epsilon_P Q_{EP}}\right) \tag{38}$$

$$F_2 = -\frac{\epsilon_P Q_{EP}(C_{EPH})^{1-\beta_2}}{(\rho-\alpha)\beta_2} \tag{39}$$

It can be verified that $C_{EPH} = C_{EP}^{cap}$ (see Equation 28), i.e., the decision to upgrade capacity when no further options are available is the same whether a HX is installed or not. Analogously, solving Equations

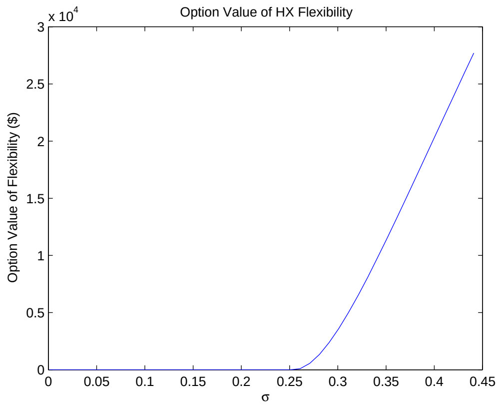

Figure 11: Option Value of HX Flexibility

24 and 25 simultaneously yields:

$$C_{HP} = \left(\frac{(\rho-\alpha)\beta_1}{\beta_1-1}\right)\frac{I_H}{\min\{Q_H,\gamma Q_{EB}\}} \tag{40}$$

$$D_1 = \frac{(C_{HP})^{1-\beta_1}}{(\rho-\alpha)\beta_1}\min\{Q_H,\gamma Q_{EB}\} \tag{41}$$

Again, it is immediate that $C_{HP} = C_{HB}^{hx}$ (see Equation 34), i.e., the decision to upgrade to a HX when there are no further options available is unaffected by the existence of a peak DG unit. The remaining investment and upgrade thresholds, C_{EB}, C_{EP}, and C_{HB}, are solved numerically for a range of σ and plotted in Figure 12. Since strategy 4 is not feasible for $\sigma < 0.26$, we also plot the investment threshold prices for strategies 1 through 3.

As mentioned in Section 2.3, strategy 1 involves direct investment in the peak DG system with a HX, i.e., the microgrid proceeds directly from state 0 to state 4 when the natural gas price drops below C_{IP}. Here, its value in state 0 is $V_0^1(C) = A_2^1 C^{\beta_2}$, if $C \geq C_{IP}$, where A_2^1 is a positive endogenous constant, while its value in state 4 is simply the PV of all the installed equipment: $V_4^1(C) = PV_B(C) + PV_P(C) + PV_H(C)$. We then solve analytically for both C_{IP} and A_2^1 using appropriate value-matching and smooth-pasting conditions. If it uses strategy 2, then the microgrid proceeds directly from state 0 to state 3 by installing a base DG unit with a HX and then waits for the natural gas price to decrease further to C_{EPH} before upgrading its capacity. Therefore, its value in state 0 with strategy 2 is $V_0^2(C) = A_2^2 C^{\beta_2}$, if $C \geq C_{IB}$, where A_2^2 is a positive endogenous constant and C_{IB} is the natural gas investment threshold price in a base DG unit combined with a HX. In state 3, the value of the microgrid is $V_3^2(C) = PV_B(C) + PV_H(C) + F_2 C^{\beta_2}$, if $C \geq C_{EPH}$, where C_{EPH} and F_2 are defined in Equations 38 and 39, respectively, while its value in state 4 is $V_4^2(C) = PV_B(C) + PV_P(C) + PV_H(C)$. Finally, if the microgrid follows strategy 3, then it first installs a peak DG system and then waits for the opportunity to upgrade to a HX, i.e., it proceeds from state 0 to state 2 initially. Hence, its value in state 0 is $V_0^3(C) = A_2^3 C^{\beta_2}$, if $C \geq C_{EPB}$, where A_2^3 is a positive endogenous constant and C_{EPB} is the natural gas investment threshold price in a peak DG system, while its values in states 2 and 4 are $V_2^3(C) = PV_B(C) + PV_P(C) + D_1 C^{\beta_1}$, if $C \leq C_{HP}$ and $V_4^3(C) = PV_B(C) + PV_P(C) + PV_H(C)$, respectively, where C_{HP} and D_1 are defined in Equations 40 and 41, respectively.

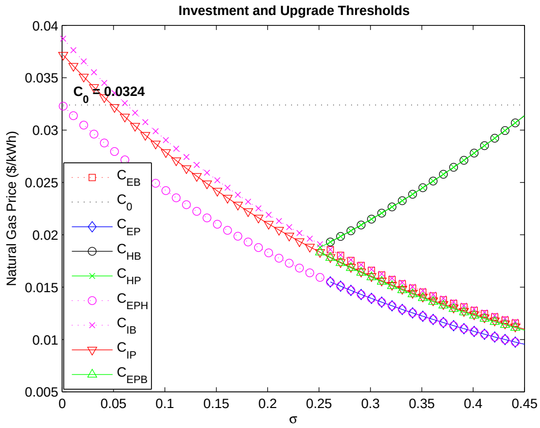

Figure 12: Natural Gas Price Investment and Upgrade Thresholds for Combined Capacity and CHP Problem

σ	C_{EB}	C_{EP}	C_{EPH}	C_{HB}	C_{HP}	C_{IP}	C_{IB}	C_{EPB}
0.25	-	-	0.0159	-	0.0188	0.0183	0.0191	0.0183
0.30	0.0166	0.0139	0.0139	0.0215	0.0215	0.0160	0.0167	0.0160
0.35	0.0145	0.0122	0.0122	0.0245	0.0245	0.0141	0.0147	0.0140
0.40	0.0128	0.0108	0.0108	0.0278	0.0278	0.0124	0.0129	0.0123
0.45	0.0113	0.0095	0.0095	0.0315	0.0314	0.0110	0.0114	0.0109

Table 4: Base and Peak DG Unit Natural Gas Investment Threshold Prices ($/kWh)

From the discussion in Section 4.2, for low natural gas price volatility, there is little incentive to wait before installing a HX since the natural gas price is not likely to increase in the future. For this reason, the microgrid's desire to reduce exposure to the natural gas price dominates the desire to optimise the timing of HX installation. Consequently, for $\sigma < 0.25$, neither strategy 3 nor 4 is feasible as it is preferable to install the HX directly. When σ increases, however, the optimal timing of HX installation becomes more important as higher natural gas prices are more likely, from which the microgrid may benefit in the future via CHP applications. Using the data from Section 3, we explore the dominance of each investment strategy for a range of σ. From Figure 12, we note that the DG investment thresholds decrease with natural gas price volatility while the HX upgrade thresholds increase with natural gas price volatility. Again, with greater uncertainty, the microgrid becomes more hesitant to act because the value of information associated with delaying decisions increases. Table 4 indicates more precisely how the different thresholds vary with σ. In comparing strategies 1 and 2, we note that $C_{IB} > C_{IP}$ since investment in a base DG unit with a HX is less risky than investment in a peak DG system with HX. Next, we compare strategies 1 and 3 to discover that $C_{IP} \geq C_{EPB}$, i.e., investment in a peak DG system with HX is less risky than one without a HX. For higher levels of volatility, however, the sequential approach of strategy 3 is preferred to the direct approach of strategy 1 as there is greater chance of higher natural gas prices in the future, which indicates the importance of the timing of HX adoption. If strategies 2 and 3 are compared, then we find that $C_{IB} > C_{EPB}$ since investment in a base DG unit with a HX is less exposed to the natural gas price than a peak DG system without a HX. Finally, if we analyse strategy 4, then we find that $C_{IP} \leq C_{EPB} < C_{EB} < C_{IB}$, i.e., the initial investment occurs sooner than in strategies 1 and 3, but later than in strategy 2 since the latter has less net exposure to the natural gas price. Furthermore, the peak DG unit upgrade thresholds are the same whether the capacity or HX upgrade occurs first, i.e., $C_{EP} = C_{EPH}$, and the HX upgrade thresholds do not depend on how much DG capacity is installed, i.e., $C_{HB} \approx C_{HP}$.

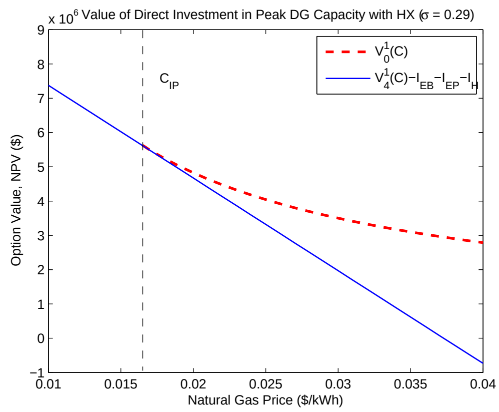

Figure 13: Value of Option to Invest with Strategy 1 ($\sigma = 0.29$)

The investment and upgrade value curves for given values of σ provide a snapshot of the microgrid's decision making under each strategy (see Figures 13 to 16). The values of flexibility relative to strategy 1 associated with strategy 2 (which relies on capacity upgrade) and strategy 3 (which relies on CHP upgrade) evolve in opposite directions with σ, viz., the former decreases as the value of DG upgrade timing diminishes while the latter increases since the microgrid is better able to take advantage of fu-

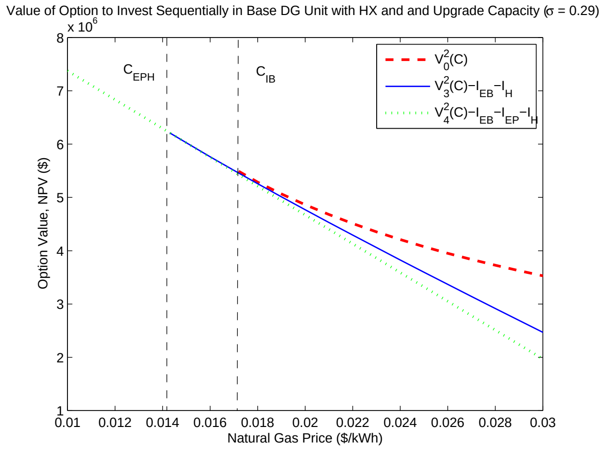

Figure 14: Value of Option to Invest with Strategy 2 ($\sigma = 0.29$)

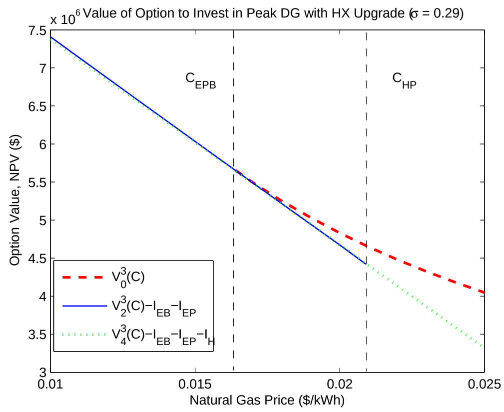

Figure 15: Value of Option to Invest with Strategy 3 ($\sigma = 0.29$)

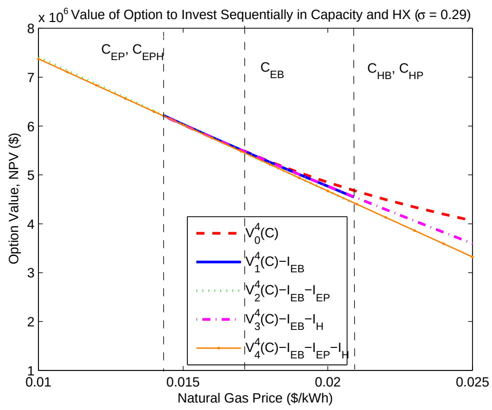

Figure 16: Value of Option to Invest with Strategy 4 ($\sigma = 0.29$)

ture natural gas price increases for the HX upgrade. However, since strategy 4 is able to use these two advantages more precisely depending on market conditions, it is the preferred investment approach once $\sigma > 0.36$ (see Figure 17). It is important to note the value of flexibility with strategy 4 is discontinuous at $\sigma = 0.26$ because once the strategy becomes feasible, a capacity upgrade option is included regardless of how the microgrid proceeds from state 1. Hence, for a commercial microgrid operating in a deregulated environment with uncertain natural gas prices, the real options approach indicates the optimal investment strategy to follow based on the level of natural gas price volatility.

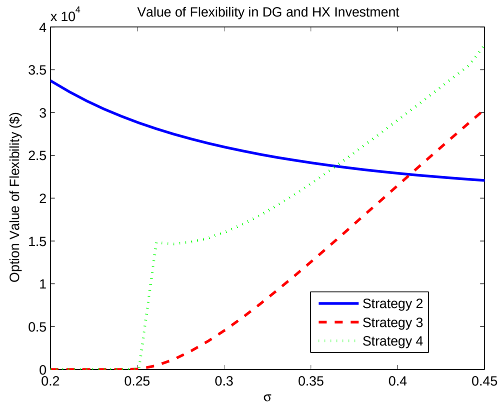

Figure 17: Option Value of Capacity and HX Flexibility

5 Conclusions

As the deregulation of the electricity industry continues, both new opportunities and challenges will become apparent to decision-makers. On the one hand, with functional markets to relay price signals, the industry may be able to realise gains in economic efficiency by matching production of electricity with its subsequent consumption more precisely both in the short run (e.g., via real-time pricing) and long run (e.g., via investment in new generation and transmission capacity). However, on the other hand, greater price volatility or regulatory uncertainty is something that decision-makers will have to internalise, i.e., they can no longer proceed with investment or operations on the basis of a risk-free state of the world. Both of these consequences of deregulation also apply to the adoption of DG by microgrids. In particular, microgrids are motivated by a profit-maximising (or, cost-minimising) incentive to meet on-site loads in the most efficient manner possible, which may be via DG and HX; yet, they remain wary of exposing a large DG system to volatile natural gas prices.

We have explored this tradeoff by allowing a hypothetical California-based microgrid with electric and heat loads to have the perpetual option to proceed sequentially with its investment decision. Our model provides the following insights:

- When only a capacity upgrade is available, investment is triggered sooner with the sequential investment strategy than with the direct strategy because there is less exposure to natural gas price volatility with only a base DG unit. Moreover, since the sequential investment strategy allows for better timing of the installation of the peak DG unit, it offers the microgrid the opportunity to increase cost savings even more. Therefore, the sequential strategy is better than the direct one both in terms of risk and cost reduction. As the natural gas price volatility increases, however, the latter advantage of the sequential strategy diminishes as higher natural gas prices become more likely, which implies that there is less chance that any DG unit will ever be installed.
- In examining the HX upgrade, we find that initial investment is delayed with the sequential strategy compared to the direct strategy because there is less net exposure to natural gas price volatility with a combined DG-HX system as the microgrid can offset any price increases by cost savings from heat production and capture. Nevertheless, the sequential strategy has more value at a high level of natural gas price volatility because it allows for better timing of the HX upgrade. Indeed, it is at a high level of natural gas price volatility that higher natural gas prices become more likely, thereby increasing future cost savings. By contrast, at a low level of natural gas price volatility, when the value of this benefit is also low, the direct strategy's benefit from lower net exposure to the natural gas price dominates.
- An evaluation of both capacity and HX upgrades is undertaken by comparing four possible investment strategies. We find that for a low level of natural gas price volatility, strategy 2 (based on initial investment in a base DG-HX system followed by a capacity upgrade if the natural gas price decreases sufficiently) dominates as it is less exposed to the natural gas price than the other three strategies. For a relatively high level of natural gas price volatility, investment in DG capacity becomes less likely, unless it is offset by a compensating subsequent opportunity to install a HX should the natural gas price increase. Hence, strategy 4 (based on initial investment in a base DG unit followed by either a capacity or a HX upgrade depending on the diffusion of the natural gas price) is the preferred one as the timing issue dominates.

For future work in this area, we would like to examine more realistic time-varying loads as well as the options to sell electricity back to the grid and to upgrade to absorption chillers to meet cooling loads. Additionally, it would be natural to explore investment in DG when the utility electricity price is also stochastic and correlated with the natural gas price. In reality, investment decisions are rarely made on the basis of one or two risk factors. However, the real options approach cannot handle multiple risk factors analytically. It is advisable, therefore, to test the robustness of the decision-making insights via sensitivity analysis as we have attempted to do and perhaps to focus on a single risk factor at a time (see Wickart and Madlener (2007)). By doing so, we find the effectiveness of various strategies in mitigating risk, which is an important practical consideration for a microgrid facing uncertain natural gas prices. A straightforward extension to this paper would be the consideration of economies of scale in direct investment strategies. Finally, from a policymaking perspective, we would like to explore alternative capacity-expansion strategies, i.e., involving mutually exclusive investment in either central-station generation and transmission capacity or DG.

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Appendix A: Solution to the Sequential Strategy for Numerical Example 1

The two value-matching and two smooth-pasting conditions imply the following:

$$V_0^{cap}(C_{EB}^{cap}) = V_1^{cap}(C_{EB}^{cap}) - I_{EB}$$
$$\Rightarrow A_2^{cap}(C_{EB}^{cap})^{\beta_2} = B_2^{cap}(C_{EB}^{cap})^{\beta_2} + PV_B(C_{EB}^{cap}) - I_{EB} \quad \text{(A-1)}$$

$$V_0^{cap'}(C_{EB}^{cap}) = V_1^{cap'}(C_{EB}^{cap})$$
$$\Rightarrow \beta_2 A_2^{cap}(C_{EB}^{cap})^{\beta_2-1} = \beta_2 B_2^{cap}(C_{EB}^{cap})^{\beta_2-1} - \frac{1}{\rho-\alpha}\epsilon_B Q_{EB} \quad \text{(A-2)}$$

$$V_1^{cap}(C_{EP}^{cap}) = V_2^{cap}(C_{EP}^{cap}) - I_{EP}$$
$$\Rightarrow B_2^{cap}(C_{EP}^{cap})^{\beta_2} + PV_B(C_{EP}^{cap}) = PV_B(C_{EP}^{cap}) + PV_P(C_{EP}^{cap}) - I_{EP}$$
$$\Rightarrow B_2^{cap}(C_{EP}^{cap})^{\beta_2} = PV_P(C_{EP}^{cap}) - I_{EP} \quad \text{(A-3)}$$

$$V_1^{cap'}(C_{EP}^{cap}) = V_2^{cap'}(C_{EP}^{cap})$$
$$\Rightarrow \quad \beta_2 B_2^{cap}(C_{EP}^{cap})^{\beta_2-1} = -\frac{1}{\rho-\alpha}\epsilon_P Q_{EP} \quad \text{(A-4)}$$

Equation A-4 can be solved for B_2^{cap} to yield:

$$B_2^{cap} = -\frac{\epsilon_P Q_{EP}(C_{EP}^{cap})^{1-\beta_2}}{(\rho-\alpha)\beta_2} \quad \text{(A-5)}$$

Next, by substituting Equation A-5 into Equation A-3, we obtain the upgrade threshold, C_{EP}^{cap}:

$$C_{EP}^{cap} = \left(\frac{(\rho-\alpha)\beta_2}{\beta_2-1}\right)\left(\frac{P}{\rho\epsilon_P} + \frac{2D_E}{8760\rho\epsilon_P} + \frac{X_E}{\rho\epsilon_P Q_{EP}} - \frac{I_{EP}}{\epsilon_P Q_{EP}}\right) \quad \text{(A-6)}$$

Similarly, if Equation A-5 is substituted into Equation A-2, then we obtain an expression for A_2^{cap}:

$$A_2^{cap} = B_2^{cap} - \frac{\epsilon_B Q_{EB}(C_{EB}^{cap})^{1-\beta_2}}{(\rho-\alpha)\beta_2}$$
$$\Rightarrow A_2^{cap} = -\frac{\epsilon_B Q_{EB}(C_{EB}^{cap})^{1-\beta_2}}{(\rho-\alpha)\beta_2} - \frac{\epsilon_P Q_{EP}(C_{EP}^{cap})^{1-\beta_2}}{(\rho-\alpha)\beta_2} \quad \text{(A-7)}$$

Finally, by substituting Equation A-7 into Equation A-1, we obtain the investment threshold for the base DG unit, C_{EB}^{cap}:

$$C_{EB}^{cap} = \left(\frac{(\rho-\alpha)\beta_2}{\beta_2-1}\right)\left(\frac{P}{\rho\epsilon_B} + \frac{D_E}{8760\rho\epsilon_B} - \frac{I_{EB}}{\epsilon_B Q_{EB}}\right) \quad \text{(A-8)}$$

Appendix B: Solution to the Direct Strategy for Numerical Example 1

$$V_0^{cap,D}(C_I^{cap,D}) = V_2^{cap,D}(C_I^{cap,D}) - I_{EB} - I_{EP}$$
$$\Rightarrow A_2^{cap,D}(C_I^{cap,D})^{\beta_2} = PV_B(C_I^{cap,D}) + PV_P(C_I^{cap,D}) - I_{EB} - I_{EP} \quad \text{(B-1)}$$

$$V_0^{cap,D'}(C_I^{cap,D}) = V_2^{cap,D'}(C_I^{cap,D})$$
$$\Rightarrow \beta_2 A_2^{cap,D}(C_I^{cap,D})^{\beta_2-1} = -\tfrac{1}{\rho-\alpha}\epsilon_B Q_{EB} - \tfrac{1}{\rho-\alpha}\epsilon_P Q_{EP} \quad \text{(B-2)}$$

Solving Equations B-1 and B-2 simultaneously yields:

$$C_I^{cap,D} = \left(\tfrac{(\rho-\alpha)\beta_2}{\beta_2-1}\right)\left(\tfrac{\frac{P(Q_{EB}+Q_{EP})}{\rho}+\frac{D_E(Q_{EB}+2Q_{EP})}{8760\rho}+\frac{X_E}{\rho}-I_{EB}-I_{EP}}{\epsilon_B Q_{EB}+\epsilon_P Q_{EP}}\right) \quad \text{(B-3)}$$

$$A_2^{cap,D} = -\tfrac{(\epsilon_B Q_{EB}+\epsilon_P Q_{EP})(C_I^{cap,D})^{1-\beta_2}}{(\rho-\alpha)\beta_2} \quad \text{(B-4)}$$

Appendix C: Solution to the Sequential Strategy for Numerical Example 2

The value-matching and smooth-pasting conditions imply:

$$V_0^{hx}(C_{EB}^{hx}) = V_1^{hx}(C_{EB}^{hx}) - I_{EB}$$
$$\Rightarrow A_2^{hx}(C_{EB}^{hx})^{\beta_2} = B_1^{hx}(C_{EB}^{hx})^{\beta_1} + PV_B(C_{EB}^{hx}) - I_{EB} \quad \text{(C-1)}$$

$$V_0^{hx'}(C_{EB}^{hx}) = V_1^{hx'}(C_{EB}^{hx})$$
$$\Rightarrow \beta_2 A_2^{hx}(C_{EB}^{hx})^{\beta_2-1} = \beta_1 B_1^{hx}(C_{EB}^{hx})^{\beta_1-1} - \frac{1}{\rho-\alpha}\epsilon_B Q_{EB} \quad \text{(C-2)}$$

$$V_1^{hx}(C_{HB}^{hx}) = V_2^{hx}(C_{HB}^{hx}) - I_H$$
$$\Rightarrow B_1^{hx}(C_{HB}^{hx})^{\beta_1} + PV_B(C_{HB}^{hx}) = PV_B(C_{HB}^{hx}) + PV_H(C_{HB}^{hx}) - I_H$$
$$\Rightarrow B_1^{hx}(C_{HB}^{hx})^{\beta_1} = PV_H(C_{HB}^{hx}) - I_H \quad \text{(C-3)}$$

$$V_1^{hx'}(C_{HB}^{hx}) = V_2^{hx'}(C_{HB}^{hx})$$
$$\Rightarrow \beta_1 B_1^{hx}(C_{HB}^{hx})^{\beta_1-1} = \frac{1}{\rho-\alpha}\min\{Q_H, \gamma Q_{EB}\} \quad \text{(C-4)}$$

As in Näsäkkälä and Fleten (2005), the upgrade threshold defined by Equations C-3 and C-4 states that the value lost must equal the value gained. The former in this case is the sum of the upgrade option and the PV of a DG unit without a HX, whereas the latter is the PV of an active DG unit with a HX minus the capital cost of a HX. The endogenous constant, B_1^{hx}, and HX investment threshold, C_{HB}^{hx}, may be solved analytically using Equations C-3 and C-4:

$$C_{HB}^{hx} = \left(\frac{(\rho-\alpha)\beta_1}{\beta_1-1}\right)\frac{I_H}{\min\{Q_H,\gamma Q_{EB}\}} \tag{C-5}$$

$$B_1^{hx} = \frac{(C_{HB}^{hx})^{1-\beta_1}}{(\rho-\alpha)\beta_1}\min\{Q_H,\gamma Q_{EB}\} \tag{C-6}$$

Appendix D: Solution to the Direct Strategy for Numerical Example 2

$$\begin{aligned} V_0^{hx,D}(C_I^{hx,D}) &= V_2^{hx,D}(C_I^{hx,D}) - I_{EB} - I_H \\ \Rightarrow A_2^{hx,D}(C_I^{hx,D})^{\beta_2} &= PV_B(C_I^{hx,D}) + PV_H(C_I^{hx,D}) - I_{EB} - I_H \end{aligned} \tag{D-1}$$

$$\begin{aligned} V_0^{hx,D'}(C_I^{hx,D}) &= V_2^{hx,D'}(C_I^{hx,D}) \\ \Rightarrow \beta_2 A_2^{hx,D}(C_I^{hx,D})^{\beta_2-1} &= -\frac{1}{\rho-\alpha}\epsilon_B Q_{EB} + \frac{1}{\rho-\alpha}\min\{Q_H,\gamma Q_{EB}\} \end{aligned} \tag{D-2}$$

Solving Equations D-1 and D-2 simultaneously yields:

$$C_I^{hx,D} = \left(\frac{(\rho-\alpha)\beta_2}{\beta_2-1}\right)\frac{(\frac{PQ_{EB}}{\rho}+\frac{D_E Q_{EB}}{8760\rho}-I_{EB}-I_H)}{\epsilon_B Q_{EB}-\min\{Q_H,\gamma Q_{EB}\}} \tag{D-3}$$

$$A_2^{hx,D} = \frac{(C_I^{hx,D})^{1-\beta_2}}{(\rho-\alpha)\beta_2}\min\{Q_H,\gamma Q_{EB}\} - \frac{(C_I^{hx,D})^{1-\beta_2}}{(\rho-\alpha)\beta_2}\epsilon_B Q_{EB} \tag{D-4}$$