Measurement of the $B^+ \rightarrow \omega \ell^+\nu$ and $B^+ \rightarrow \eta \ell^+\nu$ Branching Fractions

B. Aubert,1 M. Bona,1 Y. Karyotakis,1 J. P. Lees,1 V. Pire,1 E. Principe,1 X. Prudent,1 V. Tisserand,1
J. Garra Tico,2 E. Grauges,2 L. Lopez,a6,3 A. Palano6,3 M. Pappagallo6,3 G. Eigen,4 B. Stigu,4 L. Sun,4
G. Lynch,5 I. L. Ospienkov,5 M. T. Ronan,5 K. Tackmann,5 T. Tanabe,5 C. M. Hawkes,6 N. Soni,6 A. T. Watson,3
H. Koch,7 T. Schroeder,7 D. Walker,8 D. J. Asgeirsson,9 B. G. Fulsom,9 C. Hearty,9 T. S. Mattison,9
J. A. McKenna,9 M. Barrett,10 A. Khan,10 V. E. Blinov,11 A. D. Buki,11 A. R. Buzykaev,11 V. P. Druzhinin,11
M. Bondioli,12 S. Curry,12 I. Eschrich,12 D. Kirkby,12 A. J. Lankford,12 P. Lund,12 M. Mandelkern,12
E. C. Martin,12 D. P. Stoker,12 S. Abschi,13 C. Buchanan,13 J. W. Gary,14 F. Liu,14 O. Long,14 B. C. Shen,14
G. M. Vitug,14 Z. Yasin,14 L. Zhang,14 V. Sharma,15 C. Campagnari,16 T. M. Hong,16 D. Kovalskyi,16
W. S. Lockman,17 A. J. Martinez,17 T. Schalk,17 B. A. Schumm,17 A. Seiden,17 M. G. Wilson,17 L. O. Winston,17
C. H. Cheng,18 D. A. Doll,18 B. Echenard,18 F. Fang,18 D. G. Hitlin,18 I. Narsky,18 T. Piatenko,18 F. C. Porter,18
R. Calabrese26 26 A. Cecchi8,26 G. Gabinetto8,26 P. Franchini8,26 E. Luppi8,26 M. Negri8,26 A. Petrella8,26
L. Piemontese,26 V. Santoro,26 26 R. Baldini-Ferroli,27 A. Calcaterra,27 R. de Sangro,27 G. Finocchiaro,27
S. Facetti,27 P. Patteri,27 I. M. Peruzzi,27 M. Piccolo,27 M. Rama,27 A. Zaldo,27 A. Buzzo,27 R. Contri,27 28
A. Santroni28 28 S. Tosi28 28 K. S. Chaisanguntham,29 M. Mori29 A. Adamez,30 C. Anders,30 C. Langenbruch,30
J. Marks,30 S. Schen,30 U. Uwer,30 V. Klose,31 H. M. Lacker,31 D. J. Bard,32 P. D. Dauncey,32 J. A. Nash,32
J. Serrano,38 V. Sordini,38 A. Stocchi,38 G. Wormser,38 D. J. Lange,39 M. D. Wright,39 I. Binham,39
C. Tournaman,38 J. B. Janan,39 C. K. Clarke,39 K. A. George,39 F. Di Lodovico,39 R. Sacco,39 M. Sigamani,39
G. Cowan,40 H. U. Fiecher,40 D. A. Hopkins,40 S. Paramesvaran,40 F. Salvatore,40 A. C. Wren,40 D. N. Brown,41
C. L. Davis,41 A. G. Denig,42 M. Fritsch,42 W. Gradd,42 G. Schott,42 G. E. Dutt,42 R. J. Barlow,43
Y. M. Chia,43 C. L. Edgar,43 G. Jackson,43 G. Lafferty,43 T. J. West,43 J. I. Yi,43 J. Anderson,44 C. Chen,44
A. Jawahery,44 D. A. Roberts,44 G. Simi,44 J. M. Tuggle,44 D. Dallapiccola,44 X. Li,45 E. Salvati,45 S. Saremi,45
P. M. Patel,47 S. H. Robertson,47 A. Lazzaro,48 V. Lombardo,48 F. Palombo,48 J. M. Bauer,49 L. Cremaldi,49
R. Godang,49 R. Kroege,49 D. A. Sanders,49 J. D. Summers,49 R. H. Zhao,49 M. Simard,50 P. Taras,50
F. B. Vlaid,50 H. Nicholson,51 G. De Nardo,51 D. Lista,52 D. Monorchio,52 G. Onorato,52 C. Sciacca,52
R. Rahmat,56 N. B. Sinev,56 D. Strom,56 J. Strube,56 E. Terrence,56 G. Castelli,56 N. Gagliardi,56
M. Margoni,56 M. Morandin,56 M. Possoccio,57 M. Rotondo,57 F. Simonetto,57 R. Strollo,57 C. Voci,57
P. del Amo Sanchez,58 E. Ben-Haim,58 H. Briand,58 G. Calderini,58 J. Chauveau,58 P. David,58 L. Del Buono,58

Published in the PRD-RC
Work supported in part by US Department of Energy contract DE-AC02-76SF00515
We present a study of the charmless semileptonic $B$-meson decays $B^+ \to \omega \ell^+\nu$ and $B^+ \to \eta \ell^+\nu$.

The analysis is based on 383 million $B\overline{B}$ pairs recorded at the $\Upsilon(4S)$ resonance with the BABAR detector. The $\omega$ mesons are reconstructed in the channel $\omega \rightarrow \pi^+\pi^-\pi^0$ and the $\eta$ mesons in the channels $\eta \rightarrow \pi^+\pi^-\pi^0$ and $\eta \rightarrow \gamma \gamma$. We measure the branching fractions $B(B^+ \to \omega \ell^+\nu) = (1.14 \pm 0.10_{\text{stat}} \pm 0.08_{\text{syst}}) \times 10^{-4}$ and $B(B^+ \to \eta \ell^+\nu) = (0.31 \pm 0.06_{\text{stat}} \pm 0.08_{\text{syst}}) \times 10^{-4}$. 

(Dated: August 27, 2008)
Measurements of branching fractions of charmless semileptonic $B$ decays can be used to determine the magnitude of the Cabibbo-Kobayashi-Maskawa matrix element $V_{ub}$ and thus provide an important constraint on the Unitarity Triangle. Studies of exclusive decays allow for more stringent kinematic constraints and better background suppression than inclusive measurements. Furthermore, the predictions for exclusive decay rates depend on calculations of hadronic form factors and are thus affected by theoretical uncertainties different from those involved in inclusive decays. The description of semileptonic decays requires one or three form factors for final states with a pseudoscalar or a vector meson, respectively, if lepton masses are neglected. Currently, the most precise determination of $|V_{ub}|$ with exclusive decays, both experimentally and theoretically, comes from a measurement of $B \to \pi \ell \nu$ decays\cite{6}. It is important to study other semileptonic final states with a pseudoscalar or a vector meson to perform further tests of theoretical calculations and to improve the knowledge of the composition of charmless semileptonic decays.

In this paper, we present measurements of the branching fractions $B(B^+ \to \omega \ell^+ \nu)$ and $B(B^+ \to \eta \ell^+ \nu)$, where $\ell = e, \mu$ and charge-conjugate modes are included implicitly. These decays have previously been studied by the CLEO\cite{3} and BABAR\cite{4,5} collaborations ($B^+ \to \eta \ell^+ \nu$) and by the Belle\cite{2} collaboration ($B^+ \to \omega \ell^+ \nu$). The $\omega$ meson is reconstructed in its decay to three pions ($B(\omega \to \pi^+\pi^-\pi^0) = (89.2 \pm 0.7)\%$ \cite{1}), while for the $\eta$ meson the decays to three pions and to two photons ($B(\eta \to \pi^+\pi^-\pi^0) = (22.68 \pm 0.35)\%$, $B(\eta \to \gamma\gamma) = (39.39 \pm 0.24)\%$ \cite{2}) are used. In contrast to earlier $B^+ \to \eta \ell^+ \nu$ analyses from BABAR\cite{4,5}, the second $B$ meson in the event is not reconstructed; this yields a much larger candidate sample.

The results presented here are based on a data sample of 383 million $B\bar{B}$ pairs recorded with the BABAR detector\cite{7} at the PEP-II asymmetric-energy $e^+e^-$ storage rings at the Stanford Linear Accelerator Center (SLAC). The data correspond to an integrated luminosity of 347 fb$^{-1}$ collected at the $\Upsilon(4S)$ resonance. In addition, 35 fb$^{-1}$ of data collected about 40 MeV below the resonance (off-resonance) are used for background studies. Simulated $B\bar{B}$ events are used to estimate signal efficiencies and shapes of signal and background distributions. Charmless semileptonic decays are simulated as a mixture of three-body decays $B \to X_\gamma \ell \nu$ ($X_\gamma = \pi, \eta, \eta', \rho, \omega$) and have been reweighted according to the latest form-factor calculations from light-cone sum rules\cite{8,9,10}. Decays to non-resonant hadronic states $X_\gamma$ with masses $m_{X_\gamma} > 2m_\pi$ are simulated using the differential decay rate given in Ref.\cite{11}, which produces a smooth $m_{X_\gamma}$ spectrum. The GEANT4 package\cite{12} is used to model the BABAR detector response to simulated events.

The reconstruction of the signal decays $B^+ \to \omega \ell^+ \nu$ and $B^+ \to \eta \ell^+ \nu$ requires the identification of a charged lepton ($e$ or $\mu$) and the reconstruction of an $\omega$ or $\eta$ meson. The two dominant sources of background are semileptonic decays with a charm meson in the final state, $B \to X_c \ell \nu$ ($X_c = D, D^*, D^{**}, D^{(*)}\pi$, and $e^+e^- \to q\bar{q}$ ($q = u, d, s, c$) continuum events. Other backgrounds include charmless semileptonic decays that are not analyzed as signal and $B\bar{B}$ events with lepton candidates from secondary decays or from misidentification of hadrons as leptons. The center-of-mass momentum of the lepton is restricted to $|p_\ell^Z| > 1.6 (1.0)$ GeV\cite{4} for the $\omega$ ($\eta$) final state. This requirement significantly reduces the background events with hadrons misidentified as leptons and rejects a large fraction of leptons from secondary decays or photon conversions. For the reconstruction of the $\omega$ or $\eta$ meson, charged (neutral) pions are required to have a momentum in the laboratory frame above 200 (400) MeV to reduce combinatorial background. Neutral pion candidates are formed from two photons with energies above 100 MeV and an invariant mass, $m_{\gamma\gamma}$, in the range 100 < $m_{\gamma\gamma}$ < 160 MeV. A three-pion system is accepted as an $\omega$ ($\eta$) candidate if its invariant mass, $m_{\pi\pi\pi}$, is in the range 760 < $m_{\pi\pi\pi}$ < 806 MeV for $\omega$ candidates and 540 < $m_{\pi\pi\pi}$ < 555 MeV for $\eta$ candidates. The $\eta$ meson is also reconstructed via its decay into two photons, each with an energy above 50 MeV, with a two-photon invariant mass in the range 520 < $m_{\gamma\gamma}$ < 570 MeV. To reduce the combinatorial background, two-photon combinations are rejected as possible $\eta$ candidates if either of the photons can be combined with any other photon of the event to form a system with an invariant mass close to the $\pi^0$ mass, 110 < $m_{\gamma\gamma}$ < 160 MeV.

Event-shape variables that are sensitive to the topological differences between jet-like continuum events and more spherical $B\bar{B}$ events are used to suppress backgrounds from $e^+e^- \to q\bar{q}$ and other QED processes. The normalized second Fox-Wolfram moment $R_2$\cite{12} is required to be less than 0.5 and a loose requirement on the second Legendre moment $L_2$\cite{11} of $L_2 < 3.0$ GeV is imposed. In addition, the event must contain at least four charged tracks.

The charged lepton is combined with an $\omega$($\eta$) candidate to form a so-called $Y$ pseudo-particle candidate, whose four-momentum is defined as the sum of the corresponding lepton and hadron four-momenta. All charged tracks belonging to the $Y$ are fit to a common vertex. This vertex fit must yield a $\chi^2$ probability of at least 0.1%. Multiple $Y$ candidates per event are possible and all candidates are retained. The $Y$ multiplicity is well described by the Monte-Carlo simulation. About 96% (98%) of simulated $B^+ \to \omega \ell^+ \nu$ ($B^+ \to \eta \ell^+ \nu$) signal events and more than
90% of all selected data events, which include both signal and background, contain only one $Y$ candidate.

The momentum of the candidate neutrino is calculated from the difference between the momenta of the colliding-beam particles and the vector sum of the momenta of all detected particles in the event. The energy of the candidate neutrino is obtained as the magnitude of its momentum, since this is less susceptible to bias from lost particles or additional tracks than the missing energy, $E_{\text{miss}}$, of the event. The magnitude of the missing-momentum vector must be at least 500 MeV. The effect of losses due to detector acceptance on the reconstruction of the neutrino candidate is reduced by requiring the missing-momentum vector in the laboratory frame to point into the polar-angle range $0.3 < \theta_{\text{miss}} < 2.2$ rad. If the missing energy and momentum in the event come from a single undetected neutrino and the rest of the event is correctly reconstructed, the missing mass, $m_{\text{miss}}$, measured from the whole event should be compatible with zero. Because the missing-mass resolution varies linearly with the missing energy, only events with $|m_{\text{miss}}^2/(2E_{\text{miss}})| < 2.5$ GeV are selected.

If the $Y$ candidate originates from a signal decay that has been correctly reconstructed, the cosine of the angle between the $B$ meson and the $Y$ candidate can be calculated as

$$\cos \theta_{BY} = (E_B' \cos \gamma - m_B^2 - m_Y^2) / (2 \sqrt{s} m_B).$$

Here $m_B, E_B, \vec{p}_B, m_Y, E_Y, \vec{p}_Y$ refer to the masses, energies, and momenta of the $B$ meson and the $Y$ candidate, respectively. In the calculation of $\cos \theta_{BY}$, the $B$-meson energy $E_B'$ and momentum $\vec{p}_B'$ are not measured event by event. Instead, $E_B' = \sqrt{s}/2$ is given by the center-of-mass energy of the colliding beam particles, $\sqrt{s}$, and the magnitude of the $B$ momentum is calculated as $|\vec{p}_B'| = \sqrt{E_B'^2 - m_B^2}$. Signal candidates are required to satisfy $-1.2 < \cos \theta_{BY} < 1.1$, which allows for detector resolution and photon radiation.

To reduce backgrounds without significant loss of signal, the momenta of the lepton, $p_\ell$, and of the hadron, $p_h^\pi_\eta$, that make up a $Y$ candidate are restricted. For $B^+ \to \omega \ell^+ \nu$, the momenta are required to satisfy $|p_\ell| > 1.3$ GeV or $|p_h^\pi_\eta| > 2.0$ GeV or $|p_\ell| + |p_h^\pi_\eta| > 2.65$ GeV. In the case of $B^+ \to \eta \ell^+ \nu$, the conditions $|p_\ell| > 1.3$ GeV or $|p_h^\pi_\eta| > 2.1$ GeV or $|p_\ell| + |p_h^\pi_\eta| > 2.8$ GeV have to be fulfilled.

The kinematic consistency of the reconstructed $Y\nu$ system with a signal $B$ decay is verified using the two variables $\Delta E = (P_B - P_{\text{beam}} - s/2)/\sqrt{s}$ and $m_{\text{ES}} = \sqrt{(s/2 + P_B \cdot P_{\text{beam}})^2/E_B' - m_B^2}$, where $P_{\text{beam}} = (E_{\text{beam}}, p_{\text{beam}})$ is the four-momentum of the colliding beam particles and $P_B = (E_B, \vec{p}_B)$ is the $B$-meson four-momentum computed as the sum of the four-momenta of the $Y$ and the $\nu$ candidates. These variables are later used to extract the signal yields in a fit to the two-dimensional $\Delta E$ vs. $m_{\text{ES}}$ distribution. Only candidates that fulfill the loose requirement $|\Delta E| < 0.95$ GeV and $m_{\text{ES}} > 3.095$ GeV (fit region) are retained.

<table>
<thead>
<tr>
<th>Fit region</th>
<th>$\epsilon_{\text{signal}}$ (%)</th>
<th>$S/B$</th>
<th>Signal region</th>
<th>$S/B$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B^+ \to \omega \ell^+ \nu$</td>
<td>1.00</td>
<td>0.15</td>
<td>0.46</td>
<td></td>
</tr>
<tr>
<td>$B^+ \to \eta \ell^+ \nu, \eta \to \pi^+ \pi^- \pi^0$</td>
<td>1.62</td>
<td>0.10</td>
<td>0.35</td>
<td></td>
</tr>
<tr>
<td>$B^+ \to \eta \ell^+ \nu, \eta \to \gamma \gamma$</td>
<td>4.90</td>
<td>0.04</td>
<td>0.15</td>
<td></td>
</tr>
</tbody>
</table>

At this stage of the selection, the signal-to-background ratio, $S/B$, where $S$ and $B$ denote the expected signal and background yields, respectively, is small. It amounts to 1.5% for $B^+ \to \omega \ell^+ \nu$, and 1.8% (1.0%) for $B^+ \to \eta \ell^+ \nu$ with $\eta \to \pi^+ \pi^- \pi^0(\gamma \gamma)$. The signal efficiencies for the sum of decays with electrons and muons, estimated from simulation, are 2.8% for $B^+ \to \omega \ell^+ \nu$ and 4.0% (9.4%) for $B^+ \to \eta \ell^+ \nu$ with $\eta \to \pi^+ \pi^- \pi^0(\gamma \gamma)$.

For further discrimination between the signal and the background, a multivariate selection based on neural networks is used. For each of the three signal channels under study, neural networks with two hidden layers (four and two neurons, respectively) are applied consecutively to separate the signal from the two main backgrounds. A first neural network discriminates the signal against $q\overline{q}$ continuum events; a second network is used to further distinguish the signal from the $B \to X,\ell\nu$ background. The neural-network decision is based on the following input variables: $m_{\text{ES}}^2/(2E_{\text{miss}})$, $\cos \theta_{\text{BY}}$, $R_2$, $L_2$, $\cos \Delta \theta_{\ell\nu}$, the cosine of the polar-angle difference between the thrust axes of the $Y$ candidate and of the rest of the event, and $\cos \theta_{W\ell}$, the cosine of the lepton “helicity angle” measured in the rest frame of the virtual $W$ (calculated using the lepton and neutrino candidates) relative to the $W$ direction in the $B$ rest frame. For the three-pion final states, the Dalitz amplitude, the magnitude of the vector product of the $\pi^+$ momentum and the $\pi^-$ momentum in the $\omega/\eta$ rest frame, normalized to its maximum value, serves as an additional input variable to separate $\omega \to \pi^+ \pi^- \pi^0$ and $\eta \to \pi^+ \pi^- \pi^0$ decays from combinatorial background.

The training of the neural networks is done using the corresponding simulated signal and background samples for each of the three signal channels separately. Independent simulated event samples are used to validate the training. Based on Monte-Carlo simulation, the selection criterion for each of the output discriminants is chosen to maximize the quantity $S/\sqrt{S+B}$. The signal efficiencies and the $S/B$ ratios after the neural-network selection are given in Table 4 for the fit region and for the signal region delimited by $-0.2 < \Delta E < 0.4$ GeV and $m_{\text{ES}} > 5.255$ GeV.

For the determination of the signal branching fractions, the $\Delta E$ vs. $m_{\text{ES}}$ distributions of the simulated signal and backgrounds are fit to the data distribution for the three signal channels independently. The fits are based on an extended binned maximum-likelihood method.
and take statistical fluctuations of both the data and the Monte-Carlo samples into account. The binning of the \( \Delta E \) vs. \( m_{ES} \) distributions used in the fits contains a total of 50 bins with smaller sizes in the signal region to resolve the signal shape and larger sizes in the part of the fit region outside of the signal region to determine the background normalizations from data. The shapes of the signal and background distributions are taken from simulation. The fits determine the relative fractions of the signal and some of the background samples in the data.

The free parameters of the fits are the normalizations of the signal and the \( B \to X_s \ell \nu \) background, and for the \( B^+ \to \omega \ell^+ \nu \) channel also the overall normalization of the continuum background. The \( B \to X_s \ell \nu \) normalization is left free to account for a slight discrepancy between the \( B \to X_s \ell \nu \) yields in data and Monte-Carlo simulation. The relative contributions of events with electrons or muons to the continuum background have been determined using off-resonance data. Compared to the \( B^+ \to \omega \ell^+ \nu \) channel, the \( B^+ \to \eta \ell^+ \nu \) channels suffer from a larger continuum background and the fit shows larger correlations between signal and this background component. The normalization of the continuum background in the \( B^+ \to \eta \ell^+ \nu \) channel is therefore taken from off-resonance data and is not varied in the fit. All other background contributions, mainly other \( B \to X_s \ell \nu \) decays and \( B \) decays with secondary or misidentified leptons, are fixed to their Monte-Carlo predictions. The fit procedure has been validated with simulated signal and background samples.

The resulting signal yields and branching fractions for the three signal channels are presented in Table II. The scale factors between the \( B \to X_s \ell \nu \) background yields predicted by the simulation and the values determined by the fits are 1.06 ± 0.07 for \( B^+ \to \omega \ell^+ \nu \) and 0.96±0.07 (1.12±0.03) for \( B^+ \to \eta \ell^+ \nu \) with \( \eta \to \pi^+ \pi^- \pi^0 \) (\( \eta \to \gamma \gamma \)). The correlations between the signal and the background parameters determined by the fits are 0.08 for \( B^+ \to \omega \ell^+ \nu \) and -0.60 (-0.46) for \( B^+ \to \eta \ell^+ \nu \) with \( \eta \to \pi^+ \pi^- \pi^0 \) (\( \eta \to \gamma \gamma \)). The correlation between the signal and the continuum parameters for the \( B^+ \to \omega \ell^+ \nu \) channel is -0.55 and the continuum background normalization is adjusted by a factor of 0.89±0.12 with respect to the normalization obtained from the off-resonance data sample. The goodness of fit is evaluated using a \( \chi^2 \)-based comparison of the fitted \( \Delta E \) vs. \( m_{ES} \) distributions of the simulated and data samples and is shown in Table II. In addition, the combined branching fraction for the two \( B^+ \to \eta \ell^+ \nu \) channels has been obtained from a fit to the sum of the \( \Delta E \) vs. \( m_{ES} \) distributions for \( \eta \to \pi^+ \pi^- \pi^0 \) and \( \eta \to \gamma \gamma \).

Table II shows the projections of the fitted distributions on \( \Delta E \) and \( m_{ES} \) for the three signal channels and the combined \( B^+ \to \eta \ell^+ \nu \) channel. For a better visibility of the signal, the \( \Delta E \) projections are shown for \( m_{ES} > 5.255 \) GeV and the \( m_{ES} \) projections are shown for \( -0.2 < \Delta E < 0.4 \) GeV.

The systematic errors on the measured branching fractions are listed in Table III. They are estimated by varying the detection efficiencies or the parameters that impact the modeling of the signal and the background processes within their uncertainties. The complete analysis is then repeated and the differences in the resulting branching fractions are taken as the systematic errors. The total systematic error is obtained by adding in quadrature all listed contributions.

Uncertainties due to the reconstruction of charged particles and photons are evaluated by varying their reconstruction efficiencies and the energy depositions of photons in the simulation. The neutrino reconstruction is affected by background with long-lived \( K_L^0 \), which often escape detection and contribute to the measured missing momentum of the event. The uncertainty arising from \( K_L^0 \) production and interactions is estimated by varying their production rate as well as their detection efficiency and energy deposition in the simulation. For lepton identification, relative uncertainties of 1.4% and 3% are used for electrons and muons, respectively. A 3% uncertainty is assigned to the \( \pi^0/\eta \to \gamma \gamma \) reconstruction efficiency.

The uncertainty due to the \( B \to X_s \ell \nu \) background is evaluated by varying the \( B \to D(D^*/D^{**}) \ell \nu \) branching fractions [7] and the \( B \to D^* \) form factors [19]. Prior to the neural-network selection, the background level is high and discrepancies between data and Monte-Carlo distributions are observed at roughly the 10% level for some of the neural-network input variables. To estimate the effect of these discrepancies on the measured branching fractions, the dominant background component \( (B \to X_s \ell \nu) \) is reweighted to match the data. The weights are determined from a \( B \to X_s \ell \nu \)-enhanced sample which is obtained by selecting only events that are otherwise rejected by the \( B \to X_s \ell \nu \) neural-network selection and keeping all other selection criteria unchanged.

For the \( B \to X_s \ell \nu \) background, the branching fractions of the exclusive decays that are not analyzed as signal are varied within their uncertainties [20]. The non-resonant part is varied within the range allowed by the uncertainty of the total \( B \to X_s \ell \nu \) branching fraction [20]. The uncertainty due to the normalization of the continuum background has been determined with off-resonance data for events with electrons or muons.
FIG. 1: (color online) Projected $\Delta E$ distributions for $m_{ES} > 5.255$ GeV (top), and $m_{ES}$ distributions for $-0.2 < \Delta E < 0.4$ GeV (bottom). From left to right: $B^+ \to \omega \ell^+ \nu$ channel, $B^+ \to \eta \ell^+ \nu$ channel with $\eta \to \pi^+ \pi^- \pi^0$, $B^+ \to \eta \ell^+ \nu$ channel with $\eta \to \gamma \gamma$, and combined $B^+ \to \eta \ell^+ \nu$ channel. The error bars represent the statistical uncertainties on the data. The histograms show simulated distributions for signal (white), $B \to X_{s} \ell \nu$ decays (light shaded/yellow), $q \bar{q}$-continuum (dark shaded/blue) and all other backgrounds (hatched) and have been summed up. The distributions of the simulated signal and $B \to X_{s} \ell \nu$ background (and the $q \bar{q}$ background for $B^+ \to \omega \ell^+ \nu$) have been scaled to the results of the fits.

separately. Since the overall normalization of the continuum background is adjusted for $B^+ \to \omega \ell^+ \nu$ in the fit, the resulting error in this channel is smaller than for the $B^+ \to \eta \ell^+ \nu$ channels. For the normalization of secondary-lepton background, an uncertainty of 6-8%, depending on the signal channel, has been estimated from a detailed study of the composition of this background.

Uncertainties in the modeling of signal decays due to the imperfect knowledge of the form factors affect the shapes of kinematic spectra and thus the acceptances of signal decays. The errors on the measured branching fractions are estimated by varying the parameters of the form-factor calculations with their uncertainties [10, 11].

The branching fractions of the decays $\omega / \eta \to \pi^+ \pi^- \pi^0$ and $\eta \to \gamma \gamma$ are also varied within their uncertainties [2]. The uncertainty on the number of produced $B$ mesons is 1.1% [21]. The uncertainty on the ratio of the $\Upsilon(4S) \to B^+ B^-$ and $\Upsilon(4S) \to B^0 \bar{B}^0$ branching fractions, $f_{B^+}/f_{B^0} = 1.065 \pm 0.026$ [22], is taken into account.

The total systematic errors on the measured branching fractions are 7.2% and 25.1% for the $B^+ \to \omega \ell^+ \nu$ and the combined $B^+ \to \eta \ell^+ \nu$ channels, respectively.

In summary, we have measured the branching fractions of $B^+ \to \omega \ell^+ \nu$ and $B^+ \to \eta \ell^+ \nu$ decays to be

$$B(B^+ \to \omega \ell^+ \nu) = (1.14 \pm 0.16 \pm 0.08) \times 10^{-4},$$

$$B(B^+ \to \eta \ell^+ \nu) = (0.31 \pm 0.06 \pm 0.08) \times 10^{-4},$$

where the errors are statistical (data and simulation) and systematic, respectively.

The $B^+ \to \omega \ell^+ \nu$ result significantly improves the current knowledge of this decay. For comparison, the previously most precise measurement by the Belle collaboration yielded $B(B^+ \to \omega \ell^+ \nu) = (1.3 \pm 0.4_{stat} \pm 0.4_{syst}) \times 10^{-4}$. The $B^+ \to \eta \ell^+ \nu$ result is compatible with and comparable in relative precision to the current best measurement by BABAR based on semi-leptonic tags of the second $B$ in the event, $B(B^+ \to \eta \ell^+ \nu) = (0.64 \pm 0.20_{stat} \pm 0.03_{syst}) \times 10^{-4}$.

The two analyses are statistically independent and complement each other. The analysis presented here is statistically more precise but has larger systematic uncertainties, as expected for an untagged measurement. The improved measurements of $B^+ \to \omega \ell^+ \nu$ and $B^+ \to \eta \ell^+ \nu$ decays are important ingredients to the understanding of the composition of the inclusive charmless semileptonic decay rate. The size of the candidate samples in the data is sufficient to perform a measurement in intervals of the momentum transfer, $q^2$, of the decay, which would be necessary to determine $|V_{ub}|$ with an adequate precision.

We are grateful for the excellent luminosity and machine conditions provided by our PEP-II colleagues, and for the substantial dedicated effort from the computing organizations that support BABAR. The collaborating institutions wish to thank SLAC for its support and kind hospitality. This work is supported by DOE and NSF (USA), NSERC (Canada), IHEP (China), CEA and CNRS-IN2P3 (France), BMBF and DFG (Germany), INFN (Italy), FOM (The Netherlands), NFR (Norway), MIST (Russia), and PPARC (United Kingdom). Indi-
TABLE III: Relative systematic errors of the branching fractions $\mathcal{B}(B^+ \to \omega \ell^+ \nu)$ and $\mathcal{B}(B^+ \to \eta \ell^+ \nu)$. For the $B^+ \to \eta \ell^+ \nu$ channel, the systematic errors for the three-pion and the two-photon final states as well as for the combined result are shown. The total error in each column is the sum in quadrature of all listed contributions.

<table>
<thead>
<tr>
<th>Error source</th>
<th>$\delta \mathcal{B}(B^+ \to \omega \ell^+ \nu)$ (%)</th>
<th>$\delta \mathcal{B}(B^+ \to \eta \ell^+ \nu)$ ($\eta \to \pi^+ \pi^- \pi^0$, $\eta \to \gamma \gamma$) combined (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tracking efficiency</td>
<td>1.9</td>
<td>4.9</td>
</tr>
<tr>
<td>Photon reconstruction</td>
<td>2.1</td>
<td>1.8</td>
</tr>
<tr>
<td>$K_L^2$ production and interactions</td>
<td>2.6</td>
<td>4.8</td>
</tr>
<tr>
<td>Lepton identification</td>
<td>1.9</td>
<td>3.3</td>
</tr>
<tr>
<td>$\pi^0/\eta$ identification</td>
<td>3.8</td>
<td>6.9</td>
</tr>
<tr>
<td>Neural-net input variables</td>
<td>0.6</td>
<td>0.8</td>
</tr>
<tr>
<td>$D^*$ form factors</td>
<td>0.4</td>
<td>1.0</td>
</tr>
<tr>
<td>$\mathcal{B}(B \to X_u \ell\nu)$</td>
<td>2.1</td>
<td>5.5</td>
</tr>
<tr>
<td>$\mathcal{B}(B \to X_s \ell\nu)$</td>
<td>2.8</td>
<td>4.4</td>
</tr>
<tr>
<td>Secondary leptons</td>
<td>0.3</td>
<td>0.3</td>
</tr>
<tr>
<td>Continuum scaling</td>
<td>0.7</td>
<td>15.8</td>
</tr>
<tr>
<td>Signal form factor(s)</td>
<td>1.8</td>
<td>5.9</td>
</tr>
<tr>
<td>$\mathcal{B}(\omega/\eta \to \pi^+ \pi^- \pi^0)$, $\mathcal{B}(\eta \to \gamma \gamma)$</td>
<td>0.8</td>
<td>1.5</td>
</tr>
<tr>
<td>$N_{\pi\eta}$</td>
<td>1.1</td>
<td>1.1</td>
</tr>
<tr>
<td>$f_{\pi\pi}/f_{\eta\eta}$</td>
<td>1.2</td>
<td>1.4</td>
</tr>
<tr>
<td>Total systematic error</td>
<td>7.2</td>
<td>21.1</td>
</tr>
</tbody>
</table>

Individuals have received support from CONACyT (Mexico), A. P. Sloan Foundation, Research Corporation, and Alexander von Humboldt Foundation.

* Deceased
† Now at Temple University, Philadelphia, Pennsylvania 19122, USA
‡ Now at Tel Aviv University, Tel Aviv, 69978, Israel
§ Also with Università di Perugia, Dipartimento di Fisica, Perugia, Italy
¶ Also with Università di Roma La Sapienza, I-00185 Roma, Italy
** Now at University of South Alabama, Mobile, Alabama 36688, USA
†† Also with Università di Sassari, Sassari, Italy
[14] All variables denoted with a star (e.g. $p^*$) are given in the $Y(4S)$ rest frame; all others are in the laboratory frame except if differently specified.
[16] $L_2 = \sum \langle \vec{p}_\ell^* \rangle \cos (\theta^*_{\ell})$, where the sum over all tracks in the event not used to form the $Y$ candidate and $\vec{p}_\ell^*$ are their momenta and angles with respect to the thrust axis of the $Y$ candidate, respectively.