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In an experimental program recently completed at the 184-inch synchrocyclotron in Berkeley, data were obtained on elastic π^+-p scattering at a laboratory energy of 310 Mev. Quantities measured were the differential cross section, the total cross section, and the polarization of the recoil protons as a function of center-of-mass angle. We have analysed the data in terms of S, P, and D waves and have obtained only one acceptable solution. The resultant set of phase shifts is of the Fermi type. The D-wave phase shifts are small but definitely needed to obtain an adequate fit to the data.[1] Owing to the relatively high accuracy of the cross-section data and the inclusion of the results of the polarization experiment, the errors on the small phase shifts have been reduced to less than 1°. The differential-cross-section and polarization data are given in Tables I and II.

We performed the phase shift analysis with the aid of an IBM 704 electronic computer, using a search program that obtained a least-squares fit to the data.[2] The computer was able to accept and vary a set of phase shifts until it had located a relative minimum for the quantity M, where

$$M = \Sigma_i \left(\frac{X_i^{(c)} - X_i^{(e)}}{\sigma_i(e)} \right)^2 ,$$

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*This work was performed under the auspices of the U. S. Atomic Energy Commission.

$X_i^{(e)}$ = the quantity X_i as obtained from experiment,

$\sigma_i^{(e)}$ = the experimental error (standard deviation) on $X_i^{(e)}$,

$X_i^{(c)}$ = the quantity X_i as calculated by the computer from a given set of phase shifts.

The sum is taken over all the experimental quantities.

In order to obtain every minimum that might lie in the neighborhood of the true solution, random sets of phase shifts were fed into the computer and the resultant fits examined. From 244 random sets, 27 unique clusters of solutions were found. The solutions in each cluster agreed to within a few tenths of a degree in every phase shift. All the 27 minima were obtained at least five times (except for a few with very large M values). Assuming that the relative minima are randomly spaced and can be entered with equal ease, the probability of having missed an acceptable set is less than 1%.

Early in the analysis it became evident that a good fit to the data could not be obtained by using S and P waves only. Thus, D waves were also allowed in the random search while the phase shifts relating to higher-order orbital angular-momentum states were assumed negligible. Coulomb scattering was included in the analysis by assuming that the nuclear and Coulomb phase shifts could be added to give a total phase shift.[3]

Of the 27 solutions found in the random search, all but five have negligible probabilities of lying in the vicinity of the true solution. This conclusion assumes that the errors on the experimental points are independent and normally distributed so that the M values of the solutions are statistically significant. The five possible solutions are presented in Table III. The corresponding curves of the differential-cross-section and polarization are given in Figs. 1 and 2. Also shown is the inadequate fit with only S and P waves.

Of these five solutions, all but solution a can be eliminated. Our recent experimental differential-cross-section data at small angles (not included in the random search or listed in Table I) definitely indicate that the interference between the Coulomb and nuclear scattering is constructive. This rules out solutions b and e. Set c is of the Minami type and is unreasonable because of its large δ_{33}, the low-energy behavior of its phase shifts, and its disagreement with the requirements of the dispersion relations.[4] This leaves only solutions a (Fermi type) and d (Yang type). When tentative values of the recently obtained cross-section data are included in the analysis, the Yang set is found to be approximately 1/10 as probable as the Fermi set. Furthermore, the Yang-type solution does not satisfy the dispersion relations.[5] We therefore conclude that a is the only allowed solution.

The errors (standard deviations) associated with this solution were derived from the error matrix and are presented in Table IV. The lack of knowledge of the total inelastic cross section at this energy results in additional uncertainty in the phase shifts. Using recent theoretical[6] and experimental[7] results concerning pion production by pions, we estimate that the total inelastic cross section at 310 Mev is less than 1 millibarn. Even this small amount of inelastic scattering can cause variations in the phase shifts listed in Table III. However, our calculations indicate that these changes would probably be within the limits set by the errors in Table IV.

We can compare our final set of phase shifts with the results of other experiments and with theory. The real part of the forward elastic scattering amplitude, calculated by using solution a, agrees with the results of the dispersion relations when the value of 0.08 is used for the renormalized, unrationalized, pion-nucleon coupling constant. Our value of a_{33} is consistent with other experiments at energies above the resonance region in that it also falls below the straight line passing through the low-energy points on a Chew-Low plot.[9] The small P-wave phase shift a_{31} is now known quite accurately at 310 Mev. Its sign is negative, in agreement with the effective-range approach of Chew and Low[9] and with other experimental results.[10] The S-wave phase shift a_3 has a more negative value than is indicated by a linear extrapolation of the low-energy data, but the discrepancy is not so great as that found when only S and P waves are allowed.

Finally, we compare our experimentally determined D-wave phase shifts with the predictions of Chew, Goldberger, Low, and Nambu[11] based on the dispersion relations. They predict $\delta_{33} = 0.3^\circ$ and $\delta_{35} = -2.5^\circ$ at our energy. How good are these predictions? Chew[12] estimates that the errors introduced in these theoretically calculated phase shifts should be less than 30% if one assumes that the effects of the pion-pion interaction are negligible. Thus the differences between our D-wave phase shifts and those obtained from theory suggest that the pion-pion interaction may be significant in describing pion-nucleon scattering.

We would like to acknowledge the invaluable assistance given us during the experimental work by Mr. James T. Vale and the rest of the 184-inch cyclotron personnel. We greatly appreciate suggestions concerning the phase-shift analysis by Mr. Kent K. Curtis and Mr. Edwin M. Towster of the Mathematical and Computing Section of the Theoretical Group.

Footnotes

1. The D-wave phase shifts agree with those found by E. L. Grigoriev and N. A. Mitin at 307 Mev. See Proceedings of 1959 International Conference on Physics of High-Energy Particles (at Kiev), summary by B. Pontecorvo on Pion-Nucleon Scattering and Single Pion Production in Pion-Nucleon and Nucleon-Nucleon Interactions (p. 35 of the unpublished report).
2. The experimental methods used to obtain the data given here and the details of the analysis will be described fully in the Physical Review at a later date. Data recently obtained on the small-angle differential cross section and the total cross section will also be presented then, along with the completed analysis. When these recently obtained data are included, the errors on all the phase shifts are expected to be less than 1°.
3. This method of including Coulomb scattering is essentially that used by Stapp, Ypsilantis, and Metropolis, Phys. Rev. 105, 302 (1957). First order relativistic corrections to the Coulomb shifts were obtained using formulas (3) of F. T. Solmitz, Phys. Rev. 94, 1799 (1954). Discussions with Dr. Stapp clarifying the Coulomb scattering problem are gratefully acknowledged.
4. S. J. Lindenbaum and R. M. Sternheimer, Phys. Rev. 110, 1174 (1958).
5. W. C. Davidon and M. L. Goldberger, Phys. Rev. 104, 1119 (1956); W. Gilbert and G. R. Screaton, Phys. Rev. 104, 1758 (1956).
6. L. S. Rodberg, Phys. Rev. Letters 3, 58 (1959).
7. W. J. Willis, π^{+}-p Interactions at 500 Mev, Phys. Rev. (to be published).
8. T. D. Spearman, Dispersion Relation Predictions for π, p Scattering (to be published).
9. G. F. Chew and F. E. Low, Phys. Rev. 101, 1570 (1956).

10. For example, Mukhin, Ozerov, Pontekorvo, Grigoriev, and Mitin, Proceedings of the CERN Symposium on High-Energy Accelerators and Pion Physics, Geneva, 1956 (European Organization of Nuclear Research, Geneva, 1956), Vol. II, p. 221.
11. Chew, Goldberger, Low, and Nambu, Phys. Rev. **106**, 1337 (1957).
12. We wish to thank Professor Chew for several enlightening discussions.

Table I. Experimental differential-cross-section measurements. Statistical and independent systematic errors are included. Not shown is an error of +7% and -5% in the absolute differential-cross-section scale.

Center-of-mass scattering angle (degrees)	Differential cross section in the center-of-mass system (millibarns)
34.5	12.80 ± 0.35
36.3	12.07 ± 0.45
43.9	10.11 ± 0.25
56.6	7.62 ± 0.25
59.8	6.67 ± 0.20
69.1	4.84 ± 0.14
74.9	3.78 ± 0.12
81.2	2.96 ± 0.13
97.6	1.77 ± 0.10
107.8	1.71 ± 0.07
120.9	2.21 ± 0.09
135.0	3.00 ± 0.19
144.5	3.81 ± 0.16
151.9	4.12 ± 0.33
156.0	4.57 ± 0.18
165.2	4.96 ± 0.20

Table II. Experimental measurements of the polarization of the recoil protons. All errors are shown and are assumed independent. The sign of the polarization is said to be positive when a preponderance of the recoil protons have their spins pointing in the direction of $\vec{p}_i \times \vec{p}_f$, where this quantity is the cross product of the initial and final momentum vectors of the pi mesons.

Center-of-mass scattering angle (degrees)	Polarization of the recoil protons
113.5	$+0.053 \pm 0.078$
124.0	-0.198 ± 0.075
133.5	-0.189 ± 0.063
145.5	-0.185 ± 0.055

Table III. The statistically probable solutions found in the random search. They were obtained by using the data in Tables I and II. The orbital and total angular momentum states represented by each shift are also given. The last column refers to the types of solutions that can arise in this kind of analysis.

Solution	Nuclear phase shifts (deg)					M	Type of Solution
	α_3	α_{31}	α_{33}	δ_{33}	δ_{35}		
	(L=0	1	1	2	2)		
	(J= $\frac{1}{2}$	1/2	3/2	3/2	5/2)		
a	-17.7	-3.5	133.2	2.4	-5.0	7.1	Fermi
b	23.2	-119.8	-158.2	-2.2	3.0	11.9	Similar to d except all signs reversed.
c	-6.4	-22.6	-2.1	134.1	0.9	20.2	Minami
d	-23.2	121.9	158.3	8.0	-5.0	25.0	Yang
e	24.0	8.0	-134.6	3.1	-0.4	25.2	Similar to a except that signs of S- and P-wave shifts are reversed.

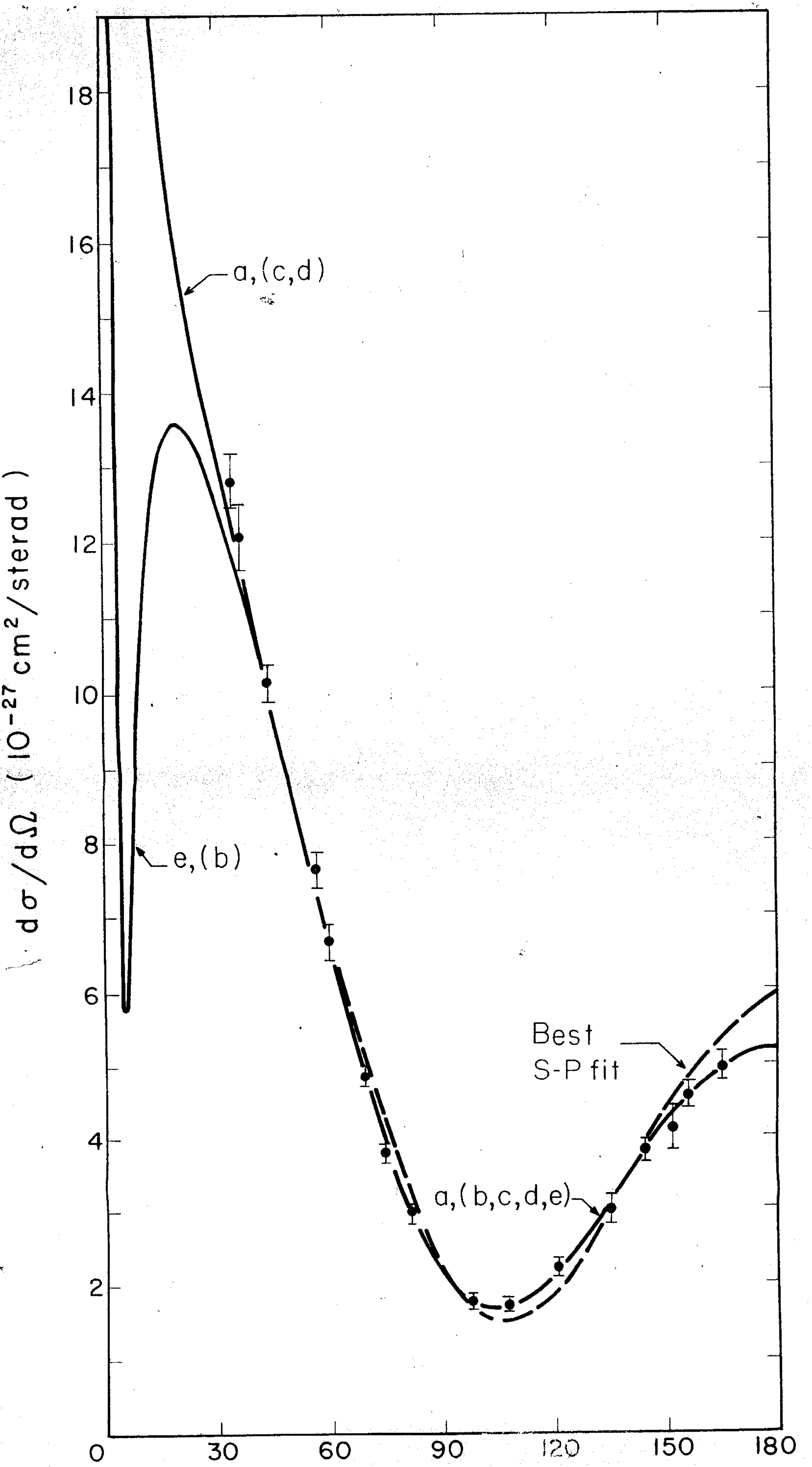
Table IV. The errors (standard deviations) in the phase shifts of solution a. The data in Tables I and II were used to obtain these errors.

Phase Shift	Error (deg)
α_3	1.2
α_{31}	0.8
α_{33}	1.7
δ_{33}	0.5
δ_{35}	0.6

Figure Legends

Fig. 1. Experimental differential-cross-section measurements given in Table I. Solid curves represent the S-P-D phase-shift fits to the data as determined by the solutions in Table III. Shown in the figure are the entire curve for solution a and the small-angle behavior of solution e. Letters in parentheses indicate solutions that give curves very similar to the ones plotted. The Minami (c) and Yang (d) solutions are slightly poorer fits to the data than is the Fermi (a). The S-P fit is shown only in the region where it noticeably deviates from the data.

Fig. 2. Experimental recoil-proton polarization measurements given in Table II. Solid curves represent the S-P-D phase-shift fits to the data as determined by the solutions in Table III. To avoid confusion, sets b and e are not shown; they give results similar to curves a and c respectively. The best S-P fit is indicated by the dashed curve.



18
16
14
12
10
8
6
4
2
0
30
60
90
120
150
180
a,(c,d)
e,(b)
Best
S-P fit
a,(b,c,d,e)
dσ/dΩ (10^{-27} cm^{2}/sterad)
θ$_{c.m.}$ (deg)

56162-

Fig 1

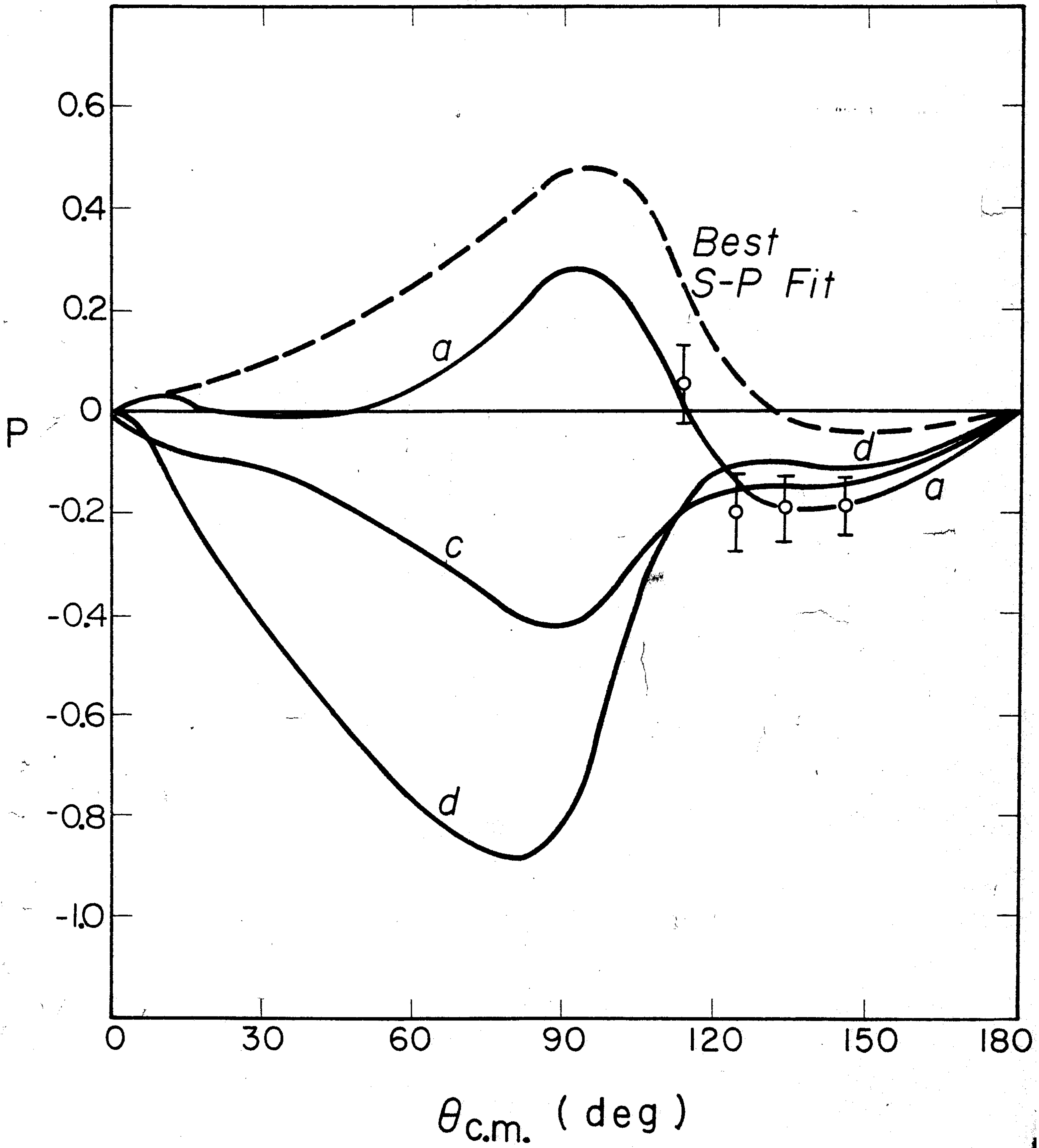

0.6
0.4
0.2
0
P
-0.2
-0.4
-0.6
-0.8
-1.0
0
30
60
90
120
150
180
$\theta_{c.m.}$ (deg)
Best
S-P Fit
a
a
c
d
d

Fig 7 561L1-1