Dynamics of an Isolated Blob in the Presence of the X-Point

R. H. Cohen, D. D. Ryutov

October 11, 2005

Contributions to Plasma Physics
Disclaimer

This document was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor the University of California nor any of their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise, does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or the University of California. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or the University of California, and shall not be used for advertising or product endorsement purposes.
Dynamics of an isolated blob in the presence of the X-point

R.H. Cohen, D.D. Ryutov

Lawrence Livermore National Laboratory, Livermore, CA 94551, USA

Abstract

The interplay of X-point shearing and axial plasma redistribution along a moving flux tube is discussed. Blobs limited to the main scrape-off-layer and the blobs entirely confined in the divertor region are identified. A strong effect of the radial tilt of the divertor plate on “divertor” blobs is found.

1. Introduction

It was suggested a few years ago that nonlinear structures (“blobs”) driven by the magnetic field curvature and strongly elongated along field lines may exist in the tokamak scrape-off-layer (SOL), e.g., [1]. In the simplest version, the blob moves in the electric field generated by the curvature drift. The contact of the blob end with the divertor plate leads to a partial short-circuiting of the electric field and slows down the motion. This simple picture does not take into account the strong shearing of the flux-tube near the X-point [2], an effect that makes impossible direct electrical contact of the perturbation in the upper SOL and the divertor leg (for typical divertor parameters). In this case, the sheath boundary condition (BC) has to be replaced by a BC imposed near the X point. A specific, approximate, version of this BC applicable at small distances from the separatrix was suggested in Ref. [3] and applied to studies of blobs in Ref. [4]. At larger distances from the separatrix connection between the upper SOL and the divertor plate may be re-established, and the sheath BC becomes again relevant (a notion made in passing in Ref. [4]).

In our present paper we study several effects associated with such dynamics. The problem that we are solving is quite complex and includes a subtle interplay of the radial motion of the blob and the parallel plasma dynamics. Therefore, in order to identify the key effects, we use the simplest possible model, assuming that the blob has already been ejected beyond the SOL and is therefore surrounded by a plasma of a negligible density and conductivity. We call such a blob an “isolated blob.” [This model was earlier used in Ref. [4].] Additionally, we assume that are dealing with a double-null divertor and concentrate on the outboard part of the tokamak. We consider only zero-beta effects and entirely neglect magnetic field perturbations.

When the blob is not too far from the separatrix, strong X-point sheering largely eliminates its electrical contact with the divertor. Therefore the blob moves radially without causing a substantial response in the divertor plasma. When the blob reaches distances from the separatrix large enough that X-point sheering no more terminates it, its ends are “dangling” in free space, not connected to the divertor plate. The parallel plasma flow turns out to be too slow to re-establish the contact. Therefore, the blob freely accelerates towards the first wall.

Conversely, there can exist blobs entirely limited to divertor legs. They differ from those present in the main SOL in that they are in an electric contact with the
divertor plate, so that their dynamics is strongly affected by sheath boundary conditions. We use these conditions in their most general form, accounting for the radial tilt of the divertor plate [5, 6] and find that this effect is important and can be used to control the blob behavior. We find that for divertors with long-enough legs blobs can exist in the private flux region.

There are many aspects of blob physics not covered in this paper. In particular, Ref. 7 considers the possible role of the electron temperature gradient and parallel shear of the EXB flow and Ref. 8 analyzes X-point boundary conditions that go beyond the heuristic model [3]. have been analyzed. Papers [7, 8] contain also an extensive bibliography.

2. The basic model

The “isolated” blob is represented as a flux tube with a circular cross-section along most of its length (except, possibly, in the vicinity of the X-point; see below), with the plasma pressure and density inside the tube being substantially higher than the pressure and density of the ambient plasma, so that the latter has little effect on the blob motion. Such an isolated blob is, of course, an idealization but a study of such a structure may provide insights into more realistic cases. This model may also be of direct relevance to isolated “spills” of the plasma beyond the region immediately adjacent to the separatrix.

Initially we neglect the plasma parallel motion and assume that the ends of a blob do not reach the X-point. We return to this issue later in the paper. An isolated blob can be created in a thought experiment where one fills up an isolated flux-tube with a plasma with desired parameters and then sets this object free. An example of such a blob situated at the outboard part of the SOL (in the region of unfavorable curvature) is shown in Fig. 1a.

Following the discussion of Refs. [1,4] we note that the magnetic drifts inside the blob generate a current in the direction of the major axis of the tokamak (see Fig. 1b). The corresponding current density is

$$ j_{\perp}^{(\text{grad})} \sim \frac{cp}{BR} \sim \text{env}_{\text{Ti}} \frac{\rho_i}{R} $$

(1)

where $p$ is the plasma pressure, $c$ is the speed of light, $B$ is the magnetic field, $R$ is the curvature radius of the flux tube (of order of the major radius of a tokamak), $n$ is the particle density, $v_{\text{Ti}}$ is the ion thermal velocity, and $\rho_i$ is the ion gyro-radius. Throughout the paper we use CGS (Gaussian) units. In order to minimize the number of “external” parameters, we assume that the electron and ion temperatures are approximately equal.

The ensuing charge separation causes a build-up of the electric field which gives rise to radial E×B acceleration; the polarization current,

$$ j_{\perp}^{(\text{polar})} \sim -\frac{m_i n c}{B} \dot{R}, $$

(2)

must neutralize the current (1) to maintain quasineutrality. From this condition one finds the radial acceleration:

$$ \ddot{R} \sim \frac{p}{nmR} \sim \frac{v_{\text{Ti}}^2}{R} $$

(3)
The cross-field motion of the flux-tube is related to the aforementioned electric field (directed vertically on Fig. 1b) by virtue of equation:

\[ E_\perp \sim B \frac{\dot{R}}{c} \]  \hspace{1cm} (4)

The corresponding potential difference is \( \delta \varphi \sim aE_\perp \), where \( a \) is the flux-tube radius. In other words,

\[ \delta \varphi \sim \frac{aB\dot{R}}{c} \]  \hspace{1cm} (5)

This discussion repeats the discussion of Refs. [1,4] We reproduce it here to establish a link with these earlier studies and to introduce notation.

If the flux-tube moved according to Eq. (3), it would have reached the first wall within the time

\[ t_\perp \sim \frac{\sqrt{2l_\perp R}}{v_{ti}} \]  \hspace{1cm} (6)

where \( l_\perp \) is the distance between the plasma and the first wall (Fig. 1a). Here, however, we have to return to our assumption that the plasma initially fills only a part of the flux tube length, with the ends of the plasma separated by vacuum gaps from the vicinity of the X point. The question then arises regarding the parallel redistribution of the plasma along the flux tube. Within the time \( t_\perp \) the plasma expands along the flux tube by the distance

\[ \delta L \sim v_{ti}t_\perp \sim \sqrt{2l_\perp R} \]  \hspace{1cm} (7)

This distance is typically much less than a connection length \( L \) between the equatorial plane and the X-point, which is \( \sim (3-4)R \). So, even if the initial gap between the blob ends was substantially smaller than \( L \) (but greater than \( \delta L \) the accelerated motion determined by Eq. (3) would indeed continue until the blob reaches the wall.

3. X-point boundary conditions

Thus far we have considered the situation where the plasma occupied only a fraction of the flux-tube between the X-points. Consider now a case where the plasma occupies the whole flux-tube between the divertor plates. In this case we have to impose a boundary condition at the divertor plates. This is the sheath BC which we mentioned in the Introduction. If, however, the geometrical parameters of the system are such that strong X-point shearing of the flux-tube takes place, so that the flux-tube thickness at the divertor plate becomes less than the ion gyroradius, then a disconnection occurs in the transition zone near the X-point; in this case we use a “heuristic” BC at the control surface just above the X-point [3]. What regime we are in depends on many factors, in particular, on the divertor geometry, the distance \( x \) of the flux-tube from the separatrix, and the flux-tube radius \( a \). By \( x \) we mean the distance mapped to the equatorial plane. We designate by \( x_{\text{crit}} \) the distance at which the disconnection occurs. In the model of the divertor considered in Ref. 9, the distance \( x_{\text{crit}} \) is determined by the equation:

\[ x_{\text{crit}} \sim l_d \rho_i / a, \]  \hspace{1cm} where \( l_d \) is the length of the divertor leg (Fig. 2). At \( x<x_{\text{crit}} \) the shearing is strong, and vice versa.

Consider first the blob in the zone of a strong shearing (i.e., not very far from the separatrix). Here the boundary condition [3] relates the parallel current and the potential
perturbation. Written in the form of a rough estimate (as was done in Ref. [4]), this condition reads as:

\[ j_\parallel \sim \sigma_h \delta \varphi / a , \tag{8} \]

where \( \sigma_h \) is a “heuristic” conductivity:

\[ \sigma_h = G \frac{\omega^2_{pe}}{4 \pi a \omega_{ce}} . \tag{9} \]

Here \( G \) is a numerical coefficient of order unity. The parallel current is related to the perpendicular current via the continuity equation which, in order of magnitude, yields:

\[ \delta j_\parallel \sim \frac{L}{a} \left( \delta j_\perp^{(\text{polar})} - \delta j_\perp^{(\text{grad})} \right) \tag{10} \]

This current has a “dipole” structure, flowing in opposite directions on opposite sides of the flux tube (Fig. 1b).

Using Eqs. (1), (2), (5) and (8)-(10), one then finds the following dynamical equation for the flux-tube radius:

\[ \ddot{R} = \frac{v^2_{Ti}}{R} - \frac{\dot{R}}{\tau} \tag{11} \]

where

\[ \tau \sim \frac{L}{G a \omega_{ce}} \tag{12} \]

If the blob starts at a zero velocity, then, at \( t \sim \tau \), the initially accelerated motion is changed to a constant-velocity outward motion, with a velocity [4]:

\[ \dot{R} \sim v_{Ti} \frac{L \rho_i}{G R a} \tag{13} \]

The time \( \tau \) can be longer than the time \( t_\perp (7) \); in such a case, the blob hits the wall moving according to Eq. (3). The condition \( \tau > t_\perp \) can be rewritten as \( \rho_i / a > \sqrt{l_\perp R / L^2} \) and is satisfied for sufficiently small values of \( l_\perp \). For \( l_\perp = 15 \text{ cm}, R = 150 \text{ cm}, \) and \( L \sim 4R \), this condition becomes 0.08 - a condition which is easy to satisfy; the opposite situation is, however, also possible.

In the vicinity of the separatrix, the disconnection of the upper SOL and the divertor region leads to the situation where the blob moves radially outward without entraining its lower, divertor part (Fig. 3 a and b). The further dynamics depends on the relation between the connection length \( L_d \) between the blob and the divertor plate, on the one hand, and the distance \( \delta L \) defined by Eq. (7). Consider first the situation where the divertor legs are long enough, so that condition \( L_d > \delta L \) holds. Then, in the course of the radial motion, a structure of the type shown in Fig. 3 is formed: As the plasma from the upper part of the blob does not have time to fill in the divertor part of the flux tube, the blob in the main SOL ends in a large vacuum gap between the control surface and the divertor plate. This, in turn, leads to a surprising effect: even in the outer SOL where the electrical connection of the blob with the divertor plate might be re-established (because of reduced shearing), the flux-tube is accelerated radially according to Eq. (3). A numerical example: For \( l_\perp = 15 \text{ cm}, R = 150 \text{ cm}, \) one has \( \delta L \sim 70 \text{ cm}, \) i.e., the disconnection can occur even in a relatively short-legged divertor. In our discussion we assumed that condition \( \tau > t_\perp \) holds. The opposite situation can be treated in analogous way.
Conversely, if the divertor has short legs, so that connection is re-established in the outer SOL (field lines 3 and 4 in Fig. 2), the sheath BC then have to be imposed, thus leading to slowing-down of the blob (see Ref. 4 and discussion at the end of Sec. 4 of our paper). A variety of scenarios for the blob motion is illustrated by Fig. 4.

4. Blobs localized in the divertor region

In the zone not far from the separatrix, where the shearing is strong, the radial motion of the flux-tube in the divertor region is not accompanied by the corresponding motion in the upper SOL. The situation here is just reverse to that shown in Fig.3: the divertor part of the flux tube moves radially without causing the same displacement in the upper SOL. There is, however, a substantial difference: the blob in the divertor is electrically connected to the divertor plate and, therefore, the sheath boundary condition has to be imposed.

We will discuss these conditions in the form relevant to a single flux-tube filled with a plasma and surrounded by vacuum (an isolated blob). The potential difference $\delta \varphi$ across the blob (see Fig. 1b) causes a “dipole” parallel current (e.g., [10,11]),

$$\delta j \sim env, (e \varphi / T)$$

The parallel current will also flow towards the “control surface”, where it will be determined by Eq. (8). One more source of the parallel current is related to the tilt of the divertor plate [5,6]: closing of the electron diamagnetic current that flows as shown by a dashed arrow in Fig. 1b. In the presence of tilt, this current is “converted” to parallel current. The corresponding contribution is

$$\delta j \sim env, \frac{\rho_i}{\alpha} \tan \alpha$$

where $\alpha$ is the angle between the normal to the divertor plate and the poloidal magnetic field at the plate. We assume that the electron and the ion temperatures are approximately equal and express this current in terms of the ion gyro-radius and the ion thermal velocity. Collecting all the contributions, we obtain, instead of Eq. (11), the following dynamic equation:

$$\ddot{R} \approx \frac{v_T^2}{R} \left( \frac{1 + \tan \alpha}{2L_d} \right) - \dot{R} \left( 1 + \frac{a}{G \rho_i} \right)$$

where

$$\tau_d \sim \frac{L_d}{G a \omega_c i}.$$ 

We skip an onerous discussion of the relative signs of various terms and provide only the final result. The convention about the sign of $\alpha$ for the outboard part of the divertor is that $\alpha$ is positive when the normal to the divertor plate is tilted in the outward direction from the poloidal field line; for the orientation of Fig. 2 the sign of $\alpha$ is negative.

One can note in passing that, in addition to the effects that drive a “dipole” component of the parallel current, there are also effects driving a “monopole” component, which is unrelated to the closure of the perpendicular current and does not directly affect the dynamics of the radial motion of the blob.

Eq. (16) shows that a sufficiently strong positive radial tilt of the divertor plate can lead to a substantial acceleration of the blob in the divertor region and therefore, to an
increased transport in this area. Conversely, negative tilt suppresses the blob transport in the outer region.

For a divertor with sufficiently long legs, the shearing near the X-point may lead to a decoupling of the plasma dynamics in the inner and the outer divertor legs of the private flux region. When the tilt is strong and negative, “strange” blobs can be generated here, which propagate from the outboard side of the divertor to the inboard side, against the usual direction determined by the field line curvature.

Eq. (16) can be applied to blob dynamics in the outer part of the main SOL when electrical connection with the divertor plate is re-established and condition $L_d < \delta L$ is satisfied. One has only to replace $L_d$ by $L$ and $\tau_d$ by $\tau$ and neglect $G$ compared to $a/\rho_i$. We note that, since $L$ is substantially longer than $L_d$, the effect of the tilt will play a relatively less important role (unless the tilt is very strong). The steady-state expansion velocity will then become (Cf. Ref. 4):

$$\dot{R} \sim v_{ti} \frac{L \rho_L^2}{Ra^2}$$

(18)

5. Summary

We have considered the dynamics of “isolated” blobs in a tokamak with divertor. It turned out that parallel plasma dynamics can play a substantial role in the blob behavior at larger distances from the separatrix. For a divertor with sufficiently long-enough legs, the blob gets detached from the divertor plate and accelerates freely to the first wall. Conversely, for a short-legged divertor, the electrical connection to divertor plates does happen and the blob slows down.

The X-point shear leads to the possibility of blobs limited to the divertor legs and not extending into the main SOL (beyond the X-point). These “blobs” are sensitive to the radial tilt of the divertor plate and can be accelerated or decelerated depending on the sign of the tilt. For divertors with long-enough legs, there may exist “strange” blobs limited to the private flux region.

6. Acknowledgment

This work was performed under the auspices of the U.S. Department of Energy by University of California Lawrence Livermore National Laboratory under contract No. W-7405-Eng-48.
References

Fig. 1. An isolated blob in the main SOL: a) A part of an equatorial cross-section of a tokamak. The inner circle represents the position of the separatrix; the shaded area is the first wall. Shown in light pink is a projection on the equatorial plane of an isolated flux-tube filled with a plasma over some part of its length. This model corresponds to a blob that has already moved beyond the near SOL. b) The cross-section of the blob at the A-A plane (blown up). Vertical arrows represent the direction of the grad-B current; if at the ends the blob contacts a conducting surface, the parallel current whose directions are shown by a dot and a cross forms; in other words, the parallel current has a “dipole” nature. The electric field that causes the radial motion of the blob is vertical (for the orientation of panel b). Dashed arrows show the direction of the electron diamagnetic current.
Fig. 2. Shown is a set 1-4 of poloidal field lines at the outboard side of the divertor region. The zone where the shearing of the initially circular flux-tube down to the scale of less than the ion gyro-radius occurs is sketched in yellow. The potential perturbations imposed, say, at the upper boundary of this region, decrease exponentially to the lower boundary (and vice versa). The yellow zone corresponds roughly to the e-folding length. So, the potential perturbation imposed in the upper part of SOL on a flux-tube #1 does not reach the divertor plate; the same is true for the field line #2. On the field line #3 some influence of the divertor plate is already present; for field line #4 the perturbation is decisively connected with the divertor plate and can be affected by the BC on this plate. Conversely, if we consider a potential perturbation on a certain field line in the divertor region, then it is decoupled from the upper SOL on the field line 1, and is connected with the upper SOL on the field line 4.
Fig. 3 Decoupling of the blob formed in the upper divertor from the divertor plate. Initially the blob fills the whole flux tube extending from the upper divertor plate through the main SOL to the lower divertor (shown is only a part of the system: the outboard part of the lower divertor, and the adjacent part of the main SOL above the X point). Because of a strong shear in the zone close to the separatrix, the upper part of the blob moves radially outward, leaving the lower part intact. The plasma left behind in the transition zone does not have time to expand along the field lines, leaving the upper part of the blob electrically isolated from the divertor plate along the field line. Because of high cross-field electrical resistivity, connection through the plasma left behind in the transition zone is weak. Accordingly, even at a large distance from the separatrix (the last frame), where one might think that the electrical conductivity is re-established, it actually is not, and the upper part of the blob proceeds to the first wall as described by Eq. (3).
Fig. 4 Several possible scenarios of the blob motion in the main SOL: 1 – no connection to the X-point, pure acceleration; connection to the X-point present but connection with the divertor plate is absent (by virtue of condition $\delta L < L_d$); connection to the divertor plate re-established at $x > x_{\text{crit}}$. 