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Plasma liner with an intermediate heavy shell and thermal pressure drive

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Abstract

One of the challenging problems of Magnetized Target Fusion is development of the ways of transporting energy to the target situated at a large-enough distance from the energy source: the distance should be such as to prevent damage to the permanent parts of the source. Several schemes have been considered in the past, including the use of particle beams coupled with the inverse diode, mechanical projectiles in combination with magneto-compressional generators, and the plasma liner. In this paper, a possible modification of the original concept of the plasma liner (Y.C.F. Thio, C.E. Knapp, R.C. Kirkpatrick, R.E. Siemon, P.J. Turchi. *J. Fusion Energy*, **20**, 1, 2001) is described. The modification consists in creating a thin, higher density shell made of a high-Z plasma and accelerating it onto an MTF target by a thermal pressure of a hydrogen plasma with the temperature ~ 10 eV. We discuss constraints on the parameters of this system and evaluate convergence ratio that can be expected.

1. Introduction

Adiabatic compression of a magnetized plasma on the time-scale short compared to the heat loss time, may allow obtaining a fusion-grade plasmas for a moderate energy input (see Refs.[1-3] and references therein). This approach is commonly called “magnetized target fusion,” or MTF [4], and is attractive as a way of demonstrating the fusion ignition in a low-cost set of experiments [1-3]. Previous study [1] has shown that reaching fusion gain Q of the order of unity may be possible for the initial energy deposition to the plasma of order of 100 kJ, if one relies on a three-dimensional compression. To make a commercial fusion reactor, one would have, however, to reach Q 's of at least ~ 10 . The Q here is defined as the ratio of the fusion yield and the total energy deposited to the plasma at the time of the maximum compression. As the analysis of Ref. [1] has shown, this would require much higher initial energy deposition, of order of 10 - 20 MJ. This, in turn, means that in every shot, an energy of the order of a few hundred megajoules would be released, of which 80% will be carried by 14-MeV neutrons, and the rest by charged particles and radiation. Therefore, an issue of protecting the permanent part of the power supply becomes a critical issue for the whole approach.

The solution to this problem will probably lie in using a large chamber, a few meters in radius, very much like that envisaged for the inertial confinement fusion (e.g., [5, 6]). However, unlike the case of laser and heavy-ion-beam fusion, magnetized target fusion has been conceived to be driven by a magnetic implosion of the material liner thus making it necessary to deliver electric current as high as 10 MA or more to the center of

the chamber [1, 2]. A possible solution to this problem is the use of disposable material electrodes [7]. But as the process of inserting such electrodes is relatively slow, it is compatible only with a relatively low shot rate, say, once in 10 s. Accordingly, in order to have a ~ 0.5 GW (thermal) power plant, one would have to generate explosions with the energy release ~ 5 GJ, which, in turn, drives up the size of the chamber.

Potentially more attractive solution is offered by the concept of the plasma liner [8,9], in which very high Mach number jets would be generated by plasma guns situated on the walls of the explosion chamber and merge near the target, to form a 3D pusher for the target. This approach, in principle, allows avoiding the use of any material structures connecting the target with the “external world.” A recent analysis of this concept has been carried out by Parks and Thio [10].

The present paper describes a variation of the concept of the plasma liner where the energy would be delivered to the target by a thin gaseous shell made of a high-Z material; this shell would be accelerated by $p dV$ work of a thermal hydrogen plasma expanding at a subsonic speed. This approach allows one to substantially reduce technological demands placed on the performance of the plasma jets. We provide a broad scoping study based mostly on simple analytical models and order-of-magnitude estimates. We identify main difficulties of this approach and speculate regarding the ways of overcoming them.

The present paper does not contain any engineering details; in particular, we do not provide any details regarding the design of plasma sources and gas injectors. We just outline the physics aspects of the concept and present a set of equations required for more detailed parametric studies. We understand that the associated engineering issues are extremely challenging and need an appropriate attention. A lot of creative thinking would be needed before solutions to the engineering problems would be found!

This paper is organized as follows: Sec. 2 contains a qualitative description of the concept. Sec. 3 deals with a detailed analysis of a particular case where the parameters of the pusher plasma are held constant during the implosion. In Sec. 4, we present a broad parameter scan for this model summarized in Table 1. Sec. 5 is concerned with the interaction of a pusher plasma and a heavy shell (a liner): liner heating by the radiation from the pusher plasma and maintaining the compactness of the shell by “effective gravity.” In Sec. 6, we discuss a way of improving the efficiency of the liner acceleration by tailoring the pusher plasma injection power. We show that this technique can provide efficiencies as high as 60%. In Secs. 3-6 a relatively hot pusher plasma was considered, $T \sim 10 - 15$ eV. Creating large volumes of such a plasma is a challenging problem. Therefore, in Sec. 7, we discuss problems associated with the use of a lower-temperature pusher plasma ($T \sim 5$ eV or less). The efficiency in this case becomes lower because of the increase of the radiative losses and of a relative cost of the ionization. Sec. 8 contains a brief discussion of the Rayleigh-Taylor instability of the liner during the acceleration stage. In Sec. 9 possible variations of the concept are described, including the ones that can lead to small-scale simulation experiments. Sec. 9 contains a discussion of the results. Some of the more lengthy calculations are moved to Appendices 1 – 3.

2. General description of the concept.

In this section, we provide a qualitative description of the concept, to give the reader an orientation for the subsequent sections where a semi-quantitative analysis is presented. We consider a spherical chamber, a few meters in radius (Fig. 1a). We assume that, prior to firing the plasma guns, a gaseous shell is formed near the surface of the chamber (Fig. 1b). This shell can be formed by using a large number of pulsed high-pressure valves distributed over the surface of the chamber. The injected material could be some high-Z gas (e.g., xenon) or a mixture of a lighter gas (e.g., nitrogen) with microscopic flakes of some high-Z substance (e.g., gold, lead, etc.). The latter (“dusty gas”) approach is interesting in that it allows using a broad range of substances unavailable in the gaseous form at room temperature. The sound speed in xenon at room temperature is $\sim 2 \cdot 10^4$ cm/s. So, in order to create a 10-cm thick shell, the valves have to be opened within the time shorter than $(10 \text{ cm}) / (2 \cdot 10^4 \text{ cm/s}) = 500 \text{ } \mu\text{s}$, which is well within the reach of the pulsed valve technology (e.g., [11]). The sound speed in a dusty gas of a comparable average mass density is even lower, thereby allowing for even slower valves. After the shell is formed, a number of plasma sources, also uniformly distributed over the chamber surface, are fired to form a thermal “pusher” consisting of a hydrogen plasma, with the temperature ~ 10 or so eV (Fig. 1c). The sources may stay “on” for some time, thereby providing a control over the pressure and temperature of the “pusher” plasma in the course of accelerating the gaseous shell. There is no need in providing a large (supersonic) velocity to the hydrogen streams filling the space between the heavy gaseous shell and the wall. Quite the contrary, we want this plasma to be thermal, and have a uniform pressure. In order to reduce possible effect of the directional motions, one can inject the plasma predominantly in the tangential directions.

The heavy shell will be accelerated towards the axis by the $p dV$ work of the expanding hydrogen pusher (Fig. 1d). As we show below, the radiation from the shell keeps its temperature low during the acceleration and this guarantees that the shell, in the spherically-symmetric model, remains thin (does not expand radially in the course of acceleration).

Eventually, the shell reaches the target (Fig. 1 e) that is supposed to have been placed in the center of the chamber before the aforementioned manipulations have started. In particular, the pre-manufactured target can be dropped into the chamber. We do not discuss here the ways of forming the initial plasma and initial magnetic field in the target. Some of the contactless techniques have been mentioned in Ref. 1. Our heavy shell may then be used for driving the magneto-compressive generator, in a way mentioned in Ref. 1. In this paper, however, we do not discuss any details of the interaction of the heavy shell with a target.

3. An example: the constant-pressure pusher

In this section, we present numerical estimates for one particular case that can be easily treated analytically. Specifically, we assume that the pressure p of the pusher plasma is maintained constant, by tailoring appropriately the rate at which the pusher plasma is delivered to the chamber. This case is certainly not the optimum one in terms of

the energy efficiency, but even so it yields results that seem to be compatible with obtaining fusion-grade plasmas.

3.1 Liner dynamics

As the gaseous shell is thin (we will quantify this statement in the further sections), its position can be adequately characterized by its radius $r(t)$. The Newton equation then yields:

$$M \frac{d^2 r}{dt^2} = -4\pi r^2 p \quad (1)$$

where M is the mass of the heavy liner. For the constant pressure, it is easy to integrate Eq. (1) to obtain:

$$\frac{M \dot{r}^2}{2} = \frac{4\pi p r_0^3}{3} \left(1 - \frac{r^3}{r_0^3} \right) \quad (2)$$

where r_0 is the initial radius of the liner (which is assumed to be close to the initial radius of the plasma chamber). The final velocity, just before the gaseous liner reaches the target, can be found from the equation

$$\frac{M \dot{r}_{final}^2}{2} \equiv \frac{M v_f^2}{2} \equiv W_f = \frac{4\pi p r_0^3}{3} \quad (3)$$

Here W_f is the final energy of the liner. We have neglected the cube of the ratio of the target radius (a few centimeters) and the chamber radius (a few meters) compared to unity. Eq. (2) shows that velocities smaller than the final velocity v_f by a mere 5% are reached when the liner radius is $\sim 0.45 r_0$, i.e., halfway to the target.

An integration of Eq. (2) allows finding the time-history of the liner radius. It is shown in Fig. 2. The run-in time is:

$$\tau_{run-in} = \left(\frac{3M}{8\pi p r_0} \right)^{1/2} \int_0^1 \frac{dx}{\sqrt{1-x^3}} \approx 0.48 \left(\frac{M}{p r_0} \right)^{1/2} \approx 1.4 \frac{r_0}{v_f} \quad (4)$$

Note (Fig. 2) that the liner stays close to its initial radius for quite a long time, before rapidly taking-off and “falling” onto the center. This is a typical feature of liner implosions, including magnetically-driven liners.

We will present numerical estimates for the following set of parameters:

- Liner mass $M=50$ g;
 - Initial liner radius $r_0=200$ cm;
 - Hydrogen plasma density $n=0.93 \cdot 10^{17}$ cm⁻³;
 - Hydrogen plasma temperature $T=10$ eV.
- (5)

These parameters correspond to the final liner energy of $W_f=10$ MJ, the final velocity $v_f=2\cdot 10^6$ cm/s, and the run-in time (Eq. (4)) $\tau_{run-in}=150$ μ s. The broader set of parameters will be considered in Sec. 4.

3.2 Limitations on the velocity of the heavy shell.

In the previous analysis, we made an assumption that the liner moves slowly enough so that the hydrogen pusher remains subsonic (with respect to the sound speed in the pusher). For the fully ionized hydrogen plasma, the sound speed s is:

$$s = \sqrt{\gamma \frac{2T}{m_p}} = \sqrt{\frac{10T}{3m_p}}, \quad (6)$$

where $\gamma=5/3$ is the adiabatic index for a monatomic gas, and m_p is the proton mass. In “practical” units, $s(km/s) \approx 18\sqrt{T(eV)}$. Accordingly, we impose the following constraint on the final velocity of the liner:

$$v_f(km/s) < 18\sqrt{T(eV)} \quad (7)$$

In all our examples this condition is satisfied.

3.3 Heat loss from the pusher plasma.

As we assume that parameters of the pusher are kept constant, the radiation power density Q_{rad} also remains constant. This allows one to evaluate easily radiative losses from the pusher plasma:

$$W_{rad} = \frac{4\pi}{3} Q_{rad} \int_0^{\tau_{run-in}} (r_0^3 - r^3) dt = \frac{4\pi r_0^4}{3v_f} Q_{rad} \int_0^1 dx \sqrt{1-x^3} = \frac{4\pi r_0^3}{5} Q_{rad} \tau_{run-in} \quad (8)$$

At a temperature exceeding a few electron-volts, the hydrogen plasma of 10^{17} cm⁻³ density is fully ionized. The radiation intensity per unit volume, Q_{rad} , can then be evaluated as [12]:

$$Q_{rad}(W/cm^3) \approx 1.69 \cdot 10^{-32} [n(cm^{-3})]^2 \left[\sqrt{T(eV)} + \frac{13.6}{\sqrt{T(eV)}} \right] \quad (9)$$

For the parameters defined by Eq. (5), one has $Q_{rad}=1.3$ kW/cm³, and, according to Eq. (8), $W_{rad}=3.3$ MJ. Note that, in a broad range of temperatures, from 3 eV to 30eV, the temperature-dependent factor varies in a narrow range, from 7.38 to 9.58. To simplify the computations in Sec. 6 and 7, we replace the last multiplier in Eq. (9) by a constant median value of 8.4, which yields the following approximate expression for the radiative loss:

$$Q_{rad}(W/cm^3 s) \approx 1.4 \cdot 10^{-31} [n(cm^{-3})]^2 \quad (10)$$

One can also characterize the radiative heat loss in terms of the radiation cooling time,

$$\tau_{rad} = \frac{3nT}{Q_{rad}}, \quad (11)$$

which can be evaluated as (see Eq. (9)):

$$\tau_{rad}(s) = \frac{2.84 \cdot 10^{13} \sqrt{T(eV)}}{n(cm^{-3})} \left(1 + \frac{13.6}{T(eV)} \right)^{-1} \quad (12)$$

For $n=10^{17} cm^{-3}$, and $T=10$ eV it is $\sim 400 \mu s$. Equation (8) for the total radiated energy loss can be presented as:

$$W_{rad} = \frac{6\pi r_0^3}{5} p \frac{\tau_{run-in}}{\tau_{rad}} = \frac{9}{10} W_f \frac{\tau_{run-in}}{\tau_{rad}} \quad (13)$$

One more mechanism of the heat loss from the plasma pusher is electron thermal conductivity to the heavy liner. The electron thermal diffusivity can be evaluated as (Cf. [13], with the Coulomb logarithm taken to be 10):

$$\chi_e(cm^2/s) \approx 6.6 \cdot 10^{19} [T(eV)]^{5/2} / n(cm^{-3}). \quad (14)$$

For the set of parameters (5), the thermal conduction time over the characteristic length-scale $D \sim 100$ cm is $\sim D^2/2\chi \sim 20$ ms, i.e., much longer than the radiation cooling time (which in turn is longer than the implosion time). We also note that the driving plasma injected by the arc sources will almost certainly carry with it some tangled magnetic field (Fig. 3). Even quite a weak field of the order of 100-200 G is already sufficient to magnetize the plasma electrons and make connection length for the electron much longer than the “line-of-sight” distance D . This would further suppress electron heat loss from the pusher plasma. Therefore, we will ignore the heat conduction loss to the gaseous liner.

3.4 Energy required to create the pusher plasma

In order to create a pusher plasma, one has to ionize initially neutral hydrogen gas. This would require spending some energy I per hydrogen ion. Therefore, the additional energy that has to be invested into generation of the pusher plasma can be evaluated as

$$W_i = \frac{4\pi r_0^3}{3} nI \equiv \frac{2\pi r_0^3}{3} p \frac{I}{T} = W_f \frac{I}{2T} \quad (15)$$

For the hydrogen gas, I is ~ 16 eV; we have included into I the dissociation energy 2.3 eV/atom. For the set of parameters listed in (5), the ionization energy is $W_i=8$ MJ.

The total mass M_p of the pusher plasma reached at the end of the acceleration process, is

$$M_p = \frac{4\pi r_0^3 n m_p}{3} \quad (16)$$

where m_p is the proton mass. For the set of parameters (5) it is ~ 5 g, i.e., $M_p \ll M$. One can therefore expect that the kinetic energy of the pusher plasma is small. In order to make a quantitative estimate, we note that the velocity distribution in the spherically-symmetric flow with a uniform density scales as $1/\hat{r}^2$ where $\hat{r}$ is the distance from the center (we use this notation in order to distinguish a running radial coordinate $\hat{r}$ from $r(t)$, the position of a heavy shell at a certain instance of time). At the liner surface the flow velocity is equal to that of the liner, i.e.,

$$v(\hat{r}) = \dot{r}(t) \frac{r(t)^2}{\hat{r}^2}, \quad r < \hat{r} < r_0 \quad (17)$$

The pusher mass density is $3M_p/4\pi r_0^3$. Accordingly, the kinetic energy $W^{(kin)}$ of the pusher material is

$$W_p^{(kin)} = (3M_p/4\pi r_0^3) \int_r^{r_0} \frac{\hat{v}^2}{2} 4\pi \hat{r}^2 d\hat{r} = 3W_f \frac{M_p}{M} \frac{r^3}{r_0^3} \left(1 - \frac{r^3}{r_0^3}\right) \left(1 - \frac{r}{r_0}\right) \quad (18)$$

The maximum of this function is reached at $r/r_0=0.65$ and is equal to

$$\max W_p^{(kin)} = 0.21 W_f \frac{M_p}{M} \quad (19)$$

i.e., does not exceed 2% of the liner energy. By the end of the acceleration process, at $r \sim 0.1r_0$, the kinetic energy of the pusher becomes very small, $W_{pf}^{(kin)} \approx 0.003 W_f (M_p/M)$, and does not make any noticeable contribution to the energy balance.

3.5 Overall energy balance of the run-in phase

The total hydrodynamic efficiency of the system defined as the ratio of the final liner kinetic energy (3) to all the energy invested into generation of the pusher plasma is:

$$\eta = \frac{W_f}{W_f + W_{pf} + W_{rad} + W_i} \quad (20)$$

Where W_{pf} is the total gas-kinetic energy of the pusher plasma at the time when the liner has reached the target. For the pusher plasma with constant plasma parameters, obviously,

$$W_{pf} = 2\pi r_0^3 p = \frac{3}{2} W_f \quad (21)$$

Using Eqs. (3), (13), (15), and (21), one finds:

$$\eta_{hydro} = \frac{1}{1 + 3/2 + 9\tau_{run-in}/10\tau_{rad} + I/2T} \quad (22)$$

In the denominator of Eq. (22), we have written separately the terms corresponding to various sources of inefficiency. For the parameters of Eq. (5), the largest contribution to inefficiency arises from the large volume of the pusher plasma at the instance of the implosion, and the fact that the energy remaining in this plasma does not give rise to any significant liner acceleration after this instance. Still, even in this scenario, which is by far non-optimum, the overall hydrodynamic efficiency remains at a decent level of 27%.

We will discuss more favorable scenarios of creating a pusher plasma in Sec. 6. Here we just note that a slight increase of the pusher temperature from 10 to 15 eV, with reducing the pusher density from 10^{17} cm^{-3} to $0.66 \cdot 10^{17} \text{ cm}^{-3}$ (so as to maintain the pressure of the pusher plasma and the final liner energy constant) gives rise to a substantial increase of the efficiency, to 31%.

3.6 Instantaneous power input from the plasma sources

In the case of the constant pressure drive, the instantaneous power input to form and sustain the pusher plasma that has to be supplied by the plasma sources is:

$$P = P_0 \left[\left(1 + \frac{I}{5T} \right) \frac{r^2}{r_0^2} \sqrt{1 - \frac{r^3}{r_0^3}} + \frac{\tau_{run-in}}{7\tau_{rad}} \left(1 - \frac{r^3}{r_0^3} \right) \right] \quad (23)$$

where

$$P_0 = \frac{15W_f v_f}{2r_0} \quad (24)$$

This result follows readily from our expressions for the instantaneous liner energy, instantaneous thermal and ionization energy of the pusher, and instantaneous radiated power; we have also used Eq. (2) for the liner velocity and Eq. (4) for τ_{run-in} . The plot of the power input is shown in Fig. 4 for the parameters of the 1st data column in TABLE 1. The peak power is equal to $0.62P_0 = 0.46 \text{ TW}$.

4. A broader set of possible parameters

In this section we make a parameter scan for a 50 g liner. The results are presented in TABLE 1, which is built in the following way. We start from specifying the liner final energy (10 or 20 MJ, line 1). For a given liner mass of 50 g, this specifies the final liner velocity (line 2). Line 3 contains the residual thermal energy of the pusher plasma (Eq.(21)). In line 4 we present two values for the chamber radius, 2 and 3 m. This

TABLE 1. Basic parameters of the system for the mass of the heavy liner $M=50$ g

1	Parameter	$W_f=10$ MJ				$W_f=20$ MJ			
2	v_f , km/s	20				28.3			
3	W_{pp} , MJ	15				30			
4	r_0 , cm	200		300		200		300	
5	τ_{run-in} , ms	0.14		0.21		0.1		0.15	
6	p_0 , 10^{18} eV/cm ³	1.87		0.55		3.7		1.1	
7	T , eV	10	15	10	15	10	15	10	15
8	n , 10^{17} cm ⁻³	0.93	0.62	0.28	0.18	1.87	1.2	0.56	0.37
9	M_p , G	5.2	3.4	5.2	3.4	10.5	6.8	10.5	6.8
10	Q_{rad} , kW/cm ³	1.1	0.45	0.1	0.04	4.4	1.8	0.4	0.17
11	τ_{rad} , ms	0.41	1.02	1.36	3.2	0.2	0.48	0.68	1.6
12	W_{rad} , MJ	3.3	1.3	1.5	0.65	9.7	4.1	3.8	1.8
13	$maxW_p^{(kin)}$, MJ	0.22	0.14	0.22	0.14	0.88	0.59	0.88	0.59
14	W_i , MJ	8	5.3	8	5.3	16	10.6	16	10.6
15	η , %	27	31	29	32	26	31	29	32
16	T_{HL} , eV	0.84	0.69	0.52	0.41	1.22	0.97	0.74	0.60
17	$h(r=r_0/3)$, cm	3.7	3.1	4.1	3.4	2.7	2.2	2.4	2.0
18	h_{final} , cm	8.5	7.5	9.8	8.4	6.8	5.9	7.2	6.3
19	$C=r_0/h_{final}$	23.5	26.5	30.5	35.7	29.4	33.9	41.6	47.6

uniquely specifies the run-in time (line 5) and the required pressure of the pusher plasma (line 6), see Eq. (3). For a given pressure, we then choose two possible values of the pusher temperature, 10 and 15 eV (line 7). This yields, via equation $p=2nT$, the pusher density (line 8) and, via Eq. (16), the final pusher mass (line 9). Lines 10 - 12 present parameters related to the radiative losses from the pusher plasma, Eqs. (9), (11), (13). In line 13, we present, for the reference purpose, the maximum kinetic energy of the pusher plasma (Eq. (19)). The final kinetic energy is by a factor of few tens smaller and is ignored in the final energy balance (and in the efficiency η). Line 14 contains the ionization energy of the pusher plasma, Eq. (15)). Line 14 contains the hydrodynamic efficiency (Eq. (22)). Lines 16-18 contain parameters of the heavy liner that are derived

in Sec. 5: its temperature T_{HL} , its thickness at the end of the acceleration phase, at $r=r_0/3$, and its final thickness h_f , when it starts interacting with the target. Finally, line 19 contains the ratio $r_0/h_{final} \equiv C$, the radial convergence at the end of acceleration. The reference set of parameters (Eq.(5)) corresponds to the left-most data column in the Table.

Main conclusions that can be drawn from this table are that the radiative losses are relatively small and do not affect efficiency in any substantial way, and that the system parameters (both the efficiency and the final thickness of the liner) are improved with the increased temperature of the pusher plasma. We also see that, even without any attempts to tailor the shape of the pusher pressure, the efficiency is quite decent, in the range of 30%. In Sec. 6 we show that, with a moderate shaping of the pressure pulse, one can increase the hydrodynamic efficiency to ~60%.

Of course, in order to close the energy balance of the fusion reactor, one would have to take into account various other losses, in particular, finite efficiency of the plasma guns (“grid-to-pusher”), and the fact that the liner will not convert 100% of its energy into the target plasma. Although an analysis of these issues is very important in determining the feasibility of the MTF reactor based on the plasma liner concept, it goes well beyond the scope of this paper, whose modest goal is to assess possible parameters of the plasma liner drive.

5. Interaction of a heavy shell with the pusher plasma.

In Sec. 3, we considered the heavy liner just as a thin structureless shell accelerated by the pressure of the pusher. In this section, we provide a more detailed description of the interaction between the pusher plasma and the liner.

5.1 The temperature of the heavy liner during acceleration phase

At the most interesting stage when the liner radius is smaller than, roughly, a half of its initial radius, the radiation energy flux through a unit surface of the heavy liner is equal to, roughly, $Q_{rad}r_0/4$ (see Appendix 1 for more detail). As we have shown, the electron heat flux is negligibly small. The energy reaching the surface of the liner is re-radiated by the liner as a black-body radiation. Therefore, the temperature of the heavy liner T_{HL} can be estimated from equation:

$$Q_{rad}r_0/4 = \sigma T_{HL}^4, \quad (25)$$

where σ is the Stefan-Boltzmann constant. For the set of parameters (5), using Eq. (9), one finds that $T_{HL} \sim 1$ eV. Because of the strong dependence of the r.h.s. on the liner temperature, this temperature weakly depends on the other parameters. At the temperature of 0.5-1 eV, the heavy liner will be a mixture of neutral and singly ionized atoms (their relative amount may vary depending on the ionization potential of the liner material). Therefore, the liner will be very opaque to the radiation and would indeed behave as a black body.

5.2 Mutual diffusion of the heavy and the light species.

One has to check that the pusher plasma does not “fall through” the heavy liner, i.e. that the liner material is dense enough to be impermeable for the pusher ions. The liner ion line-density (in the radial direction) is equal to

$$N = \frac{M}{4\pi r^2 A m_p} \quad (26)$$

where m_p is the proton mass, and A is atomic weight of the liner ions. The smallest line density corresponds to the initial liner radius r_0 . For the example of Eq. (5), and $A \sim 130$ (Xe), one has $N \sim 3 \cdot 10^{17} \text{ cm}^{-2}$. Collisional cross-sections σ range from $\sim 10^{-15} \text{ cm}^2$ for the collisions between neutral Xe and H atoms to $\sim 10^{-14} \text{ cm}^2$ for the Coulomb collisions between the singly-charged ions with the temperature of a few electron-volt. Consequently, the ratio of the thickness of the shell to the mean-free-path, which is given by the dimensionless product $N\sigma$, is $\sim 300\text{-}3000$. In other words, the hydrogen pusher cannot simply “fall through” the heavy shell and will exert its full pressure on the shell material. One can also show that the diffusion time of the hydrogen through the heavy shell is very long compared to the run-in time.

5.3 Shell thickness

During the acceleration phase, the shell thickness is determined by the Boltzmann-type hydrostatic equilibrium,

$$\frac{1}{n} \frac{dn}{dz} = \frac{\ddot{r} \bar{A} m_p}{T_{HL}}, \quad (27)$$

where we have introduced the coordinate z directed towards the center of the chamber, with the origin at the pusher-liner interface. The other symbols are: n is the volume density of the heavy particles, and $\bar{A}$ is the average atomic weight of the liner ($\bar{A} = A/2$ for the singly ionized atoms, with A being the atomic weight of the heavy gas). The solution of Eq. (27), sets the e-folding length h for the density distribution (which is $\sim \exp(-z/h)$). Using Eq. (1) and (27) we find that

$$h = -\frac{T_{HL}}{\ddot{r} \bar{A} m_p} = \frac{M T_{HL}}{6 \bar{A} M_p T} \frac{r_0^3}{r^2} \quad (28)$$

(we recall that $\ddot{r} < 0$).

At the distance $r = r_0/3$, i.e., at the point where the liner has almost reached its final velocity, we find that

$$\frac{h(r = r_0/3)}{r_0} = \frac{3M}{2\bar{A}M_p} \frac{T_{HL}}{T} \quad (29)$$

For the set of parameters (5), using Eqs. (9), (15), and (19), and taking $\bar{A}=A/2$ for xenon ($A=131$), one finds that $h \sim 3.5$ cm, i.e., the liner is indeed quite thin.

At the later stage, the liner is essentially coasting with no acceleration. At this stage, it starts to expand with the sound velocity corresponding to the liner temperature (25), i.e., with the velocity

$$s_{HL} \sim \sqrt{\frac{5}{3} \frac{T_{HL}}{Am_p}} \quad (30)$$

Accordingly, we estimate the liner thickness at the time when it reaches the target as

$$h_f = h(r = r_0/3) + s_{HL} \frac{r_0}{3v_f} \quad (31)$$

Line 18 in Table 1 shows that the final liner thickness is in the range of ~ 10 cm. According to Ref. 1, the initial target size for 10-20 MJ targets is in the same range. This means that a substantial fraction of the liner energy can be converted to the plasma heating.

Note that, when the target compression begins, acceleration of the heavy liner changes sign. At this stage, the liner “leans” on the target and is decelerated by it, converting its kinetic energy into the target energy. As the deceleration is very large, the liner thickness becomes substantially smaller than h_f . The analysis of this stage is, however, inseparable from the analysis of the target implosion and goes beyond the scope of this article.

6. Improving the efficiency of the acceleration phase.

6.1 Evaluating the liner energy in the improved scenario

It is clear from the discussion of Sec. 3 that, maintaining the constant pressure of the plasma pusher during the whole acceleration process is not optimum from the viewpoint of the energy efficiency: at the end of the acceleration process, at $t=\tau_{\text{run-in}}$, the plasma pusher still has a large thermal energy. To improve this situation, one should apply a higher pressure early in the acceleration process and then stop the further injection of the pusher plasma, allowing the injected pusher plasma to adiabatically cool down in the course of its further expansion. In this scenario, the residual thermal energy of the pusher plasma can be substantially reduced.

We provide a quantitative analysis for the following scenario: the pusher plasma parameters are maintained constant until the heavy shell reaches some radius $r_l < r_0$ and since that moment the plasma sources are turned off and the pusher works on the liner without further energy and particle influx into the system.

In other words, we assume that the pressure of the pusher is:

$$p(r) = p_1, \quad r_1 < r < r_0;$$

$$p(r) = p_1 \left(\frac{r_0^3 - r_1^3}{r_0^3 - r^3} \right)^\gamma, \quad r < r_1. \quad (32)$$

In writing down the second expression in the above equation, we have neglected the correction due to the conversion of some of the thermal energy of the expanding plasma into kinetic energy. The correction is small provided the plasma sound speed is sufficiently large compared to its flow speed. We evaluate this energy in Appendix 1 and show that it is small. To avoid misunderstanding, we emphasize that the pressure of the pusher remains uniform at any instant of time, consistent with our assumption of a large plasma sound speed; the r variable in Eq. (32) is the position of the liner at this instant of time. It goes without saying that, after solving for the function $r(t)$, one would be able to relate the pressure and the time at $t_I < t < \tau_{run-in}$, where $t = t_I$ is the time when the plasma sources are turned off.

By substituting the function $p(r)$ defined by Eq. (30) into Eq. (1) and performing elementary integrations, one finds that

$$W_f = \frac{4\pi r_0^3 p_1}{3} g(r_1/r_0); \quad g(r_1/r_0) = \left(1 - \frac{r_1^3}{r_0^3}\right) \left\{ 1 + \frac{1}{\gamma-1} \left[1 - \left(1 - \frac{r_1^3}{r_0^3}\right)^{\gamma-1} \right] \right\} \quad (33)$$

At $r_I=0$ one recovers Eq. (3). The plot of the function g is shown in Fig. 5. The residual thermal energy of the pusher plasma is

$$W_{pf} = \frac{4\pi r_0^3 p_1}{3(\gamma-1)} \left(1 - \frac{r_1^3}{r_0^3}\right)^\gamma = W_f \frac{\left(1 - \frac{r_1^3}{r_0^3}\right)^{\gamma-1}}{\gamma - \left(1 - \frac{r_1^3}{r_0^3}\right)^{\gamma-1}} \quad (34)$$

6.2 Evaluating the hydrodynamic efficiency

The ionization energy required to generate an appropriate amount of the hydrogen plasma is, obviously (Cf. Eq. (15)),

$$W_i = \frac{2\pi r_0^3}{3} \left(1 - \frac{r_1^3}{r_0^3}\right) p_1 \frac{I}{T_1} = W_f \frac{(\gamma-1)I}{2\gamma T_1} \left[1 - \frac{1}{\gamma} \left(1 - \frac{r_1^3}{r_0^3}\right)^{\gamma-1} \right]^{-1} \quad (35)$$

Neglecting for a while the radiative losses (we evaluate them in Appendix 2 and show that they are small), and the inefficiency related to the residual kinetic energy of the plasma pusher, one obtains the following expression for the hydrodynamic efficiency:

$$\eta = \frac{W_f}{W_f + W_{pf} + W_i} = \frac{1 - \frac{1}{\gamma} \left(1 - \frac{r_1^3}{r_0^3}\right)^{\gamma-1}}{1 + \frac{\gamma-1}{2\gamma} \frac{I}{T_1}} \quad (36)$$

The plot of the function η for $\gamma=5/3$ and $T_1=15$ eV is presented in Fig. 5. One sees that, in order to reach the maximum possible efficiency, which is equal to $\eta_{\max} = \left(1 + \frac{\gamma-1}{2\gamma} \frac{I}{T}\right)^{-1} \approx 0.82$, one has to make r_1 as close to r_0 as possible. However, in order to reach the same final energy of the liner, with the same chamber radius r_0 , one would then have to create a higher (than in the case of the constant plasma parameters considered in Sec. 2) initial pressure p_1 . Indeed, Eqs. (2) and (33) show that

$$\frac{p_1}{p} = \frac{1}{g(r_1/r_0)} \quad (37)$$

The plot of the r.h.s. of this equation is presented in Fig. 5. The plot of the efficiency η vs. the ratio p_1/p is presented on Fig. 6. The temporal dependence of the shell radius is shown in Fig. 7.

The use of large ratios of p_1/p , i.e., the use of r_1 that would be very close to r_0 , is limited as well, by the fact that there will certainly be some channels connecting the plasma sources with the chamber, and these channels will have some finite volume V_{ch} . If the volume of the plasma pusher at the cut-off time becomes less than this “dead” volume V_{ch} , the hydrodynamic efficiency stops increasing at further decrease of the distance $r_0 - r_1$.

Therefore, as an example, we consider a modest pressure increase, $p_1/p=2$, which corresponds to $r_1/r_0 \approx 0.9$ (Fig. 5). The “dead” volume must be a few times less than $(1 - r_1^3/r_0^3)(4\pi r_0^3/3) \approx 0.28(4\pi r_0^3/3)$. We assume that this pressure increase is achieved by increasing the temperature of the plasma pusher to 15 eV compared to the reference case (5), and the density to $1.24 \cdot 10^{17} \text{ cm}^{-3}$. One then finds from Fig. 6 that the corresponding efficiency is $\eta=0.62$. A summary of the parameters corresponding to this example is shown in Table 2, which includes the results of Appendices 2 and 3.

The mass of the plasma pusher in the case of improved efficiency is

$$M_p = \frac{4\pi(r_0^3 - r_1^3)}{3} \times \frac{p_1 m_p}{2T_1} \quad (38)$$

(Cf. Eq. (16)). We denote here the pusher temperature at the first stage (the constant parameters stage) as T_1 (15 eV). In the example presented at the end of the previous section ($p_1=2p$) and corresponding to the final liner energy of 10 MJ, the mass of the pusher plasma is ~ 1.9 g, i.e., smaller than in our “nominal” case (Eq. (5)), where it was 5.3 g.

The kinetic energy of the plasma pusher remains small (Appendix 2): for the aforementioned case of $p_i/p=2$ and the initial temperature of the hydrogen plasma $T=15$ eV, even the *maximum* kinetic energy is less than 1 % of the final energy of the heavy shell (Fig. 8). Accordingly, we ignore its contribution to the hydrodynamic efficiency.

Adiabatic expansion of the hydrogen plasma at the second stage causes the gradual decrease of its temperature. By the end of the accelerator pulse it becomes

$$T_f = T_i \left(1 - \frac{r_i^3}{r_0^3} \right)^{2/3} \quad (39)$$

For the aforementioned example, T_f is 0.45 of the initial temperature, i.e., 6.7 eV. However, the sound speed still remains substantially larger than the implosion velocity: 46 km/s vs. 20 km/s, thereby justifying the use of a subsonic model.

TANLE 2 Parameters of the system for the improved efficiency case

<u>Input parameters</u>			
Final liner energy:	$W_f=10$ MJ;	Pusher temperature at $r_i < r < r_0$: $T_i=15$ eV;	
Chamber radius:	$r_0=2$ m;	Pusher density at $r_i < r < r_0$:	$n_i=1.24 \cdot 10^{17} \text{ cm}^{-3}$.
<u>Derived parameters</u>			
Liner radius at the injection cut-off ($r_i/r_0=0.9$)	$r_i=1.82$ m	The cut-off time	$\tau_i=0.026$ ms
Run-in time:	$\tau_{run-in}=0.13$ ms	Mass of the plasma pusher	$M_p=1.9$ g
Final pusher temp.	$T_f=6.7$ eV	Final pusher density	$n_f=0.34 \cdot 10^{17} \text{ cm}^{-3}$
Residual thermal energy of the pusher plasma	$W_{pf}=3.4$ MJ	Radiated energy	$W_{rad}=0.8$ MJ
Ionization energy	$W_i=2.8$ MJ	Max. kin. energy of the pusher plasma	$max W_p^{(kin)}=0.16$ MJ
Hydrodynamic efficiency*	$\eta=59\%$		

* Here we include radiative loss to the evaluation of η .

Radiative losses are smaller than in the reference case (Eq. (5)), mainly because of a density decrease during the adiabatic expansion stage (Appendix 3). For the input parameters of Table 2, radiative losses are 0.8 MJ. Adding them to the denominator of Eq. (34), one finds that the hydrodynamic efficiency decreases from $\eta=0.62$ to $\eta=0.59$ (the value quoted in Table 2), i.e., relatively insignificantly.

Thus far, we have been considering radiation from a pure hydrogen plasma. The presence of the heavier impurities in the temperature range of interest in our problem will show itself up mostly in the appearance of line radiation. To have a significant effect on the performance of the system, the impurities should increase the total radiation significantly. Indeed, even if, in the just considered example, the radiative losses are

greater than the assumed 0.8 MJ loss by a factor of 5, the overall efficiency will still remain at the level of 0.49. Despite this rather substantial margin in radiative losses, the purity of the pusher plasma may become an issue if special care is not taken regarding the design of plasma sources.

7. Operating at a lower pusher temperature.

Although the pusher temperature is quite low by the fusion standards, $T \sim 10$ eV, the task of generating large volumes of such plasmas may be by far non-trivial. One conceivable approach to generating such plasmas by mixing up a cold plasma produced by arc sources ($T \sim 3$ eV), and a much hotter plasma streams produced by plasma guns – we expand on this issue in Sec. 10.

Meantime, we discuss whether our concept can work at lower pusher temperatures, $T \sim 5$ eV, that would bring us closer to the temperatures achievable in the arc sources (see Ref. 14 for the recent survey). We start from the simplest scenario identical to that considered in Sec. 3, assuming that the pusher parameters are maintained constant during the acceleration process. We notice that the sound speed (Eq. (6)) is still a couple of times higher than even the final velocity of the heavy shell, thereby making our sub-sonic approximation valid.

One of the problems of the lower-temperature pushers is related to the substantial increase of the radiative losses. Indeed, in order to reach the same final energy of the liner, for the same radius of the chamber, one has to increase the pusher density in proportion to $1/T$. According to a simplified expression (10), this leads to the increase of the radiative losses $\sim 1/T^2$. To somewhat mitigate this problem, one can increase the radius of the chamber. Then, in order to reach the same liner energy for a given temperature T , one should reduce the density as $1/r_o^3$. The radiated power density will then scale as $1/r_o^6$, and the radiated power as $1/r_o^3$. The run-in time, for a given final energy of the liner scales as r_o , meaning that the radiative energy loss during the run-in phase scales as $1/r_o^2$.

Following this line of reasoning, we suggest that chambers of the larger size should be used for the low-temperature pusher. Table 3, analogous to Table 1, shows the set of parameters for $T=5$ eV, and $r_o=3$ m and $r_o=4$ m.

The use an improved efficiency scenario (Sec. 6) should still be beneficial in the case of lower temperatures. However, the temperature in the adiabatically expanded hydrogen plasma will become quite low, ~ 2 -3 eV, making the sound speed in this plasma close to the final speed of the heavy shell. Accordingly, the analysis of Sec. 6 and Appendices 1 and 2 would lose its quantitative validity. We leave an investigation of the corresponding issues for the future work.

TABLE 3. Parameters of the system for the constant pusher parameters at lower pusher temperature ($T=5$ eV). The liner mass $M=50$ g.

1	Parameter	$W_f=10$ MJ		$W_f=20$ MJ	
2	v_f , km/s	20		28.3	
3	W_{pp} MJ	15		30	
4	T , eV	5		5	
5	r_0 , cm	300	400	300	400
6	τ_{run-in} , ms	0.21	0.28	0.15	0.2
7	p_0 , 10^{18} eV/cm ³	0.55	0.23	1.1	0.46
8	n , 10^{17} cm ⁻³	0.55	0.23	1.1	0.46
9	M_p , G	10.4	10.4	20.8	20.8
10	Q_{rad} , kW/cm ³	0.5	0.09	2	0.36
11	τ_{rad} , ms	0.27	0.64	0.14	0.33
12	W_{rad} MJ	7	3.9	16.8	11
13	$max W_p^{(kin)}$, MJ	0.44	0.44	1.76	1.76
14	W_i MJ	16	16	32	32
15	η , %	21	22	20	21

8. Hydrodynamic stability

The heavy shell can experience a Rayleigh-Taylor (RT) instability at the early stage of the acceleration process. The general theory of this instability in the relevant setting has been discussed in great detail in the review paper [16], where the further references can also be found. Of the main concern is the shell stability at an early stage of acceleration, when acceleration is high and the shell is relatively thin. For perturbations with the wave number satisfying condition $kh < 1$, the growth rate can be evaluated as

$$\Gamma = \sqrt{kg} \approx \sqrt{kg_{\max}} = \frac{v_f}{r_0} \sqrt{\frac{3kr_0}{2}} \quad (40)$$

where g is the effective gravity acceleration. The temporal dependence of the acceleration is presented on Fig. 2c. For some time, the acceleration remains essentially constant, and then rapidly decreases. Of a concern, therefore, is the instability relatively early in the pulse, at $t < \tau_{run-in}/2$.

The number of exponentiations occurring during this time is $\Gamma\tau_{run-in}/2$. Assuming that the initial perturbations are small-enough to allow for 4 exponentiations, and using Eq. (34), one concludes that only perturbations with the wave number exceeding some critical wave-number k_0 can play a role:

$$k > k_0 \equiv 20/r_0 \quad (41)$$

These, relatively short-wavelength perturbations, would cause the broadening of the shell. But, when the thickness h reaches the scale $\sim 1/k_0$ the further instability growth becomes much slower. The estimate (41) yields then the shell thickness ~ 10 -15 cm.

Another way of evaluating the shell thickness is to assume the self-similar model (e.g., Young [17], and references therein), in which the shell thickness grows as, roughly,

$$h \sim 0.07gt^2 \quad (42)$$

Substituting $t = \tau_{run-in}/2$, one finds that at the end of the acceleration phase $h \sim 0.05 r_0$, i.e., about the same as in the previous estimate.

The thickness of ~ 10 –15 cm is only marginally acceptable for our purpose, so, some ways of improving the situation have to be looked for. The most radical way would be to introduce a feedback control over the pusher plasma. What helps in this respect is that the instability has to be controlled mostly early in the pulse, when the shell is still not very far from the plasma sources. So, by collecting the information about the formation of bumps or dimples on the shell, processing it in a real time, and sending the corresponding signal to the power supplies of the individual plasma sources, seems to be feasible, at least in principle. Deviation of the heavy liner shape from the spherical shape, would be detected by several optical and/or UV imagers surrounding the chamber. Note that the heavy liner moves with a velocity which is much less than the sound velocity of the hydrogen plasma in a plasma pusher. Therefore, a changed inflow from a certain set of guns will cause change of the pusher pressure in the desired zone of the heavy shell.

In the version of a “dusty gas” pusher, the presence of thin flakes of heavy material dispersed in the ambient gas gives rise to an increased viscous dissipation and may cause a decrease of the growth rate. This stabilization mechanism will be analyzed elsewhere.

9. Variations on the theme of a heavy plasma liner.

The proposed scheme, potentially, provides a substantial degree of control of the heavy liner shape late in the acceleration process. This can be accomplished by giving the liner initial “kick” that varies over its surface produced by the plasma sources early in the pulse. At this stage the gap between the liner and the wall is still small compared to the liner radius, and small lateral variations of the pusher pressure are allowed. As the radial convergence C (the last line in Table 1) that can be achieved in our approach is in the range of 25, the variation of the initial radial velocity of the order of $1/C$ would give rise

to the order-of-unity variation of the final shape of the liner. One can envisage a control over separate groups of plasma sources that would create this variation and make the final shape of the liner oblate or prolate, to match the particular shape of the target.

Another interesting possibility is to generate an initial rotation or shear flow in the heavy liner. As we envisage the use of pulsed valves as the gas puffs, one could create a directional azimuthal flow at some latitudes. Moreover, by controlling the temporal dependence of the injection velocity, one might create a velocity shear across the heavy liner. Of course, this tangential velocity will not exceed the sound velocity, which is quite small for heavy gases (for Xe at room temperature it is ~ 120 m/s). However, in the course of implosion, because of the angular momentum conservation, this velocity will grow as $1/r$, and, accordingly, would reach $\sim 1/10$ of the implosion velocity. The presence of a modest velocity shear in the liner may improve its stability with respect to RT modes at the deceleration phase. Shear flow instability may turn to be more benign in terms of the mix process. This whole issue (briefly analyzed in a context of laser fusion [18]) remains a challenge to the theory and experiment.

If so desired, one could also generate rotational flow in the pusher plasma.

It might be interesting to perform small-scale experiments to test the basic features of the concept. As was pointed out to one of the authors (D.R.) by Dr. A. Velikovich, this small-scale experiments can be performed in a cylindrical geometry, where a cylindrical, well collimated annular shell of a heavy gas could be created by a single supersonic nozzle (e.g., [19]). It would then be compressed radially by a thermal pressure of a plasma injected into the annulus between the gaseous shell and the chamber wall.

One can even conceive of a table-top experiment directed towards simulation of our concept, in particular, for the study of the RT stability and hydrodynamic efficiency, where the pusher will be just a light gas, like hydrogen or helium.

We may add that cylindrical geometry may allow a realization of 3D implosions, by virtue of creating a mass-density variation over the length of the cylindrical gaseous shell. With the density higher near the equator and lower near the ends, one would reach a formation of a cavity imploding in a 3D fashion [1].

10. Discussion

We have shown that the heavy plasma liner with a sub-sonic (thermal) pusher can provide quite high hydrodynamic efficiencies, which can exceed 60%. The heavy liner would then convert its kinetic energy into the energy of the fusion fuel, via the interaction with a target situated near the center of the imploding liner. The characteristic range of parameters considered in this paper is: the final liner energy 10-20 MJ, the chamber radius 2-3 m, the liner run-in time 150-200 μ s.

An advantage of the plasma liner approach in general [8-10], and the heavy liner approach in particular (this paper) is that there is no need for any direct material connections of the primary energy source and the target and, therefore, the “stand-off” problem is solved.

The quoted high efficiency requires the use of a pusher hydrogen plasma with the temperature ~ 10 eV and the density 10^{16} - 10^{17} cm^{-3} . Creating large volume of such a plasma may be a non-trivial task. For the lower temperatures ~ 5 eV (approaching those generated in the arc sources), the efficiency of the drive with constant parameters

decreases to 20 %. A switch to a more efficient scheme where the injection occupies a fraction of the run-in time (Sec. 6) should lead to a substantial improvement of the efficiency. However, for the initial temperature of the pusher plasma ~ 5 eV, the temperature of the adiabatically cooled pusher will become quite low, ~ 2 eV, bringing the sound speed in the pusher plasma too close to the final speed of the heavy shell. The analysis of this situation would require solving a full set of hydrodynamic equations — a task that we leave for the future work. On the other hand, based on the discussion presented in Sec. 6 and Appendices 1 and 2, there is a reason to believe that the efficiency of as high as 40 % is feasible.

Returning to the higher-temperature pushers, $T \sim 10$ eV, one can suggest the use of much higher temperature and lower current plasma streams, with the energy per proton ~ 100 -200 eV, to mix up with a cold arc plasma, creating a desired set of the pusher parameters. The streams could be directed tangentially, so as not to perturb too strongly the shell boundary.

The heavy liner could be used for generating a high current in a canonical MTF setting by driving a magneto-compressional generator of the type mentioned in Ref. 1. In this case, the outer dimension of the target assembly (that would include the magneto-compressional generator, some circuitry, and the MTF target proper) could be quite large, ~ 30 -40 cm, with the MTF magnetically-compressed target of a diameter of a few centimeters sitting in the center of the target assembly. The means of generating a bias magnetic field, as well as creating initial magnetized plasma inside the liner have been outlined in Ref. 1. This approach fits well into the concept of a local quasi-spherical lithium blanket surrounding the target [20]. This approach may turn out to be preferential for the plasma liner concept compared to the thick liquid wall approach [5], because in the latter case one would have to arrange for a complex hydrodynamic flow of lithium around the holes required for the injection of gas and plasma.

Direct mechanical interaction of the heavy liner with an outer shell of the MTF target is also possible, for the targets with initial radii ~ 5 -6 cm. To generate an initial magnetized plasma configuration inside the shell some material electrodes connecting the target with the external power supply might still be needed, but in this case they would have to deliver only a small fraction (a few percent) of the energy that would be eventually needed to implode the target. Contactless introduction of the initial magnetized plasma might be possible by merging of compact toroids launched in a stand-off manner from the periphery of the chamber. Alternatively, lasers or charged particle beams may also be considered for driving currents in a preformed target plasma to produce the initial magnetic field.

It is also worthwhile to look for possible non-fusion applications of the heavy liner, in particular, in the area of high energy density physics: the liner possesses a nontrivial energy and can be created by relatively inexpensive tools. One possibility is to use it for the studies of equation of state, by observing the behavior of the liner material imploding into the empty chamber and coming to a stagnation near the center. The resulting pressure can be in the range of 1 Mbar, in the volume of tens of cm^3 . One can also use the liner to adiabatically compress any other material in question, situated in a capsule in the center of the chamber (the compression can be made adiabatic because of a relatively long rise-time of the pressure, in the range of a few micro-seconds).

One more non-fusion application is the generation of the line radiation and/or black-body radiation. Total radiated energy will be nearly equal to the liner energy, if the parameters of the gas are chosen in such a way that the radiation time in the imploded state is shorter than the dwell time. Radiation time in the imploded state is controlled by the composition of the liner.

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Appendix 1. Radiative energy flux through the surface of the heavy liner

One can check that, for the parameters of the plasma pusher accepted in our paper, the pusher is transparent to its bremsstrahlung and recombination radiation (see [12]). Therefore, the radiation energy flux through the spherical surface of the radius r will be simply

$$I = \frac{Q_{rad}}{2} \int_r^{r_0} d\hat{r} \int_0^{\arccos(r/\hat{r})} \frac{\hat{r}^2 \sin\theta (\hat{r} \cos\theta - r) d\theta}{(\hat{r}^2 - 2r\hat{r} \cos\theta + r^2)^{3/2}}, \quad (A1.1)$$

where θ is the polar angle. Switching to a new integration variable $\xi = \cos\theta$, one finds that

$$I = \frac{Q_{rad}}{2} \int_r^{r_0} \hat{r}^2 d\hat{r} \int_{r/\hat{r}}^1 \frac{(\hat{r}\xi - r) d\xi}{(\hat{r}^2 - 2r\hat{r}\xi + r^2)^{3/2}} \quad (A1.2)$$

For r small compared to r_0 one readily obtains that

$$I \approx \frac{Q_{rad}}{2} \int_r^{r_0} d\hat{r} \int_0^1 \xi d\xi \approx \frac{Q_{rad}}{4} \quad (A1.3)$$

This is an expression used in Sec. 5.1.

Appendix 2. Evaluating the kinetic energy of the pusher plasma for the case of improved efficiency

At the first stage, where the injectors are on, and the plasma parameters are kept constant, the kinetic energy of the plasma pusher varies according to equation analogous to Eq. (18):

$$W_p^{(kin)} = \frac{3M_p}{4\pi(r_0^3 - r_1^3)} \int_r^{r_0} \frac{\hat{v}^2}{2} 4\pi\hat{r}^2 d\hat{r} = 3W_f \frac{M_p}{M} \frac{x^3(1-x^3)(1-x)}{g(x_1)(1-x_1^3)}; \quad x_I < x < 1, \quad (A2.1)$$

where we use the notation

$$x = \frac{r}{r_0}; \quad x_1 = \frac{r_1}{r_0}. \quad (A2.2)$$

The cut-off time is:

$$\tau_{cut-off} \approx \frac{r_0}{v_f} \frac{2\sqrt{g(x_1)}}{\sqrt{3}} \sqrt{1-x_1} \left[1 + \frac{1-x_1}{6} + \frac{(1-x_1)^2}{24} \right] \quad (A2.3)$$

This expression (which we present without derivation) has an accuracy of better than 1% for $x_I > 0.5$. For $x_I = 0.9$, one has: $\tau_{cut-off} = 0.26r_0/v_f$.

At the stage of the adiabatic expansion that follows this initial stage, i.e., at $r < r_l$, the pusher plasma density decreases, remaining uniform over the volume. At this stage the velocity of the plasma pusher varies so as to make $(1/\hat{r}^2)\partial(\hat{r}^2\hat{v})/\partial\hat{r}$ constant over the volume (although varying in time) and to satisfy the boundary condition of the zero velocity at $\hat{r} = r_0$ (there is no more plasma inflow). The other boundary condition is that the velocity is equal to the heavy shell velocity at $\hat{r} = r$. The velocity distribution corresponding to these conditions is:

$$\hat{v} = \dot{r} \frac{r^2(r_0^3 - \hat{r}^3)}{\hat{r}^2(r_0^3 - r^3)} \quad (\text{A2.4})$$

The mass density at this stage is:

$$\rho = \frac{3M_p}{4\pi(r_0^3 - r^3)} \quad (\text{A2.5})$$

The kinetic energy of the hydrogen pusher is:

$$W_p^{(kin)} = \int_r^{r_0} \frac{\rho \hat{v}^2}{2} 4\pi \hat{r}^2 d\hat{r} = \frac{3M_p \dot{r}^2}{2} \times \frac{x^4}{(1-x^3)^3} \int_x^1 \frac{(1-\xi^3)^2 d\xi}{\xi^2} = \frac{3M_p \dot{r}^2}{2} \times \frac{x^3(1-1.8x+x^3-0.2x^5)}{(1-x^3)^3}. \quad (\text{A2.6})$$

The liner velocity at the adiabatic expansion stage is:

$$|\dot{r}| = \left(\frac{8\pi p_l r_0^3}{3M} G(x, x_1) \right)^{1/2}, \quad (\text{A2.7})$$

where

$$G(x, x_1) = (1-x_1^3) \left\{ 1 + \frac{1}{\gamma-1} \left[1 - \left(\frac{1-x_1^3}{1-x^3} \right)^{\gamma-1} \right] \right\}. \quad (\text{A2.8})$$

Substituting Eq. (A2.7) into Eq. (A2.6), one finds:

$$W_p^{(kin)} = 3W_f \frac{M_p}{M} \frac{G(x, x_1)}{g(x_1)} \times \frac{x^3(1-1.8x+x^3-0.2x^5)}{(1-x^3)^3}; x < x_l. \quad (\text{A2.9})$$

The plot of the function $W_p^{(kin)}$ normalized by $W_f(M_p/M)$ is shown in Fig .8 for $x_l=0.9$. The maximum kinetic energy is reached at the end of the injection pulse. The drop in the energy at the end of the pulse is related to that the flow velocity at the stage where the plasma sources are turned off is zero at the surface of the chamber. An actual transition occurs within a couple of acoustic transit times over the pusher thickness, which is quite short: $2 \times 0.1 r_0 / s \sim 6 \mu s$.

Appendix 3 Evaluating radiative losses for the case of improved efficiency

During the stage of the constant pusher parameters, the radiated energy can be found from the equation analogous to Eq. (8):

$$W_{rad}^{(1)} = \frac{4\pi r_0^3 Q_{rad}^{(1)}}{3} \frac{r_0}{v_f} \sqrt{g(x_1)} \int_{x_1}^1 dx \sqrt{1-x^3}. \quad (\text{A3.1})$$

Here we will use an approximate expression (10) for Q_{rad} . During the second stage, where the pusher experiences an adiabatic expansion, the radiative losses per unit volume per unit time, according to Eq. (10), are:

$$Q_{rad}^{(2)} = Q_{rad}^{(1)} \left(\frac{r_0^3 - r_1^3}{r_0^3 - r^3} \right)^2, \quad (\text{A3.2})$$

whereas the liner velocity $v(r)$ varies according to Eq. (A2.6). Accordingly, the energy radiated during the second stage is

$$W_{rad}^{(2)} = \frac{4\pi}{3} \int_0^{r_1} (r_1^3 - r^3) Q_{rad}^{(2)} \frac{dr}{v(r)} = \frac{4\pi r_0^3 Q_{rad}^{(1)}}{3} \frac{r_0}{v_f} \sqrt{g(x_1)} \int_0^{x_1} \frac{(1-x_1^3)^2}{1-x^3} \frac{dx}{\sqrt{G(x, x_1)}}. \quad (\text{A3.3})$$

The total radiated energy is

$$W_{rad} = W_{rad}^{(1)} + W_{rad}^{(2)}. \quad (\text{A3.4})$$

As we assume the adiabatic model for the pusher plasma to evaluate $v(r)$, this estimate is correct only in the case where radiative losses are small (as is the case, see below).

For $x_1=0.9$, the integral in Eq. (A3.1) is equal to 0.035, and the integral in Eq. (A3.3) is equal to 0.13. Taking into account that $g(0.9)=0.5$, we find the following expression for radiative losses:

$$W_{rad} = 0.12 \frac{4\pi r_0^3 Q_{rad}^{(1)}}{3} \frac{r_0}{v_f}. \quad (\text{A3.5})$$

Using Eq. (10) and substituting the input parameters from Table 2, one finds that $W_{rad} \approx 0.8$ MJ.

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Fig. 1

Fig. 2

Fig. 3

Fig. 4

Fig. 5

Fig. 6

Fig. 7.

Fig. 8.