Noninvariance of Space/Time-Scale Ranges under a Lorentz Transformation. Implications for the Study of Relativistic Interactions.

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Special relativity

Lorentz transformation (v along x)

\[ t' = \gamma (t - vx/c^2) \]
\[ x' = \gamma (x - vt) \]
\[ y' = y \]
\[ z' = z \]

\[ \gamma = (1 - v^2/c^2)^{-1/2} \]

Time dilation/space contraction

at rest: \( \Delta t, \Delta x=0 \) → in motion: \( \Delta t' = \gamma \Delta t \)
\[ \Delta x, \Delta t=0 \]
\[ \Delta x' = \Delta x/\gamma \]

Lorentz invariant (invariant to change of reference frame)

\[ \Delta s^2 = \Delta x^2 + \Delta y^2 + \Delta z^2 - c^2 \Delta t^2 = \Delta x'^2 + \Delta y'^2 + \Delta z'^2 - c^2 \Delta t'^2 \]
Range of space and time scale of a simple system

two identical objects crossing each other

same event as seen in two frames

<table>
<thead>
<tr>
<th></th>
<th>$F_0$-center of mass frame</th>
<th>$F_B$-rest frame of &quot;B&quot;</th>
</tr>
</thead>
<tbody>
<tr>
<td>space</td>
<td>A</td>
<td>B</td>
</tr>
<tr>
<td></td>
<td>$\gamma_0$</td>
<td>$\gamma_0$</td>
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</table>

- $\Gamma$ is not invariant under the Lorentz transformation.
- There exists an optimum frame which minimizes ranges.
- For PIC, Vlasov, fluid methods, $\text{cost} \propto \Gamma \Rightarrow$ huge penalty if calculation not performed in optimum frame!

Boosted frame calculation sample
proton bunch through a given e\textsuperscript{-} cloud*  

This is a proof-of-principle computation: hose instability of a proton bunch

Proton energy: $\gamma = 500$ in Lab
  * $L = 5$ km, continuous focusing

Code: WARP (Particle-In-Cell)

CPU time:
  * lab frame: $>2$ weeks
  * frame with $\gamma^2 = 512$: <30 min

Speedup $\times 1000$

“This sounds like a free lunch”. How is this possible?

Conventional scientific wisdom: the “complexity” of a system is invariant to a change of reference frame. Is that so?

In order to respond to this question, one needs a definition of complexity which allows quantification, i.e. units of complexity.
One possible definition

Complexity = range of spatial scales \times range of time scales

The complexity ($\Gamma$) of a system of total length $L$, shorter space scale $l$, shorter time scale $\delta t$, evolving for a total time $T$, is then given by

$$\Gamma = \frac{L}{l} \cdot \frac{T}{\delta t}$$

This definition has some practical merit: difficulty in experiments or calculations often scales with ratio of scales to cover.
A few systems of interest

In the laboratory

HEP accelerators (e-cloud)
- p+ bunch
- e- cloud
- FODO lattice
- 2cm

Free electron lasers
- e- bunch
- wiggler
- 1.5mm

Laser-plasma acceleration
- laser
- plasma
- 3cm
- 150µm

10km

Vay - June 07
The Heavy Ion Fusion Science Virtual National Laboratory
A few systems of interest

Scaling longitudinally by x100...

HEP accelerators (e-cloud)

Free electron lasers

Laser-plasma acceleration
A few systems of interest

Scaling longitudinally by $\times 1000$...

HEP accelerators (e-cloud)

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Laser-plasma acceleration
A few systems of interest

... so-called "multiscale" problems = very challenging to model!
Use of approximations (quasi-static, eikonal, ...).

HEP accelerators (e-cloud)

Free electron lasers

Laser-plasma acceleration

3cm/1µm=30,000.

10km/10cm=100,000.

10km/10cm=100,000.

10m/1nm=10,000,000,000.

10m/1nm=1,000,000.
Optimum frame => large level of compaction of scale ranges

HEP accelerators (e-cloud)
frame $\gamma \approx 22$
$450m/4.5m = 100$
compaction $\times 10^3$

Laser-plasma acceleration
frame $\gamma \approx 19$
$1.6mm/30\mu m = 53$
compaction $\times 560$

Free electron lasers
frame $\gamma \approx 4000$
$2.5mm/4\mu m = 625$
compaction $\times 3.10^7$

Hendrik Lorentz
Demonstration on the modeling of a toy 2-D FEL problem

- e- and e+ beams
  - current: 160kA
  - energy: 76MeV (γ≈150)

- wiggler in lab frame:
  - period: 43.6cm
  - field: 37.5mT

- seed pulse (in bucket frame):
  - wavelength: 4.3mm
  - length: 12.cm
  - Max electric field: 1.e6V/m

Calculation required 1000 time steps in the “bucket” frame (performed on this laptop). In the lab frame, the same calculation would have required 1000x2γ²≈45,000,000 time steps.

Speedup ≈ 45000

Backward emitted wave is neglected in Eikonal approximation: we get more physics!
Demonstration on the modeling of a toy 1-D LPWA problem

self-modulation instability in capillary discharge channels.

(courtesy D. Bruhwiler, J. Cary, Tech-X)
Summary

• The range of scales $\Gamma$ of a system is not a Lorentz invariant and can vary greatly for some systems.

• There exists an optimum frame which minimizes $\Gamma$.

• We demonstrated speedup of x1000 for PIC simulation of relativistic beam interacting with electron background.

• It is not in contradiction with the conventional scientific wisdom that “complexity” is an invariant.

• We identified three domains of application (laser-plasma acceleration, e-cloud in HEP accelerators, free electron lasers) for which speedup ranging from 2 to 4 orders of magnitude were demonstrated on toy problems.
Outlook

• Update codes to accommodate calculations independently of the choice of frame.

• Apply to the modeling of actual experimental facilities for the three identified cases.

• Develop methods which costs do not depend on the range of scales, based on argument that “complexity”, based on a more topological definition, can be made Lorentz invariant.

• Explore other applications: astrophysics,...