BNL-81770-2008-IR

NSLS-II: Control of Dynamic Aperture

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October 31, 2008

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1.0 Summary

1. Analytical framework and a control theory approach:

We elaborate on the underlying physics for the analytic framework and principles, originally developed to control the SLS dynamics¹ [3], which we have adopted and improved for the NSLS-II. In particular, how come brute force numerical minimizations with these formula, corresponding to a highly overdetermined nonlinear system of algebraic equations (up to 2nd order) for the sextupole strengths converge to similar “solutions” (in a least square sense) for the resonance- and tune shift terms. And, even though the choice of governing equations is based on intuition, rather than a theory², it does provide a systematic approach³ to control the medium term stability (10^3 turns) for a strongly nonlinear periodic Hamiltonian system. Actually, the former is to be expected, given the: complete treatment of the sextupole effects to 2nd order in the sextupole strength, symmetry of the NSLS-II lattice, using a cap for the sextupole strength, and number of sextupole families. Basically, since the governing algebraic system is highly over constrained and the 1st order terms largely controlled by the lattice symmetry, the only significant difference of the obtained “solutions” for the sextupole strengths is the magnitude of the residual 2nd order terms, i.e., the size of the tune footprint for the bare⁴ lattice; ultimately the limiting factor for the on- and off momentum dynamic aperture.

2. Analysis of the sextupole response matrix and guideline for number of chromatic knobs:

We evaluate the controllability of the resonance- and tune shift terms by the sextupole families by analyzing the Jacobian (“sextupole response matrix”) of the nonlinear algebraic system. We also explain the origin of the considerable 2nd order dispersion and how it drives the 2nd order horizontal chromaticity. We then elaborate on why it should⁵ be controlled directly; by a third chromatic sextupole family⁶. Roughly, to avoid interference with the (highly over constrained⁷) geometric families which would compromise the residual tune footprint. Intuitively (considering the limiting factors for modern synchrotron light sources) it may appear as a “no-brainer”, by observing the momentum dependence of the linear optics (i.e., a “wobbling foundation” for an elaborate sextupole scheme) see Figs. 6-8 and the resulting

-
1. Eventually prompting a redesign from a 7-Bend-Achromat [1] to a Triple-Bend-Achromat [2].
 2. Reflecting the fact that there is no general theory for stability for nonlinear dynamics.
 3. The approach has only one free parameter, i.e., the relative weight for resonance vs. tune shift terms. Also, since predictions of the nonlinear performance depends critically on the optics for a real ring, we are taking a heuristic approach to the size of the tune footprint for stable beam.
 4. Without engineering tolerances and insertion devices but with damping wiggler; since these are an integral part of the design.
 5. If adhering to a design philosophy.
 6. Which were compromised when accommodating the Three-Pole-Wigglers.
 7. 16 constraints for 7 parameters.

residual horizontal chromaticity, see Fig. 24. For a quantitative, analytic approach see Eqs. (15)-(18).

3. Selection of geometric families:

We analyze the “solution space” of the governing algebraic system. In particular, while the resonances up to 2nd order in sextupole strength (13 terms) can be cancelled over a wide range of working points with 6 geometric sextupole families, adequate control of the amplitude dependent tune shift (7 terms), nonlinear chromaticity (4 terms), and cross terms (6 terms), requires 3 chromatic families.

4. Illustrate sensitivity to sextupole placement:

By an explicit example, we show how the sextupole placement (by ~ 10 cm changes) can be used to control the nonlinear chromatic effects (originating from the linear optics); for a given working point. The sensitivity to such small changes is explained by Figs. 3-4, and Eq. (15); something to be considered. Regardless, when the working point or optics is changed, during the life cycle of the facility, the configuration is no longer optimal, unless an adequate number of knobs can be accommodated. Note, retrofits as compared to upgrades, by not being an integral part of the original design and philosophy, are in general neither cost effective nor of high performance.

5. Sextupole optimization to achieve a realistic tune footprint for robust operation for 4 different working points:

As a somewhat conservative guideline, we are using ~ 0.1 as the tune footprint for stable beam in a modern, medium energy, third generation synchrotron light source with high chromaticity per cell (e.g. SLS and SOLEIL), and no crossing of resonances up to 2nd order in the sextupole strength (very conservative). For a robust design, we are therefore striving for a significantly smaller footprint for the bare lattice (~ 0.05), i.e., to provide leeway for the contribution from Damping Wigglers (DWs), insertion devices, systematic higher multipole errors, and perturbations of the linear optics. We demonstrate how this can be done by a sextupole scheme with 3 chromatic and 6 geometric (reduced from 7) sextupole families; for 4 different working points. Conservative or not, the balance (risk) is: ease of commissioning (“turn it on and then push the limits” or “struggle from the start”) the robustness of the top-up injection process (e.g. radiation losses) and the Touscheck life time (top-up rate) for the main ring at full capacity.

6. Illustrate how the dynamic aperture is bounded by the 1st and 2nd order sextupolar resonances for 4 different working points:

We show that the on- and off-momentum dynamic aperture is determined by the tune footprint surrounded by the 1st and 2nd order sextupolar resonances. Therefore, it is ultimately limited by the symmetry of the lattice and smoothness of the linear optics, i.e., the peak values of the optics functions and momentum dependence. We obtain about ± 15 mm and $\pm 2.5\%$, respectively for a realistic lattice; for 4 somewhat arbitrarily chosen working points.

7. Control impact of linear effects from DWs by a hybrid local/global approach:

We present a hybrid approach to control the impact on the optics from DWs, i.e., local matching of the optics and global correction of the tune shift. Due to the correction algorithm based on the inverted¹ “quadrupole response matrix” (Jacobian of the governing algebraic system) implemented from the start [4], this is simply a matter of parameter choice. It also straightforward to implement on-line.

8. Control nonlinear impact of DWs by re-optimizing sextupoles:

As a refinement, we also show how the nonlinear dynamics can be restored; by including the leading order nonlinear effects from the DWs and optics correction to the sextupole optimization. As a result, the DWs are now transparent.

9. Impact of relaxing constraints on linear chromaticity:

While a small tune footprint is an assurance for robust top-up injection and adequate Touschek life time, there is a strong desire to operate at large positive chromaticities (+4) to control the head-tail instability driven by wake fields. While it is known from existing medium energy third generation light sources that this strongly affects the Touschek life time. However, due to overly simplified models the absolute impact has not been well predicted. Regardless, assuming that adequate control of the NSLS-II dynamics have been established (from first principles), we then back off from zero chromaticity. Since this increases the off-momentum tune footprint considerable, we also evaluate how this could be reduced by introducing higher order chromatic multipoles (a future upgrade).

10. Trade-offs from introduction of TPW on the tune footprint:

The introduction of Three-Pole-Wigglers (TPWs) to the lattice has, as a side-effect, increased the peak values of the beta functions and dispersion. To avoid increasing them even further, the number of chromatic sextupole families have been compromised from 3 to 2, i.e., only the linear chromaticity can now be controlled. Since the second order effects of sextupoles scales with $\beta_{x,y}^2$, this has increased the tune footprint for the bare lattice. It has also increased the sensitivity to systematic higher order multipole components (e.g. b_6 , b_{10} , and b_{14}) from the quadrupoles in the dispersive regions (due to terms scaling with $\beta_x^2 \eta_x^2$, $\beta_x^4 \eta_x^2$ and $\beta_x^6 \eta_x^2$).

11. Nota bene on notation:

A note on order. Throughout this text we are using order to refer to analytic expressions for the effects of multipoles (Lie generators), rather than the order of the phase-space terms in the expanded equation of motions from which they originate. In particular, the 1st order effects ($\sim b_3$) from sextupoles originates from third order terms in the equations of motion, the 2nd order effects ($\sim b_3^2$) from fourth order terms, and so on. Contrariwise, while the effects from the systematic higher order

1. By singular value decomposition.

multipoles mentioned above are linear in the multipole strengths, they originate from sixth and tenth order terms in the equations of motion, see Section 6.0; something worth contemplating.

12. Studies of a realistic lattice:

The results presented for a realistic lattice includes: DWs, random and systematic higher order multipoles, and mechanical mis-alignments with and girder correlations, see Appendix A; since it is an integral part of the design philosophy [2,5]. Still, as experience shows, the calibration of a nonlinear model against a real lattice is nontrivial. In particular, a prerequisite is that the (linear) optics of the real lattice is known.

2.0 Background

The Hamiltonian

$$H(s) = \frac{p_x^2 + p_y^2}{2(1 + \delta)} + \frac{b_2(s)}{2}(x^2 - y^2) + \frac{b_3(s)}{3}(x^3 - 3xy^2) \equiv H_2(s) + V(s) \quad (1)$$

where $H_2(s)$ describes the linear optics and $V(s)$ the effects from the sextupoles. The introduction of (chromatic) sextupoles to control the leading order (linear) chromaticity for a high performance lattice leads to a whole slew of undesirable effects: nonlinear resonances, amplitude dependent tune shift, nonlinear chromaticity, and cross terms. In fact, if care is not taken to control the latter, a high performance (linear) “optics design” collapses when engineering tolerances, etc. are included to the model [1]. For a robust design, the undesirable nonlinear effects must be considered from the start. In particular, it is not sufficient to first design the (linear) optics $H_2(s)$ and then attempt to determine a suitable distribution of sextupoles/multipole around the lattice $V(s)$; aka a perturbative approach. Rather, the linear optics needs to be tailored to the latter. Simply put, the system must be designed as a whole $H(s)$. And, since $V(s)$ can be chosen quite freely (and generalized to include higher order multipoles), ultimately, the nonlinear performance of the lattice is a reflection of the smoothness- and momentum dependence of the linear optics; described by $H_2(s)$. To avoid the traditional (perturbative) approach and pathological linear optics “designs”, guidelines on e.g. the max (linear) chromaticity per cell and (linear) dispersion have been provided [6].

Note, in the following the order n of a resonance $n_x v_x + n_y v_y = k$ refers to the resonances driven by a certain order of sextupole strength $(b_3 L)^n$, rather than the traditional $n \equiv |n_x| + |n_y|$, see Tab. 1.

n	Sextupolar Resonances								
1	v_x	$3v_x$	$v_x + 2v_y$	$v_x - 2v_y$					
2	$2v_x$	$2v_y$	$4v_x$	$4v_y$	$2v_x + 2v_y$	$2v_x - 2v_y$			
3	v_x	$3v_x$	$v_x + 2v_y$	$v_x - 2v_y$	$5v_x$	$v_x + 4v_y$	$v_x - 4v_y$	$3v_x + 2v_y$	$3v_x - 2v_y$

TABLE 1. Resonances to 3rd Order in the Sextupole Strength.

For the bare lattice, we will avoid crossing 1st and 2nd order resonances; for a conservative approach towards Touschek lifetime and injection efficiency. Furthermore, when DWs and engineering tolerances are introduced, we have found that some of the 3rd order resonances must to be avoided as well; due to the reduction from 15- to 3-fold periodicity.

3.0 Theoretical Framework

In the linear case the one-turn matrix can be diagonalized into block diagonal¹ form

$$M = ARA^{-1} \quad (2)$$

where M , A and R are symplectic matrices. For 1 D.O.F. ($\bar{x} \equiv [x, p_x]$) one obtains

$$A_x = \begin{bmatrix} \sqrt{\beta_x} & 0 \\ -\frac{\alpha_x}{\sqrt{\beta_x}} & \frac{1}{\sqrt{\beta_x}} \end{bmatrix}, \quad R_x = \begin{bmatrix} \cos(2\pi\nu_x) & \sin(2\pi\nu_x) \\ -\sin(2\pi\nu_x) & \cos(2\pi\nu_x) \end{bmatrix}, \quad \hat{x} \equiv A_x^{-1}x \quad (3)$$

with the action-angle variables $[J_x, \phi_x]$

$$2J_x = \hat{x}^T \hat{x}, \quad \phi_x = \text{atan}\left(\frac{p_x}{x}\right) \quad (4)$$

This can be generalized to 2.5 $\bar{x} \equiv [x, p_x, y, p_y, \delta]^2$ and 3 D.O.F. $\bar{x} \equiv [x, p_x, y, p_y, \delta, c_0 t]$: For the latter one obtains

$$A = \begin{bmatrix} A_x & 0 & 0 \\ 0 & A_y & 0 \\ 0 & 0 & A_z \end{bmatrix}, \quad R = \begin{bmatrix} R_x & 0 & 0 \\ 0 & R_y & 0 \\ 0 & 0 & R_z \end{bmatrix} \quad (5)$$

in the case of mid-plane symmetry. The longitudinal tune ν_z is the synchrotron tune.

In the nonlinear case, the (symplectic³) one-turn (functional) map can be (formally) represented by a Lie series [7]

$$\mathcal{M} \equiv \mathcal{A}^{-1} e^{:h(\bar{x}):} \mathcal{R} \mathcal{A} \quad (6)$$

where $\mathcal{A}$, $\mathcal{R}$ and $:h:$ are (functional) operators⁴, and the Lie derivative $:h(\bar{x}):$ of a function $f(\bar{x})$ of the phase-space variables is defined by the Poisson bracket

-
1. Diagonal with complex (conjugate pairs) of eigen values.
 2. With δ treated as a (slowly) varying parameter, i.e., the adiabatic approximation.
 3. $\Rightarrow$ volume preserving. Note, since the dimension of the linear symplectic group is $(2n+1)n$ but $(2n)^2 - 1$ for the unimodular matrix group: symplectic $\Leftrightarrow$ volume preserving for 1 D.O.F. only.
 4. Transforming functions into functions.

$$:h:f(\bar{x}) \equiv \sum_{i=1}^n \frac{\partial h}{\partial x_i} \frac{\partial f}{\partial p_{xi}} - \frac{\partial f}{\partial x_i} \frac{\partial h}{\partial p_{xi}}, \quad (7)$$

The *generator* h is a power series in the phase-space variables

$$h = \sum_{j,k,l,m,n} h_{jklmn} J_x^{(j+k)/2} J_y^{(l+m)/2} \delta^n e^{i[(j-k)\phi_x + (l-m)\phi_y]} + \text{c.c.}, \quad (8)$$

and, to 1st order in the sextupole strength, the *driving terms* h_{jklmn} are [3]

$$h_{jklmn} \sim \sum_{a=1}^N (b_3 L)_a \beta_{x,a}^{(j+k)/2} \beta_{y,a}^{(l+m)/2} \eta_{x,a}^n e^{i[(j-k)\mu_{x,j \rightarrow k} + (l-m)\mu_{y,l \rightarrow m}]} + \text{c.c.} \quad (9)$$

Higher order terms are generated by cross terms, i.e., commutators, of these. The driving terms to 2nd order in sextupole strength (and leading order in momentum dependence) are summarized in Tab. 2. Note, the resonance terms have additional momentum dependant terms that have not been listed¹.

1st Order	h_{10001}	h_{20001}	h_{00201}	h_{10110}	h_{21000}	h_{30000}	h_{10200}	h_{10020}
Effect	$d\eta_x/d\delta$	$d\beta_x/d\delta$	$d\beta_y/d\delta$	v_x	v_x	$3v_x$	$v_x + 2v_y$	$v_x - 2v_y$
2nd Order	h_{31000}	h_{00310}	h_{20110}	h_{11200}	h_{40000}	h_{00400}	h_{20200}	h_{20020}
Effect	$2v_x$	$2v_y$	$2v_x$	$2v_y$	$4v_x$	$4v_y$	$2v_x + 2v_y$	$2v_x - 2v_y$

TABLE 2. Driving Terms to 2nd Order in the Sextupole Strength.

The map can be (recursively²) transformed into normal form³ (perturbation theory) [8]

$$\mathcal{M} = \mathcal{A}^{-1} e^{:h:} \mathcal{R} \mathcal{A} \rightarrow e^{:-g(\bar{J}, \bar{\Phi}):} e^{:k(\bar{J}):} \mathcal{R} e^{:g(\bar{J}, \bar{\Phi}):} \quad (10)$$

where $\exp(:-g(\bar{J}, \bar{\Phi}):)$ is a canonical transformation and $:k(\bar{J}):$ a phase-independent generator⁴ of the form

$$k(\bar{J}) = \sum_{j,k,l,m,n} k_{jklmn} J_x^{(j+k)/2} J_y^{(l+m)/2} \delta^n \quad (11)$$

-
1. Which drive synchro-betatron resonances.
 2. And hence automated to arbitrary order.
 3. The equivalent of solving the Hamilton-Jacobi equation.
 4. $\Rightarrow$ an amplitude dependent rotation in Floquet space.

from which we obtain the tune shift

$$\bar{\nu}(\bar{J}) = -\frac{1}{2\pi} \frac{\partial k(\bar{J})}{\partial \bar{J}} \quad (12)$$

In particular, the *tune shift terms* to 4th order in the sextupole strengths are

$$\begin{aligned} \nu_x(\bar{J}, \delta) &= \nu_{0,x} - \frac{1}{2\pi} [2k_{11001}\delta + 4k_{22000}(2J_x) + 2k_{11110}(2J_y) + 2k_{11002}\delta^2 \\ &\quad + 4k_{22001}(2J_x)\delta + 2k_{11111}(2J_y)\delta + 4k_{22002}(2J_x)\delta^2 + 2k_{11112}(2J_y)\delta^2 \\ &\quad + 6k_{33000}(2J_x)^2 + 4k_{22110}(2J_x)(2J_y) + 2k_{11220}(2J_y)^2 + 2k_{11003}\delta^3] \\ \nu_y(\bar{J}, \delta) &= \nu_{0,y} - \frac{1}{2\pi} [2k_{00111}\delta + 4k_{00220}(2J_y) + 2k_{11110}(2J_x) + 2k_{00112}\delta^2 \\ &\quad + 4k_{00221}(2J_y)\delta + 2k_{11111}(2J_x)\delta + 4k_{00222}(2J_y)\delta^2 + 2k_{11112}(2J_x)\delta^2 \\ &\quad + 6k_{00330}(2J_y)^2 + 4k_{11220}(2J_x)(2J_y) + 2k_{22110}(2J_x)^2 + 2k_{00113}\delta^3] \end{aligned} \quad (13)$$

i.e., chromatic-, amplitude dependent tune shift-, and cross terms. Analytic expressions for k_{22000} , k_{11110} , and, k_{00220} are shown in Eq. (19), Section 4.4. The corresponding generator is given by Eqs. (99) and (100) in ref. [3]. Having gained insight into the generic structure of these terms, there is little to be learnt from grinding out analytical expressions¹ for the general case, so these are computed by numerical techniques [9].

1. Typically, valid only for the bare lattice and real magnets approximated with one thin kick; for simplicity.

4.0 Control of the Nonlinear Dynamics

4.1 Control of the Resonance Terms by Symmetry

The resonance terms up to 2nd order in the sextupole strength can be cancelled by symmetry¹. In particular, by:

1. implement a unit cell with two chromatic sextupole families,
2. repeat it N_{cell} times (15 for NSLS-II),
3. and choose the cell tune to e.g. $\bar{\nu} = (33.5/N_{\text{cell}}, 15.5/N_{\text{cell}})$

see Tab. 3, which shows the accumulative normalized phase advance for each cell and the 1st and 2nd order sextupolar resonances. Mathematically, this corresponds to a set of phasors that adds up to null; for each resonance. Clearly, there are other choices with partial-, full-, or even multiple cancellations, see Tab. 4.

Cell	ν_x	ν_y	$2\nu_x$	$3\nu_x$	$2\nu_y$	$\nu_x - 2\nu_y$	$\nu_x + 2\nu_y$	$4\nu_x$	$4\nu_y$	$2\nu_x + 2\nu_y$	$2\nu_x - 2\nu_y$
1	2.2	1.0	4.5	6.7	2.1	0.2	4.3	8.9	4.1	2.4	6.5
2	4.5	2.1	8.9	13.4	4.1	0.3	8.6	17.9	8.3	4.8	13.1
3	6.7	3.1	13.4	20.1	6.2	0.5	12.9	26.8	12.4	7.2	19.6
4	8.9	4.1	17.9	26.8	8.3	0.7	17.2	35.7	16.5	9.6	26.1
5	11.2	5.2	22.3	33.5	10.3	0.8	21.5	44.7	20.7	12.0	32.7
6	13.4	6.2	26.8	40.2	12.4	1.0	25.8	53.6	24.8	14.4	39.2
7	15.6	7.2	31.3	46.9	14.5	1.2	30.1	62.5	28.9	16.8	45.7
8	17.9	8.3	35.7	53.6	16.5	1.3	34.4	71.5	33.1	19.2	52.3
9	20.1	9.3	40.2	60.3	18.6	1.5	38.7	80.4	37.2	21.6	58.8
10	22.3	10.3	44.7	67.0	20.7	1.7	43.0	89.3	41.3	24.0	65.3
11	24.6	11.4	49.1	73.7	22.7	1.8	47.3	98.3	45.5	26.4	71.9
12	26.8	12.4	53.6	80.4	24.8	2.0	51.6	107.2	49.6	28.8	78.4
13	29.0	13.4	58.1	87.1	26.9	2.2	55.9	116.1	53.7	31.2	84.9
14	31.3	14.5	62.5	93.8	28.9	2.3	60.2	125.1	57.9	33.6	91.5
15	33.5	15.5	67.0	100.5	31.0	2.5	64.5	134.0	62.0	36.0	98.0

TABLE 3. Control of up to 2nd Order Sextupolar Resonances by Symmetry.

1. There is also a cancellation of the contribution to the amplitude dependent tune shift from the phase dependent driving terms.

Cell	v_x	v_y	$2v_x$	$3v_x$	$2v_y$	$v_x - 2v_y$	$v_x + 2v_y$	$4v_x$	$4v_y$	$2v_x + 2v_y$	$2v_x - 2v_y$
1	2.2	1.1	4.5	6.7	2.1	0.1	4.4	8.9	4.3	2.3	6.6
2	4.5	2.1	8.9	13.4	4.3	0.2	8.7	17.9	8.5	4.7	13.2
3	6.7	3.2	13.4	20.1	6.4	0.3	13.1	26.8	12.8	7.0	19.8
4	8.9	4.3	17.9	26.8	8.5	0.4	17.5	35.7	17.1	9.3	26.4
5	11.2	5.3	22.3	33.5	10.7	0.5	21.8	44.7	21.3	11.7	33.0
6	13.4	6.4	26.8	40.2	12.8	0.6	26.2	53.6	25.6	14.0	39.6
7	15.6	7.5	31.3	46.9	14.9	0.7	30.6	62.5	29.9	16.3	46.2
8	17.9	8.5	35.7	53.6	17.1	0.8	34.9	71.5	34.1	18.7	52.8
9	20.1	9.6	40.2	60.3	19.2	0.9	39.3	80.4	38.4	21.0	59.4
10	22.3	10.7	44.7	67.0	21.3	1.0	43.7	89.3	42.7	23.3	66.0
11	24.6	11.7	49.1	73.7	23.5	1.1	48.0	98.3	46.9	25.7	72.6
12	26.8	12.8	53.6	80.4	25.6	1.2	52.4	107.2	51.2	28.0	79.2
13	29.0	13.9	58.1	87.1	27.7	1.3	56.8	116.1	55.5	30.3	85.8
14	31.3	14.9	62.5	93.8	29.9	1.4	61.1	125.1	59.7	32.7	92.4
15	33.5	16.0	67.0	100.5	32.0	1.5	65.5	134.0	64.0	35.0	99.0

TABLE 4. Control of up to 2nd Order Sextupolar Resonances by Symmetry.

For an complete determination of suitable cell tunes, we introduce the norm

$$\|h\| \equiv \sum_{j+k+l+m+n \leq 4} h_{jklmn}^2 \quad (14)$$

and compute it for a 11×11 grid of cell tunes with acceptable optics, see Fig. 1. We conclude that there are several regions for which the cell tune is such that resonances up to 2nd order in sextupole strength are cancelled by symmetry. Because there are phase independent terms, e.g. h_{22000} , h_{11110} and h_{00220} , driving the amplitude dependent tune shift, there is no such cancellation for these terms, see Fig. 2. Note that the divergence in the upper left corner of both figures due to the resonance $v_x - 2v_y = 0$, i.e., is systematically driven.

FIGURE 1. Norm of the Resonance Terms ($\log(\|h\|)$) for a Cell with two Chromatic Sextupole Families [sm1, sm2].

FIGURE 2. Norm of the Tune Shift Terms ($\log(\|K\|)$) for a Cell with two Chromatic Sextupole Families [sm1, sm2].

The next step is to control the amplitude dependent tune shift, but first we must address the fact that the resulting working point is close to integer or half-integer resonances.

Clearly, while the resonance terms are cancelled by symmetry for these working points, the dynamics is not robust to imperfections, e.g. from engineering tolerances. So, the lattice must be detuned, which reduces the cancellations. However, this can be avoided by introducing additional sextupole families [3]. Ideally, full control of the:

- linear chromaticity (2 terms),
- momentum dependence of the linear optics (3 terms),
- 1st order resonances (5 terms),
- and leading (2nd) order amplitude dependent tune shift (3 terms)

would require 5 chromatic and 8 geometric families¹. However, the former is not realistic for an achromatic lattice².

4.2 The Origin of 2nd Order Chromaticity

The main effect of h_{10002} , h_{20001} and h_{00201} is the generation of 2nd order chromaticity. In particular, by

$$\begin{aligned}
 \xi_x^{(2)} &\equiv \left. \frac{\partial \xi_x}{\partial \delta} \right|_{\delta=0} \\
 &= -\frac{1}{2} \xi_x^{(1)} + \frac{1}{8\pi} \sum_{j=1}^N \left\{ 2(b_3 L)_j \frac{\partial \eta_{x,j}}{\partial \delta} \beta_{x,j} - [(b_2 L)_j - 2(b_3 L)_j \eta_{x,j}] \frac{\partial \beta_{x,j}}{\partial \delta} \right\}, \\
 \xi_y^{(2)} &\equiv \left. \frac{\partial \xi_y}{\partial \delta} \right|_{\delta=0} \\
 &= -\frac{1}{2} \xi_y^{(1)} - \frac{1}{8\pi} \sum_{j=1}^N \left\{ 2(b_3 L)_j \frac{\partial \eta_{y,j}}{\partial \delta} \beta_{y,j} - [(b_2 L)_j - 2(b_3 L)_j \eta_{y,j}] \frac{\partial \beta_{y,j}}{\partial \delta} \right\}
 \end{aligned} \tag{15}$$

The driving terms around the lattice are shown in Figs. 3-4. Note that all the (geometric and chromatic) sextupoles contributes to the first driving term.

1. That are not degenerate (nontrivial).
2. Required by the use of DWs.

FIGURE 3. First Driving Term for 2nd Order Chromaticity: no Sextupoles.

FIGURE 4. Second Driving Term for 2nd Order Chromaticity: no Sextupoles.

The second order dispersion (generated by h_{00002}) is given by

$$\begin{aligned}\eta_{x,i}^{(2)} &\equiv \left. \frac{\partial \eta_{x,i}(\delta)}{\partial \delta} \right|_{\delta=0} \\ &= -\eta_{x,i}^{(1)} + \frac{\sqrt{\beta_{x,i}}}{2} \sum_{j=1}^N [(b_2 L)_j - (b_3 L)_j \eta_{x,j}^{(1)}] \eta_{x,j}^{(1)} \sqrt{\beta_{x,j}} \text{Res}(1, 0)\end{aligned}\quad (16)$$

where

$$\text{Res}_{i \rightarrow j}(n_x, n_y) \equiv \frac{\cos(|\bar{n} \cdot \bar{\mu}_{i \rightarrow j}| - \pi \bar{n} \cdot \bar{v})}{\sin(\pi \bar{n} \cdot \bar{v})}, \quad \bar{n} \equiv (n_x, n_y), \quad \bar{v} \equiv (v_x, v_y) \quad (17)$$

and the momentum dependence of the beta functions (generated by h_{20001} and h_{00201}) are

$$\begin{aligned}\beta_{x,i}^{(2)} &\equiv \left. \frac{\partial \beta_{x,i}}{\partial \delta} \right|_{\delta=0} = \frac{\beta_{x,i}^{(1)}}{2} \sum_{j=1}^N [(b_2 L)_j - 2(b_3 L)_j \eta_{x,j}^{(1)}] \eta_{x,j}^{(1)} \beta_{x,j}^{(1)} \text{Res}(2, 0), \\ \beta_{y,i}^{(2)} &\equiv \left. \frac{\partial \beta_{y,i}}{\partial \delta} \right|_{\delta=0} = -\frac{\beta_{y,i}^{(1)}}{2} \sum_{j=1}^N [(b_2 L)_j - 2(b_3 L)_j \eta_{x,j}^{(1)}] \eta_{x,j}^{(1)} \beta_{y,j}^{(1)} \text{Res}(0, 2)\end{aligned}\quad (18)$$

Note that dispersive quadrupoles will systematically drive 2nd order dispersion if separated by 2π , or momentum dependence of the beta functions if separated by π . The former is a limiting factor for DBA lattices¹ (due to the phase advance between the DBA cells). This is the source of the considerable amount of residual 2nd (and higher) order horizontal chromaticity, see Fig. 24. Also, the chromatic sextupoles contribute as well. The residual values, after (linear) chromatic correction with 2 chromatic families, are shown in Figs. 5-8 and the corresponding driving terms in Figs. 9-10. Clearly, with only two chromatic families, these 3 driving terms can not be controlled.

1. And the latter for TBA lattices (due to the phase advance between adjacent quadrupoles within the TBA cell [3]).

FIGURE 5. Second Order Dispersion with Two Chromatic Families.

FIGURE 6. Momentum Dependence of the Orbit with Two Chromatic Families.

FIGURE 7. Momentum Dependence of the Horizontal Beta Function with Two Chromatic Families.

FIGURE 8. Momentum Dependence of the Vertical Beta Function with Two Chromatic Families.

FIGURE 9. First Driving Term for 2nd Order Chromaticity with Two Chromatic Families.

FIGURE 10. Second Driving Term for 2nd Order Chromaticity with Two Chromatic Families.

4.3 Control of 2nd Order Chromaticity by Sextupole Placement

From Figs. 3-4 and Eq. (15) it is clear that the second order chromaticity is quite sensitive to the sextupole placement, i.e., the driving terms depend on derivatives of the linear optics functions with the momentum δ . Therefore, it is not surprising that changes as small as ~ 10 cm are significant; as observed during numerical optimizations for other medium energy synchrotron light sources [10]. As an example, we show how the sextupole scheme for the baseline lattice can be improved by optimizing the sextupole placement¹ for e.g. the working point $\bar{\nu} = (33.47, 15.68)$, see Tab. 5 and Fig. 11.

Sextupole Family	sl1	sl2	sl3	sh1	sh2	sh3	sh4	sm1
Δs [m]	0.13	0.03	0.15	-0.14	0.14	0.03	-0.07	0.01

TABLE 5. Optimized Sextupole Placement for $\bar{\nu} = (33.47, 15.68)$.

Note that, while all the sextupoles contribute (chromatic as well as geometric), the 2nd order chromaticity originates from the momentum dependence of the linear optics, which is determined by the: dispersive quadrupoles and sextupoles, and cell tune, see Eqs. (15)-(18). In other words, unless an adequate number of (linear) chromatic sextupole families are provided, the control of nonlinear chromaticity (2+2 terms) will interfere² with the geometric sextupoles; introduced to control the nonlinear resonances (8+8 terms) and amplitude dependent tune shift (3+4 terms), see Fig. 12. In particular, compromise on the tune footprint for the bare lattice which will reduce the tolerance of tune shift from: DWs, insertion devices, systematic higher multipole errors, and perturbations of the linear optics (due to symmetry reduction). Simply put, make the design less robust.

Interestingly, the tune footprint for the baseline lattice can also be improved, apart from a considerable residual 2nd order horizontal chromaticity, by eliminating one of the 4 geometric families in the long matching sections, see Fig. 13. In other words, these families are degenerate. Contrariwise, given the sensitivity of the tune footprint on the sextupole placement, clearly, an adequate number of effective knobs are required for robust control of the nonlinear dynamics for a real lattice.

1. Sm2 is not moved since it is at the center of the double-bend-achromat.

2. Must be (indirectly) “controlled” by the 7 geometric sextupoles.

FIGURE 11. Tune Footprint (on/off momentum red/blue) and Distance from Resonances for Baseline Sextupole Scheme and Optimized Sextupole Placement for $\bar{\nu} = (33.47, 15.68)$.

FIGURE 12. Tune Footprint (on/off momentum red/blue) and Distance from Resonances for the Baseline Lattice for $\bar{\nu} = (33.47, 15.68)$.

FIGURE 13. Tune Footprint (on/off momentum red/blue) and Distance from Resonances with 2 Chromatic [sm1, sm2] and 6 Geometric [sl1, sl2, sl3, sh1, sh3, sh4] Families for $\bar{\nu} = (33.47, 15.68)$.

4.4 Control of Resonance- and Tune Shift Terms

As a compromise, we use 3 chromatic [sm1, sm2, sm+]¹ and 6 geometric¹ [sl1, sl2, sl3, sh1, sh3, sh4] families to control the:

- 1st and 2nd order chromaticity (4 terms),
- momentum dependence of the linear optics (3 terms),
- 1st order resonances (5 terms),
- and leading order amplitude dependent tune shift (3 terms),

governed by a 15×9 nonlinear system of (up to 2nd order) algebraic equations for the sextupole strengths. The controllability of the dynamics can be evaluated by computing

1. Since we have shown that the 4 families in the long matching section are degenerate, see Section 4.3.

the *sextupole response matrix*, i.e., the Jacobian of the system. The result is summarized in Tab. 6: where (sl1, sl2, sl3, sh1, sh2, sh3, sh4) are the geometric and (sm1, sm2, sm+) the 2+1 chromatic families. Shaded cells shows the primary terms. However, minimizing these terms will drive the higher order terms in Tab. 2 and Eqs. (13), so they must be included as well¹. In particular the:

- 2nd order resonances (8 terms),
- 3rd order chromaticity (2 terms),
- next (4th) order of amplitude dependent tune shift (4 terms),
- and chromatic cross terms (6 terms),

leading to a 35×9 nonlinear system (up to 4th order). The result is shown in Section 4.5, Figs. 16-17 for a range of working points. The control of the tune shift is diverging towards the lower part of the graph due to $v_y = 15$.

	sl1	sl2	sl3	sh1	sh2	sh3	sh4	sm1	sm2	sm+
k_{11001}	1.0E-06	3.1E-06	1.2E-06	6.3E-06	8.1E-06	4.1E-06	9.8E-07	-9.0E-03	-1.6E-02	-9.6E-03
k_{00111}	-1.1E-06	-1.3E-06	-1.9E-06	-3.3E-06	-3.9E-06	-5.4E-06	-4.6E-06	1.0E-02	5.3E-03	2.4E-02
h_{10002}	-6.7E-15	-1.3E-14	-3.4E-15	-1.5E-13	-2.0E-13	-1.0E-13	-2.4E-14	2.7E-07	5.0E-07	3.3E-07
h_{20001}	-1.8E-11	-6.2E-11	-2.6E-11	8.2E-11	1.2E-10	6.8E-11	1.9E-11	3.5E-08	6.4E-08	-2.2E-08
h_{00201}	5.4E-11	8.9E-11	1.6E-10	-5.9E-11	8.7E-12	8.8E-11	1.0E-10	-7.9E-07	-4.3E-07	-2.2E-06
h_{10110}	7.1E-08	8.7E-08	1.2E-07	2.9E-07	3.6E-07	5.2E-07	4.7E-07	-4.1E-07	-2.1E-07	-1.1E-06
h_{21000}	-3.3E-08	-1.0E-07	-3.9E-08	-2.8E-07	-3.8E-07	-2.0E-07	-5.0E-08	1.8E-07	3.2E-07	2.1E-07
h_{30000}	2.1E-08	7.1E-08	2.9E-08	-2.5E-09	-1.9E-08	-1.6E-08	-7.2E-09	-6.1E-08	-1.1E-07	-4.7E-08
h_{10200}	-7.1E-08	-9.5E-08	-1.2E-07	8.6E-08	6.8E-08	5.6E-08	2.0E-08	4.8E-08	2.7E-08	4.0E-07
h_{10020}	7.8E-08	8.7E-08	6.1E-08	-8.0E-08	-1.5E-07	-2.5E-07	-2.1E-07	4.3E-07	2.3E-07	8.8E-07
k_{22000}	-1.9E-07	-5.1E-07	-1.8E-07	-2.2E-06	-2.9E-06	-1.5E-06	-4.0E-07	1.9E-06	2.9E-06	2.5E-06
k_{11110}	1.3E-06	1.0E-06	1.9E-06	5.4E-07	1.9E-07	2.1E-06	2.6E-06	-3.8E-07	1.9E-06	-5.2E-06
k_{00220}	2.8E-07	3.7E-07	6.2E-07	9.0E-07	1.2E-06	1.8E-06	1.5E-06	-2.9E-06	-1.5E-06	-7.3E-06
k_{11002}	-1.6E-07	-4.8E-07	-1.8E-07	-9.7E-07	-1.3E-06	-6.4E-07	-1.5E-07	5.6E-06	8.8E-06	6.8E-06
k_{00112}	1.7E-07	2.0E-07	2.9E-07	5.0E-07	6.0E-07	8.4E-07	7.2E-07	-1.1E-06	1.0E-06	-5.0E-06

TABLE 6. The Sextupole Response Matrix (Jacobian of the System).

4.5 Properties of the “Solution Space”

The only reason that this highly over constrained, nonlinear algebraic system has a satisfactory “solution” (in a least-square sense), i.e., that an adequate minimum can be obtained by a brute force numerical minimization, is due to the:

1. 15-fold symmetry of the lattice,
2. and because the higher order terms originate from cross terms of the lower order.

1. More typically, this is avoided by a brute-force approach, i.e., by introducing an ad hoc cap on the sextupole strengths.

In general the leading (2nd) order amplitude dependent tune shift

$$\begin{aligned}
\left. \frac{\partial v_x}{\partial J_x} \right|_{\bar{J}, \delta=0} &= -\frac{4}{\pi} k_{22000} = -\frac{1}{16\pi} \sum_{j=1}^n \sum_{k=1}^n (b_3 L)_j (b_3 L)_k \beta_{x,j}^{3/2} \beta_{x,k}^{3/2} [3 \text{Res}_{j \rightarrow k}(1, 0) + \text{Res}_{j \rightarrow k}(3, 0)], \\
\left. \frac{\partial v_x}{\partial J_y} \right|_{\bar{J}, \delta=0} &= \frac{\partial v_y}{\partial J_x} \bigg|_{\bar{J}, \delta=0} = -\frac{2}{\pi} k_{11110} \\
&= \frac{1}{8\pi} \sum_{j=1}^n \sum_{k=1}^n (b_3 L)_j (b_3 L)_k \sqrt{\beta_{x,j} \beta_{x,k}} \beta_{y,j} \\
&\quad \times [2\beta_{x,k} \text{Res}_{j \rightarrow k}(1, 0) - \beta_{y,k} \text{Res}_{j \rightarrow k}(1, 2) + \beta_{y,k} \text{Res}_{j \rightarrow k}(1, -2)], \\
\left. \frac{\partial v_y}{\partial J_y} \right|_{\bar{J}, \delta=0} &= -\frac{4}{\pi} k_{00220} \\
&= \frac{1}{8\pi} \sum_{j=1}^n \sum_{k=1}^n (b_3 L)_j (b_3 L)_k \sqrt{\beta_{x,j} \beta_{x,k}} \beta_{y,j} \beta_{y,k} \\
&\quad \times [4 \text{Res}_{j \rightarrow k}(1, 0) + \text{Res}_{j \rightarrow k}(1, 2) + \text{Res}_{j \rightarrow k}(1, -2)]
\end{aligned} \tag{19}$$

scales with the square of the number of sextupoles n . However, due to the symmetry, it scales linearly with the number of cells N_{cell}

$$\begin{aligned}
\left. \frac{\partial v_x}{\partial J_x} \right|_{\bar{J}, \delta=0} &= -\frac{N_{\text{cell}}}{16\pi} \sum_{j=1}^{n^{\text{cell}}} \sum_{k=1}^{n^{\text{cell}}} (b_3 L)_j (b_3 L)_k \beta_{x,j}^{3/2} \beta_{x,k}^{3/2} [3 \text{Res}_{j \rightarrow k}^{\text{cell}}(1, 0) + \text{Res}_{j \rightarrow k}^{\text{cell}}(3, 0)], \\
&\text{etc.}
\end{aligned} \tag{20}$$

where

$$\bar{\mathbf{v}} \equiv (\mathbf{v}_x^{\text{cell}}, \mathbf{v}_y^{\text{cell}}) \tag{21}$$

To summarize, symmetry is the only way to fully control the tune footprint. Still, due to the relatively large number of cells, to obtain a satisfactory tune footprint for the bare lattice, <0.1 , is somewhat of a challenge. In particular due to the relatively large peak values of the beta functions¹, see Figs. 14-15, which also makes the lattice somewhat more sensitive to systematic higher multipole errors (see Section 6.0); as compared to other medium energy synchrotron light sources.

Since both the resonance- and tune shift terms depend strongly on the cell tune, the numerical optimization is done over e.g. a 11×11 grid of working points with satisfactory optics², see Figs. 16-17. For comparisons, corresponding results for the baseline lattice are shown in Figs. 18-19. We note that, while the control of the resonance terms is comparable

-
1. A side-effect of relocating the TPWs from the center of the double bend achromat to adjacent of the dipoles (to reduce the horizontal beam size).
 2. Obtained from the optics optimizations [11].

(due to the symmetry of the lattice), the control of the tune footprint is compromised. Moreover, Figs. 20-22 shows the performance of the other three permutations when one family out of the 4 (degenerate) is removed from the long matching sections. We have chosen to remove sh2, but sh3 or sh4 could be considered as well. But, note the influence of $2(v_x - v_y)$; from the 4th order tune shifts. Apart from that, this is confirmed by the singular values¹ for the sextupole response matrix for the different configurations, see Tab. 7.

[sm1, sm2, sm+] [sl1, sl2, sl3] [sh1, sh2, sh3, sh4]	no sh1	no sh2	no sh3	no sh4	no sm+ (baseline)
3.1×10^{-2}	3.1×10^{-2}	3.1×10^{-2}	3.1×10^{-2}	3.1×10^{-2}	2.1×10^{-2}
1.2×10^{-2}	1.2×10^{-2}	1.2×10^{-2}	1.2×10^{-2}	1.2×10^{-2}	5.7×10^{-3}
4.1×10^{-6}	3.2×10^{-6}	2.8×10^{-6}	3.6×10^{-6}	3.7×10^{-6}	4.6×10^{-6}
1.4×10^{-6}	1.2×10^{-6}	1.3×10^{-6}	1.3×10^{-6}	1.2×10^{-6}	1.9×10^{-6}
1.1×10^{-6}	8.1×10^{-7}	1.2×10^{-6}	1.2×10^{-6}	7.2×10^{-7}	8.8×10^{-7}
6.6×10^{-7}	6.8×10^{-7}	7.1×10^{-7}	6.8×10^{-7}	5.3×10^{-7}	3.3×10^{-7}
4.5×10^{-7}	4.9×10^{-7}	2.6×10^{-7}	4.5×10^{-7}	4.7×10^{-7}	2.2×10^{-7}
2.0×10^{-7}	1.3×10^{-7}	1.3×10^{-7}	2.1×10^{-7}	1.4×10^{-7}	1.7×10^{-7}
1.4×10^{-7}	8.7×10^{-8}	1.0×10^{-7}	1.4×10^{-7}	1.2×10^{-7}	6.6×10^{-8}
5.3×10^{-8}					

TABLE 7. Singular Values for the Sextupole Response Matrix for Various Sextupole Configurations.

1. For the first working point.

FIGURE 14. Linear Optics.

FIGURE 15. Linear Dispersion.

FIGURE 16. Norm of the Resonance Terms for the Sextupole Scheme with 3 Chromatic [sm1, sm2, sm+] and 6 Geometric [sl1, sl2, sl3, sh1, sh3, sh4] Families.

FIGURE 17. Norm of the Tune Shift Terms for the Sextupole Scheme with 3 Chromatic [sm1, sm2, sm+] and 6 Geometric [sl1, sl2, sl3, sh1, sh3, sh4] Families.

FIGURE 18. Norm of the Resonance Terms for the Sextupole Scheme with 2 Chromatic [sm1, sm2] and 7 Geometric [sl1, sl2, sl3, sh1, sh2, sh3, sh4] Families.

FIGURE 19. Norm of the Tune Shift Terms for the Sextupole Scheme with 2 Chromatic [sm1, sm2] and 7 Geometric [sl1, sl2, sl3, sh1, sh2, sh3, sh4] Families.

FIGURE 20. Norm of the Tune Shift Terms for the Sextupole Scheme with 3 Chromatic [sm1, sm2, sm+] and 6 Geometric [sl1, sl2, sl3, sh1, sh2, sh3] Families.

FIGURE 21. Norm of the Tune Shift Terms for the Sextupole Scheme with 3 Chromatic [sm1, sm2, sm+] and 6 Geometric [sl1, sl2, sl3, sh2, sh3, sh4] Families.

FIGURE 22. Norm of the Tune Shift Terms for the Sextupole Scheme with 3 Chromatic [sm1, sm2, sm+] and 6 Geometric [sl1, sl2, sl3, sh1, sh2, sh4] Families.

5.0 Proof-of-Concept

For a control theory approach to the medium term (10^3 turns) stability of the nonlinear dynamics, we graph the 1st and 2nd order resonances from Tab. 1 and, for a given working point, compute the tune footprint for $(\Delta x, \Delta y) = (\pm 15, \pm 6.5)$ mm on-momentum and $(\Delta x, \Delta\delta) = (\pm 10 \text{ mm}, \pm 2.5\%)$ off-momentum; at the center of the long straight. Note that the tune footprints have been shoehorned into areas surrounded by these resonances, i.e., these are our estimates for the on- and off-momentum aperture. The main result from this section is summarized by Figs. 23, 32, 40, 42, and 51, i.e., essentially the same on- and off-momentum dynamic aperture for a realistic lattice, see Appendix A.

FIGURE 23. 1st and 2nd Order Sextupolar Resonances and Tune Footprint (on/off momentum red/blue) for the 4 Somewhat Arbitrarily Chosen Working Points.

5.1 First Working Point: $\bar{\nu} = (33.47, 15.68)$

The tune footprint for the bare lattice is shown in Fig. 24; the green lines are 3rd order resonances. From Fig. 26 we note that the rather excessive momentum dependence of the vertical beta function in Fig. 8 has been reduced to an adequate level. Frequency maps for the bare¹ lattice without DWs are shown in Fig. 27. For comparison, we also show the results for 2 chromatic and 6 geometric families in Fig. 28. In particular, the broadening of the off-momentum tune footprint due to residual 2nd order chromaticity. Since the improved tune footprint barely fits within the 1st and 2nd order resonances, this reduces the leeway from other contributions.

FIGURE 24. Tune Footprint (on/off momentum red/blue) and Distance from Resonances.

1. Without engineering tolerances.

FIGURE 25. Momentum Dependence of the Horizontal Beta Function.

FIGURE 26. Momentum Dependence of the Vertical Beta Function.

FIGURE 27. Frequency Maps for the Bare Lattice without DWs.

FIGURE 28. Tune Footprint (on/off momentum red/blue) and Distance from Resonances with 2 Chromatic- and 6 Geometric Families.

5.2 Control of Impact from Damping Wigglers

We now introduce 3 DWs and correct the impact of the linear vertical focusing by a hybrid approach, i.e., local matching of the optics and global correction of the resulting tune shift. Due to the correction algorithm based on the “*quadrupole response matrix*” (Jacobian of the governing algebraic system) inverted by singular value decomposition implemented from the start [4], this is simply a matter of parameter choice. Moreover, since it is using the beam response matrix as observable, it is straightforward to implement on-line. The result is shown in Figs. 29-30. We note that the tune footprint is not restored. This is due to the gradient/optics changes in the matching sections, which affects the sextupole contributions, and the nonlinear contributions from the DWs.

FIGURE 29. Beta- and Normalized Phase Advance Beat after Optics Correction.

We therefore include these effects into our optimization and re-optimize the sextupole strengths for a lattice with DWs and linear corrections. The result is shown in Tab. 8 and Fig. 31. We note that the tune footprint now has been restored. Frequency maps for a realistic lattice, see Appendix A, are shown in Fig. 32. The extended yellow regions shows some minor tune modulation due to weakly excited resonances by the engineering tolerances. To summarize the introduction of damping wigglers is now transparent.

To illustrate the impact of 3rd order resonances we make a minor adjustment to the working point, i.e., from $\bar{\nu} = (33.47, 15.68)$ to $(33.45, 15.64)$. The result is shown in Figs. 33-34. This can be understood by noting the crossing of $\nu_x + 4\nu_y = 96$, a systematic effect due to the 3-fold symmetry¹ with DWs.

1. The impact on the optics could be reduced by using a 5-fold symmetry.

FIGURE 30. Tune Footprint (on/off momentum red/blue) and Distance from Resonances after Optics Correction.

(b_3L)	Sextupole Family								
	sl1	sl2	sl3	sh1	sh3	sh4	sm1	sm2	sm+
w.o. DWs	0.83	2.17	-2.65	2.63	-1.44	-0.81	-2.72	3.47	0.71
w. DWs	0.65	2.36	-3.00	3.00	-2.03	-0.40	-2.54	3.40	0.40

TABLE 8. Sextupole Strengths with and without DWs.

FIGURE 31. Tune Footprint (on/off momentum red/blue) and Distance from Resonances After Re-optimization with DWs.

FIGURE 32. Frequency Maps for a Realistic Lattice.

FIGURE 33. Tune Footprint (on/off momentum red/blue) and Distance from Resonances for $\bar{\nu} = (33.45, 15.64)$.

FIGURE 34. Frequency Maps for $\bar{\nu} = (33.45, 15.64)$.

5.3 Impact of IVUs

We now also include 5 non-symmetrically distributed IVUs. While these mainly contribute to the on-momentum tune footprint, they do not have a major impact, see Fig. 35, because adequate leeway has been established; by design. The reduction in the vertical plane is due to the vertical aperture, i.e., ± 2.5 mm. In other words, the vertical gap for the DWs is not the limiting factor; again by design.

FIGURE 35. Frequency Maps for a Realistic Lattice with 3 DWs and 5 IVUs.

5.4 Impact of EPUs

It is well known that EPUs are quite nonlinear devices that might require special shimming [12] to not have a major impact on e.g. the Touschek life time in medium energy rings. For a preliminary evaluation, we have inserted one into a short straight and computed the frequency map for a bare lattice without DWs for three different polarizations modes, see Figs. 36-38. Comparing with the baseline, i.e., Fig. 27 in Section 5.1, apart from a reduction in the vertical due to the reduced vertical aperture, there is not much impact. However, a more detailed study should be done for a realistic lattice.

FIGURE 36. Frequency Maps for a Bare Lattice with no DWs and 1 EPU in Helical Mode.

FIGURE 37. Frequency Maps for 1 EPU in Vertical Linear Mode.

FIGURE 38. Frequency Maps 1 EPU 45° Inclined Mode.

5.5 Second Working Point: $\bar{\nu} = (33.15, 15.68)$

The results are summarized by Figs. 39-40. Note, $\nu_x + 4\nu_y = 96$ needs to be avoided due to the 3-fold symmetry from DWs.

FIGURE 39. Tune Footprint (on/off momentum red/blue) and Distance from Resonances.

FIGURE 40. Frequency Maps for a Realistic Lattice.

5.6 Third Working Point: $\bar{\nu} = (33.14, 16.27)$

The results are summarized by Figs. 41-42. Note, $3\nu_x + 2\nu_y = 132$ needs to be avoided due to the 3-fold symmetry from DWs.

FIGURE 41. Tune Footprint (on/off momentum red/blue) and Distance from Resonances.

FIGURE 42. Frequency Maps for a Realistic Lattice (DWs w. 80 mm pole width).

5.7 Validation of the Damping Wiggler Pole Width

The baseline pole with for the DW is 80 mm, which was used for the results shown in e.g. Fig. 42 in Section 5.6. The effect of changing the pole width to 90, 65, and 48 mm are shown in Figs. 43-45. Clearly, within the limits of the model, e.g. no manufacturing errors, the choice is good.

FIGURE 43. Frequency Maps for 3 DWs with 90 mm Pole Width.

FIGURE 44. Frequency Maps for 3 DWs with 65 mm Pole Width.

FIGURE 45. Frequency Maps for 3 DWs with 48 mm Pole Width.

5.8 Impact of Non-Zero (linear) Chromaticity

At high bunch current, there is a desire to stabilize the head-tail instability by operating at positive chromaticities. In particular, in the vertical plane [13]: $\xi_{x,y} = (1.0, 4.0)$. We anticipate that the increased tune footprint from our model, see Fig. 46, will have a significant impact on the Touschek lifetime for a real ring. In fact, a stop band is starting to appear at $\delta = +2.6\%$ due to $2\nu_y + 2\nu_y = 99$, see Fig. 47. We also expect another stop band at $\delta = -0.5\%$ due to $4\nu_y = 65$; when IVUs are added. However, the tune footprint could be reduced by deliberately driving the vertical third order chromaticity e.g. by introducing a pair of chromatic decapoles per supercell, see Fig. 48. In particular, while the off-momentum tune footprint has been reduced it is still not quite within $2\nu_y + 2\nu_y = 99$ and $4\nu_y = 65$, and the on-momentum has been increased. We are also driving $5\nu_x$, $\nu_x \pm 4\nu_y$, and $3\nu_x \pm 2\nu_y$ resonances. While the frequency maps Fig. 49 confirms that, to improve the on- and off-momentum dynamic aperture, the tune footprint needs to be squeezed somewhat further, i.e., until it is within the leading order resonances, the fact that they did not collapse is already encouraging. Note that, conceptually, we are now attempting to improve the stability by introducing nonlinear effects; after an elaborate effort to cancel them¹.

1. Of course, this is the norm rather than exception in biology, i.e., nonlinear systems far away from thermal equilibrium.

FIGURE 46. Tune Footprint (on/off momentum red/blue) and Distance from Resonances for $\xi_{x,y} = (1.0, 4.0)$.

FIGURE 47. Frequency Maps for $\xi_{x,y} = (1.0, 4.0)$.

FIGURE 48. Tune Footprint (on/off momentum red/blue) and Distance from Resonances for $\xi_{x,y} = (1.0, 4.0)$ and 1 Family of Chromatic Decapoles.

FIGURE 49. Frequency Maps for $\xi_{x,y} = (1.0, 4.0)$ and 1 Family of Chromatic Decapoles.

5.9 Fourth Working Point: $\bar{\nu} = (33.45, 16.35)$

The results are shown in Figs. 50-51.

FIGURE 50. Tune Footprint (on/off momentum red/blue) and Distance from Resonances.

FIGURE 51. Frequency Maps for a Realistic Lattice.

6.0 Impact of Higher Order Systematic Multipoles

Pure multipoles e.g. quadrupoles and sextupoles do not exist¹. In particular, intrinsic terms compatible with the geometry of an n th order multipole are

$$(k + 1/2) \cdot n \quad k = 1, 2, 3, \dots \quad (22)$$

And, since the magnitude of the components for a given magnet type are essentially the same, they contribute systematically. Hence, the main impact on the dynamics is their contribution to the tune footprint. An inventory of the main contribution for an off-momentum ($\sim 2.5\%$) particle with large horizontal amplitude (~ 15 mm), i.e., a Touschek event in a dispersive region, is shown in Tab. 9.

Type	Quadrupole			Sextupole		
Multipole Component	b_6	b_{10}	b_{14}	b_9	b_{15}	b_{21}
Tune Shift Terms	k_{11004}	k_{11008}	$k_{1100, 12}$	k_{11007}	$k_{1100, 13}$	$k_{1100, 19}$
	k_{22002}	k_{22006}	$k_{2200, 10}$	k_{22005}	$k_{2200, 11}$	$k_{2200, 17}$
		k_{33004}	k_{33008}	k_{33003}	k_{33009}	$k_{3300, 15}$
		k_{44002}	k_{44006}	k_{44001}	...	...
			k_{55004}		k_{77001}	$k_{10, 10, 0, 0, 1}$

TABLE 9. Leading Order Chromatic Terms from Higher Order Systematic Multipole Components in the Dispersive Sections.

The horizontal tune shift terms for the bare lattice without damping wigglers are (for one super period)

$$\begin{aligned}
 v_x^{\text{cell}} = & 2.23 - 2.4 \times 10^2 (2J_x) + 3.8 \times 10^1 (2J_y) - 5.7 \times 10^0 \delta^2 - 6.6 \times 10^1 \delta^3 \\
 & + 2.7 \times 10^5 (2J_x) \delta^2 + 2.3 \times 10^3 \delta^4 \\
 & + 2.5 \times 10^{14} (2J_x)^3 \delta^2 + 3.4 \times 10^{12} (2J_x)^2 \delta^4 - 5.5 \times 10^9 (2J_x) \delta^6 \\
 & + 5.8 \times 10^{23} (2J_x)^5 \delta^2 + 1.7 \times 10^{22} (2J_x)^4 \delta^4 + 2.0 \times 10^{20} (2J_x)^3 \delta^6 + \dots
 \end{aligned} \quad (23)$$

whereas if (1.0, 4.5, 4.0) units of $(\Delta B_6, \Delta B_{10}, \Delta B_{14})$ are added to the quadrupoles in the dispersive sections (see Appendix A) we obtain

1. On a fundamental level, they are inconsistent with Maxwell's equations. Therefore, at some point the impact of e.g. fringe fields needs to be checked.

$$\begin{aligned}
v_x^{\text{cell}} = & 2.23 - 2.4 \times 10^2 (2J_x) + 3.8 \times 10^1 (2J_y) - 5.7 \times 10^0 \delta^2 - 6.6 \times 10^1 \delta^3 \\
& + 2.7 \times 10^4 (2J_x) \delta^2 + 5.1 \times 10^4 \delta^4 \\
& + 5.6 \times 10^{14} (2J_x)^3 \delta^2 + 3.1 \times 10^{13} (2J_x)^2 \delta^4 + 2.3 \times 10^{11} (2J_x) \delta^6 \\
& + 1.5 \times 10^{24} (2J_x)^5 \delta^2 + 5.3 \times 10^{23} (2J_x)^4 \delta^4 + 1.2 \times 10^{22} (2J_x)^3 \delta^6 + \dots
\end{aligned} \tag{24}$$

For a $\beta_x \sim 20$ m we have $2J_x = 1.1 \times 10^{-5}$ and the contribution to the tune shifts for the full lattice is a few parts in 10^{-3} for each term. Care is therefore required for the field tolerances for the quadrupoles and sextupoles in the dispersive sections, see ref. [14] and Appendix A.

To leading order in the multipole strength, the tune shifts are generated by

$$k_{ii00m} = -\frac{1}{n2^{n-m}} \binom{n}{m} \binom{2i}{i} \sum_{k=1}^N (b_n L)_k \beta_{x,k}^i \eta_{x,k}^m, \quad n = 2i + m \tag{25}$$

and from Eq. (12) we obtain the corresponding tune shift terms

$$\Delta v_x = -\frac{1}{2\pi} \frac{\partial(k_{ii00m} (2J_x)^i \delta^m)}{\partial J_x} = \frac{i}{\pi n 2^{n-m}} \binom{n}{m} \binom{2i}{i} \sum_{k=1}^N (b_n L)_k \beta_{x,k}^i \eta_{x,k}^m (2J_x)^{i-1} \delta^m \tag{26}$$

Correspondingly, the tune shift terms per unit of multipole error can be computed for a given lattice. And, by providing a max tune shift contribution for each component, e.g. 10% of the acceptable tune footprint (0.05), the tolerance can be estimated. Note that the contributions from the quadrupoles/sextupoles are even/odd in δ . The result is summarized in Tab. 10; which is consistent with Tab. 12 in Appendix A.

Type	Quadrupole			Sextupole		
Multipole Component	ΔB_6	ΔB_{10}	ΔB_{14}	ΔB_9	ΔB_{15}	ΔB_{21}
Tolerance	1.5	1.2	0.9	4.2	2.6	1.6

TABLE 10. Estimated Higher Order Systematic Multipole Tolerances in the Dispersive Sections for a Max Tune Shift Contribution of 0.005.

7.0 Conclusions

We have outlined how, by an intuitive approach, the on- and off-momentum dynamic aperture for a synchrotron light source can be estimated from a nonlinear system of algebraic equations for the sextupole/multipole strengths. The approach has only two free parameters: the relative weight for resonance vs. tune shift terms and the tune footprint for stable trajectories in a modern third generation synchrotron light source. In other words, we have established a control theory approach for the medium term (10^3 turns) stability for a dynamic system described by a nonlinear system of ordinary differential equations.

Equipped with a predictive, quantitative model for stability, we have then evaluated how to improve the control of the dynamics by analyzing and modifying the properties of the corresponding algebraic system. In particular, by changing the number- and characteristics of the parameters, i.e., we have not evaluated how the underlying (linear) optics could be improved. We have also validated our conjectures by numerical simulations with a realistic model.

Presumably, our conclusions, summarized in Section 1.0, are a direct result of the presented analysis and observations.

Appendix A: Summary of Engineering Tolerances

During the ALS conceptual design, one found that the on- and off-momentum dynamic aperture were not robust when magnet alignment errors and the, presumably, nonlinear effects from the insertion devices were introduced [15]. However, during the construction phase we demonstrated that these effects are due to the symmetry breaking¹ of the linear optics and hence can be corrected; by tweaking all the quadrupoles independently² [16]. In the mean time, this correction scheme has been successfully implemented³ [17] and perfected [18].

For the SLS conceptual design we therefore distributed the BPMs so that the orbit is maintained at the center of the sextupoles and, by including the girder correlations in our model, also showed that the girder alignment tolerances could be relaxed relative to the magnet alignment tolerances on the girder [2]. As a result, when the orbit is corrected, the symmetry of the optics is automatically restored [19]. This philosophy has been applied to the NSLS-II [20]. It can be further improved by using automated quad centering, e.g. by using the algorithm developed for the ALS [21]; which we have ported to and tested at the NSLS X-Ray Ring [22]. The magnet alignment tolerances are summarized in Tab. 11.

The initial magnetic field tolerances were based on experience from the SLS [23] and parametric studies [24]. Moreover, two spare quadrupoles and sextupoles were borrowed and re-measured/studied locally [25]. Also, reference magnet designs were pursued by the NSLS-II Magnet Design- [26] and Accelerator Physics [14] groups leading to improved field quality specifications and corresponding manufacturing tolerances. The results are summarized in Tabs. 12 and 13. Clearly, this work is on-going and further refinements⁴ are expected and will eventually be validated by measurements.

All random errors have been generated from a normal distribution with a 1 sigma cut.

-
1. Due to the residual orbit in the sextupoles and vertical linear focusing from insertion devices.
 2. Aka “the 48-knob scheme”.
 3. By chance the quadrupoles were independently powered for cost reasons.
 4. Random errors for “allowed terms”, see Tab. 12, were lumped with the systematic; for now. Also, dipole- and gradient errors have been excluded since they would be corrected by control of: orbit, beta- and phase advance beat, and vertical beam size.

	Δx RMS [μm]	Δy RMS [μm]	Roll RMS [mrad]
Girder	100	100	0.5
Quadrupoles	30	30	0.2
Sextupoles	30	30	0.2

TABLE 11. Magnet Alignment Tolerances.

Magnet Type	ΔB_6	ΔB_{10}	ΔB_{14}	ΔB_9	ΔB_{15}	ΔB_{21}
Standard quadrupoles	1.0	4.5	4.0			
Large aperture quadrupoles	1.0	0.5	0.1			
Standard sextupoles				1.0	1.0	4.0
Large aperture sextupoles				0.5	0.5	0.5

TABLE 12. Systematic Higher Order Multipoles [10^{-4}] ($R_0 = 25$ mm).

n	$(\Delta B_n, \Delta A_n)$	
	Quadrupoles	Sextupoles
1	0.0, 1.0	10.0, 1.0
2	0.0, 0.0	0.0, 1.0
3	3.0, 1.0	0.0, 1.0
4	1.0, 0.1	1.0, 1.0
5	0.1, 0.1	0.5, 0.1
6	0.0, 0.1	0.5, 0.1
7	0.1, 0.1	0.5, 0.1
8	0.1, 0.1	0.1, 0.1
9	0.1, 0.1	0.0, 0.1
10	0.0, 0.1	0.2, 0.1
11	0.1, 0.1	0.2, 0.1
12	0.1, 0.1	0.2, 0.1
13	0.1, 0.1	0.2, 0.1
14	0.0, 0.1	0.2, 0.1
15	0.1, 0.1	0.0, 0.1
16	0.1, 0.1	0.1, 0.1
17	0.1, 0.1	0.1, 0.1
18	0.1, 0.1	0.1, 0.1
19	0.1, 0.1	0.1, 0.1
20	0.1, 0.1	0.1, 0.1

TABLE 13. Random Higher Order Multipoles for Standard- and Large Aperture Quadrupoles and Sextupoles RMS [10^{-4}] ($R_0 = 25$ mm).

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