Ship Dynamics for Maritime ISAR Imaging

Armin W. Doerry
Ship Dynamics for Maritime ISAR Imaging

Armin Doerry
SAR Applications Department
Sandia National Laboratories
PO Box 5800
Albuquerque, NM 87185-1330

Abstract
Ship motions are analyzed for their impact on range-Doppler imaging using Inverse Synthetic Aperture Radar (ISAR). A framework for analysis is developed, and limitations of simple ISAR systems are discussed.
Acknowledgements

A special thanks to Capt. Norbert Doerry, USN, SEA05DT, Technical Director, Future Concepts and Surface Ship Design, SEA 05D, NAVSEA, for recommendations and insight into Naval Architecture principles, and for review of an early draft of this report.

This work was performed as part of the ISAR Laboratory Directed Research and Development (LDRD) project.
# Contents

Foreword ............................................................................................................................. 6

1 Introduction .................................................................................................................... 7

2 A Brief Review of Ship Motion .................................................................................... 9
   2.1 Basic Terms ........................................................................................................... 10
   2.2 Sea-State.............................................................................................................. 13
   2.3 Dynamic Motions .............................................................................................. 14
   2.4 Examples ............................................................................................................ 18

3 Implications for ISAR Imaging .................................................................................. 19
   3.1 Mapping Range and Doppler ............................................................................ 19
   3.2 Resolution ......................................................................................................... 22
   3.3 ISAR Focusing .................................................................................................. 24
   3.4 Bonus Discussion .............................................................................................. 26

4 Conclusions ................................................................................................................. 27

References ....................................................................................................................... 29

Distribution ...................................................................................................................... 32
Foreword

Demand is increasing for imaging ships at sea. Conventional SAR fails because the ships are usually in motion, both with a forward velocity, and other linear and angular motions that accompany sea travel. Because the target itself is moving, this becomes an Inverse-SAR, or ISAR problem. Developing useful ISAR techniques and algorithms is considerably aided by first understanding the nature and characteristics of ship motion. Consequently, a brief study of some principles of naval architecture sheds useful light on this problem. We attempt to do so here.
1 Introduction

Normal Synthetic Aperture Radar (SAR) assumes the target scene is stationary, and calculates enhanced angular, or cross-range resolution by analyzing subtle differences in range-rates due to the motion of the radar. In SAR, range rates are synonymous with Doppler. Consequently SAR is often referred to as range-Doppler imaging.

Inverse Synthetic Aperture Radar (ISAR) is in fact SAR but also allows that the target itself is moving, and often with unknown velocities, both linear and angular. It is a much more difficult problem due to the unknown motions. ISAR is also range-Doppler imaging, but the Doppler includes target motion, too.

ISAR of moving vehicles on land is complicated by interference of land clutter and is still an area of research. ISAR of airborne and space-borne vehicles is also reported in the literature. ISAR has enjoyed some success in imaging maritime targets, in particular ships. In fact, a number of maritime ISAR systems have been operational for a number of years. Examples include the following.

- Raytheon SeaVue™ Maritime and Overland Surveillance Radar
- Raytheon AN/APS-137B(V)5 Radar System (Note: The latest generation of the AN/APS-137 has been granted a new nomenclature, the AN/APY-10, by the U.S. Navy)
- Telephonics RDR-1700B Search, Surveillance, Tracking, Imaging and Weather Avoidance Radar System
- Telephonics APS-143C(V)3/OceanEye™ Maritime Surveillance Imaging and Tracking Radar System
- IAI Elta Systems EL/M-2032 Multimode Airborne Fire Control Radar

Although operational and useful, such systems still typically suffer image quality problems in comparison to terrain imaging SAR. Operators of such system routinely refer to ISAR image analysis as “blobology”. Figure 1 shows an example of an ISAR image of a ship.

Nevertheless, system requirements that include a maritime ISAR mode are increasingly popular. This report presents some preliminary analysis of expected ship motions, and their likely impact on the ISAR problem.
Figure 1. Optical and ISAR image of ship. (Images courtesy of US Naval Research Laboratory)
2 A Brief Review of Ship Motion

It is difficult to imagine taking advantage of ship motion without first understanding the nature of the ship motion we are likely to encounter. Towards that end we now briefly review ship motion characteristics.

Designing ships for specific motion characteristics is the domain of naval architecture. This field enjoys a history as long as human’s interest in building water craft. This field has a language all its own, and many empirical rules for predicting ship characteristics. Rules tend to be empirical because of the difficulty in achieving closed form solutions for highly non-linear phenomena in hydrodynamics.

This section is not about turning the reader into a naval architect. Rather, we wish to examine a few key concepts that are relevant to designing an ISAR system to image ships at sea.

Much literature exists in the field of naval architecture. We cite several recommended sources here, and borrow heavily from them subsequently.

Definitions of key terms and concepts are standardized in a DoD Interface Standard.\(^1\)

A high-level description of surface warship design principles and issues is discussed by Gates.\(^2\)

Fundamental concepts in Naval Architecture are discussed by Gillmer and Johnson.\(^3\)

A fairly technical Naval Architecture text is edited by Comstock.\(^4\)

Another popular text with Naval engineers is edited by Lewis.\(^5\)

While much of the material presented here is applicable to all ship types, this report was written for the displacement monohull which is the ship type most often encountered at sea. Examples of displacement monohulls include destroyers, aircraft carriers, containerships, and oil tankers. (Note: multi-hull, planing hulls, semi-planing hulls, SWATH (Small Waterplane Area Twin Hull), hydrofoils, and hovercraft have somewhat different considerations.)
2.1 Basic Terms

We now define some basic terms.

**Longitudinal axis** – the axis that runs fore and aft.

**Transverse axis** – the axis that runs side to side.

**Vertical axis** – the axis that runs up and down.

**Starboard** – the right side of a ship when looking forward.

**Port** – the left side of a ship when looking forward.

The following oscillatory motions are illustrated in Figure 2.

**Roll** – an oscillatory angular motion of the ship about its longitudinal axis.

**Pitch** – an oscillatory angular motion of a ship about its transverse axis.

**Yaw** – an oscillatory angular motion of a ship about its vertical axis.

**Surge** – an oscillatory linear motion of a ship in the direction of its longitudinal axis.

**Sway** – an oscillatory linear motion of a ship in the direction of its transverse axis.

**Heave** – an oscillatory linear motion of a ship in the direction of its vertical axis.

The following motions are related, but not oscillatory.

**Heel** – a temporary non-oscillatory angular displacement about the longitudinal axis. Heel is often due to a turn or wind.

**List** – a steady angular displacement about the longitudinal axis.

**Trim** – a steady angular displacement about the transverse axis.
The following ship dimensions and locations are important to its stability. Many of these have more precise definitions for which we refer the reader to texts on naval architecture. Some of these are illustrated in Figure 3.

**Beam** – the width of the ship in the direction of the transverse axis.

**Length Between Perpendiculars (LBP)** – the length of a ship between its perpendiculars, specific locations on the hull, that is typically the distance between forward and aft extremities of the designed waterline.

**Center of Gravity** – the location through which all the weights constituting the ship and its contents may be assumed to act.

**Center of Buoyancy** – the geometric centroid of the submerged volume of the ship through which the total buoyancy may be assumed to act. When a ship is upright, the center of buoyancy is directly below the center of gravity.

**Metacenter** – transverse/longitudinal – the intersection of a vertical (in space) line through the center of buoyancy with a vertical (to the ship) line through the

---

**Figure 2. Ship oscillatory motion definitions.**
(Image courtesy of Northrop Grumman Newport News)
center of gravity at a particular angular displacement about the longitudinal/transverse axis. More precisely, the Metacenter is the limit of the intersection location as the roll angle approaches zero.

**Metacentric Height** ($GM$) – the distance between the metacenter and the center of gravity, with a positive number indicating that the metacenter is above the center of gravity in an upright ship. A positive metacentric height indicates that buoyancy acts to upright a ship, making the ship initially stable. A negative metacentric height will act to capsize the ship. More specifically, the Metacentric Height is the slope of the righting arm curve at zero roll angle.

**Righting Arm** – the lever that acts to restore the ship to an upright orientation. The restoring moment for a ship at a given roll angle is the righting arm times the ship’s weight (or displacement). In Figure 3, the righting arm is $GM$ times the sine of the roll angle.

![Figure 3. Illustration of Metacenter and Metacentric Height.](image)
Figure 4. The Metacentric Height is the slope of the righting arm curve at zero roll angle.

In subsequent analysis we will treat a ship as a rigid body. This is not strictly true, as a ship will bend, flex, stretch, and twist. However, to first order a ship will not bend, flex, stretch, or twist too much to the extent that it changes shape to the point of unrecognizability. Consequently we will presume that the rigid body model is adequate for our purposes, or at least for this initial study.

In general, a ship will exhibit a combination of roll, pitch, and yaw. Treating the corresponding angular motions as rotational vectors, we can add them together to create a net instantaneous rotational vector (axis) for any point in time. Roll, pitch and yaw angular velocities are then merely projections of the rotational vector along the longitudinal, transverse, and vertical axes respectively. This rotational axis will change in both magnitude and orientation with time due to the various angular oscillations.

2.2 Sea-State

The forcing function driving ship oscillatory motions is waves in the sea surface, mainly gravity waves or swells. The nature of the waves is typically described in terms of ‘sea-state’. Statistics of sea-state occurrence depends on location and time of year. Sea-states are characterized as follows.
Table 1. Annual Sea States in the Open Ocean, Northern Hemisphere.

<table>
<thead>
<tr>
<th>Sea-State</th>
<th>Significant Wave height (m)</th>
<th>Sustained wind speed (kts)</th>
<th>Percent probability</th>
<th>Modal wave period range (sec)</th>
<th>Modal wave period most probable (sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0-1</td>
<td>0 – 0.1</td>
<td>0 - 6</td>
<td>0</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>2</td>
<td>0.1 – 0.5</td>
<td>7 – 10</td>
<td>5.7</td>
<td>3 – 15</td>
<td>7</td>
</tr>
<tr>
<td>3</td>
<td>0.5 – 1.25</td>
<td>11 – 16</td>
<td>19.7</td>
<td>5 – 15.5</td>
<td>8</td>
</tr>
<tr>
<td>4</td>
<td>1.25 – 2.5</td>
<td>17 – 21</td>
<td>28.3</td>
<td>6 – 16</td>
<td>9</td>
</tr>
<tr>
<td>5</td>
<td>2.5 – 4.0</td>
<td>22 – 27</td>
<td>19.5</td>
<td>7 – 16.5</td>
<td>10</td>
</tr>
<tr>
<td>6</td>
<td>4.0 – 6.0</td>
<td>28 – 47</td>
<td>17.5</td>
<td>9 – 17</td>
<td>12</td>
</tr>
<tr>
<td>7</td>
<td>6.0 – 9.0</td>
<td>48 – 55</td>
<td>7.6</td>
<td>10 – 18</td>
<td>14</td>
</tr>
<tr>
<td>8</td>
<td>9.0 – 14.0</td>
<td>56 – 63</td>
<td>1.7</td>
<td>13 – 19</td>
<td>17</td>
</tr>
<tr>
<td>&gt;8</td>
<td>&gt;14.0</td>
<td>&gt;63</td>
<td>0.1</td>
<td>18 – 24</td>
<td>20</td>
</tr>
</tbody>
</table>

Figure 5. On the left: Sea-state 4. Small waves 1.25 – 2.5 m, numerous whitecaps. On the right: Sea-state 7. Sea heaps up, waves 6 – 9 m, white foam streaks off breakers. (Images courtesy of US Coast Guard)
2.3 Dynamic Motions

As previously stated, the main forcing function driving ship oscillatory motions is waves in the sea surface, mainly gravity waves or swells. These waves interact with a ship’s buoyancy and hydrodynamic forces to induce oscillatory motions, both linear and angular. Oscillatory motions tend to be sinusoidal. Other lesser forces include wind, and non-symmetrical vortex shedding off of the hull and control surfaces.

The nature of the various dynamic components depend on a number of factors, including wave characteristics such as sea-state, wind, control surfaces, maneuvers, ship mass, ship loading, and hydrostatic and hydrokinetic pressures that depend on ship motion due to propulsion.

The most obvious motions are typically roll and pitch. Roll is of great interest because with a typical hullform, it is the least damped of the motions, hence the roll angle can be quite large. Pitch is of interest because in heavy sea-states it can cause the bow to come out of the water, then slam hard on the water in a very nonlinear way, causing great stress on the ship structure. Next include yaw, surge, and heave. These motions are typically heavily damped. Yaw is somewhat of interest to naval engineers designing the rudder and other control surfaces. Heave and pitch tend to be more closely coupled. Sway seems to often be of lesser concern. For displacement mono-hulls, sway accelerations are usually not too severe. Fast Multi-hulls can experience unexpected sway and yaw motions when a wave of the right wavelength interacts with the multiple hulls. A ride on the HSV-2 Swift (a non-commissioned catamaran leased by the United States Navy as a mine countermeasures and sea basing test platform) in bad weather at high speed was described by a naval observer as being on an airplane in bad turbulence.

Of greatest significance to ISAR imaging are roll and pitch. As these are oscillatory, their characteristics include significant respective oscillatory periods, as well as substantial peak angular displacements (amplitudes).

Figure 6. Various hullforms have substantially different dynamic motion characteristics as a function of sea-state. (Image courtesy of GlobalSecurity.org)
**Roll**

The maximum roll angle depends on sea-state, frequency of encounter of the waves, and the ship’s righting arm curve which is heavily influenced by its beam. The maximum angle will depend on how well the wave energy couples with the hull. Waves hitting the ship at the hull’s resonant frequency can cause the roll angle to be quite large. Having a bigger beam usually helps in increasing the righting arm, thereby allowing a lesser maximum roll angle. Typical roll angles may be single digit degrees in sea-state 4, to several tens of degrees in sea-state 8. US Navy design limits are $45^\circ$ for most surface ships, and $30^\circ$ for aircraft carriers.

Roll period can be calculated from the empirical formula

$$T_r = \frac{CB}{\sqrt{GM}} \text{ seconds}$$

where

- $C$ = roll constant typically between 0.69 and 0.89 sec/$\sqrt{\text{m}}$ for large ships,
- $B$ = maximum beam at or below the water line, and
- $\frac{GM}{\sqrt{\text{m}}}$ = maximum metacentric height.

The exact nature of $C$ is based on experimental results from similar ships. Maximum metacentric height is often designed to be on the order of low-single-digit meters. If roll periods are too short, the ship is called ‘stiff’ and is uncomfortable for crew and passengers due to the high accelerations generated. This isn’t particularly good for equipment either. Ferries and passenger ships are often designed for longer roll periods to facilitate human comfort. Longer than typical roll periods make a ship ‘tender’. This, of course makes them less stable, more prone to capsize. Typical roll periods are on the order of 10 to 20 seconds.

Rolling is exacerbated when the wave periods as seen by the ship match the ship’s natural roll period. Common wave periods are taken into account by naval architects during ship design. Roll stabilization techniques are increasingly being adopted to reduce this motion.
**Pitch**

The maximum pitch angle depends on sea-state and the ships length between perpendiculars. A longer ship allows less pitch angle. Typical pitch angles may be 1° to 2° in sea-state 4, and perhaps 5° to 11° in sea-state 8. US Navy design limits are 10° for most surface ships.

Typical pitch periods can be determined from the Table 2.

<table>
<thead>
<tr>
<th>Length Between Perpendiculars (m)</th>
<th>Pitch Period (seconds)</th>
</tr>
</thead>
<tbody>
<tr>
<td>&lt; 46</td>
<td>3.5</td>
</tr>
<tr>
<td>46 - 76</td>
<td>4</td>
</tr>
<tr>
<td>76 - 107</td>
<td>5</td>
</tr>
<tr>
<td>107 - 152</td>
<td>6</td>
</tr>
<tr>
<td>152 - 213</td>
<td>7</td>
</tr>
<tr>
<td>&gt; 213</td>
<td>8</td>
</tr>
</tbody>
</table>

These table entries can be fit to a curve, allowing pitch period to be calculated as approximately

\[ T_p = 2.44 + 0.032L - 0.000036L^2 \]

where \( L \) is the length between perpendiculars of the ship in meters.

The undamped natural pitch period is typically between 1/3 and 2/3 the value of the natural roll period.
**Yaw**

Yaw represents temporary bearing changes, and tends to be damped by rudder control. Yaw periods will generally be equal to wave periods.

**Sidebar Comment:**

We contrast ship motion with land-vehicle motion, which we expect to exhibit large linear motion, lateral accelerations in a turn, perhaps a slight heel, and perhaps with some yaw, but relatively little pitch or roll. From a vehicle motion standpoint, it is a different animal.

### 2.4 Examples

In Table 3 we present some typical warship characteristics.

<table>
<thead>
<tr>
<th></th>
<th>Typical Destroyer</th>
<th>Typical Aircraft Carrier</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length Between Perpendiculars (m)</td>
<td>152.4</td>
<td>304.8</td>
</tr>
<tr>
<td>Max. Beam Below Waterline (m)</td>
<td>15.2</td>
<td>38.1</td>
</tr>
<tr>
<td>Roll Constant (sec/√m )</td>
<td>0.82</td>
<td>0.725</td>
</tr>
<tr>
<td>Max. Metacentric Height (m)</td>
<td>1.52</td>
<td>3.05</td>
</tr>
<tr>
<td>Roll Period (sec)</td>
<td>10.1</td>
<td>15.8</td>
</tr>
<tr>
<td>Pitch Period (sec)</td>
<td>6.5</td>
<td>8.8</td>
</tr>
</tbody>
</table>

We stress that these periods denote the natural or ‘resonance’ periods of the ship. The actual periods of roll and/or pitch may depart from these depending on the forcing function of the waves, noting that the frequency of the wave encounter is heavily influenced by the forward motion of the ship.
3 Implications for ISAR Imaging

While the nature of the expected ship motion is discussed in the previous section, its impact on radar data will be addressed below. This section is not meant to be a detailed derivation of an imaging algorithm, but rather an overview of the impact of unknown ship motion on the ISAR imaging problem. This places context for the preceding section.

3.1 Mapping Range and Doppler

Conventional ISAR makes two kinds of measurements

1) range – due to time delay

2) Doppler – due to an angular velocity (actually an angular perspective change)

While we use the concept of Doppler, this is really a range-rate. In fact what we are interested in are variations in range-rate for different parts of the target ship as a function of angular perspective. Range rate is measured as a pulse-to-pulse microwave signal phase variation.

A three-dimensional object maps into a range-Doppler map due to relative motion between radar and target. The nature of the relative motion defines the nature of how the target maps into the range-Doppler map. Generally, the radar’s position is fairly well known, but the target ship’s location, orientation, and motion are not. Consequently, the relative motion between radar and target ship is not well known. A well focused ISAR image requires knowledge of the relative motion between radar and target. This is the challenge with ISAR. Many current operational ISAR systems make presumptions of this relative motion, and then merely occasionally provide a useful product when the presumptions happen to be adequately correct.

Since ultimately we wish to image the target, it makes sense to define a coordinate frame that is target centric. Doing so means the coordinate frame will move dynamically with the ship. Consequently, of interest is the location of the radar with respect to the ship’s orientation. This is of course unknown. We define the radar location with respect to the target as a vector

\[ \mathbf{r}_c = \text{vector from the target reference location to the radar phase center.} \]

This is illustrated in Figure 7.
The vector \( r_c \) can be described in polar coordinates with a magnitude and a direction, with the direction being a pair of angles, for example azimuth and elevation. We refer to the magnitude of this vector \( |r_c| \) as simply ‘range’. Now for some observations.

- If the radar and ship are both static, with no motion at all, then this vector \( r_c \) is a constant.
- If the radar maintains constant range \( |r_c| \), and radar position has the same rotational vector as the ship, then the radar will appear to be stationary with respect to the ship. The vector \( r_c \) will be constant in the ship’s coordinate frame. This is analogous to a satellite’s synchronous orbit.
- If the ship is rotating with respect to the radar, then the radar is in motion with respect to the ship. Consequently, angular motion of a ship with respect to the radar will cause an apparent motion of the radar to trace an arc at some range from the ship.

Let the angular motion of the ship be described with rotational vector \( \mathbf{p}_s \), and the orbit of the radar be described with rotational vector \( \mathbf{p}_c \). We can then calculate the apparent angular motion of the radar with respect to the ship’s coordinate frame as
If a radar is stationary in non-rotating 3-D space, then from an observer on the ship, the
radar seems to move in an opposite angular direction from its own motion. For example,
if a ship is pitching in an upward direction, then a radar located forward of and above the
ship will appear to decline in elevation with respect to the longitudinal axis of the ship.

From a ship’s perspective, as the ship rotates, even an otherwise stationary radar seems to
move across the sky. Furthermore, oscillatory motions of the ship cause the radar to
exhibit apparent oscillatory trajectories across the sky. Since a ship can roll, pitch, and
yaw, the ship’s instantaneous rotation vector can point to any direction, implying that
\( \rho_{c,\text{apparent}} \) itself can point in any direction. This means that the arc that the radar traces
can in fact wander in any tangential or angular direction. That is, the radar may
apparently move in elevation as well as azimuth, with any ratio of the two, positive or
negative.

The data record of radar soundings during the course of its apparent wandering across the
ship’s sky define the ISAR equivalent of a ‘synthetic aperture’ in SAR. It is stressed that
the synthetic aperture is defined by angular rotations, and not linear translations of the
radar itself, as is customary for airborne SAR. Consequently, perhaps a more descriptive
phrase for ISAR is borrowed from Moving Target Indicator (MTI) radar nomenclature,
i.e. Coherent Processing Interval (CPI). We shall use CPI henceforth.

If a short segment of the radar’s apparent path were extracted as a CPI, and that path
were short enough to be considered linear, then the orientation of that linear segment
could be in any direction. The linear segment along with the ship’s coordinate frame
origin define a slant plane, just as in SAR. Also just as in SAR, this slant plane identifies
the planar projection of range and Doppler. Furthermore, the direction normal to this
plane identifies the nominal local layover direction, that is, the direction of projection of
the 3-D target object into a 2-D image (at least for objects that aren’t too far away from
this plane).

The orientation of the slant plane, and hence what Doppler represents, and the projection
direction, vary all over the place as a ship rolls, pitches, and yaws. This is discussed by
Wehner.\(^7\) For example, Table 4 identifies how range and Doppler ‘image’ a ship with
different motions.

Note that ISAR can form useful images of the ship even with a radar directly above the
ship looking straight down.

In fact, any reasonably linear trajectory CPI will give us some fairly constant (during the
CPI), and most likely focused, projection of the ship. However, since the orientation of
the linear segment will fluctuate with time, so too will the ship’s projection fluctuate with
time. Typical ISAR systems seem to hope that at some time during a dwell on a ship,
that the ship’s projection will eventually be oriented to provide a focused and useful
rendering, often desired to be a broadside profile. Musman, et al.\(^8\), advocate a multiframe
processing technique whereby “Classification should be based on information obtained from multiple image frames in a time series of ISAR images.” Furthermore, these multiple frames may be constructed from highly overlapped CPIs to yield “movie-like” presentation.

Table 4. Ship image renderings as a function of radar direction and ship motion.

<table>
<thead>
<tr>
<th>Direction of $r_c$</th>
<th>Ship rotation axis</th>
<th>Doppler resolution axis</th>
<th>Image perspective</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\hat{x}$ (forward)</td>
<td>$\hat{y}$ (pitch)</td>
<td>$\hat{z}$ (height)</td>
<td>Broadside profile</td>
</tr>
<tr>
<td>$\hat{x}$ (forward)</td>
<td>$\hat{z}$ (yaw)</td>
<td>$\hat{y}$ (transverse)</td>
<td>Plan view</td>
</tr>
<tr>
<td>$\hat{y}$ (broadside)</td>
<td>$\hat{x}$ (roll)</td>
<td>$\hat{z}$ (height)</td>
<td>Front profile</td>
</tr>
<tr>
<td>$\hat{y}$ (broadside)</td>
<td>$\hat{z}$ (yaw)</td>
<td>$\hat{x}$ (longitudinal)</td>
<td>Plan view</td>
</tr>
<tr>
<td>$\hat{z}$ (above)</td>
<td>$\hat{x}$ (roll)</td>
<td>$\hat{y}$ (transverse)</td>
<td>Front profile</td>
</tr>
<tr>
<td>$\hat{z}$ (above)</td>
<td>$\hat{y}$ (pitch)</td>
<td>$\hat{x}$ (longitudinal)</td>
<td>Broadside profile</td>
</tr>
</tbody>
</table>

3.2 Resolution

Range resolution is achieved by the usual methods, modulating the pulse to provide sufficient bandwidth.

Cross-range resolution is a function of the aspect change viewed by a radar during the CPI. This is of course equal to the angular width of the arc that the radar apparently traverses from the point of view of the target ship during a CPI. Since a radar doesn’t know the motion of the ship a priori, this is a difficult quantity to control. Furthermore, since the slant plane orientation is also unknown, other than cross-range resolution being orthogonal to range, we do not know if it is oriented in azimuth, elevation, or something in between.

Recall that the angular width of a CPI is related to cross-range resolution by

$$\rho_{cr} = \frac{a_w \lambda}{2\theta_{CPI}}$$

where
\[ \lambda = \text{nominal wavelength of the radar,} \]
\[ \theta_{\text{CPI}} = \text{the angular width of the apparent arc during a CPI,} \]
\[ a_w = \text{Impulse Response (IPR) broadening factor due to data tapering.} \]

This brings to question “How fine should resolution be to identify a ship?”

The answer to this is beyond the scope of this report, except that we note that typical warships might be many tens of meters to perhaps 300 meters long, allowing even relatively coarse resolutions of perhaps 1 meter to provide many pixels on the target to aid recognition.

**Simple ISAR Example**

As example, we note that at Ku-band, with \( \lambda = 0.018 \text{ m} \), and with \( a_w = 1.2 \), then \( \theta_{\text{CPI}} = 0.62^\circ \) will yield 1 m of cross-range resolution.

A simple ISAR system would require this angular width of a CPI, but during motion that is at an acceptably constant angular velocity and acceptably planar with the ship’s coordinate frame origin.

It seems reasonable to presume that a sinusoid is acceptably linear around it’s zero crossing, to perhaps \( \frac{1}{\sqrt{2}} \) of its peak value. This corresponds to an angular segment of 90° of its cycle, at two different parts of the wave. Consequently, we might expect at least two opportunities per ship motion period to achieve 0.62° of aspect change if the motion amplitude exceeds 0.9° peak-to-peak, or 0.45° peak. Previous discussion showed this to be typically exceeded at sea-states of 4 and greater, both in pitch, and especially in roll. It is likely that roll will exceed this even in much calmer seas, more so than pitch.

We emphasize that for a constant CPI time, cross-range resolution is sea-state dependent, and ship design dependent (roll and pitch periods).

If we assume an oscillatory pitch period of perhaps 6 seconds, with a longer roll period, then we would want to limit our CPI to collect no more than for 90° of the shorter period, or 1.5 seconds.

Furthermore, simply taking 1.5 second increments for CPIs will yield something focused no more than 50% of the time, but likely less due to any angular accelerations in other axes. Recall that for this simple ISAR system the apparent radar motion should be acceptably planar, with acceptably constant angular increments between radar pulses.

In addition, achieving a particular desired projection (i.e. the right slant plane) is likely to occur considerably less often yet. Hence the frequent casting of ISAR as “blobology”.
3.3 ISAR Focusing

If the components of $r_c$ were known in the ship’s coordinate frame, then a matched filter could be constructed for every location on the ship, and focusing could be accomplished, regardless of the linearity of the apparent trajectory. Note that because the Fourier Transform preserves angles, vector $r_c$ defines the direction of samples of the Fourier space of the target ship itself. The radar pulse’s frequency content defines the radial distance and width of the data at those angles in Fourier Space. Furthermore, if $r_c$ were known, the 2-D projection of the ship would then also be well understood. This would increase opportunities for target ship characterization and identification.

Sufficient components to completely describe $r_c$ are its magnitude, azimuthal angle, and elevation angle.

Sufficient information to focus the ISAR image are a relatively coarse notion of $|r_c|$, sub-wavelength precision and accuracy of pulse-to-pulse increments in $|r_c|$ and the increments in azimuth and elevation angles, not necessarily the actual angles themselves.

Unknown variations in $|r_c|$ can be mitigated with autofocus techniques, especially those that mitigate large residual range migration errors. Li-bo, et al. describe a two-stage range migration correction based on a “Short Time Fourier Transform” to directly focus moving ship targets.

Robustly and reliably mitigating unknown variations in azimuth and elevation angles is more problematic, and requires further research and development. Jain and Patel propose dividing the CPI into smaller intervals (i.e. subapertures) where the rotation rate over the smaller interval is assumed uniform and processed accordingly before being combined into a final image. Chen suggests time-frequency analysis to deal with non-constant angular velocities. Pastina, et al. address angular motion estimation directly and optimum time selection for maritime ISAR. Gibbins, et al. also estimate ship motion based on variations in Doppler signatures.

It is expected that multiple antenna phase-centers might offer advantage to solving this problem by providing an independent source of signal angle-of-arrival information. This is in fact proposed in a pair of papers by Given and Schmidt.

However, as previously stated, it is presumed that most ISAR systems themselves presume a constant angular velocity (implying a planar collection geometry) and live with the consequences. This is presumed by Ward who relates ISAR imaging to the Polar Format Algorithm for SAR.

While few sources describe operational ISAR system algorithms, a paper by Rapsilber discusses some details of a specific X-band ISAR processor design. Melendez and Bennett describe a series of coordinated algorithms used by the SAIC ISAR processor.
for “identifying target scattering points, measuring advanced motion parameters, and producing movie frames.”

3-D Imaging

Another important note is that due to the multiple rotation axes, that is, the oscillatory nature of the arc that the apparent radar direction traces in the sky over the ship, we are in fact collecting data at various combinations of elevation and azimuth angles in the ship’s coordinate frame. This implies that we are sampling the ship’s Fourier space over a 3-D volume, and can therefore reconstruct a 3-D rendering of the ship. The hindrance is merely knowing just where the Fourier samples we collect are in fact located in the Fourier space of the target ship, that is, an accurate description of all components of $r_c$ in the ship’s coordinate frame.

Indeed, Aleksoff and Subotic\textsuperscript{22} also state that “ISAR processing is fundamentally a 3-D process. The collected data is present over a 3-D volume.” Cooke\textsuperscript{23} further describes a “numerical algorithm for retrieving 3-D scatterer locations and ship motion parameters from a series of Doppler measurements extracted from ISAR imagery.”

![Diagram](image)

Figure 8. A ship’s roll, pitch, and yaw combine to cause the apparent radar path to generate a 2-D angular aperture in Fourier space, that along with bandwidth create a 3-D volume of data in Fourier space, allowing 3-D target reconstruction.
3.4 Bonus Discussion

Ultimately, the purpose of ISAR is typically to facilitate ship classification, or perhaps even identification. Although a well-focused ISAR image can significantly assist this, a well-focused ISAR image may not be absolutely necessary for useful information to be collected about a ship target.

An interesting question is, for example, whether measures of ship roll and pitch periods might themselves be useful towards ship classification or identification. Capt. Norbert Doerry, USN,† opines as follows.

“The roll and pitch periods are influenced by the wave encounter frequency (linear theory says they'll be the same). In linear theory, the ship is just a filter to the excitation force provided by the incoming waves. The natural frequencies of the "filter" for roll and pitch are definitely functions of the mass distribution of the ship (which can be changed somewhat for a warship, but not tremendously) For pitch, the wave encounter frequency is heavily dependent on the forward speed of the ship. The pitch period could be an indicator of speed. (or, if you knew the speed, heading, and wave direction / wavelength, you could determine the filter transfer function magnitudes)

If the transfer function(s) could be determined over time, that could be used to identify a ship -- definitely to a class.

In any case, results don't have to be perfect. It may be sufficient to be able to differentiate between two tracks and/or to be able to foil attempts by an opponent to spoof or jam our sensors. War at sea is a lot about fusing uncertain data from multiple sources to gain a higher confidence understanding of the battle space.”

† Excerpts from an email from Capt. Norbert Doerry, USN, SEA05DT, Technical Director, Future Concepts and Surface Ship Design, SEA 05D, NAVSEA, sent on 24 January 2008.
4 Conclusions

The following points are worth repeating.

- A fundamental problem in ISAR imaging results from not knowing the relative motion between ship and ISAR instrument.

- Assuming a planar angular perspective change of the target ship with constant angular velocity will allow focused images some of the time. Doing this and getting focused images with a desired projection will happen less often.

- An improvement in focused image yield requires achieving better estimates of the relative motion. This is a challenge for ISAR. Success in this might even allow 3-D imaging of the target ship.

- Multiple phase centers in the ISAR instrument might facilitate achieving better estimates of the relative motion.

---

Figure 9. US Coast Guard Cutter ANDROSCOGGIN in heavy seas while deployed in Viet-Nam; Nov. 1967--Sept. 1968 [Cruise Book], page 96. (Image and caption courtesy of US Coast Guard)
Figure 10. Rollin', rollin', rollin'. USS Devastator (MCM 6) battles heavy seas during a refueling at sea operation with USS Inchon (MCS 12) in the Atlantic, Mar. 16, 1999. The ships are enroute to the Sixth Fleet area of operations. (Image and caption courtesy of U.S. Navy, photo by Photographer's Mate 3rd Class Sean Jordan. [990316-N-7949J-007] Mar. 16, 1999)
References


Technology presumes there's just one right way to do things and there never is.

-- Robert M. Pirsig
Distribution

Unlimited Release

1  MS 1330   S. C. Holswade  5340
1  MS 1330   B. L. Burns    5340
1  MS 0519   T. J. Mirabal  5341
1  MS 1330   M. S. Murray   5342
1  MS 1330   W. H. Hensley 5342
1  MS 1330   T. P. Bielek   5342
1  MS 1330   A. W. Doerry   5342
1  MS 1330   D. W. Harmony  5342
1  MS 1330   J. A. Hollowell 5342
1  MS 1330   S. Nance      5342
1  MS 1330   B. G. Rush    5342
1  MS 1330   D. G. Thompson 5342
1  MS 0501   P. R. Klarer   5343
1  MS 1330   K. W. Sorensen 5345
1  MS 0529   B. C. Brock   5345
1  MS 1330   D. F. Dubbert 5345
1  MS 1330   G. R. Sloan   5345
1  MS 1330   S. M. Becker  5348
1  MS 1330   O. M. Jeromin 5348
1  MS 1330   S. A. Hutchinson 5349
1  MS 0519   D. L. Bickel   5354
1  MS 0899   Technical Library 9536 (electronic copy)

;-)