SLAC-PUB-11853
May 2006

Performances of Induction System for Nanosecond Mode Operation

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Abstract- **An induction system comprises an array of single turn pulse transformers. Ferromagnetic cores of transformers are toroids that are stacked along the longitudinal core axis. Another name for this array is a fraction transformer or an adder. The primary and secondary windings of such a design have one turn. The step up mode is based on the number of primary pulse sources. The secondary windings are connected in series. Performances of such a system for the nanosecond range mode operation are different in comparison to the performances of traditional multi-turn pulse transformers, which are working on a 100+ nanosecond mode operation. In this paper, the author discusses which aspects are necessary to take into account for the high power nanosecond fractional transformer designs. The engineering method of the nanosecond induction system design is presented.**

I. INTRODUCTION

Performances fraction 1:1 transformer (x-fmr) or adder in the nanosecond range mode operation are different in comparison to the performances of a traditional multi-turn pulse x-fmrs, which work on a 100+ nanosecond mode. Which aspects are necessary to take into account for the nanosecond high power x-fmr design?

- If the pulse width goes down to the nanosecond range, the magnetic field *H* vector in the core becomes the same magnitude as the magnetization *M* vector. The effect of magnetization is to induce the bound current densities inside of the ferromagnetic core and, as a result, to induce a bound surface current. From the engineering point of view, the effect looks like there is a shunt current, or the core material possesses an active part of impedance. The pulser in this case will need additional coulombs to conduct the magnetic flux through the core. In the traditional high power x-frms for milli- and sub-milli second range this effect is not noticeable. However, a designer of the nanosecond high power x-fmrs should not ignore the value of the magnetic field when the volt-second integral is calculated.
- The core shunt impedance may be estimated from the dynamic magnetization, which can be written in form of the Landau–Lifshitz–Gilbert equation [1]. The equation that describes the dynamics of magnetization vs. time is as follows

$$\frac{\partial \vec{M}}{\partial t} = -|\gamma| \cdot (\vec{M} \times \vec{H}) + \frac{\alpha}{M_s} \cdot (\vec{M} \times \frac{\partial \vec{M}}{\partial t}) \qquad (1)$$

Here the first term on the right of equation describes the gyro-magnetic precession, where $\gamma = \frac{e}{m}\mu_0 = 2.22 \cdot 10^5$ [m/C] is the gyro-magnetic ratio of the free electron spin. The second term describes the dissipation of energy. It causes the magnetization to become aligned parallel to the effective field as the system proceeds towards equilibrium. The dimensionless damping parameter α is called as the Gilbert damping parameter. This formula describes the time evolution of magnetization, if the damping parameter α is adequately estimated. The simplified equation (1) for a cylindrical coordinate system is as follows:

$$\frac{dm}{dt} = \frac{\gamma\alpha}{1+\alpha^2} \cdot \left[1 - m(t)^2\right] \cdot H(t) \qquad (2)$$

here $m=M(t)/M_s$

Assuming $M(t) \cong \mu_0 B(t)$ and $M_s \cong B_s$ =const the following formula for the normalized dynamic core resistance *r (B(t))* may be derived:

$$r(B(t)) = r_{\max} \cdot \left[1 - m(t)^2\right] \qquad (3)$$

where

$$r_{\max} = \left[\frac{\alpha}{1+\alpha^2} \cdot \gamma \cdot B_s\right]_{\max} = \frac{\gamma}{2} \cdot B_s$$

Function of *r(B)* vs. *m* is a parabola for $-1<m<+1$. The Gilbert damping parameter α may be evaluated from the r_{max}/B_s ratio by experimental means. The precessional frequency λ of vector *M* around *H* is correlated with a normalized dynamic core resistance as follows $\lambda=r_{max}/\mu_0$, where $\mu_0=4\pi 10^{-7}$ H/m. A number of the *M* oscillations around *H* is inversely proportional to the Gilbert damping parameter, i.e. $N = \frac{1}{\pi \cdot \alpha}$. Thus, the relaxation time is:

$$t_r = \frac{2\pi}{\omega} N = \frac{2}{\omega_L} \cdot \frac{1+\alpha^2}{\alpha}$$

where ω_L is the Larmor frequency. For the whole oscillatory dumping $\alpha=1$, the number of turns is N ~ 0.3, and the relaxation time in the ferromagnetic media is 2/3 of the Larmor frequency. This is for an ideal ferromagnetic. For such an ideal ferromagnetic and for $H=2M_s$, the precessional frequency and the relaxation time are $\lambda=25$ GHz and $t_r=40$ psec accordingly. There is not ferromagnetic media faster than an ideal ferromagnetic.

Contributed to IEEE International Power Modulator Conference 2006, 14-18 May 2006, Washington, DC, USA

Work supported by Department of Energy contract DE-AC02-76SF00515

Experimental results of λ for µm metal ferromagnetic ribbons gives a value of ~0.5 GHz. The precessional frequency λ for thin ferromagnetic alloys is ~10^3 lower than the Larmor frequency and ~50 times less than ferrites. The expected relaxation time for alloys is ~ 2 nsec, as a result. It is necessary to take into account an increasing factor 1.5-2 due to the skin effect for alloy ferromagnetic with a ribbon thickness of 5-10 µm. Thus, the soft ferrite material is a more adequate material for high power nanosecond x-fmrs. Additionally, the core shunt effect is expected to be less for ferrites.

II. FORMULAS FOR A DESIGN OF THE NANOSECOND PULSE FRACTION TRANSFORMERS

Let us consider the case of a transmission of rectangular pulse power with t_p width through 1:1 x-fmr (see Fig. 1). The input and output impedances are the same value (Z_0). The rise and fall times of the incident pulse are much smaller in comparison to the pulse width. An equivalent simplified circuit is shown in Fig. 2. The set of equations for this case is as follows [2]:

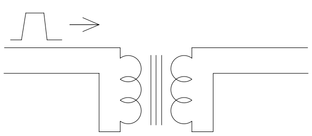


Fig. 1.

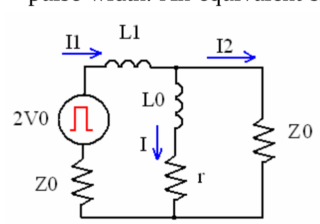


Fig. 2.

$$
\begin{aligned}
V0(t) &= \frac{d\Phi_{l1}}{dt} + \frac{d\Phi}{dt} \\
V(t) &= -\frac{d\Phi_{l2}}{dt} + \frac{d\Phi}{dt} \\
\Phi(t) &= \mu_0 \cdot [H(t) + M(t)] \cdot A \\
\frac{dm}{dt} &= \frac{\lambda}{M_s} \cdot [1 - m(t)^2] \cdot H(t)
\end{aligned} \qquad (4)
$$

here

V0(t) is the amplitude of step voltage which is applied on the primary; $\Phi_{l1,2}$ is the leakage magnetic flux for primary and secondary windings; Φ is the magnetic flux for both windings; A is the core cross section; H(t) is the acting magnetic field; M(t) is the magnetization of a ferromagnetic. The set of equation (4) can be modified in the following form:

$$
\begin{aligned}
V0(t) &= L_{s1}\frac{dI_1}{dt} + L_0\frac{dI}{dt} + L_0\lambda \cdot \left[1 - m^2(t)\right] \cdot I \\
V(t) &= -L_{s2}\frac{dI_2}{dt} + L_0\frac{dI}{dt} + L_0\lambda \cdot \left[1 - m^2(t)\right] \cdot I \\
I_1(t) &= I(t) + I_2(t)
\end{aligned} \qquad (5)
$$

where

$L_{s1,2} = \frac{\Phi_{s1,2}}{I_{1,2}}$, $L_0 = \frac{\Phi}{I} = \mu_0 \cdot \frac{AH}{I}$ are leakage and air core inductances accordingly. The active resistance $r(t) = L_0\lambda \cdot \left[1 - m^2(t)\right]$ is a core shunt resistance. It is a parabolic function of *r(t)/*$L_0\lambda$ vs. *m(t)*. Essentially, *r(m)* is changed twice during the flat top pulse transmission (see Fig. 3).

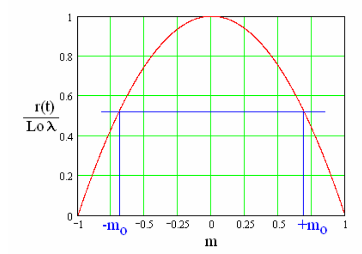


Fig. 3.

III. RISE AND FALL TRANSMISSION THROUGH THE CORE

The set of equations (5) is a non-linear system. The Laplace method can not be used to get a solution. However, the transmission of a nanosecond range pulse through 1:1 induction system can be divided into two parts. The first part is a transmission of the rise and fall parts of the pulse. At this interval the initial core shunt resistance essentially does not change and it can be written as

$r_0 = L_0\lambda \cdot \left[1 - m_0^2\right]$ =const

The second part is a transmission of the pulse plateau. Hence, the set of equations, which describe the transmission of rise and fall times, is as follows.

$$
\begin{aligned}
&Z_0 \cdot I_1(t) + L_s \cdot \frac{dI_1(t)}{dt} + Z_0 \cdot I_2(t) = 2V_0 \\
&Z_0 \cdot I_2(t) - r_0 \cdot I(t) - L_0 \cdot \frac{dI(t)}{dt} = 0 \\
&I_1(t) = I(t) + I_2(t)
\end{aligned} \qquad (6)
$$

The image of a transfer function for the output voltage can be received if the step function is applied on the primary.

$$
\frac{V(s)}{V_0} = \frac{1}{s} \cdot \frac{1 + \tau_0 s}{n\tau_0^2 s^2 + (\frac{k + kn + n}{k})\tau_0 s + \frac{2k+1}{2k}}
$$

Here the next terms (the circuit time constants) are introduced and normalized

$$
\begin{aligned}
&\tau_0 = \frac{L_0}{r_0}, \quad \tau_2 = \frac{L_0}{Z_0} = k\tau_0, \quad \tau_1 = \frac{L_s}{2Z_0} = n\tau_0 \\
&k = \frac{r_0}{Z_0}, \quad \frac{L_s}{L_0} = \frac{2n}{k}
\end{aligned} \qquad (7)
$$

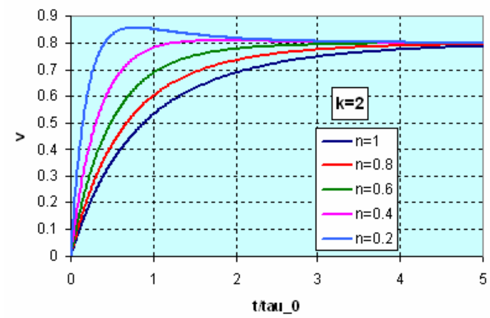
The normalized original of the output voltage is

$$V(t)=\frac{1}{b}-\frac{1}{b}\cdot\frac{a^2}{a^2-4bn}\cdot e^{-\frac{at}{2\tau_0 n}}\cdot\cos(\frac{\omega t}{2})-\frac{2n\tau_0}{a^2-4bn}\cdot e^{-\frac{at}{2\tau_0 n}}\cdot\omega\cdot\sin(\frac{\omega t}{2})+\frac{2an\tau_0}{(a^2-4bn)b}\cdot e^{-\frac{at}{2\tau_0 n}}\cdot\omega\cdot\sin(\frac{\omega t}{2})+\frac{4n}{a^2-4bn}\cdot e^{-\frac{at}{2\tau_0 n}}\cdot\cos(\frac{\omega t}{2}) \quad (8)$$

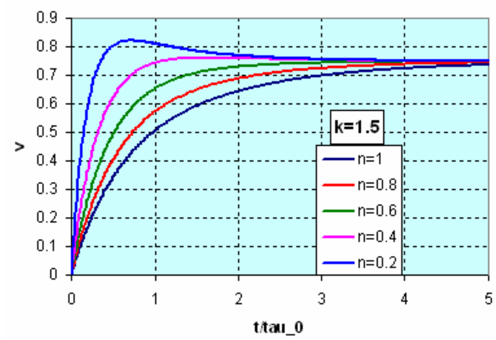
where

$$\omega=\sqrt{\frac{4bn-a^2}{n^2\tau_0^2}}, \quad a=\frac{k+kn+n}{k}, \quad \frac{1}{b}=\frac{2k}{2k+1}=\frac{1}{1+\frac{Z_0}{2r_0}}$$

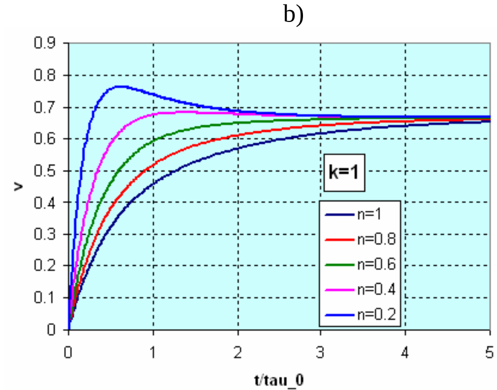
The normalized output voltages *v(t)* vs. normalized time t/τ_0 are shown in Fig. 4 a, b, and c for different *k* and *n*.



a)



b)



c)

Fig. 4 a), b), and c). Output voltage as a function of t/τ_0

The rise and/or fall time is controlled by $n=\frac{\tau_1}{\tau_0}$ (*i.e.* $\frac{L_s}{2Z_0}$ time constant). As could be seen from Fig. 4 a), b) and c), the relationship between τ_1 and τ_0 for the optimal case is as follows:

$$\tau_{1\_opt}\cong 0.4\tau_0 \quad (9)$$

Another parameter *k* controls the value of “steady state” output voltage during transient time, i.e. the shunting effect. As it can be seen from the previous pictures, the core (ferromagnetic) shunts the output voltage stronger for the smaller *k* values.

IV. A FLAT TOP TRANSMISSION THROUGH THE CORE

During the flat top transmission, the core shunt resistance *r(m)* is changed by a factor of two as shown in Fig. 3. The equivalent circuit for the computation of the output voltage response on Z_0 is shown in Fig. 7.

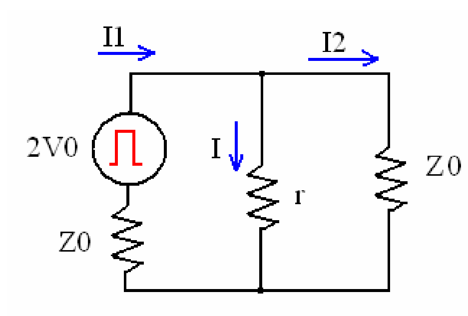


Fig. 7.

The dynamic resistance *r(m)* shunts the secondary impedance. The output voltage is less than the primary voltage. If a deformation of the pulse flat top $\Delta=\frac{V_0-V}{V}$ is small, then $V\cong V_0$ and $I\cong\frac{V_0}{r(m)}$. In this case:

$$I(t)=\frac{V_0}{L_0\lambda\cdot\left[1-m^2(t)\right]}=\frac{V_0}{L_0\lambda\cdot\left[1-(m_0+\frac{V_0 t}{\mu_0 M_s A})^2\right]} \quad (10)$$

$$m(t)=m_0+\frac{V_0 t}{\mu_0 M_s A}$$

If a symmetry cycle (from $-m_0$ to $+m_0$) takes place, then the maximum pulse deformation occurs at $t=0$ and $t=t_p$.

For the desired distortion Δ and $t=0$, the core geometry time constant is:

$$\tau_2 = k \cdot \tau_0 = \frac{L_0}{Z_0} = \frac{1}{\lambda \cdot \Delta \cdot (1 - m_0^2)} \qquad (11)$$

The necessary core cross section for the given V_0 and pulse width t_p is:

$$A = \frac{V_0 \cdot t_p}{2 \cdot \mu_0 \cdot M_s \cdot (-m_0)} \qquad (12)$$

Therefore, formula (12) shows a traditional relationship between a volt-second integral and ferromagnetic cross-section. The magnetic path length of a ferromagnetic is not restricted here. The set of formulas (9), (11), and (12) is a complete set of equations where the core size is uniquely determined. The set of formulas answers the question: What is the core cross-section and what is the magnetic path length needed in order to pass the nanosecond pulse through x-fmr for a given value of the flat top distortion? The precessional frequency λ in (11) can be obtained either from experiments or in Reference [3]. The air core induction L_0 could be measured if the ferromagnetic core was replaced by the dielectric core mockup. It should be noted that the calculation of L_0 uses the following formula $L_0 = \mu_0 \cdot \frac{A}{l} \cong \mu_0 \cdot h \cdot \ln\left(\frac{D}{d}\right)$ and may give a result that does not correspond to the experiment (especially for the single turn x-fmrs). This is why information about L_0 is better obtained from the test bench experiment.

V. CONCLUSION

Performances of the induction system in the nanosecond mode operation are linked with performances of single turn pulse transformers. The features of single turn x-fmr are discussed. If the pulse width goes down to the nanosecond range, the magnetic field *H* vector in the core becomes the same magnitude as the magnetization *M* vector. The ferromagnetic magnetization is taken into account for a design of single turn nanosecond pulse x-fmrs. The method of the design includes two parts. The first part is a transmission of the rise and fall times. The second part is the flat top transmission though the ferromagnetic core. There are optimal transformer time constants when the front and flat top are transmitted with a small pulse distortion. It was shown that ferrite material is a more preferable ferromagnetic for the nanosecond range. The precessional frequency λ for thin ferromagnetic alloys is ~10^3 lower than the Larmor frequency and ~50 times less than ferrites. As a result, the expected relaxation time for alloys is ~ 2 nsec. The expected relaxation time for ferrite in the strong magnetic acting fields is ~ 40 psec and this is a physical limitation of the core material. The complete set of equations answers the question: What is the core cross-section and what is the magnetic path length needed in order to pass the nanosecond pulse through 1:1 fraction transformer or adder?

ACKOWLEDGMENT

Work supported by Department of Energy contracts DE–AC02–76SF00515.

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