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Exponential growth, superradiance, and tunability of a seeded free electron laser

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Abstract: Exponential growth and superradiance regimes in a high-gain free electron laser (FEL) are studied in this paper for both a seeded FEL and a Self-Amplified Spontaneous Emission (SASE) FEL. The results are compared to the earlier superrdaince theory and the recent experimental observation. The influence of an initial energy chirp along the electron bunch on the superradiance mode is explored for the first time. With a short seed to increase the initial seed bandwidth, a tunable seeded FEL is possible.

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Exponential growth and superradiance regimes exist in a high-gain free electron laser (FEL). For a seeded FEL, in the exponential growth regime, the seed power is exponentially amplified; the pulse duration, the frequency bandwidth, and the frequency chirp are modified [1, 2, 3, 4]. The exponential growth stops when the FEL induced energy spread gets too large compared to the FEL Pierce parameter ρ [5, 6]. The exponential growth is regarded as a steady state solution, where the slippage effect is small. In contrast to this, the superradiance regime should be studied when slippage is important. Superradiance has been studied theoretically [7, 8, 9]. Recently, study showed the possibility of pulse shortening in the superradiance regime [10]. This has been verified experimentally [11]. In this paper, we first illustrate that the exponential growth mode and the superradiance mode arise from the same origin, *i.e.*, they are two saddle points in the FEL Green function with emphasis on the FEL group velocity and slippage effect. We show the pulse shortening and give some detailed temporal and spectral information in the superradiance regime to compare with the experiment [11]. We then show that by injecting an ultrashort seed pulse, the seeded FEL can be tunable over a large bandwidth. The concepts are applicable to both the seeded FEL and the Self-Amplified Spontaneous Emission (SASE) FEL, however the mathematical derivation in the following will be mostly for the seeded FEL. To analyze the start-up of a seeded FEL amplifier we use the coupled set of Vlasov and Maxwell equations which describe the evolution of the electrons and the radiation field [12, 13, 14]. The seeded FEL can be described by an integral representation of the initial seed, $A(\theta',0)$, convoluting with the FEL Green function [2]. The FEL electric field is written as $E(t,z) = A(\theta,Z)e^{i(\theta-Z)}$ with $A(\theta,Z)$ being the slow varying envelope function. The evolution is [2]

$$A(\theta,Z) = \int_c \frac{ds}{2\pi i} e^{sZ} \int_{-\infty}^{\theta} d\theta' e^{-s(\theta-\theta')+\frac{i(2\rho)^3(\theta-\theta')}{(s-i\mu\theta)(s-i\mu\theta')}} A(\theta',0), \tag{1}$$

where ρ is the Pierce parameter, $\mu = (d\gamma/dt)2/(\gamma_0\omega_s)$ is the energy chirp along the electron beam with γ_0 being the resonance energy, $Z = k_w z$, $\theta = (k_s+k_w)z - \omega_s t$, where $k_s = 2\pi/\lambda_s$, $\omega_s = k_s c$, and $k_w = 2\pi/\lambda_w$ with λ_s being the radiation wavelength, λ_w the undulator period, and c the speed of light in vacuum. The double integral in Eq. (1) can be evaluated by first performing the contour integral to obtain the Green function. Explicitly,

$$A(\hat{s},\hat{z}) = \int_0^{\infty} d\hat{\xi} e^{-i\hat{\alpha}\hat{s}(\hat{z}-2\hat{\xi})} A\left(\hat{s}-\hat{\xi},0\right) g\left(\hat{z},\hat{\xi},\hat{\alpha}\right), \tag{2}$$

with the Green function $g(\hat{z},\hat{s},\hat{\alpha})$ and the corresponding phasor $f(p,\hat{z},\hat{s},\hat{\alpha})$ defined as

$$g(\hat{z},\hat{s},\hat{\alpha}) \equiv 2\int_c \frac{dp}{2\pi i} \exp\{p(\hat{z}-2\hat{s}) + 2i\hat{s}/[p(p-i\hat{\alpha}\hat{s})]\} \equiv 2\int_c \frac{dp}{2\pi i} \exp[f(p,\hat{z},\hat{s},\hat{\alpha})]. \tag{3}$$

To compare our work with previous work [7, 8, 9], the notations are, $\hat{z} = 2\rho Z$, $\hat{s} = \rho\theta$, and $\hat{\alpha} = -\mu/(2\rho^2)$. The Green function is estimated by saddle point approximation. The saddle point p_s is found from $df(p)/dp|_{p=p_s} = 0$, and the Green function is approximated as $g(\hat{z},\hat{s},\hat{\alpha}) \approx 2\exp[f(p_s,\hat{z},\hat{s},\hat{\alpha})][2\pi f''(p_s,\hat{z},\hat{s},\hat{\alpha})]^{-1/2}$. For the un-chirped case, *i.e.*, $\hat{\alpha} = 0$, the phasor is

$f(p,\hat{z},\hat{s},\hat{\alpha}=0) = p(\hat{z}-2\hat{s}) + 2i\hat{s}/p^2$. The saddle point is found from $p^3 - 4i\hat{s}/(\hat{z}-2\hat{s}) = 0$. If p is not a function of $\hat{s}$, the ponderomotive phase, then $p\hat{z}$ is not an oscillating function; hence a steady state solution. This is determined by $\hat{z}-2\hat{s} = \eta\hat{s}$, with η being a constant. Under this condition, $p^3 - 4i\hat{s}/(\hat{z}-2\hat{s}) = 0$ becomes $p_s^3 = 4i/\eta$. At this saddle point, the phasor is $f(p_s) = (4i/\eta)^{1/3}\hat{z} - (4i/\eta)^{1/3}2\hat{s} + 2i\hat{s}\eta^{2/3}/(4i)^{2/3}$. Hence, for $f(p_s)$ not to be a function of $\hat{s}$, we need $-(4i/\eta)^{1/3}2\hat{s} + 2i\hat{s}\eta^{2/3}/(4i)^{2/3} = 0$, which sets $\eta = 4$, *i.e.*, $4\hat{s} = \hat{z}-2\hat{s}$. This gives $p_{s,0} = i^{1/3}$, which supports a steady state solution, the exponential growth mode. Notice that $4\hat{s} = \hat{z}-2\hat{s}$ gives the FEL group velocity as $v_g = \omega_s/(k_s + 2k_w/3)$. When the FEL evolves into superradiance regime, the group velocity goes back to the speed of light in vacuum [10]. This indicates that the condition $4\hat{s} = \hat{z}-2\hat{s}$ should be abandoned. Indeed, for $v_g = c$, we need $\hat{z} - 2\hat{s} = 0$. However; since $p \sim (\hat{z}-2\hat{s})^{-1/3}$, $e^{p\hat{z}}$ has an essential singularity. In summary, for $\eta \to 4$, the general condition $\hat{z}-2\hat{s} = \eta\hat{s}$ leads to the condition $4\hat{s} = \hat{z}-2\hat{s}$, which supports exponential growth mode; for $\eta \to 0$, the general condition leads to $\hat{z}-2\hat{s} = 0$ for the superradiance mode.

Superradiance has been studied using collective variables approach [7, 8, 9]. The asymptotic solution of Ref. [8] was given in Eq. (28) as $|A_{\rm SR}| \approx (b_0/\sqrt{3\pi})(z_1/y)\exp\left[3(\sqrt{3}/2)(y/2)^{2/3}\right] = (2b_0/\sqrt{6\pi})\sqrt{\hat{s}}(\hat{z}-2\hat{s})^{-1}\exp\left\{3^{3/2}2^{-4/3}\left[\sqrt{\hat{s}}(\hat{z}-2\hat{s})\right]^{2/3}\right\}$, where we have switched from their notation to ours, *i.e.*, $z_1 = 2\hat{s}$, $z_2 = \hat{z}-2\hat{s}$, $\overline{z} = \hat{z}$, $y = \sqrt{z_1}z_2 = \sqrt{2\hat{s}}(\hat{z}-2\hat{s})$, and b_0 is the initial bunching. Now, according to $p^3 - 4i\hat{s}/(\hat{z}-2\hat{s}) = 0$, the growth mode is $p_s = 4^{1/3}i^{1/3}\hat{s}^{1/3}(\hat{z}-2\hat{s})^{-1/3}$. This then gives the phasor and Green function as $f(p_s) = 3i^{1/3}\left[\sqrt{\hat{s}}(\hat{z}-2\hat{s})\right]^{2/3}/2^{1/3} \Longrightarrow g(\hat{z},\hat{s},\hat{\alpha}) \propto \exp\left\{3^{3/2}\left[\sqrt{\hat{s}}(\hat{z}-2\hat{s})\right]^{2/3}/2^{4/3}\right\}$. Comparing this Green function which arises at the saddle point $p_s = 4^{1/3}i^{1/3}\hat{s}^{1/3}(\hat{z}-2\hat{s})^{-1/3}$ to the superradiance field in Eq. (28) of Ref. [8] quoted above, we conclude that the superradiance indeed arises from yet another saddle point in the Green function in Eq. (3).

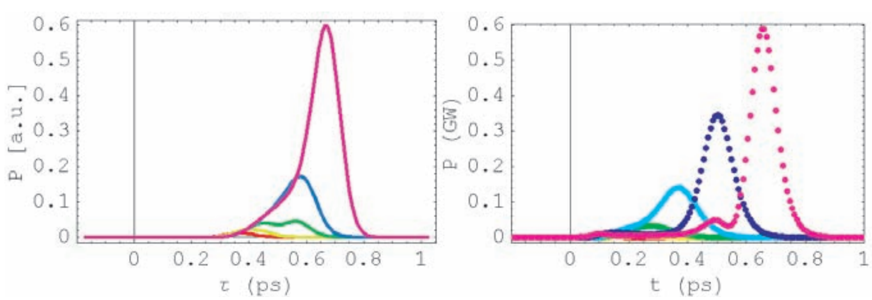

Fig. 1. (a) FEL intensity $P \equiv |A(\theta,Z)|^2$ at $Z = 0.5Z_{\rm up}$ (red), $0.6Z_{\rm up}$ (yellow), $0.8Z_{\rm up}$ (green), $0.9Z_{\rm up}$ (blue), and $Z_{\rm up}$ (purple). (b) *Genesis* simulation of the FEL power at $Z = 0$ (red), $0.2Z_{\rm up}$ (yellow), $0.4Z_{\rm up}$ (green), $0.6Z_{\rm up}$ (blue), $0.8Z_{\rm up}$ (dark blue), and $Z_{\rm up}$ (purple).

After comparing the un-chirped case to previous work [7, 8, 9], we study the effect of the energy chirp in the electron bunch on the superradiance. We rewrite Eq. (2) with the notations in Eq. (1), and perform a saddle point approximation, we have

$$A(\theta,Z) \approx \frac{i^{1/6}2^{1/6}\rho^{1/2}}{\sqrt{3\pi}}\int_0^\infty d\xi\, \frac{\xi^{1/6}A(\theta-\xi,0)}{(Z-\xi)^{2/3}} e^{i\mu(\theta-\xi/2)(Z-\xi)+3i^{1/3}2^{1/3}\rho(Z-\xi)^{2/3}\xi^{1/3}}. \qquad (4)$$

For an initial Gaussian seed, $E(t,z=0) = E_0 e^{-i\omega_s t - \alpha_0 t^2} = E_0 e^{i\theta - \theta^2\alpha_0/\omega_s^2} \Longrightarrow A(\theta,0) = E_0 e^{-\theta^2\alpha_0/\omega_s^2}$, where $\alpha_0 = 1/(4\sigma_{t0}^2)$ with σ_{t0} being the initial seed rms pulse duration.

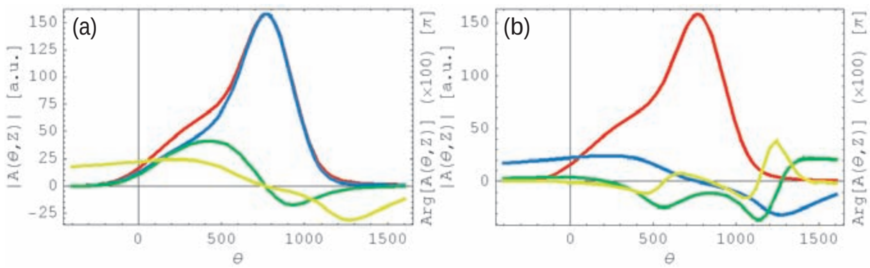

Fig. 2. (a) FEL field envelope $|A(\theta,Z)|$ (red), the real part of $A(\theta,Z)$ (blue), the imaginary part of $A(\theta,Z)$ (green), and the phase $\phi(\theta,Z)$ of $A(\theta,Z)$ (yellow) at $Z = Z_{\rm up}$. (b) Plot of the FEL field envelope $|A(\theta,Z)|$ (red), the phase $\phi(\theta,Z)$ of $A(\theta,Z)$ (blue), the $\partial\phi(\theta,Z)/(\partial\theta)$ (green), and the $\partial^2\phi(\theta,Z)/(\partial\theta^2)$ (yellow) at $Z = Z_{\rm up}$.

We study the example in Ref. [11]. The parameter set is: $\sigma_{t0} \approx 45$ fs, $\mu = 0$, $Z \in [0.5Z_{\rm up}, Z_{\rm up}]$ with $Z_{\rm up} \equiv (2\pi/0.039)10 \approx 1611$, $\rho = 10^{-3}$, and $\lambda_s = 0.8$ μm. Recall that $Z = k_w z$ and $k_w = 2\pi/\lambda_w$ with $\lambda_w = 3.9$ cm being the undulator period and $L_w = 10$ m the undulator total length. In Fig. 1(a), we plot the FEL intensity $P \equiv |A(\theta,Z)|^2$ at $Z = 0.5Z_{\rm up}, 0.6Z_{\rm up}, 0.8Z_{\rm up}, 0.9Z_{\rm up}$, and $Z_{\rm up}$. It is seen that the pulse duration gets shorter, if we trace the main pulselet. At $Z = 0.5Z_{\rm up}$, the rms pulse duration σ_t is about $\sigma_t \approx 75$ fs, and at $Z = Z_{\rm up}$, $\sigma_t \approx 35$ fs. Also we observe that the FEL pulse develops pulselets. According to our convention, the main pulselet is at the head of the FEL pulse towards the right in Fig. 1. Simulation with *Genesis* [15] for the same set of parameters and with initial seed power $P_{\rm in} = 1$ MW, shows the evolution of the FEL pulse in Fig. 1(b) for $Z \in [0, Z_{\rm up}]$. In both results, we find the pulse shortening and pulselet development. This agrees with the experimental observation that the FEL pulse temporal duration increases in the exponential growth regime and decreases in the superradiance regime [11].

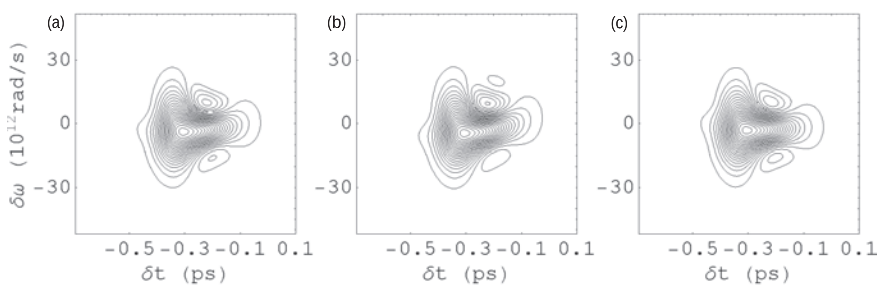

Fig. 3. Wigner function at $Z = Z_{\rm up}$ for $\mu = 0$(a), 2.0×10^{-6}(b), and -2.0×10^{-6}(c).

Details of the FEL pulse at $Z = Z_{\rm up}$ is shown in Fig. 2(a) with the amplitude as the red curve, the real part the blue, the imaginary part the green, and the phase the yellow. To get the sign of the chirp in the superradiance mode, recall that, $E(t,z) = A(\theta,Z)e^{i(\theta-Z)} = |A(\theta,Z)|e^{i\phi(\theta,Z)+i(\theta-Z)} \equiv |A(\theta,Z)|e^{-i\Phi(t,z)}$, where $\Phi(t,z) = \omega_s t - k_s z - \phi(\theta,Z) = \Phi(t_c,z) + \partial\Phi/(\partial t)|_{t=t_c}(t-t_c) + (1/2)\partial^2\Phi/(\partial t^2)|_{t=t_c}(t-t_c)^2 + \cdots$, in which t_c stands for the pulse centroid. With this, we have $\partial\Phi/(\partial t) = \omega_s + \omega_s\partial\phi/(\partial\theta)$, and $\partial^2\Phi/(\partial t^2) = -\omega_s^2\partial^2\phi/(\partial\theta^2)$. From Fig. 2(b), we find that $\partial\phi(\theta,Z)/(\partial\theta) < 0$ in the main pulselet, hence, the entire pulse gets redshifted. This agrees with Fig. 1(b) of Ref. [11]. Furthermore, the chirp flips sign at roughly the center of the main pulselet. From the center to the head $\partial^2\phi(\theta,Z)/(\partial\theta^2) < 0$, hence a positive chirp; and from the center to the tail a negative chirp. The second pulselet has

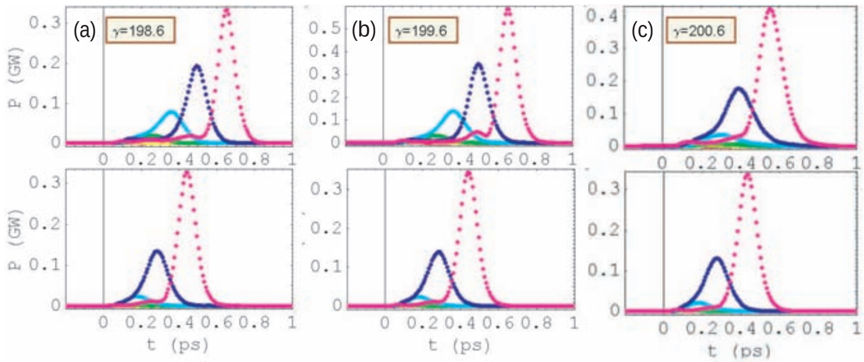

Fig. 4. *Genesis* simulation of the FEL power at $Z = 0$ (red), $0.2Z_{\mathrm{up}}$ (yellow), $0.4Z_{\mathrm{up}}$ (green), $0.6Z_{\mathrm{up}}$ (blue), $0.8Z_{\mathrm{up}}$ (dark blue), and Z_{up} (purple) for $\gamma = 198.6$(a), 199.6(b), and 200.6(c). The upper row is for $\sigma_t \sim 42.5$ fs, and the lower row $\sigma_t \sim 8.5$ fs.

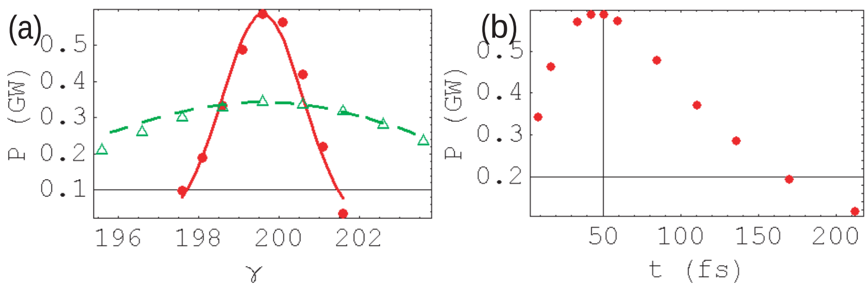

Fig. 5. (a) FEL power as a function of the electron energy and the seed duration. The dot (red) • is for $\sigma_t = 42.5$ fs with the solid curve (red) as the spectrum form factor, and the triangle (green) △ for $\sigma_t = 8.5$ fs with the dashed curve (green) as the spectrum form factor. (b) FEL power as a function of the seed duration.

a small positive chirp. To further compare with the experiment [11], as in Ref. [2] the Wigner function $W(t,\omega,z) \equiv \int_{-\infty}^{\infty} E(t-\tau/2,z)E^*(t+\tau/2,z)e^{-i\omega\tau}d\tau$ at $Z = Z_{\mathrm{up}}$ is shown in Fig. 3. In Fig. 3(a), we set $\mu = 0$, *i.e.*, no energy chirp. It is seen that the second pulselet, *i.e.*, from the pulse center to the pulse tail, bears a positive chirp, which agrees with that in Fig. 2(b). For the main pulselet, it is complicated, similar to Fig. 3 of Ref. [11]. For Fig. 3(b) and (c), the electron bunch has $\mu = \pm 2.0 \times 10^{-6}$, respectively. Phase space rotation due to this energy chirp is seen.

Having identified the saddle points in the FEL Green function for the exponential growth mode and the superradiance mode, we now study the evolution of these two modes. For a SASE FEL, the radiation starts from shot noise, and develops into many spikes. A good description for this process is to model the spikes as random $\delta-$function pulses along the electron bunch, and these spikes travel towards the head of the electron bunch. During this process, the coherence length increases and the temporal duration of the spikes increases. For a seeded FEL, as long as the seed is much short compared to the electron bunch, the seed will travel through the electron bunch in a similar way as the above described case. Even though, the seed can be so strong that the electron bunch gets energy modulated quickly and FEL process starts from these energy modulated electrons quickly. To explore this more clearly, let us take the limit of a $\delta-$function seed, *i.e.*, $A(\theta,0) \sim \delta(\theta)$. In this limit, the evolution in Eq. (4) is reduced to $A(\theta,Z) \cong \theta^{1/6}(Z-\theta)^{-2/3}e^{3i^{1/3}2^{1/3}\rho(Z-\theta)^{2/3}\theta^{1/3}}$ for superradiance, and

$e^{\rho\left(\sqrt{3}+i\right)\left[Z-9(\theta-Z/3)^2/(4Z)\right]}$ for exponential growth [2] with $\mu = 0$. The $\delta-$function seed passes through the electron bunch with speed of light in vacuum, hence we have $Z-\theta \to 0$. This supports the superradiance growth due to the singular point of $(Z-\theta)^{2/3}$ in the denominator; but the exponential growth mode is not excited since $Z - 9(\theta - Z/3)^2/(4Z) \overset{(Z-\theta)\to 0}{\Longrightarrow} 0$. However, since the energy modulated electrons will keep radiating, as long as the resonance condition is satisfied, the late emitted radiation will coherently merge with the proceeding radiation. Macroscopically, the FEL pulse grows in temporal duration and the macroscopic group velocity slows down from the speed of light in vacuum. This then is termed as the exponential growth mode.

Let us explore the detuning effect on the exponential growth and the supperradiance. For the SASE FEL, since there is no external seed, if the electron bunch energy is deviated from the resonance condition, the exponential growth mode still exists. Even though, it will have a different wavelength and different efficient. For the seeded FEL, however, the late emitted radiation will have a different wavelength from that of the seed, and it will not add coherently with the seed. Yet, if the seed is short enough, then the ultra-short seed has a very wide spectrum, hence the seeded FEL with an energy detuned electron bunch will start as efficiently as that with an electron bunch having the resonant energy, even though the central wavelength is shifted. For a transform limit Gaussian seed with rms temporal duration of σ_t, the rms frequency bandwidth is $\sigma_\omega = 1/(2\sigma_t)$. For a seeded FEL, the initial seed then supports a relative bandwidth of $\sigma_\omega/\omega_s = 1/(2\sigma_t\omega_s)$, which in turn supports a relative energy detuning of $\Delta\gamma/\gamma_0 = \sigma_\omega/(2\omega_s) = 1/(4\sigma_t\omega_s)$. Hence, with an ultrashort seed, a seeded FEL acts like a single spike SASE FEL, and exponential growth mode is excited. Indeed, a short seed having a broad bandwidth increases the tunability of the seeded FEL. For the experiment [11], the nominal radiation wavelength is $\lambda_s = 0.8\ \mu$m, $\rho = 2.5\times10^{-3}$, so in order for $\Delta\gamma/\gamma_0 > \rho$, we need $\sigma_t < 42.5$ fs. We define the tunability as $\chi \equiv 1/(4\rho\sigma_t\omega_s)$, which should be compared to the detuning $\mathscr{D} \equiv (\gamma^2-\gamma_0^2)/(2\rho\gamma_0^2)$. To further explore this tunability concept, we invoke *Genesis* simulation with results shown in Fig. 4. For the upper row, $\sigma_t = 42.5$ fs, which gives $\chi = 1$. The resonance is shown in Fig. 4(b) with $\gamma_0 = 199.6$. For Fig. 4(a) and (c), $\mathscr{D} = \mp 2$, respectively, and the peak power starts to decrease. Decreasing the seed pulse duration to $\sigma_t = 8.5$ fs, the results are in the lower row. There is essentially no difference for $\mathscr{D} \in [-2,2]$. In summary, the FEL peak power as a function of the electron bunch initial energy is shown in Fig. 5(a) for both the case of $\sigma_t = 42.5$ fs and 8.5 fs. Indeed, this amplification bandwidth, or the tunable range is determined mostly by the initial seed bandwidth. For a Gaussian seed, the power is $P(t) = e^{-t^2/(2\sigma_t^2)}$ with the spectrum form factor $\tilde{P}(\omega) = e^{-(\omega-\omega_0)^2\sigma_t^2/2}$, which is plotted in Fig. 5(a) as the curves, that agree with the simulation results well. This confirms that the tunability is mostly due to the broadband initial seed, which contains broad spectrum content to be resonant to a detuned energy. With an ultrashort seed to provide broadband spectrum content, together with a large tuning range modulator [16], a tunable cascaded seeded FEL is possible. In addition, there is an optimization of the initial seed pulse duration to get maximum FEL output power for a given system. For the parameter set in this paper, the FEL power as a function of the seed duration is shown in Fig. 5(b). The optimization detail is beyond the scope here.

In summary, we study superradiance and exponential growth in a high-gain FEL. The superradiance and exponential growth originate from two different saddle points of the FEL Green function, respectively. For the exponential growth regime, the saddle point locates at $p_{s,0} = i^{1/3}$; while for the superradiance regime, $p_s = 4^{1/3} i^{1/3} \hat{s}^{1/3} (\hat{z}-2\hat{s})^{-1/3} \overset{(\hat{z}-2\hat{s})\to 0}{\Longrightarrow} \infty$. The analysis on the superradiance, for the first time, includes an energy chirp in the electron bunch. With a short seed, the seeded FEL can have a large tunability range. The work of JW was supported by the US Department of Energy under contract DE-AC02-76SF00515. The work of JBM and XW was supported by the US Department of Energy under contract DE-AC02-98CH10886.