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**COLLEGE OF ENGINEERING
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**INTERFACIAL AREA
AND INTERFACIAL TRANSFER
IN TWO-PHASE FLOW SYSTEMS**

Volume III. Chapters 11 – 14

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EXECUTIVE SUMMARY

Advances in the study of two-phase flow increasingly require detailed internal flow structure information upon which theoretical models can be formulated. The void fraction and interfacial area are two fundamental parameters characterizing the internal flow structure of two-phase flows. However, little information was available on these parameters, and it is mostly limited to vertical flow configuration. Particularly, there was virtually no database for the local interfacial area concentration in spite of its necessity in multidimensional two-fluid model analysis.

In view of the above, a research program, which has been sponsored by the DOE/BES, has been underway at the University of Wisconsin-Milwaukee. The overall objectives of the research program were to develop instrumentation methods, an extensive database and analysis leading to predictive models for describing the internal flow structure and behaviors of two-phase flow in horizontal configurations.

Experimental efforts were directed at developing instrumentation technique for measurements of the local interfacial area concentration and void fraction in the bubbly flow, plug/slug flow, stratified smooth and stratified wavy and annular flow patterns encountered for two-phase flows in horizontal configurations. Chapters 5 through 12 describe several conductivity probe techniques that have been developed under this research program. More specifically:

- Chapters 5 and 6 present the development and utility of the two-sensor conductivity probe for a bubbly flow,
- Chapter 7 describes the utility of hot-film anemometry method for a bubbly flow whereas Chapters 9 and 10 documents the development of hot-film probe method for a plug/slug flow,
- Chapter 8 describes the four-sensor conductivity probe design for measuring the large bubble interfacial area concentration in a plug/slug flow-pattern,
- Chapter 11 summarizes the development and utility of a two-sensor parallel-wire conductivity probe for the stratified, i.e., stratified-smooth,

stratified-wavy and stratified-atomizing, flow patterns encountered in horizontal two-phase flows, finally,

- Chapter 12 involves with the development of eight-sensor parallel-wire probe method designed for a horizontal annular flow-pattern.

Analytical efforts were focused on deriving predictive mechanistic models describing the interfacial structure in various adiabatic two-phase flow patterns as well as predicting fluid particle behavior in boiling channels. Chapters 13 through 18 summarize these efforts. More specifically, they are summarized as follows:

- Chapter 3 deals with the derivation of the interfacial area transport equation and discusses the basic mechanisms affecting the source and sink terms appearing in the interfacial transport equation.
- Chapter 13 is devoted to studies of interfacial instabilities of horizontal stratified flows,
- Chapter 14 describes mechanistic scaling methodologies and derivation of similarity requirements for two-phase flow-regime transitions,
- Chapter 15 presents an analysis of the spherically symmetric phased change (moving boundary) problem to describe the bubble growth and/or collapse in boiling channels,
- Chapter 16 describes several droplet disintegration mechanisms and offers droplet size and size distributions model for an annular flow-pattern, whereas Chapter 17 is devoted to bubble break-up processes and describes a detailed mechanistic model describing the averaged bubble size and size distributions in bubbly flow, finally,
- Chapter 18 deals with modeling of void fraction and interfacial area concentration and concludes that the core break-up is a dominant break-up mechanism, which determines the interfacial area concentration in the core whereas the Taylor break-up mechanism becomes dominant around the pipe perimeter.

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11. DEVELOPMENT OF DOUBLE-SENSOR, PARALLEL-WIRE CONDUCTIVITY PROBE FOR HORIZONTAL STRATIFIED FLOW PATTERNS

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ABSTRACT

The interfacial characteristic parameters of horizontal stratified wavy flow patterns were experimentally investigated for a mixture of air and water two-phase flow by using the double-sensor, parallel wire conductance probe method. The experiments were conducted in a horizontal flow loop 15.4 m long consisting of Pyrex glass tubing of 50.3 mm i.d. The range of gas superficial velocities was from 0.85 to 31.67 m s⁻¹ and the liquid superficial velocities ranged from 0.014 to 0.127 m s⁻¹. Several interfacial wave patterns as described by Andritsos and Hanratty (Int. J. Multiphase Flow 13 (1987a) 583-603) were identified and their characteristic parameters such as wave height, most dominant frequency, mean propagation velocity and mean wavelength were investigated in terms of liquid and gas flow rates. The interfacial shear stress calculated from the experimental measurements was used to evaluate the most widely used interfacial shear models.

11.1 Introduction

A stratified two-phase flow pattern, which is characterized by the liquid moving along the bottom of the flow channel and the gas concurrently above it, exists for low gas and liquid flow rates in a horizontal flow channel. When the gas velocity is sufficiently low, the gas-liquid interface appears undisturbed. This flow structure is recognized in various horizontal flow pattern maps (Baker, 1954; Mandhane, 1974; Taitel, 1976a) by the definition of a stratified smooth flow pattern. However, as the gas velocity is increased in stratified flow, waves are formed on the gas-liquid interface, giving a stratified wavy flow pattern. At sufficiently high gas velocities, long wavelength, large amplitude waves with a highly irregular interface appear. At very high gas velocities, at

optimization occurs through the removal of wavelets riding on top of the waves by the shearing gas flow. For these stratified wavy flow patterns the structure and dynamics of the interface greatly influence the rates of mass, momentum and heat transfer as well as the stability of the system. Therefore an accurate knowledge of interfacial wave formation and wave parameters such as wavelength, speed of propagation, amplitude and more detailed wave structure is essential to the eventual understanding of the interfacial transfer mechanisms and two-phase flow pattern transition in a large number of engineering applications.

For the purpose of providing basic information on the interfacial wave parameters, horizontal, stratified, two-phase flow has been widely studied experimentally and analytically. Most of the earlier studies of wave generation in concurrent gas-liquid flows have been conducted for rectangular channels with a large aspect ratio (Hanratty, 1957, 1961, 1983). Later Andritsos and Hanratty (1987a) classified the stratified flow patterns observed in horizontal circular channels, 25.2 and 95.3 mm i.d., for a fixed liquid flow rate as the gas flow rate is increased. They are (a) smooth surface, (b) regular, small amplitude two-dimensional waves, (c) large amplitude irregular waves and (d) atomization. Linear stability theory was used by Andritsos and Hanratty to provide a physical interpretation and to predict conditions for the initiation of these stratified flow patterns. Although these stratified flow structures can be considered simple for analytical treatments, large differences have been observed between measurements and available predictive methods for pressure drop and void fraction.

A work by Lockhart and Martinelli (1949) is one of the earliest studies to predict analytically the pressure drop and the void fraction for two-phase gas-liquid flows. Though this attempt may be the simplest one and applicable to all flow patterns, as suggested by many researchers, the measured pressure drops for stratified flow patterns are always smaller than those predicted by the Lockhart- Martinelli method. Moreover, the calculated values of void fraction are also in error, sometimes by as much as 100%/G, compared with the experimental values. Taitel and Dukler (1976b) improved the Lockhart- Martinelli method for predicting frictional pressure drop and liquid height by considering separate momentum balances for fully developed gas and liquid flows. In recent years, however, Andritsos and Hanratty (1987b) demonstrated that the Taitel and

Dukler method was incorrect because of the assumption that the interfacial shear is the same as that for a smooth surface and because the Blasius equation is used to evaluate the wall shear on the liquidwall interface. Based on their 25.2 and 95.3 mm i.d. flow channel observations, Andritsos and Hanratty showed that interfacial waves can have a significant influence on the interfacial shear.

In the present paper we first describe the results of an experimental work for concurrent, stratified, air-water flow in a horizontal 50.3 mm i.d. flow channel. The purpose of this experiment is to obtain background data for the initiation of various interfacial wave patterns and to investigate the characteristic wave parameters in terms of liquid and gas flow rates. The last part of the paper deals with calculation of the interfacial shear from the experimental measurements to investigate the influence of waves on the shear and to evaluate various interfacial shear models suggested in the literature.

11.2 Experimental set-up and procedure

11.2.1 Description of the flow loop

A horizontal flow loop was designed and built for the purpose of investigating the interfacial structure of horizontal two-phase flow. The overall loop schematic is illustrated in Fig. 1. The loop basically consists of various flanged lengths of 50.3 mm i.d. circular Pyrex glass tubing with pressure tabs installed between them. However, smaller or larger diameter test sections can be easily fitted to the loop. The entire test section is about 15.4 m in length and is all transparent, so that flow visualization, high speed photography and high speed cinematography are possible. It is designed such that various local instrumentations for two-phase flow measurements and different mixing chambers can be easily accommodated.

The air and water are used as coupling fluids. The air to the test section is supplied from the central air system in the institution. It is, however, regulated through a 0.95m³ capacity high pressure storage tank and metered by a series of turbine flowmeters. The water is recirculated. It is pumped from a 1.9 m³ capacity storage tank by a stainless steel centrifugal pump and regulated from 0% to 100% of the pump capacity by a transistor inverter. The water flow rate is measured by a series of paddle-wheel

flowmeters assembled in a parallel configuration. As shown in Fig. 2, the air enters the mixing chamber from a 90° vertical leg and is injected into the water flow through a cylindrical porous medium of 100 μm porosity to achieve a uniform mixing. The two-phase mixture from the test section is directed to an air-water separator. The air is vented to the atmosphere and the water is returned to the water storage tank.

The last 1.5 m of the test section incorporates two quick-closing valves. These valves are pneumatically operated and electronically controlled and are synchronized through a common electrical switch to ensure simultaneous operation. The distance between the valves is long enough to minimize any experimental error. The system is protected against pressure surges.

Pressure transducers of the diaphragm type are utilized for both absolute pressure and differential pressure measurements and a series of U-tube manometers are also used for differential pressure measurements. The test section differential pressure is measured at six intervals with high frequency transducers, with a natural frequency of 5 kHz, located 1.55 m apart from each other. Their range is from 0 to 34.4 kPa with an accuracy of $\pm 0.3\%$ of the full scale. The absolute pressure transducers are located at two locations in the test section, 6.70 and 8.22 m downstream of the mixing chamber respectively. They have a range from 0 to 172 kPa with an accuracy of $\pm 0.25^\circ/\text{D}$ of the full scale. More details of the flow loop can be found in Shi (1991).

11.2.2 Test Section

A large number of liquid film thickness measurement techniques have been used with varying success. One method that has been particularly popular is the conductance technique, which was briefly reviewed by Hewitt (1978). All conductance probes have a basic relationship between the thickness of a liquid film and the electrical resistance between two electrodes that depends upon the location of the electrodes relative to the film. A number of electrode geometries have been utilized. Numerous investigators have used a probe that consisted of two electrodes flush mounted in the surface over which the film is formed. Coney (1973) theoretically analyzed and experimentally tested conductivity probes consisting of two parallel rectangular strips in either a flat surface or aligned with the axis of curvature on a cylindrical surface. From his results it can be

inferred that flush-mounted probes have excellent spatial resolution and thickness resolution for measurements of the order of 2 mm or less, but suffer loss of resolution for thicker films.

By changing the form of the electrodes to parallel wires mounted normal to the wall, it is possible to greatly increase the range of liquid thickness over which the instrument is useful. A probe of this form was first used by Swanson (1966) and later by Miya et al. (1971) and Andritsos and Hanratty (1987b) to measure the interfacial wave properties for horizontal, stratified flows. Recently Koskie et al. (1989) compared parallel wire probes with needle contact probes, which are another method for measuring large film thickness. It was concluded that the parallel wire probe had a distinct advantage over the needle contact probe because it provides time-resolved liquid film thickness data. A calibrated parallel wire probe yields accurate and continuous thickness measurements for thick films. This type of data is necessary for autocorrelation data and studies of wave shapes.

In view of the above, a double-sensor, parallel wire conductance probe was used in the present investigation to measure the basic characteristics of interfacial waves. The probe used for the measurement is shown in Fig. 3. It consists of two identical sensors 10.8 mm apart from each other. Each sensor consists of two parallel chromel wires 0.25 mm in diameter and with 3.5 mm spacing in between. The sensors are aligned perpendicular to the direction of the flow. It should be noted that it was essential to properly determine the distance between the two sensors. The signal-noise ratio of the output signal of each sensor may decrease with the distance between the two sensors because of larger interference. On the other hand, identification of corresponding waves in the two signals becomes more difficult as the distance increases. A series of preliminary experiments were conducted to compromise these two points and a suitable separation distance of 10.8 mm was set.

A probe manifold was constructed from Plexiglass and used to hold and align the conductance probe in place. The wires were inserted into the Plexiglass cylinders, which were plugged into the manifold. Epoxy was used to cement the wires at the lower cylinders and clay was used to seal the wire holes at the upper cylinders. There was a metal frame fixed on the manifold with four springs hung on and four nylon rings were

connected with the springs. The wires coming from the upper cylinders were led to pass through the spring rings, which could stretch the wires firmly and always keep them in tension. If the wires are stretched with sufficient tension, the noise of the signal induced by wire vibration can be eliminated. The ends of the wires were led to the terminal and then connected to cables.

The Plexiglass manifold is 14.4 cm in length, 14.4 cm in height and 7.0 cm in width. A cylindrical section was machined out of the center of the manifold. Considerable care was used to ensure that the diameter of the manifold cylinder was the same as that of the glass tubing. O-ring grooves were machined into two faces of the conductance probe manifold and O-rings were used to seal the joints. The probe manifold was aligned and bolted between two pieces of glass test section piping. The manifold was located about 50 diameters from the nearest upstream joint and about 8 diameters from the nearest downstream joint.

An analyzer circuit was built. The power source to the circuit was through a 4.5 V battery. The voltage was kept as low as possible to avoid the effects of electrolysis. The analyzer circuit sent two a.c. signals to the two sensors simultaneously. The circuit converted the signal across the probe wires to a d.c. voltage reading proportional to the conductance and therefore to the liquid height between the probe wires. The output from the analyzer circuit was designed so that it ranged from 0 to 5 V d.c., corresponding to liquid heights in the range from 0 to 50.3 mm. The output signals were monitored by a Metrabyte model DASH-16 data acquisition system and a computer.

11.2.3 Experimental procedure

There is no theoretical method available for a particular probe. Thus a calibration had to be performed to determine the exact relation between liquid heights and response signals for a given probe geometry. The static calibration procedure was performed every day and the calibration curves of liquid height vs. analyzer output voltage were then constructed for each sensor to calculate liquid heights from conductivity measurements.

For each preset experimental condition the data were recorded, including conductance probe measurements of the instantaneous liquid height in two locations at a distance of 10.8 mm and with pressure drops at six intervals and the absolute system

pressure at two locations. From the data so obtained it was possible to calculate other parameters such as time-averaged mean liquid height, basic liquid height, most dominant wave frequency, mean wave propagation velocity and space separation of waves (wavelength) as well as the interfacial shear stress. It is estimated that the uncertainty in measuring the time-averaged liquid height is less than 5%. At the end of each experiment the quick-closing valves were operated to measure the average void fraction, which was used to check the validity of the liquid thickness measurements. Experiments were interfaced with a data acquisition system utilizing a Zenith PC/ AT computer with a Metrabyte DASH-16 16 channel multifunction high speed analog-digital I/O expansion board and Labtech Notebook software.

Earlier investigators normally chose a sampling rate ranging from 500 Hz to 3 kHz and a sampling time from 2 to 20 s in their wavy and annular flow experiments. In order to choose proper values, a series of sensitivity studies were undertaken. Data were taken at 500, 800 and 1000 Hz sampling rates and from 5 to 35s sampling time. It appeared that when the sampling time was longer than 15s, the measurements in frequency, velocity, average liquid height, basic film height and average wave height all tended to converge irrespective of the sampling rate. In this study a sampling rate of 800 Hz and a sampling time of 20s were used throughout the experimentation.

The typical output signals of the double-sensor, parallel wire conductance probe at three gas superficial velocities are shown in Fig. 4. It is interesting to note that the output signals of the conductance probes do not differ markedly from each other, which means that the small amplitude regular waves as well as the large amplitude irregular waves propagate without significant changes in their shape for the short distance of 10.8 mm between the two sensors. This was also confirmed by visual observations. The front of the wave may therefore propagate with the same velocity as the wave crest.

The time-averaged liquid height $\bar{h}$ was then calculated by

$$\bar{h} = \frac{1}{t_2 - t_1} \int_{t_1}^{t_2} h(t) dt \quad (1)$$

where t_1 , and t_2 are the times indicating initiation and termination of a given experiment respectively, while h is the instantaneous liquid height. It is to be noted that either the front or rear sensor signals can be used in Eq. (1). Considering small deformations and

disturbances generated by the wires, only the front sensor signals were used for calculating $\bar{h}$.

As shown in Fig. 4, a wave passing through the channel is first touching the upstream sensor and then the downstream one. The wave propagation velocity can be determined from the signals of the front and rear sensors. The time delay between the two signals is a measure of the velocity of the wave. As a result, the wave velocity can be expressed as

$$c = \frac{l}{\Delta t} \quad (2)$$

where l is the distance between the two sensors and Δt is the time delay. Here the cross-correlation method was used to determine Δt . The cross-correlation function, which gives the most probable time delay, was computed. If $x(t)$ and $y(t)$ are two signals from the front and rear sensors respectively, then the cross-correlation function $F_{xy}(\Delta t_m)$ is given by

$$F_{xy}(\Delta t_m) = \frac{1}{t_2 - t_1} \int_{t_1}^{t_2} x(t - \Delta t_m) y(t) dt \quad (3)$$

The maximum value of $F_{xy}(\Delta t_m)$ yields the most probable time delay Δt_m , from which the interfacial wave propagation velocity is determined through the use of Eq. (2).

The frequency spectrum was used to determine the most dominant wave frequency. The Fourier transform was employed to transform spectral data from the time domain to the frequency domain. The frequency of the dominant component, i.e. the fundamental component, was considered as the wave frequency since it carried the most power.

11.2.4 Experimental conditions

The experiments were carried out under quasi-steady, fully developed flow conditions at a location of $L/D = 253$ downstream of the mixing chamber. By varying the air and water flow rates, a wide range of stratified wavy flow pattern regions was covered. The values of the superficial liquid velocity j_f were 0.018, 0.024, 0.028, 0.035, 0.043 and 0.052 m/s. Fig. 5 shows the range of basic operating conditions in the j_f - j_g

plane. The water temperature was kept at about 20°C and the system pressure at about 120 kPa during the course of each experiment.

11.3 Results and discussion

11.3.1 Interfacial wave patterns

Fig. 6 is a typical example of the dimensionless liquid thickness fluctuations recorded by the front sensor of the parallel wire probe located at a distance of $L/D = 253$ downstream of the mixing chamber. The observed flow patterns under the present experimental conditions were almost identical with those previously reported for the horizontal air-water system of Andritsos and Hanratty (1987a) in a 25.2 mm i.d. pipeline. For a fixed water flow rate the first waves appearing with an increasing gas velocity are small amplitude, two-dimensional (2D) waves that can be observed visually by light reflection from the liquid surface. It is evident from Fig. 6 that these waves are regular in appearance. However, as the gas superficial velocity gradually increases from 1.094 to 4.34 m/s, the wavelength decreases. At the higher gas superficial velocities the interfacial wave structure changes into irregular, large amplitude (LA) waves. The profile of peaks corresponding to the large amplitude wave comprises a fairly steep front face and a relatively longer rear face, which is one of the notable features of the large amplitude waves as described by Andritsos and Hanratty (1987a) and Sawai et al. (1989). Above a certain gas velocity, droplets or liquid filaments are torn from the liquid phase and deposited on the pipe wall. This flow pattern, which is identified as atomization (AT) by Andritsos and Hanratty, is defined as the gas velocity at which droplets first hit the top surface of the pipe. For the example illustrated in Fig. 6 it occurred at $j \approx 8.384$ m/s. As the liquid superficial velocity increases, the time tracings observed in Fig. 6 are similar in nature. The main difference is that the transitions from stratified smooth to 2D and from 2D to LA occur at lower gas velocities and the range of gas superficial velocities in which these wavy patterns exist covers a narrower range of gas velocities as the liquid superficial velocity increases. When j_f increases beyond 0.080.1 m/s, the stratified flow patterns are replaced by intermittent flow patterns.

Fig. 7 summarizes the experimental flow pattern transitions for the horizontal air-water flow in a 50.3 mm i.d. pipe together with Taitel and Dukler (1976a) and Mandhane

et al. (1974) flow pattern maps in the j_f - j_g plane. This figure also shows the detailed wavy flow pattern transitions of air-water flow observed by Andritsos and Hanratty (1987a) in a 25.2 mm i.d. pipeline. Several observations can be made from this figure.

(1) It is evident from the figure that the transitions to slug and annular flow patterns are parallel to those observed by Andritsos and Hanratty. However, the transitions from stratified to slug flow occur at higher liquid velocities and the annular flow transition occurs at higher gas velocities than those observed in the 25.2 mm i.d. flow channel. The same tendency is also predicted by the Taitel and Dukler and Mandhane et al. flow pattern transition criteria. A similar trend was observed for the annular flow transition by Francis and Lahey (1992). Higher gas and liquid velocities are needed to initiate the annular and slug flow respectively.

(2) The transitions from each stratified flow pattern agree approximately with those observed by Andritsos and Hanratty. The experimental observations of the present study indicate that the Mandhane et al. criterion for the initiation of stratified wavy flow nearly corresponds to the initiation of the large amplitude waves, whereas the transition to the stratified wavy flow proposed by Taitel and Dukler is closer to the initiation of regular 2D waves.

(3) Fig. 7 also illustrates the wavy flow pattern transitions as suggested by Andritsos and Hanratty. According to these authors, the criterion for predicting the initiation of regular 2D waves is given by

$$j_g \geq 2\bar{\alpha} \left(\frac{(1-\alpha)v_f(\rho_f - \rho_g)g}{s\rho_g j_f} \right)^{1/2} \quad (4)$$

where $\bar{\alpha}$ is the mean void fraction corresponding to the time-averaged liquid thickness $\bar{h}$ and s is Jeffreys' sheltering coefficient. Fig. 7 illustrates predictions from Eq. (4) by the "long-dashed" line with $s = 0.06$ as suggested by Andritsos and Hanratty. From this figure it is evident that Eq. (4) approximates the data in the average sense.

(4) Initiation of the large amplitude waves as suggested by Andritsos and Hanratty is shown in Fig. 7 by the "chain" line. The criterion is given by

$$j_g = j_{g)K-H} \left[\tan h \left(\frac{k_m \bar{h}}{10} \right) \right]^{-0.1} \frac{1}{\alpha^2} \left(\frac{\theta_w}{\theta} \right)^{0.025} \quad (5)$$

where θ is the dimensionless group to account for the effects of liquid viscosity, $j_{g(K-H)}$ is the gas superficial velocity and k_m is the characteristic wavenumber as predicted by the Kelvin-Helmholtz (K-H) theory. These terms are given by

$$\theta \equiv \frac{\rho_f}{\rho_g} \left(\frac{\bar{\alpha}\sigma}{\mu_f\mu_g} \right)^2 \quad (6)$$

$$j_{g(K-H)} = \frac{\bar{\alpha}}{1-\bar{\alpha}} j_f + \frac{1}{\alpha} \left(\frac{k_m\sigma}{\rho_g} + \frac{\Delta\rho g}{k_m\rho_g} \right)^{1/2} \quad (7)$$

$$k_m = \left(\frac{\rho_f g}{\sigma} \right)^{1/2} \quad (8)$$

The term θ_w appearing in Eq. (5) is the value of θ if the liquid is water. For the present experiments $\theta_w/\theta = 1.0$.

(5) Finally, an approximate criterion recommended by Andritsos and Hanratty for the initiation of the wavy atomization flow pattern is shown in Fig. 7 by the "short-dashed" line. The expression for this transition is

$$j_{ga} = 1.8j_g \quad (9)$$

where j_g is the superficial velocity predicted by Eq. (5). Eq. (9) shows close agreement in tendency with the experimental observations. However, the value of the coefficient required by the data must be a little larger than the value appearing in Eq. (9)

11.3.2 Wave parameters

11.3.2.1 Time-averaged liquid thickness

The dimensionless time-averaged mean liquid thickness $\bar{h}/d$ as a function of the gas superficial velocity is shown in Fig. 8, where the liquid superficial velocity is treated as a parameter. From this figure it is evident that the average liquid thickness decreases with increasing j_g for a given value of j_f and that the slope indicating the rate of decrease in liquid thickness with respect to the gas superficial velocity shows strikingly different behavior in the 2D and LA wave regions. For a fixed liquid superficial velocity a sharp decrease in $\bar{h}/d$ is observed in the 2D region, whereas the decrease is gradual in the LA region. It is also important to note that the slope has an almost constant value in the LA region unaffected by the liquid flow. As j_f increases, however, the average liquid film

thickness increases proportionally in this region. In a very close situation, $j_f = 0.024$ m/s and $j_g = 1-15$ m/s, the results observed here are comparable with those observed by Andritsos and Hanratty (1987b) in a 25.2 mm horizontal air-water experiment, indicating that the dimensionless liquid thickness is insensitive to changes in pipe diameter if comparison is made at the same liquid velocities.

Figs. 9 and 10 compare the liquid thickness measurements with the predictive correlations developed by Taitel and Dukler (1976a) and Andritsos and Hanratty (1987b). The Taitel and Dukler analysis provides a correlation similar to that suggested by Lockhart and Martinelli (1949), in that for a horizontal flow the dimensionless liquid height $\bar{h}/d$ is a unique function of the LockhartMartinelli flow parameter χ . The predictions of the Taitel and Dukler method are generally higher than the experimental values, although their error margin is much higher at the lower values of χ .

This discrepancy may be attributed to the fact that the Taitel and Dukler method treats the interface as a smooth surface and utilizes a singlephase flow friction factor for smooth surfaces. Since $\chi \propto j_g^{-n}$ with $n \geq 1.0$, it is expected that the effect of a wavy interface will be more prominent at low values of χ . This is the case observed in Fig. 9. Fig. 10 shows fairly good agreement with the Andritsos and Hanratty method. The obvious discrepancy observed in the low gas velocity region may be partially attributed to the experimental uncertainties in measuring the pressure drop.

11.3.2.2 Wave frequency

The most dominant frequency f_m of the interfacial waves, which was obtained from the wave frequency power spectrum, is shown in Fig. 11 as a function of the gas superficial velocity. This figure indicates that the values of frequency increase significantly with increasing j_g , going through a peak at the transition from two-dimensional to large amplitude waves and then descending to a stable value toward the atomization region. This may be attributed to the coalescence of large amplitude waves to form larger waves, resulting in an apparent decrease in the most dominant wave frequencies as the gas flow rate increases. These waves may be thought of as surges in the volumetric flow of the liquid rolling over small ripples, so that the dominant frequency decreases. As j_f increases, the frequency of interfacial waves decreases and the

peaks observed in Fig. 11 also drop. In the frequency power spectrum a clear peak appears at lower j_g values. This is produced by a near-sinusoidal wave pattern observed for regular 2D waves. As j_g increases, more and more disturbances appear and the power spectrum tends to broaden and flatten.

11.3.2.3 Wave propagation velocity

Mean interfacial wave propagation velocities are illustrated in Figs. 12 and 13 in dimensional and dimensionless forms respectively. At lower gas superficial velocities in the 2D wave region the wave velocities show a slight decrease. Jurman et al. (1988) observed similar tendencies in their rectangular channel experiments. However, the wave velocity linearly increases with j_g in both the LA and atomization regions. In the majority of large amplitude wave regions the wave velocities are between 0.5 and 0.8 m/s. These numerical values are about the same as those reported by Miya et al. (1971) for their air-water experiments in a rectangular flow channel.

An important common feature is that the rate of increase in the propagation velocity with j_g is not affected by the liquid flow rate in the 2D and atomization regions. As seen in Fig. 13, the dimensionless wave velocity c/j_g decreases drastically with j_f/j_g in the 2D wave region but reaches a constant value of 0.09 in the LA and atomization regions, irrespective of j_f/j_g values. This may be attributed to the coalescence of the large amplitude waves to form an even larger accelerating wave with increasing gas velocity, resulting in a constant value of c/j_g in the 2D and atomization regions.

The present data concerning the LA and AT regions for the mean propagation velocity were correlated in terms of the interfacial shear stress τ_i . Here c is proportional to the friction velocity u^* as shown in Fig. 14; the friction velocity is defined for the liquid phase as $u^* \equiv (\tau_i / \rho_f)^{1/2}$, where ρ_f is the density of the liquid phase. As shown below, the interfacial shear stress is estimated from the gas momentum balance with the measured axial pressure gradient and the time-averaged mean liquid thickness. Although a slight but systematic effect of the liquid superficial velocity can be observed from Fig. 14, a correlation in the form of

$$c = a(u^*)^n \quad (10)$$

appears to give a good average fit to the measurements. Considering the complicated nature of LA stratified wavy flow patterns, Eq. (10) represents the data quite well. It is to be noted that a similar observation was made by Sawai et al. (1989) for an air-water system at three different values of the gas-liquid density ratio, $\rho_f/\rho_g = 802, 276$ and 152 , which correspond to system pressure values of $103, 299$ and 544 kPa respectively.

11.3.2.4 Space separation of waves

Fig. 15 shows the mean value of the space separation, i.e. mean wavelength Λ , between successive waves, which is defined as the product of the mean propagation velocity c and the most dominant wave time separation $1/f_m$. In the 2D wave region the experimental results show that the mean wavelength characterizing the most dominant waves is relatively long and wave frequencies are generally low. This is because when gas flows over a liquid surface at a low flow rate, there is an attempt to minimize the interfacial curvature to maintain the equilibrium, thus resulting in a longer wavelength. As the gas flow rate increases, the interfacial shear stress increases, resulting in an increase in the energy transfer to the liquid surface. The interfacial curvature becomes larger, so the wavelengths decrease and wave frequencies increase. As indicated above with respect to the wave frequency behavior, in the large amplitude wave region the coalescence of waves results in faster-moving, larger wavelength waves with increasing gas velocity. This justifies the behavior observed in Fig. 15.

11.3.2.5 Interfacial shear

In order to investigate the influence of interfacial waves on the gas-liquid and liquid-solid interfacial stresses, these stresses are calculated by using the momentum equations for quasi-steady, fully developed flow. Under these conditions the momentum equations for the gas and liquid phases are given as

$$-A_g \frac{dP}{dz} = \tau_i \xi_i - \tau_{gw} \xi_g \quad (11)$$

$$-A_f \frac{dP}{dz} + \tau_i \xi_i - \tau_{fw} \xi_f = 0 \quad (12)$$

where dP/dz is the axial pressure gradient, A_f and A_g are the liquid and gas cross-sectional areas respectively, the ξ s are the wetted perimeters and τ_i , τ_{gw} and τ_{fw} are the shear stresses at the gas-liquid, gas-wall and liquid-wall interfaces respectively.

Eqs. (11) and (12) are solved for τ_i and τ_{fw} by using the experimentally measured values for dP/dz and the geometric variables A_f , A_g , ξ_i , ξ_g and ξ_f . These geometric variables can be easily calculated in terms of the experimentally measured time-averaged mean liquid thickness $\bar{h}$. The gas-wall interface shear τ_{gw} is calculated in terms of the friction factor f_g as

$$\tau_{gw} = f_g \frac{\rho_g u_g^2}{2} \quad (13)$$

where u_g is the actual average gas velocity; f_{gw} is calculated from the Blasius equation by using the gas phase Reynolds number $Re_g \equiv \rho_g d_{hg} u_g / \mu_g$ with the hydraulic diameter defined by

$$d_{hg} \equiv \frac{4A_g}{\xi_g} \quad (14)$$

Fig. 16 illustrates the effects of gas and liquid superficial velocities on f_i/f_g , where f_i is the calculated value of the interfacial friction factor and f_g is the friction factor based on the assumption of a smooth interface. For low gas flow rates, i.e. for the small amplitude, two-dimensional wave region, the major factor influencing f_i/f_g is the liquid flow rate. For a given liquid flow rate f_i/f_g stays constant with increasing gas flow. Even in this region there is a slight effect of interfacial waves on f_i/f_g as indicated by the numerical values of f_i/f_g . If there were no wave effects, f_i/f_g should have been close to unity. With the appearance of large amplitude waves f_i/f_g shows a rapid increase. In fact, for a given liquid flow rate f_i/f_g increases linearly with j_g in the large amplitude wave region. In this area the primary factor affecting f_i/f_g is the gas velocity. The liquid flow rate effect shows a decrease with increasing gas velocity.

There are several methods for predicting the interfacial shear stress for stratified gas-liquid flows (Lockhart, 1949; Johannesen, 1972; Russel, 1974; Taitel, 1976b; Cheremisinoff, 1979; Kowalski, 1984; Andritsos, 1987b; Jurman, 1988). Figs. 17-19 compare the interfacial friction factor data with the predictive methods suggested by

Cheremisinoff and Davis (1979), Kowalski (1984) and Andritsos and Hanratty (1987b) respectively. Fig. 17, where $Re_f^* \equiv (j_f / j_g)(\pi d^2 / 4 \xi_i)$, shows that f_i cannot be correlated in terms of Re_f^* as suggested by Cheremisinoff and Davis. It significantly over-estimates the interfacial friction factor within the range of Re_f^* covered by the present experiments.

From Fig. 18 it can be seen that Kowalski's method appears to give satisfactory agreement with the measurements at relatively large values of f_i , i.e. at the high gas flows. At the lower gas flows the predictions significantly underestimate the data.

Recently Andritsos and Hanratty (1987b) improved the existing predictive methods and suggested the empirical correlation

$$\frac{f_i}{f_g} = 1 \quad \text{for } j_g \leq \left(\frac{\rho_{go}}{\rho_g} \right)^{1/2} (5m/s)$$

$$\frac{f_i}{f_g} = 1 + 15 \left(\frac{\bar{h}}{d} \right)^{1/2} \left[\frac{j_g}{5} \left(\frac{\rho_g}{\rho_{go}} \right)^{1/2} - 1 \right] \quad \text{for } j_g \geq \left(\frac{\rho_{go}}{\rho_g} \right)^{1/2} (5m/s) \quad (15)$$

where ρ_{go} is the gas density at atmospheric pressure.

The comparison of the measurements with this correlation is given in Fig. 19. At the lower end of f_i/f_g there is an apparent data scatter in the small amplitude wave region. This may be partially due to the experimental uncertainties in measurements encountered at relatively low gas flow rates. However, even taking these discrepancies into account, Andritsos and Hanratty's method appears to predict the experimental data quite well in the wide range of operational variables covered in the present study.

11.4 Summary and Conclusions

Several interfacial stratified wavy flow patterns were identified for air-water two-phase flow in a 50.3 mm transparent horizontal flow channel. It was found that the observed wave patterns were in complete agreement with Andritsos and Hanratty's (1987a) descriptions for a 25.2 mm air-water flow in a horizontal pipeline. Transition criteria suggested by these authors gave relatively good results for describing the initiation of stratified wavy flow patterns.

The liquid height time fluctuations were traced by using a double-sensor, parallel wire conductivity probe technique. Time-averaged mean liquid thickness, most dominant frequencies, mean propagation velocities and mean wavelengths of interfacial waves were analyzed as a function of superficial volumetric fluxes of gas and liquid phases. It was found that the gas velocity was the major factor affecting these wave parameters in the large amplitude, irregular stratified wavy region. The liquid velocity had secondary effects.

The axial pressure gradient and mean liquid thickness measurements were used to evaluate the available interfacial shear stress and liquid height predictive models. It was found that the most recent models proposed by Hanratty and his coworkers were satisfactorily within the range of experimental operating conditions.

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Nomenclature

A	cross-sectional area
c	mean interfacial wave propagation velocity
d	pipe diameter
d_{hf}	hydraulic diameter for liquid phase
d_{hg}	hydraulic diameter for gas phase
f	friction factor
f_m	most dominant interfacial wave frequency
F_{xy}	cross-correlation function
g	gravitational acceleration
h	instantaneous liquid height
$\bar{h}$	time-averaged liquid height

j	superficial velocity
k_m	characteristic wavenumber
l	distance between rear and front sensors
P	pressure
Re_f	liquid Reynolds number ($Re_f \equiv \rho_f d_{hf} u_f / \mu_f$)
Re_g	gas Reynolds number ($Re_g \equiv \rho_g d_{hg} u_g / \mu_g$)
u	average velocity
u^*	friction velocity
z	coordinate in direction of flow

Greek letters

$\bar{\alpha}$	mean void fraction
Δt	time delay
θ	variable defined by Eq. (6)
λ	wavelength
μ	dynamic viscosity
ν	kinematic viscosity
ξ	wetted perimeter
ρ	density
σ	surface tension
τ	shear stress
χ	Lockhart- Martinelli parameter

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HORIZONTAL TWO-PHASE FLOW LOOP

A - Interchangeable Air-Water mixing chambers	K - Pneumatic operated ball valves
B - Water flow meters of appropriate size	L - Motor control
C - Water flow meter control valves	M - computer and data acquisition system
D - Air flow meters of appropriate size	N - 250 gal. Air tank
E - Air flow meter control valves	P - 500 gal. Water tank
F - Air flow regulating valves	Q - Air-Water separator, with internal baffles
G - Air pressure regulator	R - Water shut-off valve
H - Air filter	S - 20 hp. 750 gpm Water pump
I - Water pressure relief valves	T - Glass pipe couplings with pressure taps
J - Water flow regulating valves	

Fig. 1. Schematic of the experimental flow loop.

Fig. 2. Schematic of the air-water mixing chamber.

Fig. 3. Double-sensor, parallel-wire conductance probe.

Fig. 4. Typical output signals of (from top to bottom) low, moderate and high gas flows.

Fig. 5. Experimental conditions.

Fig. 7. Flow pattern transitions.

Fig. 6. Recordings of liquid thickness signals for a fixed liquid flow rate.

Fig. 8. Effect of liquid and gas superficial velocities on time-averaged liquid thickness.

Fig. 9. Comparison of time-averaged liquid thickness measurements with Taitel and Dukler's predictions.

Fig. 10. Comparisons of time-averaged liquid thickness measurements with those predicted by Andritsos and Hanratty's method.

Fig. 11. Variation in most dominant frequency with gas and liquid superficial velocities.

Fig. 12. Variation in mean wave propagation velocity with gas and liquid superficial velocities.

Fig. 13. Dimensionless presentation of mean propagation velocity data.

Fig. 14. Correlation of mean propagation velocity for air-water system.

Fig. 15. Variation in mean space separation with gas and liquid superficial velocities.

Fig. 16. Influence of gas and liquid superficial velocities on interfacial friction factor.

Fig. 17. Presentation of interfacial friction factor data in terms of Chermisinoff and Davis's correlation parameter.

Fig. 18. Comparison of interfacial friction factor data with Kowalski's correlation.

Fig. 19. Comparison of interfacial friction factor data with Andritsos and Hanratty's correlation.

12. DEVELOPMENT OF EIGHT-SENSOR CONDUCTIVITY PROBE AND PITOT TUBE SAMPLING PROBE FOR ANNULAR FLOW MEASUREMENTS

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Abstract

The characteristics of horizontal, annular two-phase flow were experimentally investigated for a mixture of low-pressure air and water. The experiments were conducted in a horizontal loop consisting of a 50.3 mm diameter circular pyrex glass piping with an overall length of 16 m.

For a very wide range of flow conditions, the liquid film characteristics were measured as a function of a circumferential position. The local liquid entrainment flux measurements were made in the gas core. In addition, pressure drop in the developing and fully-developed zones as well as the average void fraction were measured. The measured values of those physical quantities and the highest circumferential location from which liquid film starts to form toward the tube bottom are presented, and the effects of various flow parameters on them are examined. Results are analyzed, and the compatibility of the liquid film thickness measurements with the average liquid fraction is discussed. Simple empirical correlations for the liquid film thickness distribution and entrainment fraction are proposed.

12.1. Introduction

The annular two-phase flow-pattern can occur in a variety of important industrial operations. Several examples may be listed as follows: in the evaporators and condensers of conventional power generating plants; in pressurized nuclear reactor power plants during loss of coolant accident; in gas-oil pipelines; and in many other operations of the process industry.

The annular two-phase flow-pattern is basically characterized by a liquid film at the channel wall and a central core of gas containing liquid droplets. The droplets originate from large interfacial waves between the liquid film and the gas core. They are entrained into the gas core, accelerated by it to a much higher velocity compared with the slow moving liquid film, travel

through it, and end-up redepositing back on the liquid film. From this brief description, it may be inferred that for a given operating condition, the liquid film thickness, entrainment and deposition rates are the basic parameters which characterize the annular flow. These parameters may well describe the vertical annular flow where the radial symmetry can be assumed. However, in horizontal annular flow these parameters are dependent upon the circumferential position; and, therefore, it is important to know the variation of these parameters along the perimeter.

Despite the many studies which have been made of horizontal annular two-phase flow, only a few of them have been concerned with the details of the interfacial structure [1-9]. Most have been concerned with overall phenomena such as overall pressure drop or void fraction. There are, however, cases in which it is desirable to understand the details of the flow structure. For example, variations in conditions around the perimeter of the tube can become very important, since they give rise to circumferential variation in mass, momentum and heat transfer characteristics. Finally, most of the available experimental data in horizontal annular flow are limited by small diameter pipes about 25.4 mm or smaller.

The present study which deals with the experimental observations on the horizontal two-phase flow is basically concerned with the measurements of local liquid film thickness, local entrainment flux, the void fraction, and the pressure gradient for a relatively wide range of flow conditions in a 50.3 mm diameter pipe. From these measured parameters, it was possible to make inferences about the mechanisms operating in horizontal annular flow. In particular, it was possible to decide when one can maintain the continuous liquid film at the top of the tube without the film breakup. The data are extended to observe this important condition. The experimental results are compared with available experimental data, and simple local film thickness and entrainment fraction correlations are developed.

12.2. Experimental Setup and Procedure

12.2.1. Experimental System

The experimental system is shown in Figure 1. The horizontal two-phase flow test section consisted of flanged lengths of 50.3 mm inside diameter circular pyrex glass pipes. The entire

test section was about 15.4 m in length. It was desirable to use transparent piping for the complete test section, since this would allow visual studies and observations of the various two-phase flow patterns. Air and water are used as coupling fluids.

The air to the test section is supplied from the central air system available in the institution. It is, however, regulated through a 250 gallon-capacity air storage tank, and metered by turbine flowmeters. The water is recirculated. It is pumped from a 500 gallon-capacity storage tank by a stainless-steel centrifugal pump and regulated from 0 to 100% of the pump capacity by a transistor inverter. The water flow rate is measured by a series of paddle wheel flow meters assembled in a parallel configuration.

The air-water mixing chamber was designed to assist quick development of annular flow. However, the capabilities exist to use any desired mixing chamber design. The air enters the mixing chamber from a 900 vertical leg and then is injected to the pipe through a specially designed stainless steel nozzle. The injection nozzle is centered in the glass test section and directs the air flow in an axially aligned downstream direction. As the water enters the mixing chamber, it is guided by a centrally positioned cone to a 2 mm annular slot around the stainless steel air injection nozzle. The two-phase mixture from the test section is taken to an air-water separator. The air is being vented to the atmosphere, and the water is returned to the water storage tank. All data for the flow loop is monitored and recorded by a data acquisition system.

12.2.2. Experimental Conditions

Measurements were made for 59 flow conditions. The ranges of the basic operating conditions are summarized in Table 1, where flow rates, superficial velocities and Reynolds numbers and pressure are given. No attempt was made to regulate the air and water temperatures, and these varied over a range of about 20 - 23°C during the course of the experimentation.

The characteristic annular flow parameters were measured and tabulated for a different semi-annular and 50 different annular flow patterns. The "semi-annular" pattern refers to the breakup of liquid film on the top of the horizontal pipe. These parameters included the two-phase mixture temperature, pressure, surface tension, superficial gas and liquid velocities, pressure gradient in the developing and fully-developed zones, liquid film thickness, local entrained liquid droplet flux and average void fraction and average void fraction. These parameters along with

superficial gas and liquid Reynolds numbers, Weber number and gas and liquid densities and absolute viscosities are tabulated for annular flow and semi-annular flow patterns[8]. From this database it was possible to develop a semi-empirical correlations for liquid film thickness and entrainment in horizontal annular flow.

12.2.3. Liquid Film Thickness Measurement

A large variety of techniques have been developed for the measurement of local and average film thickness along the perimeter. These methods may be grouped as the conductance probe method [5,6,10-12], the needle contact method [1,10,13,14], fluorescent technique [15,16], a X-ray absorption technique [17]. Details of these methods can be found in a fine review y Hewitt [10].

The local liquid film thickness is measured using the electrical conductance probes. Schematics of the eight-point conductance probe, probe manifold and manifold orientation are shown in Figure 2 and 3. The principles and details of these probes are described by Kitscha [18]. The conductance probes were constructed in the form of plexiglass plugs, so that they could be easily removed for construction and calibration purposes. A total of eight probes spaced at 45° interval around the circumference of the test section are used to obtain the liquid film thickness profile. Each probe is calibrated by using a static film calibration method and checked by using a needle contact method. Finally a probe manifold is constructed from plexiglass and used to hold and align the eight conductance probes in place. The probe manifold is aligned and bolted between two pieces of glass test section piping. The manifold is located at about 50 pipe diameters from the nearest upstream joint and at 230 pipe diameters downstream from the two-phase mixing chambers.

12.2.4. Entrainment Measurement

The local entrained droplet flow is measured by sampling the droplets present in the core of annular flow using what is essentially referred to as the pitot tube sampling technique. In this method, a sample of the air-water droplet mixture taken from the core is collected for a given period of time. After the separation process, the mass of the collected liquid is measured for each corresponding radial position of the probe within the annular core. The local droplet flux is then

calculated for the corresponding probe position. The pitot tube assembly, the droplet collection system and the sampling positions are shown on Figures 4, 5 and 6, respectively.

The pitot tube probe has an outer diameter of 4.572 mm and an inner diameter of 3.81 mm. A right angle portion of the probe is extended approximately 20 mm upstream to avoid any disturbances caused by the tube entry through the glass piping wall. The probe is traversed vertically using a micrometer device. Through careful positioning of the probe, its centerline could approach to within 2.286 mm from the top and bottom wall of the test section. This droplet flux measurement procedure is performed for nine different vertically aligned positions within the test section in order to obtain the flux profiles. No attempt is made to achieve isokinetic flow for the flux measurements.

12.2.5. Average Void Fraction Measurement

As explained in the discussion of results section, the published data on the liquid film measurements indicated some inconsistencies regarding the integrated liquid film fraction. In order to ensure the reliability of data, the average void fraction measurements are also undertaken. The last 1.5 m of the glass test section is set up for the measurements of the average void fraction. This section consists of two pneumatically-operated and electrically-controlled quick closing valves. The valves are connected to a 1.5 m length of the glass tubing, one at each end. Both valves are electrically connected to a single switch to ensure simultaneous operation.

At the end of each run, the quick closing valves are operated trapping the two-phase mixture. By carefully measuring the accumulated liquid mass and volume, it was possible to determine the average liquid fraction and void fraction. Furthermore, as shown later the simultaneous measurements of the local liquid film thickness and average void fraction enabled us to calculate the liquid film fraction and the entrained liquid droplet fraction.

12.3. Analysis and Discussion of Results

This section provides an analysis of results obtained from experimental observations and comparison of results with simple correlations. The characteristic annular two-phase flow-pattern parameters are measured for 50 different annular flow patterns and 9 different semi-annular flow-pattern flow conditions. The "semi-annular pattern" refers to the breakup of liquid film on

top of the horizontal pipe. The basic parameters included superficial gas and liquid velocities, pressure gradient in the developing and fully- developed zones, local liquid film thickness, local entrained liquid droplet flux and average void fraction. The experimental observations are summarized below.

12.3.1. Circumferential Liquid Film Thickness Distribution Results

One set of sample results obtained by electrical conductance probe is illustrated in Figures 7a to 7c. These figures show the effects of gas and liquid Reynolds numbers on the local time averaged film thickness at three different circumferential locations, namely, at $\theta = 0, 90$ and 180° , respectively. The angle measurement is taken with reference to the vertical axis starting from the top of the tube. Similar results are obtained for the other experimental conditions specified in Table 1. In general, it can be observed that for the flow conditions considered, there is a large variation of film thickness characteristics with circumferential variations. The liquid Reynolds number has a profound effect on the liquid film thickness at the bottom, $\theta = 180^\circ$ and $\theta = 90^\circ$, and may be very well fitted by a straight line in a semi-log scale. However, this effect gradually reduces from the bottom toward the top, and Figure 7a does not show any significant variation at the top of the tube. It is important to note here that since the thickness is very small at the top of the tube, the experimental accuracy in measuring such small values is less. But consistency of data indicates that the liquid Reynolds number is not effective for liquid film spreading at that position. Similar observations may be made regarding the gas Reynolds number. As expected the film thickness decreases at any location as Re_g increases, and the effect gradually decreases as θ decreases.

Figure 8 shows the continuous variation of the liquid film thickness along the circumference of the pipe for a set of given conditions. A large variation in film flow characteristics with circumferential position is evident from this figure. Similar results are obtained for the other experimental conditions specified in Table 1. The solid lines shown in this figure indicate a fourth-order polynomial curve fit for a given flow condition in the following form:

$$h^* \equiv h/D = a_0 + a_1\theta + a_2\theta^2 + a_3\theta^3 + a_4\theta^4 \quad (1)$$

The coefficients a_i 's are determined from each set of flow conditions. It is found that the coefficients differ significantly for each of the 50 sets of data. It was concluded that a correlation of the coefficients in terms of flow conditions would not be practical.

After carefully studying the experimental data, an attempt is made to obtain a simple correlation. The correlation is given by

$$\frac{h_{\theta}^* - h_{\theta=0}^*}{2.1 \times 10^{11} \text{Re}_f^{0.86} \text{Re}_g^{-2.71}} = \exp\left\{(-a \text{Re}_f^m \text{Re}_g^n) \cdot \left[\ln\left(\frac{\theta}{2\pi - \theta}\right)\right]^2\right\} \left(\frac{\rho_g}{\rho_f}\right)^{1/2} \left(\frac{v_f}{v_g}\right) \quad (2)$$

where $\text{Re}_f \equiv \rho_f j_f D / \mu_f$ is the superficial liquid Reynolds number and $\text{Re}_g \equiv \rho_g j_g D / \mu_g$ is the superficial gas Reynolds number whereas a , m and n are function of θ defined as

$a = 1.05 \times 10^4 \exp(-2.8\theta)$, $m = 0.4 - 0.107\theta$ and $n = -1.081 + 0.344\theta$. $h_{\theta=0}^*$ is given by

$$h_{\theta=0}^* = 21 \times 10^4 \text{Re}_f^{0.028} \text{Re}_g^{-0.904}.$$

The dimensionless film thickness predicted by Eq. (2) is compared with the experimental measurements in Figure 9. Figure 9 also includes experimental data of Dallman [5] and Laurinat[6]. It may be observed from this figure that the predictive correlation represents the data very well at relatively high liquid film thicknesses. However, at low values the predictions are not very good when compared with experimental data. This may be attributed to the experimental error made in measuring such small film thicknesses. Including the low film thickness region, the mean error between the predicted and measured film thickness values is $\pm 20\%$.

Figures 10a and 10b illustrate one set of data taken at a semi-annular flow-pattern condition. It is to be noted here that most of the semi-annular flow-patterns observed in these flow conditions fall into the annular flow-pattern region in those flow-pattern maps suggested by Laurinat et al. [19], Taitel and Dukler [20] and Mandhane et al. [21]. A large variation in liquid film break-up characteristics with Re_f and Re_g is evident around $\theta=0^\circ$. More specifically, Figures 10a and 10b, respectively, show the effects of gas and liquid flow rates on wetting the top of the tube. Nonuniform spreading and the liquid film breakdown characteristics as a function of gas Reynolds numbers can be clearly seen in Figure 10a. When the superficial gas velocity $j_g = 14.4 \sim 19$ m/s, which corresponds respectively to $\text{Re}_g = 47,445 \sim 62,625$, a liquid film can not climb the tube wall to completely wet the surface. As the superficial gas velocity increases the dried

area at the top of the tube decreases. As noted by Butterworth [4], there are four different mechanisms to maintain the liquid film at the top. They are surface tension, redeposition, secondary motion in the gas core and wave spreading mechanisms. Of these mechanisms, the secondary flow induced in the gas phase and wave spreading mechanism models are promoted directly by the gas velocity. As j_g increased more than 14 m/s, by the action of these two effects, especially the disturbance wave like large waves, the film can be formed at the higher positions of the tube surface. This tendency clearly is seen in Figure 10a. In conclusion, the secondary motion and wave spreading models appear from the present data to be reasonable ones in preventing the liquid film breakdown at the top of the tube. The surface tension wetting mechanism was shown to be insignificant by Butterworth [4] in a 31.8 mm diameter pipe. This effect certainly becomes less significant in a 50.3 mm diameter pipe, which is the case in the present experimentations. The entrainment and redeposition mechanism will be further explored in the sections that follow.

From Figure 10b it is evident that the liquid superficial velocity, $j_f = 0.033 \sim 0.16$ m/s, ($Re_f = 1,735 - 8,425$), is not as effective as the gas superficial velocity in maintaining liquid film at the top of the tube. This region falls into the transition zone between annular and stratified-wavy regions. Although the effect is small, it may still be explained in terms of the two mechanisms that are discussed above with respect to the gas flow.

Finally, the interfacial wave characteristics are also measured and analyzed for the annular flow conditions documented in Table 1. The results on wave behavior will be documented elsewhere. However, as a general observation, it may be stated that the wave height is highest at the bottom of the tube yielding the wave height to mean film thickness ratio at about $2 \sim 2.5$. The same trend in terms of relative wave height with respect to the mean film thickness is observed for $90^\circ \leq \theta \leq 180^\circ$, although the film height is gradually reduced as θ decreases from 180° . On the other hand, more uniform film is observed for $0^\circ \leq \theta \leq 90^\circ$, and the wave height to mean film thickness ratio is much less than 2. These comments on wave behavior are illustrated in Figure 11 in a condensed form.

12.3.2. Entrained Liquid Mass Flux Results

The liquid entrainment mass flux is measured for 5D annular and 9 semi-annular flow conditions. The mass flux is measured at 9 different vertical positions within the annular core. The horizontal traverse is not attempted since the variations along the horizontal traverse were not found to be significant by Butterworth [4]. The entrained liquid mass flow rate is sampled at each position for a period of five minutes. This is done to ensure an accurate data sample, therefore reducing the possibility of any short term entrainment fluctuations affecting the overall results. The local mass flux, $\dot{m}_{fe}''$, profiles for the vertical traverse are shown in Figures 12a and 12b for relatively low and high gas flow rates, respectively. Similar results are obtained for the other experimental conditions specified in Table 1.

The following observations can be made by direct analysis of Figures 12a and 12b. For relatively low gas superficial Reynolds number, the entrainment mass flux is lowest near the top of the pipe and increases toward the bottom. This condition tends to be most predominant towards the top half of the pipe. It reaches a maximum value just above the liquid film layer at the bottom of the pipe. Careful examination of Figure 12b together with the interfacial wave characteristics presented in Figure 11 indicates that the rapid increase in the entrained liquid mass flux close to the wall is certainly due to interference from the interfacial waves. On the other hand, at high gas superficial Reynolds numbers on the order of 100,000 and higher, the mass flux approaches a constant value, independent of the position within the annular core.

The total liquid entrainment, $\dot{m}_{fe}$, is determined by integrating the liquid flux, $\dot{m}_{fe}''$, profiles as follows:

$$\dot{m}_{fe} = 2 \int_0^{\pi} \int_0^{R-h} \dot{m}_{fe}'' dr d\theta \quad (3)$$

where h_w is the wave height. The entrainment is expressed in dimensionless form as $E \equiv \dot{m}_{fe}/\dot{m}_f$. In this expression $\dot{m}_f$ is the total liquid mass flow rate.

Based on shearing off of the roll-wave crests entrainment mechanism, the entrainment fraction is correlated in terms of the modified Weber number, $We_m \equiv (\rho_g j_g^2 D / \sigma)(\Delta(\Delta\rho/\Delta\rho_g))^{1/3}$, and the superficial liquid Reynolds number as follows:

$$E = \tanh(4.33 \times 10^{-13} We_e^{2.55} Re_f^{0.25}) \quad (4)$$

The entrainment fraction predicted by Eq. (4) is compared with our entrainment data as well as with available entrainment data in Figure 13. A favorably good agreement observed in Figure 13 justifies the entrainment model used to arrive at Eq. (4).

12.3.3. Void and Liquid Fraction Results

The average void fraction data is illustrated in Figure 14. The figure is self-explanatory. However, it is interesting to note that the void fraction linearly decreases as a function of the liquid Reynolds number. The slope would be a very strong function of the gas phase Reynolds number.

The total area occupied by the liquid, A_f at any cross section consists of that occupied by liquid in the film plus that of the entrained liquid in the core. Thus,

$$A_f = A_{ff} + A_{fe} \quad (5)$$

where A_{ff} is the liquid film cross-sectional area, and A_{fe} is the cross-sectional area occupied by the entrained liquid droplets in the core. By dividing Eq. (4) by the total cross-sectional area, A , it may be expressed in terms of liquid fractions as follows:

$$\alpha_f = \alpha_{ff} + \alpha_{fe} \quad (6)$$

where $\alpha_{ff} = A_{ff}/A$, referred to as the liquid film and entrained liquid droplet fraction, respectively.

By measuring the liquid film thickness and fitting the data by Eq. (1), α_{ff} is calculated as follows:

$$\alpha_{ff} = \left(\frac{2}{\pi R^2} \right) \int_{\theta=0}^{\pi} \int_{r=R-h(\theta)}^R h(\theta) dr d\theta \quad (7)$$

where R is the inner radius of the test section. After calculating α_{ff} from Eq. (6), α_{fe} is calculated from Eq. (6) with the measured value of $\alpha_f = 1 - \alpha_v$.

The results are summarized for a sample of runs in Figure 15. For the purpose of comparison, Laurinat's [6] runs are duplicated in the present test loop, and the results are shown in Figure 16. Nine runs that are used in this comparison are listed in the figure. In this figure α_{ff} calculated from Laurinat [6] and the present data are compared with those measured α_f values. From Eq. (6) it is expected that α_{ff} must always be less than α_f . The results obtained from the present experiments confirm the expectations. In all these runs α_{ff} based on the present study is

less than of which is indicated by black columns. However, Laurinat's data shows $\alpha_{ff} > \alpha_f$ indicating some error in the film thickness measurements.

The above comparison justifies the importance of taking simultaneous data of the local film thickness and the void fraction. Laurinat's data has been used for this comparison since it was the only available data fitting the present test conditions in terms of pipe diameter. Furthermore, the consistency of the present film thickness measurements with the averaged void fraction data gives some confidence in the conductance-probe method used in the present experimentation.

12.4. Conclusion

The fundamental two-phase flow parameters are experimentally investigated for a horizontal annular flow in a 50.3 mm diameter pipe. The results are summarized as follows:

- 1) For the flow conditions considered, there were large variations in liquid film characteristics, i.e., the mean film thickness and wave height, with circumferential position.
- 2) The liquid film thickness tends to be more uniformly distributed as the gas velocity increases. However, the liquid flow rate is not as effective as the gas flow rate on the circumferential liquid film distribution.
- 3) The circumferential location from which a liquid film starts to form is very strongly affected by the superficial gas velocity. This starts to occur about $j_g \approx 19$ m/s, and the dry area at the top of the tube increases as j_g is reduced. Of the various mechanisms which have been proposed for maintaining the film at the top of the tube, the secondary motion and the wave spreading mechanisms appear from the present data to be the most reasonable.
- 4) For relatively low superficial gas Reynolds numbers, the entrained liquid flux is lowest near the top and increases toward the bottom of the pipe. However, at high gas Reynolds number on the order of 100,000 and higher, the entrained liquid flux approaches a constant value independent of the position within the annular core.
- 5) Large variations in void fraction were observed with flow conditions. With the help of the circumferential liquid film data, the average void fraction data is reduced to obtain the liquid fractions in the film and in the core. This information was used to

demonstrate the reliability of the conductance-probe method used in the present study.

- 6) The proposed correlations for (h/D) and E have proved to be quite good when compared with experimental data.

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Nomenclature

A_f	Total liquid cross sectional area
A_{fe}	Total liquid entrainment cross sectional area
A_{ff}	Total liquid film cross sectional area
d	Wire diameter
E	Entrainment flow rate
Fr	Froude number
g	Gravitational acceleration
h	Liquid film thickness
$\bar{h}$	Area-averaged film thickness
j	Superficial velocity
$\dot{m}_{fe}$	Total liquid entrainment rate
P	Static pressure
R	Pipe radius
Re	Superficial Reynolds number
Re_{LF}	Reynolds number of the liquid film
R^+	Non-dimensional radius
We	Weber number
w	Axial velocity
z	Axial direction

Greek Symbols:

- α_f Liquid volumetric fraction
- α_{fe} Volumetric fraction of the liquid in entrainment
- α_{ff} Volumetric fraction of the liquid in film
- ρ density
- θ Circumferential direction($\theta = 0$ at the top)
- ν Kinematic viscosity
- η Standard deviation in counts
- σ Surface tension

Subscripts:

- f Liquid phase
- fe Liquid in the entrainment
- ff Liquid in the liquid film

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Table 1 Flow conditions Used in Experiments

<u>Mass Flow Rate</u>	
Water	0.0595 – 0.6346 kg/s
Air	0.0334 – 0.1452 kg/s
Total	0.0929 – 0.7798 kg/s
<u>Superficial Velocity</u>	
Water	0.03 – 0.32 kg/s
Air	14.0 – 43.0 kg/s
<u>Superficial Reynolds Number</u>	
Water	1,400 – 16,700
Air	47,000 – 202, 000
<u>Nominal Pressure</u>	1.03 – 1.5 bars

HORIZONTAL TWO-PHASE FLOW LOOP

A - Interchangeable Air-Water mixing chambers	K - Pneumatic operated Ball valves
B - Water flow meters of appropriate size	L - Motor control
C - Water flow meter control valves	M - Computer and data acquisition system
D - Air flow meters of appropriate size	N - 250 gal. Air tank
E - Air flow meter control valves	P - 500 gal. Water tank
F - Air flow regulating valves	Q - Air-Water separator, with internal baffles
G - Air pressure regulator	R - Water shut-off valve
H - Air filter	S - 20 hp. 750 gpm Water pump
I - Water pressure relief valves	T - Glass pipe couplings with pressure taps
J - Water flow regulating valves	

Figure 1. Schematic of Experimental Flow Loop

Figure 2. Conductance Probe and Probe Manifold

Figure 3 Conductance Probe and Manifold Orientation

Figure 4 Pitot Tube Assembly and Orientation

Figure 5. Droplet Collection System

Figure 6. Pitot Tube Sampling Positions

Figure 7 Average Liquid Film Thickness at,
 a) $\theta = 0^\circ$, b) $\theta = 45^\circ$, and c) $\theta = 180^\circ$.

Figure 8. Liquid Film Thickness Distribution

Figure 9. Comparison of Predicted Liquid Film Thickness with Experimental Data

Figure 10. Effect of Superficial Velocities on Formation of Liquid Layer around $\theta=0^\circ$,
a) Effect Of Gas Velocity, b) Effect Of Liquid Velocity.

Figure 11. Distribution of Wave Characteristics.

Figure 12. Entrained Liquid Droplet Mass Flux Profiles at
a) Low and b) High Gas Flow Rates.

Figure 13. Comparison of Predicted Entrainment Fraction with Experimental Data.

Figure 14. Variation of Average Void Fraction with Liquid and Gas Reynolds Numbers.

Figure 15. Liquid Fraction Variations

Figure 16. Comparison of Liquid Film Fraction Results of Laurinat and Present Studies with Measured Total Liquid Fraction Data.

13. INTERFACIAL STABILITY STUDIES OF STRATIFIED FLOWS

PART1. PARAMETRIC STUDIES

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ABSTRACT

The interfacial surface of two superposed immiscible fluids which are confined between two parallel infinite planes (Poiseuille flow) and are flowing due to both an applied pressure gradient is the object of a stability study. This work is divided into two parts. Part I (the present paper) mathematically sets up the problem and obtains a solution. The solution corresponds to disturbances of long wavelengths which are considered to be the primary cause of instability in stratified flow.

With a stability solution obtained, a dimensionless parametric study is conducted which determines the effects of the viscosity, density and fluid depth ratios, the Reynolds and Froude numbers and the angle of inclination. In part II, a case study of an air-water stratified flow is done, and a parametric study is performed where the viscosities, densities and channel depths of the air and water are varied. From this study a partial air water flow region map is developed and compared to existing maps and experimental data

13.1 Introduction

Parallel flows of two fluids or of the liquid and gas phases of a single fluid are common in nature and in industry and the study of their behavior has been accelerating for more than a century.

There are many possible configurations that two fluids may achieve and these configurations (or flow patterns) strongly affect heat and mass transfer rates as well as pressure drop—significant phenomena in a wide variety of industries and hence a motivation for the extensive study to date.

The stratified flow is the most basic flow pattern and an indicating of incipient transition to another pattern (e.g., wavy, slug) can be found in the stability of the interfacial surface between the two fluids. Many such stability studies have been conducted, the vast majority for the simple Couette flow, Yih, and Boyd, Hooper and Renardy. The study of the stability of the more complex Poiseuille flow has been limited to a few selected density, viscosity or depth ratios for horizontal flow, Yih, Hooper (1a), Graebel, Yiantsios. A detailed analysis showing the effects of various parameters on interfacial stability is not available in the literature for Poiseuille flow.

This paper, considers the flow of two superposed immiscible fluids confined between two parallel infinite planes (Poiseuille flow). It has three objectives. The first objective is to examine incipient interfacial instability due to small disturbances by waves of long wavelength. The second is to carry out a parametric study to investigate the effects on stability of fluid properties (mass density and viscosity), angle of inclination of the channel, fluid depths and relative velocity between the two fluids. Finally, the third objective is to investigate a possible

two-phase flow-region change due to interfacial instability. The method of analysis is Yih's two-dimensional, long-wavelength perturbation scheme.

13.2 The Base Flow

The flow configuration under consideration is illustrated in Fig. 1. It is comprised of two superposed incompressible fluids of different viscosities and densities which are contained between two rigid plane-parallel boundaries that may be horizontal or inclined. With the interface between the fluids also being plane and parallel to the boundaries, the fluids are in motion due to an applied pressure gradient and to the influence of gravity for the case of non-horizontal flow. The base flow consists of two parabolic dimensionless velocity profiles:

$$u_1 = A_1 y^2 + a_1 \cdot y + 1 ; \quad u_2 = A_2 \cdot y^2 + a_2 \cdot y + 1 \quad (1)$$

where

$$A_1 \equiv \frac{-(m+n)}{n \cdot (n+1)} + \frac{n \cdot R \cdot F^2 \cdot \sin \theta}{2 \cdot (n+1)} ; \quad a_1 \equiv \frac{m-n^2}{n \cdot (n+1)} - \frac{n \cdot R \cdot F^2 \sin \theta}{2 \cdot (n+1)} \quad (2)$$

and

$$A_2 \equiv \frac{-(m+n)}{m \cdot n \cdot (n+1)} - \frac{R \cdot F^2 \cdot \sin \theta}{2 \cdot m \cdot (n+1)} ; \quad a_2 \equiv \frac{m-n^2}{m \cdot n \cdot (n+1)} - \frac{R \cdot F^2 \sin \theta \cdot n}{2 \cdot m \cdot (n+1)} \quad (3)$$

The velocities are non-dimensionalized by the interfacial velocity, U_0 . The subscripts 1 and 2 identify the upper and the lower fluid, respectively, and $m \equiv \mu_2/\mu_1$ is the viscosity ratio, whereas $n \equiv d_2/d_1$ is the fluid layer ratio. The Reynolds number, R , and Froude number, F , are defined as follows:

$$R \equiv \frac{d_1 \rho_1 U_0}{\mu_1} ; \quad F \equiv \sqrt{G(r-1)} \quad (4)$$

where $G \equiv d_1 g / U_0^2$ and $r \equiv \rho_2 / \rho_1$ are a dimensionless gravity number and the density ratio, respectively. The spatial coordinates are made dimensionless with the upper fluid thickness, d_1 . The velocity profiles are the same as Yih's, but with the inclusion of the depth ratio, density ratio and the angle of inclination. The equations determining the stability of the interface are the well-known Orr-Sommerfeld equations (one for each fluid):

$$\phi^{iv} - \left[2 \cdot \alpha^2 - i\alpha \cdot R \cdot (c - u_1) \right] \cdot \phi'' + \left[\phi^4 - i\alpha^3 \cdot R \cdot (c - u_1) + 2 \cdot i\alpha \cdot R u_1'' \right] \cdot \phi = 0 \quad (5)$$

and

$$\begin{aligned} \psi^{iv} - \left[2 \cdot \alpha^2 - i\alpha \cdot \frac{r}{m} \cdot R \cdot (c - u_2) \right] \cdot \psi'' \dots \\ + \left[\alpha^4 - \left[i\alpha^3 \cdot \frac{r}{m} \cdot R \right] \cdot (c - u_2) + 2 \cdot i\alpha \cdot \frac{r}{m} \cdot R \cdot u_2'' \right] \cdot \psi = 0 \end{aligned} \quad (6)$$

which are derived in detail by Yih. The terms ϕ and ψ were the amplitudes of the disturbances in the stream functions which are used to represent the perturbation velocities, u^* and v^* , for the upper and lower fluids. These are defined as follows:

$$u_1^* = \phi' \cdot e^{i\alpha(x-ct)} ; \quad u_2^* = \psi' \cdot e^{i\alpha(x-ct)} \quad (7)$$

and

$$v_1^* = -i \cdot \alpha \cdot \phi \cdot e^{i\alpha(x-ct)} ; \quad v_2^* = -i \cdot \alpha \cdot \psi \cdot e^{i\alpha(x-ct)} \quad (8)$$

The primes in (5) through (8) denote differentiation with respect to y ; α is the wave number, and c is the dimensionless complex wave speed, i.e., $c \equiv c_r + ic_i$. The quantity c_r represents the speed of the disturbance wave and αc_i is the growth factor of the wave. The stability of the perturbation is then determined by the sign on c_i , such that $c_i > 0$ is unstable.

Since the Orr-Sommerfeld equation is a fourth-order differential equation and there are two equations (one for each of the two fluids), there must be eight boundary conditions for the complete solution. Each of the two solid boundaries provides two of these conditions and four are found at the interface of the fluids. The linearized boundary conditions expressing no slip and no mass transport across the channel walls are

$$\phi'(1) = 0, \quad \phi(1) = 0, \quad \psi'(-n) = 0, \quad \text{and} \quad \psi(-n) = 0 \quad (9)$$

and the linearized boundary conditions at the interface (at $y=0$) for the continuity of velocity and normal and shear stresses are

$$\phi' - \psi' = \frac{\phi}{c'} \cdot (a_2 - a_1); \quad (10)$$

$$\phi) = \psi \phi'' + \alpha^2 \cdot \phi = m \cdot (\psi'' + \alpha^2 \cdot \psi) \quad (11)$$

and,

$$i \alpha R \cdot \left[r \cdot (c' \cdot \psi' + a_2 \cdot \psi) - (c' \cdot + a_1 \cdot \phi) + (\alpha^2 \cdot S + F^2 \cdot \cos(\theta)) \cdot \frac{\phi}{c'} \right] \quad (12)$$

$$+ m \cdot (\psi''' - 3 \cdot \alpha^2 \cdot \psi') - (\phi''' - 3 \cdot \alpha^2 \cdot \phi') = 0$$

In the above equations, $c' \equiv c - u_0$ is the relative interfacial velocity with respect to the dimensionless interface velocity, u_0 , and $S \equiv \sigma/d_1/\rho_1 U_0^2$ is the surface tension dimensionless group.

A solution of the Orr-Sommerfeld equations is possible by a method used by Yih [15] which consists of expanding the eigenvalue c and the eigenfunctions ϕ and ψ . These expansions are in powers of the wave number α as follows:

$$\phi(y) = \phi_0 + \alpha \cdot \phi_1 + \alpha^2 \cdot \phi_2 + \alpha^3 \cdot \phi_3 + \dots \quad (13)$$

$$\psi(y) = \psi_0 + \alpha \cdot \psi_1 + \alpha^2 \cdot \psi_2 + \alpha^3 \cdot \psi_3 + \dots \quad (14)$$

and

$$c = c_0 + \alpha \cdot c_1 + \alpha^2 \cdot c_2 + \alpha^3 \cdot c_3 + \dots \quad (15)$$

and are appropriate for small wave numbers which correspond to wavelengths that are long with respect to the depth of the upper layer. The expansions are substituted into the Orr-Sommerfeld equations, (5) and (6), and into the governing boundary and interfacial conditions, (9) through (12). The different powers of α are then separated out, thereby forming a series of differential equations and their corresponding boundary conditions. These are more readily solved. Since α is considered small, the series expansion is terminated after the first two terms which produces two approximations corresponding to the zeroth and first powers of α .

Using the method just outlined, the zeroth order approximations of the Orr-Sommerfeld equations are

$$\phi_0^{iv} = 0 \quad \text{and} \quad \psi_0^{iv} = 0 \quad (16)$$

Four integrations produce

$$\phi_0 = 1 + B_{\phi,0} \cdot y + C_{\phi,0} \cdot y^2 + D_{\phi,0} \cdot y^3 \quad (17)$$

and

$$\psi_0 = 1 + B_{\psi,0} \cdot y + C_{\psi,0} \cdot y^2 + D_{\psi,0} \cdot y^3 \quad (18)$$

where the final constant of integration is taken to be unity without any loss in generality. From (9), (11) and (12) the constants are found:

$$B_{\phi,0} = \frac{-4 \cdot n^3 - 3 \cdot n^2 - m}{2 \cdot n^2 \cdot (n+1)}, \quad B_{\psi,0} = \frac{4 \cdot m + 3 \cdot n \cdot m + n^3}{2 \cdot n \cdot m \cdot (n+1)} \quad (19)$$

$$C_{\phi,0} = m \cdot C_{\psi,0}, \quad C_{\psi,0} = \frac{m + n^3}{m \cdot n^2 \cdot (n+1)} \quad (20)$$

and

$$D_{\phi,0} = m \cdot D_{\psi,0}, \quad D_{\psi,0} = \frac{n^2 - m}{2 \cdot n^2 \cdot m \cdot (n+1)} \quad (21)$$

Substituting (16) through (21) into the zeroth order approximation of (10) gives.

$$c_0 = \frac{n \cdot (m-1) \cdot \left[2 \cdot (m-n^2) - n^2 \cdot R \cdot F^2 \cdot \sin(\theta) \right]}{6 \cdot n^2 \cdot m + 4 \cdot m \cdot n^3 + m^2 + 4 \cdot n \cdot m + n^4} + 1 \quad (22)$$

Since this is a real number, it is the zeroth order approximation of the dimensionless wave velocity, c_R . Since c_R is non-dimensionalized with the interfacial velocity, waves travel slower than the interface when $c_0 < 1$. This occurs for the viscosity-ratio range of

$$n^2 \cdot \left[1 + \frac{1}{2} \cdot R \cdot F^2 \cdot \sin(\theta) \right] < m < 1 \quad (23)$$

which is for a more viscous upper fluid or for

$$1 < m < n^2 \cdot \left[1 + \frac{1}{2} \cdot R \cdot F^2 \cdot \sin(\theta) \right] \quad (24)$$

which is for a more viscous lower fluid. For the horizontal case these two ranges are $n^2 < m < 1$ and $1 < m < n^2$, respectively. It will be shown later that these two ranges of m for the horizontal case are always stable.

13.3 First-Order Solution Substitution of (13), (14) and (14) into (5) and (6) and then separating out the terms containing the first-power of α yields the first-order approximations of the Orr-Sommerfeld equations:

$$\phi_1^{iv} = i \cdot R \cdot \left[\phi_0'' \cdot (u_1 - c_0) - 2 \cdot A_1 \cdot \phi_0 \right] \quad (25)$$

and

$$\psi_1^{iv} = i \cdot R \cdot \frac{r}{m} \cdot \left[\psi_0'' \cdot (u_2 - c_0) - 2 \cdot A_2 \cdot \psi_0 \right] \quad (26)$$

Integrating four times yields

$$\phi_1 = 1 + B_{\phi,1} \cdot y + C_{\phi,1} \cdot y^2 + D_{\phi,1} \cdot y^3 + \Phi(y) \quad (27)$$

and

$$\psi_1 = 1 + B_{\psi,1} \cdot y + C_{\psi,1} \cdot y^2 + D_{\psi,1} \cdot y^3 + \Psi(y) \quad (28)$$

where Φ and Ψ result from the inhomogeneous term and are given in the appendix. Following Y. L.'s reasoning, the final constants of integration are taken to be zero in the first order approximation since the solution of the eigenvalue problem is determined up to an arbitrary constant factor which was set in the zeroth order approximation.

Application of the first-order approximations of the boundary conditions, (a) through (12) in (27) and (28) yields the constants and c_1 , which is the first-order approximation of the wave speed:

$$c_1 = -c_0' \cdot \frac{\phi_1' - \psi_1'}{\phi_0' - \psi_0'} \quad (29)$$

Although this is concise in its current form, it is not very informative. Therefore, through substitution and extensive algebraic manipulation, it is rewritten in terms of the primary dimensionless groups:

$$c_1 = A + B + C + D \quad (30)$$

where

$$A = \frac{-i \cdot n^3 \cdot (n + m) \cdot R \cdot G (r - 1) \cdot \cos(\theta)}{3 \cdot (m^2 + 6 \cdot m \cdot n^2 + 4 \cdot m \cdot n^3 + 4 \cdot n \cdot m + n^4)} \quad (31a)$$

$$B = \frac{-i \cdot R \cdot (m - 1) \cdot (m - n^2) \cdot (r \cdot K + L)}{420 \cdot m^2 \cdot (n + 1)^2 \cdot (m^2 + 6 \cdot m \cdot n^2 + 4 \cdot m \cdot n^2 + 4 \cdot n \cdot m + n^4)^3} \quad (31b)$$

$$C = \frac{-i \cdot (m - 1) (r - 1) \cdot (r \cdot M + N) \cdot (n^2 \cdot R^2 \cdot G)^2 \cdot \sin(\theta)}{840 \cdot m^2 \cdot (n + 1)^2 \cdot (m^2 + 6 \cdot m \cdot n^2 + 4 \cdot m \cdot n^3 + 4 \cdot n \cdot m + n^4)^3} \quad (31c)$$

$$D = \frac{i \cdot (m - 1) (r - 1)^2 \cdot (r \cdot P + Q) \cdot n^4 R^3 G^2 \sin^2 \theta}{1680 \cdot m^2 \cdot (n + 1)^2 \cdot (m^2 + 4 \cdot m \cdot n^3 + 6 \cdot m \cdot n^2 + 4 \cdot m \cdot n + n^4)^3} \quad (31d)$$

The constants K, L, M, N, P and Q are real and are functions of only m and n (see the appendix). It can now be seen that c_1 is purely imaginary and therefore is the growth factor of the wave.

The first two terms, c_0 and c_1 , of (15) are the most significant terms of the series and are used in this paper to represent the complex wave celerity. The first term, c_0 , is real and represents the interfacial wave velocity. The second term, c_1 , is imaginary. Being imaginary, it represents the growth factor, c_i , which is the complex component of c . A positive value of the growth factor indicates wave growth, and, consequently, interfacial instability. A negative

value is indicative of wave decay and stability, and for a value of zero there is neither growth nor decay, i.e., neutral stability.

From (29) through (30d) it can be seen that the primary cause of the instability for flows with a favorable density stratification (the lower fluid is more dense than the upper fluid) is the disparity in the viscosity of the two fluids. This is observed because of the expression $(m - 1)$ which occurs in (30b) through (30d). When the viscosity ratio has the value of one of those terms which contain the expression $(m - 1)$ are eliminated from (29) leaving the stability to be entirely governed by (30a). The sign of this term, and hence the interfacial stability, is solely affected by the value of r because of the expression $(r - 1)$. If the lower fluid is more dense ($r > 1$), (30a) is negative and the interface is stable. If both fluids should have the same viscosity, (30a) is zero and a case of neutral stability occurs—but then this is simply the case of a single fluid. For $r < 1$, the upper fluid is more dense, (30a) is a positive and the interface is unstable. When the viscosity ratio has a value other than one, (30b) through (29) also affect the stability.

For fluids of equal densities, $r = 1$, (30b) is the only term in (29) which governs stability. This is due to the elimination of (30a), (30c) and (30d) which contain the expression $(r - 1)$. It is only m and n that determine the stability of the interface and then only by the simple expressions $(m-1)$ and $(m-n^2)$ since the remaining terms do not change sign for all physically practical ranges of m or n . These expressions cause (30b) to be equal to zero when $m = n^2$ or $m = 1$. For $r = 1$, these are conditions of neutral stability.

The relationships $m = 1$ and $m = n^2$ are also significant with regard to the interfacial shear stress and the velocity profiles of each fluid. (A relationship with respect to the velocity of interfacial waves will be shown later). Shear stress is linearly proportional to the slope (the first derivative) of the velocity profile for a fluid and is continuous across the interface for a

stratified flow. The slope of velocity profile, however, may be discontinuous across the interface except when $m = 1$ or $m = n^2$. When $n^2 = m$ for horizontal flow, the slopes of both velocity profiles are zero at the interface, and therefore a condition of zero shear stress exists at the interface as well as continuity in the slopes velocity profile. This zero-shear stress condition will be significant in the subsequent discussions on neutral stability.

This relationship of the shear stress to the velocity profile and the effect of the equations $m = 1$ and $m = n^2$ are illustrated in Figs. 2, 3 and 4 for horizontal flow. Figure 2 shows the discontinuity of slope of the velocity profile at the interface when $m > 1$ and $n^2 < m$. It also shows the shear stress as a function of the fluid depth. Zero shear stress occurs at the depth in the upper fluid where the velocity is maximum, which corresponds to the point at which there is a slope of zero (with respect to y) in the velocity profile. Figure 3 shows what happens when both fluids have the same viscosity — the slope of the velocity profile is continuous across the interface. As stated earlier in this section, it is under this condition, $m = 1$, that the flow is never unstable. Zero shear stress occurs where the velocity is maximum, which for this case is in the lower fluid. The slope of the velocity profile is also continuous when $m = n^2$ as seen in Fig. 4. This figure shows that the maximum velocity occurs at the interface which corresponds to the point of zero shear stress. For $r = 1$ in horizontal flow, the conditions $m = n^2$ and $m = 1$ are lines of neutral stability. For $r > 1$ in horizontal flow, the conditions $m = n^2$ and $m = 1$ are never unstable. (This will be seen later in the parametric study). These three figures show the role of viscosity in the velocity profiles for horizontal flow and indicate that if the slopes of the profiles are continuous across the interface, stability results for $r \geq 1$.

13.4 Parametric Stability Analysis

The growth factor, c_1 , which is defined by the stability equation, (29), is expressed in terms of the dimensionless groups, m , n , r , R , G and θ . Figures 5 and 7 through 17 are the results of a parametric study of (29) in which these dimensionless groups are varied in order to study their effects on the stability of interfacial waves. For horizontal flow the Reynolds number is not included in the parametric study since it has no effect on the neutral stability of horizontal flow. The reason is that it is a linear multiplication factor in (29). Because of this, all the figures for horizontal flow are valid for all $R > 0$.

An important note is that the analysis in this paper has assumed laminar flow. The following parametric studies maintain this assumption. It is possible that in reality, some of the conditions represented in portions of the figures may not be laminar. However, no attempt is made to identify these non-laminar conditions, since that is not the focus of this paper.

13.4.1 Horizontal flow The effect of the viscosity ratio, m , depth ratio, n , and density ratio, r , on interfacial stability for horizontal flow is illustrated in Fig. 5 where the neutral stability diagram is plotted in the plane of the viscosity ratio and the fluid depth ratio for $r = 1, 3, 10$ and 1000 , $G = 0.1$, $\theta = 0$ and $R > 0$. From this figure, it is observed that as the viscosity ratio moves away from the value of one, the unstable ranges in the m - n plane grow while the regions of stability decrease; that is, the interface becomes unstable over a larger range of the depth ratio. It is in this relationship with the depth ratio that the destabilizing characteristic of a viscosity stratification is most clearly seen. In a dimensional parametric study in Part II, the viscosity ratio will be shown to be both stabilizing and destabilizing, and yet remain consistent with the destabilizing characteristic found here.

Figure 5 shows that the effect of the depth ratio on the stability is dependent on the viscosity ratio. For $m > 1$, with a sufficient increase in n , an unstable fluid will become stable. In contrast to this, with a sufficient increase in n for $m < 1$, a stable fluid will be induced to become unstable. The consequence of this is that a thin viscous layer (regardless of its orientation) tends to be unstable; or conversely, if the thinner fluid is also the less viscous of the two, it tends to be stable. This thin-layer effect was noted by Hooper [17] in a semi-bounded plane Couette flow and by Renardy [21] for a bounded plane Couette flow. Yiantsios and Higgins [22] discovered this effect for plane Poiseuille flow when $n \ll 1$. This paper extends this by noting that the thin layer can be either the upper or lower layer ($n \gg 1$ or $n \ll 1$).

For the case of both fluids with the same density, $r = 1$, Fig. 5 shows that neutral stability occurs along the lines $m = 1$ and $n^2 = m$. This divides the m - n plane into four regions. For the half-plane $m < 1$, the flow is stable when $n^2 \leq m$ and unstable otherwise. For the half-plane $m > 1$, the situation reverses and the flow is unstable when $n^2 < m$ and stable when $n^2 \geq m$. With increasing r (the lower fluid increasing in density, or the upper decreasing in density), these lines of neutral stability shift into the unstable regions causing the regions of stability to grow at the expense of the unstable ones. Consequently, having the lower fluid more dense than the upper fluid will enhance stability. It is interesting to note, however, that this has a limited effect. Figure 5 shows that over most of the m - n plane, the enhancement of stability when the density ratio was increased from one to three is comparable to the enhancement of stability for the increase in density ratio from three to one thousand.

This increase in stability due to the density stratification is primarily caused by the hydrostatic pressure difference induced by a disturbance to the interface (Rayleigh-Taylor stability). Two figures are used to illustrate this; the first is Fig. 6 in which the neutral stability

curve for an adverse density stratification, $r = 1/3$, is plotted along with the curve for a favorable stratification, $r = 3$, and with the curve for $r = 1$ for $G = 0.1$, $\theta = 0$ and $R > 0$ in the m - n plane. Having the upper fluid more dense decreases the regions of stability in the m - n plane. The effect is the opposite of that for a favorable density stratification in that the lines of neutral stability are shifted into the stable areas thus reducing the stability. It is interesting to note that even with an upper fluid layer more dense than the lower, there still can be stability in the interface if the arrangement is for the less viscous fluid to be a thinner layer. This is the thin layer effect and occurs from $m > 1$ when $n \gg 1$ and for $m < 1$ when $n \ll 1$.

In the second figure there is not any Rayleigh-Taylor hydrostatic pressure effect due to the absence of gravity, $G = 0$. This is Fig. 7 in which are plotted the neutral stability curves for $r = 10$, $r = 1$ and $r = 1/10$ in the m - n plane for $G = 0$, $\theta = 0$ and $R > 0$. In this figure, all the density stratifications are slightly stabilizing; regions of stability appear within unstable areas but the lines of neutral stability, $n^2 = m$ and $m = 1$, remain unchanged. (It should be noted, however, that these stable protrusions occur for an unusual combination of thermodynamic properties — the more viscous fluid is also the less dense one). Looking back at Figs. 5 and 6, it now becomes apparent that the shift of the neutral stability lines away from $n^2 = m$ and $m = 1$ was due to the hydrostatic pressure effect; whereas, the small stable region in the $m < 1$ half-plane above the $n^2 = m$ line was due to the intrinsic stabilizing effect of a density stratification illustrated in Fig. 7.

The effect of the dimensionless gravity number, G , on the stability of the interface is seen in Fig. 8 where lines of neutral stability are plotted for $G = 0.01$, 0.1 and 1.0 for $r = 10$, $\theta = 0$ and $R > 0$. It can be seen that increasing G is stabilizing in that it enlarges the regions of stability. The reason for this effect can be found in the three terms which comprise the

definition of G : the gravitational constant, g ; the upper fluid depth, d_1 ; and the interfacial velocity, U_0 . The dimensionless gravity number can be changed by any of these three variables and each shall be considered. An increase in g increases the dimensionless gravity number. This is stabilizing and is consistent with the Rayleigh-Taylor stability for $r > 1$. Since a stronger gravitational pull will enhance the restorative hydrostatic pressure differential across the interface. Decreasing the interfacial velocity, the second term mentioned in the definition of G , also increases G and hence stability. Decreasing U_0 implies that the entire flow is slowing which, at the limit of $U_0 = 0$, would be absolutely stable. Finally, an increase in the upper fluid depth also increases G and, consequently, the stability. The increase in d_1 necessitates an increase in the entire channel depth for a given value of n . This indicates that deeper channels are more stable and is consistent with the results of Hooper and Boyd [16] who found that no long-wave instability exists for unbounded flows of the Couette type.

13.4.2 Inclined and Vertical flows In the discussion of (29), it was stated that the angle of inclination has no effect on the stability of the interface unless the two layers differ in density. Figure 9 illustrates the effect of inclination of the neutral stability of fluids in which their densities differ only slightly. In this figure, the density ratio $r = 1.5$. The angle is varied such that $\theta = 0^\circ, 45^\circ$ and 90° with $G = 0.1$ and $R = 3$. It is seen that, although increasing the angle of inclination increases the instability, its effect is small. Also noticeable is that the effect on the interfacial stability of the angle of inclination is greatest for fluids of comparable depth and comparable viscosity. As n or m becomes large or small, the angle has a diminishing effect. In the region of comparable fluid depths and viscosity, it can be seen that the effect of changing the angle from 45° to 90° is more pronounced than a change from 0° to 45° .

For a larger density ratio, the angle has a more profound effect on increasing instability in the $m < 1$ half-plane as illustrated in Fig. 10 for $r = 10$ where $\theta = 0^\circ, 45^\circ$ and 90° with $G = 0.1$ and $R = 3$. In the $m > 1$ half-plane, the increase in the density ratio from Fig. 9 to Fig. 10 produces an unstable protrusion into the stable region in the $n > 1$ area. It is interesting to note that for about $m > 2$, an increase in the angle from 0° to 45° slightly improves the stability by shifting the neutral stability curve into lower n . But as the angle increases from 45° to 90° , the curve shifts into higher n decreasing the stability.

For vertical flows the effect of the parameters r , G , and R are very similar. These are illustrated in Figs. 11-13. In general, an increase in of these parameters has several effects. First, the neutral stability line which is parallel to the $n^2 = m$ line shifts towards smaller n . Second, there is an appearance of a stable protrusion into the unstable region in the $m < 1$ half-plane, and an unstable protrusion into the stable region in the $m > 1$ half-plane. These two regions grow in length with increasing parameter and join, but with opposite stability conditions on either side of the $m = 1$ line. Finally, with all the complex shifting of the neutral stability lines with increasing parameters, there is not any clear gain or decrease in the stability of the interface.

13.4.3 Parametric Interfacial Wave Velocity Studies

Horizontal Flow. From the definition of the dimensionless wave velocity, c_0 , it is seen that for horizontal flow it is a function of only m and n . The dimensionless parameters, r , G and R do not affect the dimensionless interfacial wave velocity for horizontal flow although the dimensional interfacial wave velocity will be affected by all these dimensionless parameters. The effect of n on c_0 for $m = 0.01, 0.1, 1, 10$ and 100 is shown in Fig. 18 for horizontal flow. Since the dimensionless wave velocity is the ratio of the wave velocity, C , to the interfacial

velocity, U_0 , the wave travels slower than the interface when $c_0 < 1$ and faster for $c_0 > 1$. Figure 14 shows that the maximum of c_0 occurs when the fluid thicknesses are comparable to each other, irrespective of the viscosity ratio (except when $m = 1$).

The minimum of c_0 occurs for $m < 1$ when the bottom fluid is much thinner than the upper fluid and for $m > 1$ when the upper fluid is much thinner than the lower fluid. More specifically, c_0 is always less than one in the regions $n^2 < m$ for $m < 1$, and $n^2 > m$ for $m > 1$. As was seen in Fig. 5, these are also the ranges in the m - n plane which are always stable for $r \geq 1$ for horizontal flows. This indicates that waves which move slower than the interfacial velocity are always stable for a favorable density stratification in horizontal flow.

There are several conditions under which the wave velocity equals or approaches the velocity of the interface (c_0 equals or approaches the value of one). The first is for the simple case where both fluids have the same viscosity, $m = 1$. In this case, the dimensionless interfacial velocity equals one regardless of the depth ratio. The second case occurs when $n^2 = m$. This can be seen in Fig. 14 where the curves cross the $c_0 = 1$ line when $n^2 = m$. It is interesting to note that these first two cases, $m = 1$ and $n^2 = m$, when compared to Fig. 5, correspond to neutral stability for fluids of equal density, $r = 1$. These also correspond to the condition illustrated in Fig. 3 and 4 where slope of the velocity profile is continuous across the interface. Finally, the dimensionless interfacial wave velocity approaches one as n approaches either zero or infinity, indicating that as one of the fluids becomes much thinner than the other fluid, the wave velocity approaches the velocity of the interface.

A final observation with respect to Fig. 14 is that when the bottom fluid is significantly thinner than the upper fluid, $n \ll 1$, the dimensionless interfacial wave velocity increases as the viscosity ratio increases. However, this effect is more pronounced when $m < 1$. On the other

hand, when the bottom fluid is much thicker than the upper fluid, $n \gg 1$, the dimensionless interfacial velocity decreases as m increases. This is more pronounced for m greater than one.

13.5 Summary and Conclusions

A long-wave stability analysis of the interface of a stratified flow of two superimposed immiscible fluids confined between two parallel planes has been completed. A stability equation was derived, and a long wave instability was found to exist under specific conditions. Several studies were conducted in order to determine the conditions, which initiate the instabilities. The significance of the viscosity and density stratifications, the fluid depths, the Reynolds number, the Froude number, the average fluid velocities, the interfacial velocity, the interfacial wave velocity, the angle of inclination of the parallel planes and gravity was presented.

It was found that when the lower fluid is equally or more dense than the upper fluid, the principal cause of the instability is the change in viscosity at the interface of the two fluids. This is seen by the fact that when both fluids have the same viscosity, regardless of the other parameters, the interface is never unstable. The destabilizing characteristic of the viscosity stratification is observed when the viscosity ratio increases or decreases from a value of one. When this happens, the interfacial instability will increase such that an unstable condition will exist over a larger range of the depth ratio. This produces a thin-layer effect where a thin fluid layer, if it is the lesser viscous one, tends to be stable.

This stabilizing characteristic of a viscosity stratification is not always apparent and may seem to be contradicted. It was found that an increase in the viscosity ratio will absence of gravity, a density stratification is always, but only slightly, stabilizing regardless whether the upper or lower fluid is more dense. When the densities are equal, the interface is stable when $n^2 \leq m$ and $n^2 \geq m > 1$ and unstable otherwise. When the lower fluid becomes more dense or less dense than the upper fluid, this range increase or decrease, respectively. In terms of the overage fluid velocities, an increase in the density ratio increase the stability when the lower fluid is much more thinner than the upper but has little effect when both fluids are of comparable depth.

A relationship between the interfacial velocity and the stability of the interface was observed. It was found that interfacial waves which move slower than the interfacial velocity are always stable when the flow is horizontal and the lower fluid is more dense than the upper fluid. In terms of the average fluid velocities, the interfacial waves are slower than the interfacial velocity and are stable when the average velocity of the upper fluid is slower than the average velocity of the lower fluid.

Nomenclature

A_k	Coefficient in Velocity Profile of Eq. (1)
a_k	Coefficient in Velocity Profile of Eq (1)
$B_{\phi,n}$	Constant of Integration for the n'th approximation of the Stream function ϕ , Eqs. (17) & (18)
$B_{\psi,n}$	Constant of Integration for the n'th approximation of the Stream functions, ψ , Eqs. (17) &(18)
C	Wave Celerity
c	Dimensionless Wave Celerity
C'	Difference of the Dimensionless Wave Celerity and Interfacial Velocity
$C_{\phi,n}$	Constant of Integration for the n'th approximation of the Stream function ϕ , Eqs. (17) & (18)
$C_{\psi,n}$	Constant of Integration for the n'th approximation of the Stream function ψ , Eqs. (17) & (18)
c_i	Imaginary Part of c
c_r	Real Part of c
d_k	Fluid Layer Thickness
F	Froude Number, Eq. 94)
G	Dimensionless Gravity
g	Gravitational Acceleration
J_k	Superficial Velocity of the k'th Phase
m	Viscosity ratio
n	Fluid Layer Thickness Ratio
P_k	Pressure of the k'th Phase
R	Reyno
r	Density Ratio
S	Dimensionless Surface Tension
T	Time
t	Dimensionless Time
U_o	Interfacial Velocity

U_k	Primary axial flow Velocity of the k^{th} Phase
u_k	Dimensionless Axial Velocity
V_k	Primary Vertical flow Velocity of the k^{th} Phase
v_k	Dimensionless Vertical Velocity
X	Axial flow Direction
Y	Perpendicular to flow direction
w	Channel width

Greek Letters

α	Wave Number
β	Void Fraction
δ	Displacement of the Disturbed Interface
ϕ	Magnitude of the stream Function for u_i and v_i
ϕ_n	n^{th} term in the Expansion of the Stream Function ϕ
η	Dimensionless δ
λ	Wave length
μ_k	Dynamic viscosity of the k^{th} Phase
ρ_k	Density of the k^{th} Phase
σ	Surface Tension
θ	Angle of Inclination
ψ	Magnitude of the Stream Function for u_2 and v_2
ψ_n	n^{th} Term in the Expansion of the shear

Subscripts

0	Evaluated at the Interface
1	Upper Fluid
2	Lower Fluid
g	Upper Fluid (gas)
k	Applicable to both Upper and Lower Fluids

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Fig. 1 Base flow configuration.

Fig. 2 Typical velocity profile, (a), and shear stress, (b), for horizontal flow when $m \gg 1$ and $n^2 \ll m$. For this example $n = 4$, $m = 50$, $\mu_1 = 1 \cdot 10^{-5} \text{ N}\cdot\text{s/m}$, $r = 1000$ and $\rho_1 = 1 \text{ kg/m}^3$ in a 50 mm channel with average velocities $U_1 = 0.7 \text{ m/s}$ and $U_2 = 0.475 \text{ m/s}$.

Fig. 3 Typical velocity profile, (a), and shear stress, (b), for horizontal flow when $m = 1$ and $n^2 > m$. For this example $n = 4$, $m = 1$, $\mu_1 = 1 \cdot 10^{-5} \text{ N}\cdot\text{s/m}$, $r = 1000$ and $\rho_1 = 1 \text{ kg/km}^3$ in a 50 mm channel with average velocities $U_1 = 0.325 \text{ m/s}$ and $U_2 = 0.7 \text{ m/s}$.

Fig. 4 Typical velocity profile, (a), and shear stress, (b), for horizontal flow when $m \gg 1$ and $n^2 = m$. For this example $n = 4$, $m = 16$, $\mu_1 = 1 \cdot 10^{-5}$ N·s/m, $r = 1000$ and $\rho_1 = 1$ kg/m³ in a 50 mm channel with average velocities $U_1 = 0.6$ m/s and $U_2 = 0.6$ m/s.

Fig. 5 The effect of the density, viscosity and depth ratios on neutral stability for horizontal flow. The density ratios are 1, 3, 10 and 1000. The parameters are $G = 0.1$, $R > 0$ and $\theta = 0$.

Fig. 6 Comparison of the effect of favorable and adverse density stratifications on neutral stability in the m - n plane for horizontal flow. The density ratios are $1/3$, 1 and 3 . The flow parameters are $G = 0.1$, $R > 0$ and $\theta = 0$.

Fig. 7 Comparison of the effect of favorable and adverse density stratifications on neutral stability for horizontal flow in absence of gravity in the m - n plane. The density ratios are $1/10$, and 10 . The parameters are $G = 0$, $R > 0$ and $\theta = 0$.

Fig. 8 The effect of G , the viscosity ratio and the depth ratio on neutral stability for horizontal flow. The values of G are 0.01, 0.1 and 1. The parameters are $r = 10$, $R > 0$ and $\theta = 0$.

Fig. 9 The effect of the angle on inclination on neutral stability in the m - n plane for fluids of comparable density. The angles are 0° , 45° and 90° . The parameters are $r = 1.5$, $R = 3$ and $G = 0.1$.

Fig. 10 The effect of the angle on inclination on neutral stability in the m - n plane. The angles are 0° , 45° and 90° . The parameters are $r = 10$, $R = 3$ and $G = 0.1$.

Fig. 11. The effect on the density, viscosity and depth ratios on neutral stability for vertical flow. The density ratios are 15 and 100. The parameters are $R = 3$, $\theta = 90^\circ$ and $G = 01$.

Fig. 12. The effect of G and the viscosity and depth ratios on neutral stability for vertical flow. The values for G are 0.1 and 1.0. The parameters are $r = 10$, $R = 3$ and $\theta = 90^\circ$.

Fig. 13. The effect of R and the viscosity and depth ratios on neutral stability for vertical flow. The values for R are 3 and 10. The parameters are $r = 10$, $G = 0.1$ and $\theta = 90^\circ$.

Fig. 14. The effect of the viscosity and depth ratios on the dimensionless interfacial wave velocity for horizontal flow. The viscosity ratios are 0.01, 0.1, 1, 10 and 100. The parameters are $r > 0$, $\theta = 0$, $G > 0$ and $R > 0$.

14. SIMILARITY REQUIREMENTS FOR TWO-PHASE FLOW-PATTERN TRANSITION

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ABSTRACT

Overall scaling criteria for a natural circulation loop under single-phase and two-phase flow conditions are summarized. The phase change similarity requirement is used to establish scaled-down model operating conditions for fluid-to-fluid and reduced-pressure simulations of a high-pressure water system. Based on the overall scaling methodology, additional similarity requirements are derived from two-phase flow-pattern transition boundaries for horizontal and vertical two-phase flow configurations. It is shown that flow-pattern transition scaling requirements are all different from each other. However, it is demonstrated that a solution in the form of the axial length ratio as a function of the cross-sectional area ratio and some fluid property ratio groups is possible for each flow-pattern transition studied. Finally, it is concluded that since the constitutive relation between the two-phase flow-patterns and the void fraction depends on the actual size, a completely correct scaling may be impossible to achieve and scaling compromises must be made.

14.1 Introduction

Similarity laws and scaling criteria are important for designing, performing and analyzing simulation experiments using scale models. They are particularly important for studying phenomena which can not be easily observed in prototype models, such as those that occurs in nuclear reactor accidents and in some petrochemical experiments. In view of the inherent difficulties associated with full-scale testing of such systems, it is necessary to use scale models and simulation experiments. However, for a justifiable extrapolation of experimental model observations to a full-scale prototype, it is necessary that the scaling rationale be soundly based.

In line with the central importance in establishing similarity relations between the thermal-hydraulic behavior of a prototype and of scale models, extensive investigations have recently been undertaken to develop the similarity criteria for the natural circulation loops under single-phase and two-phase flow conditions [1-13]. The similarity criteria obtained from these investigations were further examined for their applications to existing scaled-down test facilities [1, 2, 8-10,13]. In all these investigations, however, emphasis was placed on the scaling philosophy and the overall scaling requirements. Local phenomena associated with the two-phase flow-pattern transitions have not yet been addressed.

Two-phase flow is characterized by the existence of the interfaces between phases and discontinuities of properties associated with them. The internal structures of two-phase flow are classified by two-phase flow patterns. Various transfer mechanisms between the mixture and the wall as well as between phases strongly depend on these two-phase flow patterns. This leads to the use of flow-pattern dependent correlations and closure equations

together with appropriate flow-pattern transition criteria. Therefore, before selecting the flow-pattern dependent constitutive equations and closure relations, and even before examining these relations to determine whether they properly account for the scale effects, similarity in the internal two-phase flow structure must be studied in detail. If distortions exist then those associated with such similarity should be clearly identified.

This paper, which summarizes the overall similarity requirements for single-phase flow, two-phase flow and single-phase to two-phase flow as obtained by the recent studies [1-5], has three objectives. The first is to examine the existing two-phase flow-pattern transition criteria and to verify that these criteria properly account for the effects of scale. The second is to study two-phase flow-pattern transition scale effects for horizontal and vertical two-phase flows. Finally, the third objective is to develop two-phase flow-pattern transition scaling requirements. The development includes the effects of primary fluid properties so as to be applicable to models using low pressure water or alternate fluids to simulate high pressure water systems, as well as prototypic fluid, prototypic pressure and temperature models.

14.2 Scaling Criteria

14.2.1 Single-Phase Flow Similarity

Although the major effort in this study is directed at examining the two-phase flow-pattern transition scale effects, the single-phase flow similarity requirements are first needed to simulate the single-phase to two-phase flow transition. Any natural circulation loop consists of a thermal energy source, energy sink, and connecting piping between major components. Circulating fluid goes through a phase change through the major components.

The fluid continuity, integral momentum and energy equations in one-dimensional, area-averaged forms have been used together with the appropriate boundary conditions and the solid energy equation [1-5]. From the non-dimensional form of these equations, important dimensionless groups characterizing geometric, kinematic, dynamic and energetic similarity parameters are derived. These are Richardson number R , Friction number F_i , Stanton number St_i , Time Ratio number T_i^* , Biot number B_i and Heat Source number Q_{si} . In addition to the physical similarity groups listed above, several geometric similarity groups are obtained. These are the axial length scale $L_i \equiv l_i/l_o$ and the flow area scale, $A_i \equiv a_i/a_o$. In this presentation, subscript i , o and s identify the i th component of the loop the reference state and solid, respectively.

Identifying the ratio between any model to prototype variable by the subscript R , it is required for similarity that

$$\begin{aligned} A_{iR} = 1, L_{iR} = 1, F_{iR} = 1, R_R = 1, \\ St_{iR} = 1, T_{iR}^* = 1, B_{iR} = 1, Q_{siR} = 1 \end{aligned} \quad (1)$$

As discussed in detail in [1-5], the frictional similarity requirement can be satisfied independently of the remaining scaling requirements. Hence, from the remaining scaling requirements, it was shown that the following conditions should be satisfied for a complete similarity.

$$u_{oR} = [B q_o^{\cdot\cdot\cdot} l_o^2 / (\rho_s c_{ps})]_R^{1/3} \quad (2)$$

$$\Delta T_{oR} = [q_o^{\cdot\cdot\cdot} l_o / (\rho_s c_{ps}) u_o]_R^{1/2} \quad (3)$$

$$\xi_{iR} \equiv \xi_R = (\alpha_s l_o / u_o)_R^{1/2} \quad (4)$$

$$d_{iR} \equiv d_R = [(\rho_s c_{pf})/(\rho_f c_{pf})]_R (\alpha_s l_o / \nu_o)_R^{1/2} \quad (5)$$

$$h_{iR} \equiv h_R = k_{sR} (u_o / l_o \alpha_s)_R^{1/2} \quad (6)$$

$$\tau_R \equiv (l_o / \nu_o)_R = l_{oR} [B q'' l_o^2 / (\rho_s c_{ps})]_R^{-1/3} \quad (7)$$

Here u_o , ΔT_o and l_o are the prescribed reference velocity, temperature difference, and equivalent length, respectively. Most of the symbols appearing above conform to standard nomenclature. However, for the purpose of completeness, all symbols are defined in the nomenclature. In addition to the above constraints, the following geometric and dynamic similarity requirements must be met:

$$(l_i / l_o) = (a_i / a_o)_R = 1 \quad (8)$$

$$(\sum_i F_i / A_i^2)_R = 1 \quad (9)$$

As briefly discussed above, the friction similarity requirement can be easily satisfied by introducing appropriate flow resistances in the loop. However, as demonstrated by Kocamustafaogullari and Ishii [2,4], this requirement imposes a limit on the size of a scaled-down model such as $l_R = F(a_R)$. The procedure illustrated in [2,4] must be carried out carefully, so that the resulting system geometry would be practical. In general, the heat transfer coefficient ratio criterion is difficult to satisfy in general, because h depends on single-phase flow regime. Detailed analysis on the heat transfer simulation was carried out by Zvirin and Sursock [9] and was demonstrated that additional conditions imposed on l_R would guarantee the same the single-phase flow regime (laminar or turbulent) in the model and prototype. Furthermore, it is important to note that the conduction depth

ratio and hydraulic diameter ratio given by Eqs. (4) and (5), respectively, may be difficult to meet over the entire loop. However, as was discussed in detail by Refs. [1-5], they are important only at the major heat transfer components where they can be easily met.

14.2.2 Two-Phase Flow Similarity

The similarity requirements for a natural circulation loop under two-phase flow conditions may be obtained from the integral effects of the local two-phase flow balance equations around the entire flow loop. Under natural circulation conditions, the majority of flow transients are expected to be relatively slow. Furthermore, for the development of system similarity laws, the response of the whole mixture is important rather than the detailed response of each phase and phase-to-phase interactions. Therefore, the drift-flux model formulation is more appropriate for the derivation of system similarity parameters under natural circulation conditions.

Based on the drift-flux model formulation, the resulting important dimensionless parameters that characterize the geometric kinematic, dynamic and energetic fields of a two-phase flow natural circulation loop can be summarized as follows:

$$N_{pch} \equiv (4 \dot{q}_0'' \delta l / d u_0 \rho_f h_{fg})(\Delta\rho/\rho_g) \quad (10)$$

$$N_{sub} \equiv (\Delta h_{sub}/h_{fg})(\Delta\rho/\rho_g) \quad (11)$$

$$N_{Fr} \equiv (u_0^2/g\delta_0)(\rho_f/\alpha\Delta\rho) \quad (12)$$

$$N_{di} \equiv (V_{gj}/u_0)_i \quad (13)$$

$$T_i^* \equiv (\delta_0/\mu_0)/(\delta^2/\alpha_s)_i \quad (14)$$

$$N_{thi} \equiv (\rho_s C_{ps} \delta / \rho_f C_{pf} d)_i \quad (15)$$

$$N_{fi} \equiv (f l / d)_i [(1 + \Delta x \Delta \rho / \rho_g) / (1 + x \Delta \mu / \mu_g)]^{0.25} (a_o / a_i)^2 \quad (16)$$

$$N_o \equiv K_i [1 + x^{3/2} (\Delta \rho / \rho_g)] (a_o / a_i)^2 \quad (17)$$

Details of the derivations leading to these equations can be found in [1-5].

In addition to the above-defined physical similarity parameters, geometric similarity groups (l_i / l_o) and (a_i / a_o) are obtained.

For a complete similarity, the characteristic parameters must be the same in both the prototype and the model. Thus,

$$\begin{aligned} (N_{pch})_R = 1, (N_{sub})_R = 1, (N_{Fr})_R = 1, (N_{di})_R = 1 \\ (\tau_i^*)_R = 1, (N_{thi})_R = 1, (N_{fi})_R = 1, (N_o)_R = 1 \end{aligned} \quad (18)$$

In view of Eqs. (10) through (17), solutions to the above set of similarity requirements, Eq. (18), results in the following conditions which must be met for complete similarity:

$$q_{oR} = (\rho_f \rho_g h_{fg} / \Delta \rho) (d / \delta) / l_{oR}^{1/2} \quad (19)$$

$$u_{oR} = l_{oR}^{1/2} \quad (20)$$

$$(\Delta h_{sub})_R = (h_{fg} \rho_g / \Delta \rho)_R \quad (21)$$

$$\delta_R = (l_{oR} \alpha_s^2)_R^{1/4} \quad (22)$$

$$d_R = (\rho_s C_{ps} / \rho_f C_{pf})_R (l_{oR} \alpha_s^2)_R^{1/4} \quad (23)$$

$$\tau_R = l_{oR}^{1/2} \quad (24)$$

$$x_R = (\rho_g / \Delta \rho)_R \quad (25)$$

$$\alpha_R = (\rho_f / \Delta \rho)_R \quad (26)$$

The conditions imposed by Eqs. (19) - (26) assure that the effects of each term in the conservation equations are preserved in both model and the prototype without any distortions. If they are not all satisfied, the effects of the processes observed in the model and prototype will be distorted.

14.2.3 Single-Phase to Two-Phase Transition

As demonstrated above, single-phase and two-phase flow similarity requirements are quite different from each other. This introduces difficulties in meeting geometric, power and time scaling requirements when both single-phase and two-phase flow occur simultaneously in a loop. This problem may be overcome by requiring the same power scale in both phases. Since the power scale for the two-phase flow similarity is established by Eq. (19), the simplest approach is to use this power scale for the single-phase flow also. Thus,

$$(\dot{q}_{oR})_{sp} = (\dot{q}_{oR})_{tp} = (\rho_f \rho_g h_{fg} / \Delta \rho)_R (\rho_s C_{ps} / \rho_f C_{pf})_R^{1/2} \quad (27)$$

When the two-phase flow power scale, Eq. (27), is used in the single-phase similarity requirements, Eqs. (3)-(7), it can be shown that

$$(u_{oR})_{sp} / (u_{oR})_{tp} = (\beta \rho_g h_{fg} / \Delta \rho C_{pf})_R^{1/3} \quad (28)$$

$$(\delta_R)_{sp} / (\delta_R)_{tp} = (\beta \rho_g h_{fg} / \Delta \rho C_{pf})_R^{-1/6} \quad (29)$$

$$(d_R)_{sp} / (d_R)_{tp} = (\beta \rho_g h_{fg} / \Delta \rho C_{pf})_R^{-1/3} \quad (30)$$

$$(\tau_R)_{sp} / (\tau_R)_{tp} = (\beta \rho_g h_{fg} / \Delta \rho C_{pf})_R^{-1/3} \quad (31)$$

It is interesting to note from Eqs. (28) through (31) that the single-phase to two-phase flow simulation can be preserved when

$$(B \rho_g h_{fg} / \Delta \rho C_{pf})_R = 1 \quad (32)$$

If this fluid property group requirement is met, then complete similarity can be achieved during the single-phase to or from two-phase flow transitions in a natural circulation loop. This ensures that both the time scale and power scale as well as geometric scales are matched simultaneously.

Variations of this group as a function of pressure are illustrated for water, R-11 and R-113 in Figure 1. This figure indicates that the distortions may be unavoidable in simulating a high-pressure system with a low-pressure model by using the same fluid since the single-phase to two-phase flow transition constraint can not be met. The degree of distortions depends on the model pressure. For example, for modeling 110 bar water systems with a low pressure water system at 20 bar, Figure 1 yields

$$(B \rho_g h_{fg} / \Delta \rho C_{pf})_R = 0.007/0.08 = 0.0875 \quad (33)$$

This requires scaling the single-phase and two-phase flow sections differently. For the above numerical example from Eqs. (28) - (31), it can be shown that $(u_{oR})_{sp}/(u_{oR})_{tp} = (d_R)_{sp}/(d_R)_{tp} = 0.446$ whereas $(\tau_R)_{sp}/(\tau_R)_{tp} = 2.242$ and $(\delta_R)_{sp}/(\delta_R)_{tp} = 1.50$. On the other hand for a water model at 40 bar these numbers, respectively, become 0.630, 1.587 and 1.260 indicating that the distortions are reduced.

Fluid to fluid scaling is possible without any problem. For the prototype pressure of 110 bar which corresponds to the saturation pressure of a typical PWR, it is evident from the figure that Eq. (32) can be satisfied at 23 bar from R-113 and 26 bar for R-11. Both of these values are within the

Figure 1. Single-phase to or from two-phase flow simulation for the prototypic and selected model fluids.

practical range of applicability for the selected model fluids. These pressure levels will be considered as base pressures for the flow-pattern transition scaling studies.

14.3 Flow-Pattern Scaling

14.3.1 Existing Flow-Pattern Transition Criteria

The internal structures of two-phase flow are classified by the flow regimes or flow patterns. Various transfer mechanisms between the two-phase flow mixture and the wall, as well as between two phases, depend on the flow patterns. This leads to the use of flow-pattern dependent correlations together with two-phase flow-pattern transition criteria. From this point of view alone, the scaling of two-phase flow patterns becomes one of the most important issues for the conceptual design of simulation experiments whose aim is to investigate phenomena associated with two-phase flow in a prototype.

Due to its central importance in selecting flow-pattern dependent constitutive equations and closure relations, a reasonably large amount of experimental data on flow-pattern has been accumulated, and the problem of predicting flow-pattern transitions has received a great deal of attention in vertical and horizontal two-phase flow systems. Traditionally, flow patterns are identified from flow-pattern maps obtained from experimental observations. Many flow-pattern maps have been proposed using dimensional coordinates based on the liquid and gas superficial velocities. Other investigations have attempted to correlate the transition boundaries by non-dimensional groups. All of these maps are based on experimental data; and, therefore, there is no theoretical basis to suggest that these maps can be extrapolated to a wider range of tube diameters, fluid properties and flow patterns. This uncertainty introduces difficulties in their use for the

purpose of scaling analysis. There have also been theoretical works, [14-19], based on phenomenological modeling to determine the suitable mapping parameters. Although physical transitions postulated by these modeling efforts represent the first comprehensive treatment of flow-pattern transitions in horizontal and vertical two-phase flow systems, Weisman et al. [19-22] and Serdasai [23] indicated a number of deficiencies in these models.

In view of this brief overview on existing flow-pattern maps, it is improbable that the correct and complete set of flow-pattern transition maps, which can be used for a wide range of tube diameters, fluid properties and flow parameters, has yet been identified. However, the analyses presented by Dukler et al. [14-17] for horizontal and vertical pipes and by Mishima and Ishii [18] for vertical systems appear to be the most complete to date. For the purpose of flow-pattern scaling investigations undertaken here, the discussions will be concentrated on the flow-pattern transition criteria suggested by Dukler et al. and by Mishima and Ishii.

14.3.2 Flow-Pattern Transition Scaling Studies for Horizontal Pipes

Taitel and Dukler [14] presented a generalized flow-pattern map for the case of horizontal tubes. The map proposed by these investigators uses different coordinate systems according to the transitions which are considered. However, one coordinate is common for all transitions. It is X which is recognized as the parameter introduced by Lockhart and Martinelli [24]. As demonstrated by [14], the dimensionless liquid level, $\tilde{h}_f \equiv h_f/d$, is an explicit function of the Martinelli parameter; and, since the void fraction and $\tilde{h}_f$ are uniquely related, the Taitel-Dukler map, for the purpose of convenience, can be described in terms of the void fraction. These interrelations among α , X_{tt} and $\tilde{h}_f$ are illustrated in Figure 2.

Figure 2. Variations of geometric variables used in Taitel and Dukler horizontal flow-pattern transitions.

14.3.2.1 Stratified-Smooth and Stratified-Wavy Transition

This transition model is based on the assumption that the Jeffreys model [25,26] is valid for describing the condition for transfer of energy to the liquid phase in order to generate waves, with the wave velocity estimated from the mean velocity of the liquid film and the sheltering coefficient determined from analyses of Benjamin [27, 28]. The transition criterion may be expressed as

$$K \geq 20 \alpha (1-\alpha)^{1/2} \quad (34)$$

where the dimensionless group K is defined by

$$K \equiv F (Re_{jf})^{1/2} \quad (35)$$

In this definition $F \equiv \rho_g j_g^2 / g \Delta \rho d$ is a modified Froude number and $Re_{jf} \equiv d j_f / \nu_f$ is the Reynolds number based on the superficial liquid velocity, j_f .

Equation (34) is presented by a solid line in Figure 3 for the prototype. It appears that if transition is to take place without any distortions

$$K_R \equiv F_R (Re_{jf})_R^{1/2} = \alpha_R (1-\alpha)_R^{1/2} \quad (36)$$

In view of the scaling criteria summarized in the preceding sections, it can be shown that

$$F_R = (\rho_g / \Delta \rho)_R^{1/2} j_{gR} / d_R^{1/2}; (Re_{jf})_R = d_R j_{fR} / \nu_{fR} \quad (37)$$

By definition it is evident that

$$d_R = a_R^{1/2}; j_{gR} = (\rho_f / \rho_g)_R x_R u_{oR}; j_{fR} = (1-x)_R u_{oR} \quad (38)$$

Figure 3. Horizontal flow-pattern transitions between stratified-smooth and stratified-wavy flows.

Furthermore, using Eq. (25) for x_R and (20) for u_{oR} , the superficial velocity ratios become

$$j_{gR} = (\rho_f/\Delta\rho)_R l_R^{1/2}; j_{fR} = (1-x)_R l_R^{1/2} \quad (39)$$

In view of Eqs. (38) and (39), F_R and $(Re_{jf})_R$ can be expressed as

$$F_R = (\rho_g \rho_f^2 / \Delta\rho^3)_R^{1/2} l_R^{1/2} / a_R^{1/4}; (Re_{jf})_R = (1-x)_R a_R^{1/2} l_R^{1/2} / \nu_{fR} \quad (40)$$

When Eq. (40) is used in (36) the flow-pattern scaling criteria can be expressed by

$$l_R^{3/4} = \alpha_R [(1-\alpha)_R / (1-x)_R]^{1/2} (\Delta\rho^3 \nu_f / \rho_f^2 \rho_g)^{1/2}_R \quad (41)$$

where α_R can be replaced by Eq. (26). The $(1-x)_R$ term can be found by rearranging the Martinelli parameter as defined by Taitel and Dukler and substituting the two-phase similarity requirements.

$$(1-x)_R = (x_{tt})_R^{1.11} (\rho_g / \Delta\rho)_R (\rho_f / \rho_g)_R^{4/9} (\nu_g / \nu_f)_R^{1/9} \quad (42)$$

where

$$(x_{tt})_R = (\rho_f / \rho_g)_R^{1/2} (j_f / j_g)_R^{0.9} (\nu_f / \nu_g)_R^{0.1} \quad (43)$$

By substituting Eq. (42) into Eq. (41), it may be shown that

$$l_R = f(\text{fluid properties}). \quad (44)$$

It is important to note that Eq. (41) or (44) puts an additional restriction on the length scale, and, if Eq. (41) can not be met, then the scale distortion is unavoidable. In order to be able to appreciate the scale distortions the results are also plotted in Figure 3 with l_R as the parameter

for the reduced pressure and fluid-to-fluid scalings. In constructing these curves, K_m and α_m are determined by

$$K_m = K_R K_p \quad \text{and} \quad \alpha_m = \alpha_R \alpha_p \quad (45)$$

where K_R and α_R are calculated by Eqs. (40) and (26), respectively. It is apparent from this figure that if the conditions in the prototype are such that a flow transition occurs, say from stratified-smooth to stratified-wavy, then the corresponding horizontal pipe section in the model will have stratified-wavy for the fluid-to-fluid scaling with R-11 and R-113 and stratified-smooth for the reduced pressure scaling. It is to be noted that the distortions are proportional to the length scale dictated by Eq. (41).

It can be concluded therefore, that the flow-pattern transition scaling when used in conjunction with the general scaling criteria developed in the preceding sections, leads to a distortion which precludes the possibility of simultaneously satisfying the particular flow-pattern structure transition presented here and the general criteria set in the preceding sections. If one is satisfied, the other is not and vice versa. However, considering the fact that this particular transition is not important for practical purposes, the distortion introduced by Eq. (41) can be relaxed. Furthermore, it should also be noted that as Eq. (41) requires the flow-pattern transition criterion can be satisfied by changing the linear length of the horizontal section in the model.

14.3.2.2 Stratified to Intermittent or Annular-Dispersed Liquid Transition

The transition is established from the argument that waves of finite size will form and tend to block or sweep around the pipe when the force due to the Bernoulli effect above the wave is greater than gravity force acting on the

wave. It is represented by a modified Froude number and the geometric parameters h_f and α . It is given by Taitel and Dukler as

$$F \geq \{(\pi/4) (1-\tilde{h}_f)^2 \alpha^2 / [1-(2\tilde{h}_f-1)^2]^{1/2}\}^{1/2} \quad (46)$$

Since h_f and α are uniquely related from Figure 2, this transition can be described by

$$F \geq f_F(\alpha) \quad (47)$$

where $f_F(\alpha)$ stands for the right hand side of Eq. (46).

Equation (47), or more explicitly Eq. (46), is presented in Figure 4 by the solid line for the prototype. It is evident that if the transition is to take place without any distortions

$$F_R \equiv (\rho_g \rho_f^2 / \Delta \rho^3)_R \ell_R^{1/2} a_R^{-1/4} = f_F(\alpha_R) \quad (48)$$

where Eq. (40) is used for F_R .

Equation (48) puts an additional restriction on the length scale, ℓ_R , in terms of a_R , α_R and fluid properties. Since this equation is in implicit form, the degree of distortion can not be determined directly. However, the distortions are demonstrated in Figure 4 for the fluid-to-fluid scaling as well as reduced pressure scaling. For a given $a_R = 1/200$, ℓ_R is treated as a parameter in this figure. Here it is evident that if the flow conditions in the prototype are such that a flow transition occurs from stratified to intermittent or annular as indicated by the solid line, then the corresponding model horizontal pipe section will be either intermittent or annular for the fluid-to-fluid scaling, and still stratified flow for the reduced pressure scaling. However, it may be clearly seen from this figure

Figure 4. Horizontal flow-pattern transitions between stratified, intermittent and annular-dispersed liquid flows.

that the distortions introduced by this scaling are not significant for the fluid-to-fluid scaling. Therefore, it may be considered that the transition criteria is approximately satisfied without appreciable distortions for this case, although the distortions will be significant for a scaled-down model operating at a reduced pressure of 20 bar. Other values of a_R were examined, and the same trend was observed.

The length scale requirement for a proper flow-pattern transition without any distortion may be expressed from Eq. (45) as follows:

$$l_R = a_R^{1/2} (\Delta p^3 / \rho_g \rho_f^2) f_F(\alpha_R) \quad (49)$$

Considering that α_R can be expressed by Eq. (26), Eq. (49) yields $l_R = l_R(a_R, \text{properties})$, and the more detailed transition analysis may be performed from this functional relation. It is to be noted that for the case of the representative fluid-to-fluid, R-11 and R-113, and the reduced pressure cases considered here Eq. (49) yields

$$l_R = l_R(a_R) \quad (50)$$

This requirement must be studied in detail together with the friction number scaling, which results in a similar requirement.

14.3.2.3 Intermittent or Dispersed Bubbles and Annular-Dispersed

This transition occurs at a constant value of the Martinelli parameter resulting from the argument that the growing waves will have a sufficient liquid supply to form a slug only when $h_f \geq 0.5$, and below that value they will be swept around the pipe into an annular configuration. Therefore, it can be represented in terms of α as follows

$$\alpha \leq 0.5 \quad (51)$$

In terms of the scaling ratio the transition similarity requires that

$$\alpha_R = 1 \quad (52)$$

However, in view of the scaling criteria developed in the preceding sections, it was demonstrated by Eq. (26) that

$$\alpha_R = (\rho_f/\Delta\rho)_R \quad (53)$$

Numerical evaluation of $(\rho_f/\Delta\rho)_R$ for the examples considered here reveals that this density ratio is 1.06 for R-11, 1.11 for R-113 and 0.917 for the reduced pressure simulations. Therefore, for the practical purposes this flow-pattern transition can be simulated without any appreciable distortions satisfying Eq. (52). For the purpose of convenience these transitions are also illustrated in Figures 4 and 5.

14.3.2.4 Intermittent and Dispersed-Bubble Transition

According to the Taitel and Dukler [24] this transition represents an identification of conditions where the turbulent fluctuations in the liquid become equal to the buoyant forces which tend to make the gas rise to the top of the pipe. Accordingly, this transition is represented by

$$T \geq \{ [2\pi\alpha (1-\alpha)^2/\tilde{s}_l] \cdot (\pi/\tilde{s}_f)^n \}^{1/2} \quad (54)$$

where T is defined as

$$T \equiv [2 c_f (j_f d/v_f)^{-n} \cdot (\rho_f j_f^2/g\Delta\rho d)]^{1/2} \quad (55)$$

In these equations $c_f = 16$ and $n = 1.0$ for the laminar liquid flow whereas $c_f = 0.046$ and $n = 0.2$ for the turbulent flow. For the numerical studies presented here, the latter is used throughout the analysis, which is clearly the case of greater practical interest.

Since $\tilde{\alpha}_f$ and $\tilde{\alpha}_i$ can be uniquely determined in terms of void fraction, as shown by Figure 2, this transition may be viewed as

$$T \geq f_T(\alpha) \quad (56)$$

where $f_T(\alpha)$ is defined as the right-hand side of Eq. (54).

The solution of Eq. (56) is executed for turbulent flow of liquid phase. The result is shown in Figure 5 as the solid line. All values of T below the curve represent a condition where the turbulence is insufficient to keep the gas mixed, and the elongated gas bubbles characteristic of intermittent flow will form.

Correct scaling of this transition requires that

$$T_R = f_T(\alpha_R) \quad (57)$$

where T_R is given by

$$T_R \equiv v_{fR}^{0.1} (\rho_f/\Delta\rho)_R^{0.5} (1-x)_R^{0.9} a_R^{0.45} a_R^{-0.3} \quad (58)$$

In the derivation of Eq. (58), Eqs. (38) and (39) are used for d_R and j_{fR} , respectively.

Equation (57) indicates that the flow-pattern transition can be met when the linear length scale in a horizontal pipe section satisfy the condition given by

$$a_R^{0.45} = f_T(\alpha_R) a_R^{0.3} / [v_{fR}^{0.1} (\rho_f/\Delta\rho)_R^{0.5} (1-x)_R^{0.9}] \quad (59)$$

Since $f_T(\alpha_R)$ is an implicit function of α_R , the requirement imposed by Eq. (59) can not be illustrated explicitly. Instead, the possible distortions introduced by this flow-pattern transition are illustrated in

Figure 5. Horizontal flow-pattern transitions between intermittent and dispersed-bubble flows.

Since s_f and s_i can be uniquely determined in terms of void fraction, as shown by Figure 2, this transition may be viewed as

$$T \geq f_T(\alpha) \quad (56)$$

where $f_T(\alpha)$ is defined as the right-hand side of Eq. (54).

The solution of Eq. (56) is executed for turbulent flow of liquid phase. The result is shown in Figure 5 as the solid line. All values of T below the curve represent a condition where the turbulence is insufficient to keep the gas mixed, and the elongated gas bubbles characteristic of intermittent flow will form.

Correct scaling of this transition requires that

$$T_R = f_T(\alpha_R) \quad (57)$$

where T_R is given by

$$T_R \equiv v_{fR}^{0.1} (\rho_f/\Delta\rho)_R^{0.5} (1-x)_R^{0.9} z_R^{0.45} a_R^{-0.3} \quad (58)$$

In the derivation of Eq. (58), Eqs. (38) and (39) are used for d_R and j_{fR} , respectively.

Equation (57) indicates that the flow-pattern transition can be met when the linear length scale in a horizontal pipe section satisfy the condition given by

$$z_R^{0.45} = f_T(\alpha_R) a_R^{0.3} / [v_{fR}^{0.1} (\rho_f/\Delta\rho)_R^{0.5} (1-x)_R^{0.9}] \quad (59)$$

Since $f_T(\alpha_R)$ is an implicit function of α_R , the requirement imposed by Eq. (59) can not be illustrated explicitly. Instead, the possible distortions introduced by this flow-pattern transition are illustrated in

Figure 5 by treating l_R and a_R as parameters. The figure indicates that the distortions are not significant at lower l_R values, and, with proper adjustments of the horizontal piping dimensions, this transition can be completely simulated for both the fluid-to-fluid and reduced pressure scaling.

14.3.2.5 Flow-Pattern Transition Scaling Requirements for Horizontal Pipes

Additional flow-pattern transition scaling requirements may be imposed by Eq. (41) for stratified-smooth and stratified-wavy, by Eq. (49) for stratified and intermittent or annular-dispersed liquid, and by Eq. (59) for intermittent and dispersed-bubble transitions. Although these requirements result in different geometric scales, they can be unified in the form of

$$l_R = (\text{property ratio group}) \times a_R^n \quad (60)$$

where the exponent n differs from one transition to another.

For the purpose of numerical comparison we can use the representative model fluid conditions considered here. This type of approach determines the property values for the model fluids such as used here to satisfy the single-phase to or from two-phase flow transition, Eq. (32). As noted from Figure 1, the phase change requirement can be easily met with R-113 at 23 bar and R-11 at 26 bar. Different requirements imposed by Eqs. (41), (49) and (59) are illustrated in Figure 6 in the form of Eq. (50) for the fluid-to-fluid simulation as well as for the water at reduced pressure of $P_m = 20$ bar.

Figure 6 indicates that it is impossible to satisfy all the flow-pattern transition scaling requirements simultaneously. Therefore, the designer of a model should carefully evaluate the possible flow-pattern transitions in the prototype, and establish the length scale accordingly with the least amount of distortion.

Figure 6. Flow-pattern scaling requirements for horizontal two-phase flow.

14.3.3 Flow-Pattern Scaling Studies for Vertical Pipes

Taitel, Barnea and Dukler [17] presented a flow-pattern map for upward gas-liquid flows in vertical tubes where coordinates vary in regard to possible combinations of the gas and liquid superficial velocities. For the purpose of the following scaling studies we use the criteria suggested by Taitel et al. in convenient dimensionless forms. Recently, Mishima and Ishii [18] proposed different flow-pattern maps in terms of the parameters which characterize the interfacial structure of the two-phase flow, such as α . Although these flow-patterns seem different from each other, they are very close in terms of the mechanistic modeling. Therefore, these two maps will be used in the following scaling studies parallel to each other.

14.3.3.1 Bubbly and Slug Flow Transition

As studied by Radovic and Moissis [29] and Griffith and Snyder [30], the transition from bubbly flow to slug flow in a vertical tube occurs mainly due to agglomerations and coalescences of smaller bubbles into cap bubbles. Once a cap-bubble is formed, further coalescences follow in the wake region of a cap bubble. This transition occurs at a constant value of the void fraction. Thus

$$\alpha \geq C \quad (61)$$

Taitel et al. used $C = 0.25$, whereas Mishima and Ishii obtained from a very simple geometric consideration a value of $C = 0.3$. In terms of the scaling ratio, it requires that

$$\alpha_R = 1. \quad (62)$$

Since the void fraction scale is fixed by Eq. (26), then the requirement can be expressed as follows:

$$\alpha_R = (\rho_f / \Delta \rho)_R = 1 \quad (61)$$

As demonstrated above, the numerical evaluations of $(\rho_f / \Delta \rho)_R$ for the representative examples proposed reveal that the density ratio appearing in Eq. (61) is 1.06 for R-11, 1.11 for R-113 and 0.917 for the reduced pressure simulations. Therefore, it may seem that for practical purposes this flow-pattern transition can be considered to be simulated without significant distortions.

At this point some comments might be appropriate regarding this flow-pattern transition. It is to be noted that the void fraction similarity, Eq. (26), was derived based on the premise that the contribution of the local slip is small in comparison with the slip due to the transverse velocity and void profiles [1-6]. When the local slip is the dominant factor for the relative velocity, the similarity requirement becomes $u_R = 1$ which is different from Eq. (20). This inconsistency can be illustrated by converting Eq. (61) into a conventional form based on the superficial velocities. Thus, Taitel and Dukler showed for relatively larger pipe diameters

$$j_f / j_g = 3.0 - 1.15 (g \Delta \rho \sigma / \rho_f^2)^{1/4} / j_g \quad (62)$$

whereas Mishima and Ishii proposed

$$j_f / j_g = [(3.33 / C_0) - 1] - (0.76 / C_0) (g \Delta \rho \sigma / \rho_f^2)^{1/4} / j_g \quad (63)$$

where C_0 is given by

$$C_0 = 1.2 - 0.2 (\rho_g / \rho_f)^{1/2} \quad (64)$$

for round tubes. It is to be noted that the second term on the right-hand side of Eqs. (64) and (65) accounts for the dimensionless local slip, whereas

the first term indicates the distribution slip. Furthermore, it is to be noted that the transition lines given by Eqs. (64) and (65) are essentially similar in terms of the dimensionless groupings. Differences arising are due to the numerical constants only. Therefore, only the Taitel et al. transition is considered for further scaling studies.

Equation (64) is presented in Figure 7 by the solid line. Distortions are analyzed by forming the model to prototype ratios for the basic dimensionless coordinates as follows:

$$(j_f/j_g)_R = [(1-x)_R/x_R] (\rho_g/\rho_f)_R \quad (67)$$

$$(g\Delta\rho\sigma/\rho_f)_R^{1/4}/j_{gR} = (\Delta\rho\sigma/\rho_f^2)_R^{1/4} (\Delta\rho/\rho_f)_R x_R^{-1/2} \quad (68)$$

where x_R can be replaced by Eq. (25) in terms of the fluid property ratio.

The model operating conditions are also illustrated in Figure 7 for the representative cases. At low liquid flow rates where the liquid velocity is negligible, the transition is controlled by the free rise velocity of the bubbles, i.e., by the second term on the right-hand side of Eqs. (64) and (65); and, as expected, the distortions are much greater. On the other hand, at higher gas and liquid flow rates, where the bubble rise velocity relative to the liquid velocity is negligible, i.e., the local slip is negligible, the distortions are relatively very small. This region corresponds to $(g\Delta\rho\sigma/\rho_f^2)_R^{1/4}/j_g \leq 2.0$. The latter case corresponds to the scaling requirement given by Eq. (63).

In order to obtain a good scaling of this particular flow-pattern transition, additional scaling requirements may be obtained from Eqs. (67) and (68). They are given, respectively, as follows

Figure 7. Vertical flow-pattern transition between bubbly and slug or churn flows.

$$\left(\frac{1-x}{x}\right)_R (\rho_g/\rho_f)_R = 1 \quad (69)$$

$$L_R = (\Delta\rho\sigma/\rho_f^2)_R^{1/2} (\Delta\rho/\rho_f)_R^2 \quad (70)$$

The requirement imposed by Eq. (69) which corresponds to the case of insignificant local slip can be easily met by the basic requirement of Eq. (63). However, the condition given by Eq. (70) must be carefully evaluated for the cases where the local slip effects are important. In fact, the latter puts an additional restriction on the linear length scale of the vertical section of models. For example, Eq. (70) requires $L_R = 0.22$ for R-113 simulation at $P_m = 23$ bar whereas $L_R = 0.34$ for R-11 simulation at $P_m = 26$ bar. These results are confirmed in Figure 7 by $L_R = 0.3$ curves.

14.3.3.2 Slug and Churn Flow Transition

This transition is postulated to occur when the mean void fraction over the entire region exceeds that over the slug-bubble section. Just prior to the slug-to-churn flow transition, the bubbles are lined up right next to each other and the tail of the preceding bubble starts to touch the nose of the following bubble. Under this condition, the liquid slugs become unstable and are unable to sustain their individual identities due to the strong wake effect. Then destructions and creations of liquid slugs follows. It is considered that mechanistically this corresponds to the flow-pattern transition. Although this is the basic mechanism used for both references considered here, [12] and [18], their results are quite different from each other in terms of dimensionless groupings. Taitel et al.'s transition criterion for relatively large pipes can be expressed by

$$j_f/(gd)^{1/2} = 0.245 (Le/d)^{-0.22} - (j_g/(gd)^{1/2}) \quad (71)$$

whereas Mishima and Ishii's proposed model yields

$$\alpha \geq 1 - 0.813 \{ [(C_0 - 1) j / (g \Delta \rho d / \rho_f)^{1/2}] + 0.35 / [j / (g \Delta \rho d / \rho_f)^{1/2}] + 2.25 \}^{0.75} \quad (72)$$

Results of Eqs. (71) and (72) are presented in Figures 8 and 9, respectively. In constructing these figures

$$[j_f / (gd)]_R^{1/2} = (1-x)_R \, l_R^{1/2} / a_R^{1/4} \quad (73)$$

$$[j_g / (gd)]_R^{1/2} = (\rho_f / \Delta \rho)_R \, l_R^{1/2} / a_R^{1/4} \quad (74)$$

$$[j / (\Delta \rho gd / \rho_f)]_R = (\rho_f / \Delta \rho)_R^{3/2} \, l_R^{1/2} / a_R^{1/4} \quad (75)$$

are used whereas Eq. (26) is used for α_R .

Differences between Figures 8 and 9 may stem from the void correlation used in Taitel et al. However, it is important to note that both flow-pattern transition criteria proposed by [17] and [18] require a similar axial length scale in vertical pipe sections. It is given by

$$l_R \sim a_R^{1/2} \quad (76)$$

This requirement should be evaluated carefully for the vertical sections of models if the slug-churn flow transition is encountered in the prototype.

Recalling that the friction number scaling may result in a different $l_R = l_R(a_R)$, compromises should be carefully evaluated for designing a model.

14.3.3.3 Slug/Churn and Annular Flow Transition

This transition mechanism is based on the destruction of liquid slugs or large waves by entrainment or by deformation. For relatively large pipes, which are of the greatest interest in this study, the entrainment induced flow transition is given by Taitel et al. as follows:

Figure 8. Vertical flow-pattern transition between slug and churn flows for Taitel et al. Map.

Figure 9. Vertical flow-pattern transition between slug and churn flows for Mishima and Ishii Map.

$$j_g / (\sigma g \Delta \rho / \rho_g^2)^{1/4} = 3.1 \quad (77)$$

and by Mishima and Ishii as

$$j_g / (\sigma g \Delta \rho / \rho_g^2)^{1/4} \geq N_{\mu_f}^{-0.2} \quad (78)$$

where N_{μ_f} is the liquid phase viscosity group defined by

$$N_{\mu_f} \equiv \mu_f / [\sigma \rho_f (\sigma / g \Delta \rho)^{1/2}]^{1/2} \quad (79)$$

The transition lines given by Eqs. (77) and (78) are essentially very similar. Therefore, only Taitel et al.'s transition, which is expressed by Eq. (77), will be considered. The similarity in the flow-pattern transition requires that the following relation must be met:

$$j_{gR} (\rho_g^2 / \sigma \Delta \rho)_R^{1/4} = 1 \quad (80)$$

Furthermore, using Eq. (39) for j_{gR} , it can be shown that

$$\mathfrak{L}_R = (\Delta \rho / \rho_f)_R^2 (\sigma \Delta \rho / \rho_g^2)_R^{1/2} \quad (81)$$

Equation (78) can be evaluated for the cases presented here. This equation requires that $\mathfrak{L}_R = 0.108$ for R-113, 0.212 for R-11 and 14.125 for reduced pressure simulation at $P_m = 20$ bars. This numerical evaluation shows that the fluid-to-fluid scaling may ensure that the churn to annular flow-pattern transition may be met for a scaled-down model. On the other hand it is impossible to simulate this flow-pattern transition for a scaled-down water model at reduced-pressure.

14.3.3.4 Flow-Pattern Scaling Requirements for Vertical Pipes

As demonstrated above, additional flow-pattern transition scaling requirements should be taken into account when designing a scaled-down model operating in two-phase flow. These requirements are given by Eq. (70) for bubbly and slug, Eqs. (73) and (74) for slug and churn, and Eq. (81) for churn and annular flow-pattern transitions. Even though all these requirements are different from each other, they may be unified in the form of Eq. (60).

Figure 10 summarizes these requirements for the fluid-to-fluid and reduced-pressure scaling cases considered here. It is evident from the figure that it is impossible to meet all the flow-pattern transition requirements simultaneously. However, given the fluid and flow conditions, the most likely flow patterns which may occur in the prototype can be predicted. Then the specific flow-pattern scaling requirements should be carefully evaluated in view of the overall scaling requirements. If it is necessary, proper scaling compromises can be made by adjusting the length scale and/or the cross-sectional area scale.

14.4 Summary and Conclusions

Overall scaling criteria for a natural circulation loop under single-phase and two-phase flow conditions were summarized. These criteria were obtained from the parameters needed to preserve the values of dimensionless groups in the normalized continuity, momentum and energy equations. The phase change similarity requirement between the single-phase to or from two-phase flow was used to establish scaled-down model operating conditions for fluid-to-fluid and reduced-pressure simulations of a high-pressure water system. For the fluid-to-fluid simulation, R-11 and R-113

Figure 10. Flow-pattern scaling requirements for vertical two-phase flow.

were used as representative fluids, whereas for the reduced-pressure case, water at 20 bar was used for numerical evaluations.

Based on the overall scaling methodology, two-phase flow-pattern simulations were studied in detail between high-pressure prototype and conceptual models. In addition to general scaling requirements for modeling basic thermal-hydraulic phenomena in the loop as a whole, scaling requirements were derived for each flow-pattern transition boundary. It was shown that scaling requirements are all different from each other. This observation indicates that it is impossible to simulate all flow-pattern transitions in a fixed scaled-down model. Distortions are unavoidable. Since the relation between the two-phase flow-patterns and the void fraction depends on the actual diameter, a completely correct scaling may be impossible to achieve in any modeling analysis, and compromises must be made.

Although each flow-pattern transition imposes different scaling requirements, it was demonstrated that they all can be represented in a unified form as $\ell_R \sim a_R^n$. The exponent n varies from one transition to another. Since the friction similarity requirement may result in a different requirement between ℓ_R and a_R , compromises should be made between frictional and flow-pattern requirements.

As a concluding remark, it is appropriate to state that the scaling studies presented here are not conclusive. Numerical evaluations are based on the specific flow-pattern transition map proposed by Taitel et al. and Mishima and Ishii. Since the controversy regarding the flow-pattern transition maps has not yet been resolved, it is probable that other flow-pattern transition maps may result in completely different scaling requirements from those obtained here. This is partly due to an insufficient understanding of system scaling for two-phase flow systems. There exists two scales in two-phase flow

systems. One is the global scale such as the pipe diameter or hydraulic diameter. The other is the internal scale which is characterized by the structure of flow pattern.

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Nomenclature

A	non-dimensional area
a	flow cross-sectional area
a_s	solid cross-sectional area
B	Biot number, $B_i \equiv (h\delta/ks)_i$
C_p	specific heat capacity at constant pressure
d	hydraulic diameter
F	friction number, $F_i \equiv [(fL/d) + K]_i$
f	friction factor
g	gravity
h	heat transfer coefficient
K	orifice coefficient
k	conductivity
L	non-dimensional length
l	axial length
j	volumetric flux
N_{pch}	phase change number
N_{sub}	subcooling number
N_{Fr}	Froude number
N_d	drift-flux number
N_f	friction number (two-phase)
N_o	orifice number (two-phase)
N_{th}	thermal inertia ratio
Nu	Nusselt number
P	pressure
Pr	Prandtl number

q_o''	heat generation density in solid
q''	heat flux,
Q_s	heat source number, $Q_{s,i} \equiv (q_o'' l_o / \rho_s c_{ps} u_o \Delta T_o)_i$
R	Richardson number, $R \equiv g \beta \Delta T_o l_o / u_o^2$
Re	Reynolds number
St	Stanton number, $St_i \equiv (4 h l_o / \rho_f c_{pf} u_o d)_i$
T	temperature
T^*	characteristic time ratio, $T_i^* \equiv (l_o / u_o) / (\delta^2 / \alpha_s)_i$
u	fluid velocity,
u_o	heated section representative velocity
v_{gj}	drift velocity
x	vapor flow quality
x_e	exit quality
α	void fraction
α_e	exit void fraction
α_s	solid thermal diffusivity
β	thermal expansion coefficient
δ	conduction thickness
Δh_{fg}	latent heat of evaporation
Δh_{sub}	subcooling
ΔT_o	characteristic temperature rise
$\Delta \rho$	density difference
$\Delta \mu$	viscosity difference
ξ	wetted perimeter
ρ	mass density
τ	non-dimensional time.

q''_0	heat generation density in solid
q''	heat flux,
Q_s	heat source number, $Q_{sj} \equiv (q''_0 l_0 / \rho_s C_{ps} u_0 \Delta T_0)_j$
R	Richardson number, $R \equiv g\beta\Delta T_0 l_0 / u_0^2$
Re	Reynolds number
St	Stanton number, $St_j \equiv (4 h l_0 / \rho_f C_{pf} u_0 d)_j$
T	temperature
T^*	characteristic time ratio, $T_j^* \equiv (l_0 / u_0) / (\delta^2 / \alpha_s)_j$
u	fluid velocity,
u_0	heated section representative velocity
v_{gj}	drift velocity
x	vapor flow quality
x_e	exit quality
α	void fraction
α_e	exit void fraction
α_s	solid thermal diffusivity
β	thermal expansion coefficient
δ	conduction thickness
Δh_{fg}	latent heat of evaporation
Δh_{sub}	subcooling
ΔT_0	characteristic temperature rise
$\Delta\rho$	density difference
$\Delta\mu$	viscosity difference
ξ	wetted perimeter
ρ	mass density
τ	non-dimensional time.

Subscripts

e	exit
f	liquid
g	gas or vapor
i	ith section
m	model
p	prototype
R	model-to-prototype ratio
s	solid
sp	single-phase flow
tp	two-phase flow
0	reference constant (heated section).

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