Development of a Plant Dynamics Computer Code for Analysis of a Supercritical Carbon Dioxide Brayton Cycle Energy Converter Coupled to a Natural Circulation Lead-Cooled Fast Reactor

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Development of a Plant Dynamics Computer Code for Analysis of a Supercritical Carbon Dioxide Brayton Cycle Energy Converter Coupled to a Natural Circulation Lead-Cooled Fast Reactor

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1. Introduction

1.1. Plant Design Description

The dynamics code described in this document is developed to simulate transient and accident conditions for the Secure Transportable Autonomous Reactor with Liquid Metal coolant (STAR-LM) coupled to a supercritical carbon dioxide (S-CO\textsubscript{2}) Brayton cycle.

Figure 1-1 shows the STAR-LM configuration and steady-state parameters.

1.1.1. Reactor

STAR-LM is a lead-cooled pool-type fast reactor concept operating under natural circulation of the coolant. The reactor core power is 400 MWt. The open-lattice core consists of fuel pins attached to the core support plate, (the does not consist of removable fuel assemblies). The coolant flows outside of the fuel pins. The fuel is transuranic nitride, fabricated from reprocessed LWR spent fuel. The cladding material is HT-9 stainless steel; the steady-state peak cladding temperature is 650 °C. The coolant is single-phase liquid lead under atmospheric pressure; the core inlet and outlet temperatures are 438 °C and 578 °C, respectively. (The Pb coolant freezing and boiling temperatures are 327 °C and 1749 °C, respectively). The coolant is contained inside of a reactor vessel. The vessel material is Type 316 stainless steel. The reactor is autonomous meaning that the reactor power is self-regulated based on inherent reactivity feedbacks and no external power control (through control rods) is utilized. The shutdown (scram) control rods are used for startup and shutdown and to stop the fission reaction in case of an emergency.

The heat from the reactor is transferred to the S-CO\textsubscript{2} Brayton cycle in in-reactor heat exchangers (IRHX) located inside the reactor vessel. The IRHXs are shell-and-tube type heat exchangers with lead flowing downwards on the shell side and CO\textsubscript{2} flowing upwards on the tube side. No intermediate circuit is utilized.

The guard vessel surrounds the reactor vessel to contain the coolant, in the very unlikely event of reactor vessel failure. The Reactor Vessel Auxiliary Cooling System (RVACS) implementing the natural circulation of air flowing upwards over the guard vessel is used to cool the reactor, in the case of loss of normal heat removal through the IRHXs. The RVACS is always in operation. The gap between the vessels is filled with liquid lead-bismuth eutectic (LBE) to enhance the heat removal by air by significantly reducing the thermal resistance of a gas-filled gap.

1.1.2. S-CO\textsubscript{2} Brayton Cycle

The S-CO\textsubscript{2} Brayton Cycle Energy Converter is used to convert heat removed from the reactor into electricity. The CO\textsubscript{2} heated in the IRHXs is expanded in the turbine to produce mechanical work in turbine rotational energy. The mechanical work is converted
into electricity in a generator. After the turbine, the CO₂ goes through the recuperative heat exchangers (high and low temperature recuperators – HTR and LTR). The flow from the recuperators is split into two parts. One part goes through the cooler, where heat is rejected from the cycle to the water heat sink, is compressed in Compressor No. 1 and then returns to the LTR. The other part is directly compressed by Compressor No. 2 bypassing the cooler and LTR. The two parts merge before passing through the HTR. The flow goes through the HTR and returns to the IRHXs. The recuperators and cooler are Printed Circuit Heat Exchangers (PCHEs) designed by Heatric, a subsidiary of Meggitt (UK) Ltd. The turbine and compressors are axial flow turbomachines. The turbine, generator and two compressors are installed on a single common shaft.

The Brayton cycle operates at supercritical conditions meaning that the CO₂ pressure and temperature are above the critical values everywhere in the cycle. It is beneficial for the cycle efficiency to approach the critical point at the low end of the cycle. The lowest pressure and temperature (7.4 MPa and 31.25 °C) are achieved at the cooler outlet (the critical point is 7.38 MPa and 30.98 °C). The highest pressure, 20 MPa, is achieved at the Compressor No. 1 outlet. The highest temperature, 540 °C, is the turbine-inlet temperature.

The flow split between the compressors is optimized to maximize the cycle efficiency. The net generator output is 179 MWe. The steady-state cycle efficiency is 45%.

Figure 1-1. Schematic Illustration of STAR-LM Configuration and Steady-State Conditions and Performance.
1.2. Justification of the Need for Plant Dynamics Code Development

The previously developed S-CO$_2$ Brayton Cycle Design Analysis Computer Code (which characterizes the system performance at design, i.e., full power conditions) and Load Following Code (which calculates the system parameters at steady-states other than full power) are used to characterize the system performance under steady-state conditions. However, those codes cannot be used to describe the system: i) during the transition between the steady-state conditions; and ii) under accident conditions. A special Plant Dynamics Analysis Computer Code, which solves time-dependent equations has to be used for those calculations.

The specific features of the S-CO$_2$ Brayton cycle coupled to the STAR-LM reactor could not be effectively addressed using the available codes existing at the moment. These specific features of the energy converter and reactor together with the requirements for the code are described below.

The reactor is a fast-spectrum reactor which operates under natural circulation. Since the natural circulation depends on power and the reactor power depends on coolant temperatures through the reactivity feedbacks, it is important to solve for the reactor power and coolant flow simultaneously. Also, for the fast reactor, the reactivity feedbacks are very important, so they should be incorporated in the model. Therefore, a model which combines both reactor dynamics and thermal hydraulics has to be used. Moreover, the reactor behavior in this system depends also on the conditions on the balance-of-plant so that the balance-of-plant model should be coupled with the reactor model. A simple thermal hydraulics system model was developed for the reactor with a focus on the calculation of natural circulation conditions. More detailed systems analysis code capabilities exist for liquid metal fast reactors such as the SAS4A/SASSYS-1 systems analysis computer code [1].

The S-CO$_2$ Brayton cycle has several specific features which limit the applicability of the existing codes. First, the main feature of the cycle is that it takes advantage of sharp CO$_2$ properties changes near the critical point. Therefore, accurate calculations near the critical point are very important to reliably calculate the cycle performance. At the same time, the majority of the Brayton cycle analysis codes use an ideal-gas approximation for turbomachinery design and performance. Since this approximation is not applicable to the S-CO$_2$ Brayton cycle, those codes could not be applied here. Also, as mentioned above, the cycle performance affects the reactor behavior, so the Brayton cycle model with accurate representations of the CO$_2$ properties should be highly coupled to the fast-reactor systems dynamics model.

Other incentives for the new code development were:
- to have a modular code where parts and models could be changed and replaced with other models for the design and applicability to other reactor system type; and
- to develop a code which has a sufficiently short running time such that many simulations can be performed and investigations of the effects of different changes in design on the transient performance can be carried out (the existing multi-dimensional codes are too slow for such investigations).
For the reasons described above, it was decided to develop a special dynamics code rather than modify and apply an existing code to this system. Still, the code allows implementing of additional external models, if it will be found helpful. For example, the current version of the code uses the point kinetics subroutines from the SAS4A/SASSYS-1 code to solve for the reactor power.
2. Reactor System Models

2.1. General Conservation Equations

The reactor structures as well as the selected regions and nodes on the reactor part are shown on Figure 2-1. The equations for the coolant, cladding, bond, fuel, and other structure temperatures are based on the energy conservation equation. The energy conservation equation states that the rate of change of the total energy in a volume is equal to the heat addition to the volume minus the heat transfer from the volume. The heat flow could be either by mass flow (from the flowing coolant) or by heat transfer from surrounding structures:

$$\frac{\partial (M\bar{h})}{\partial t} = (\dot{m}h)_{in} - (\dot{m}h)_{out} + Q_{in} - Q_{out}$$

(2-1)

where

- $M$ = mass of the material,
- $h$ = specific enthalpy,
- $\dot{m}$ = mass flow rate of the material (if any),
- $Q$ = heat transfer from other materials,
- $in, out$ = values at the inlet and outlet boundaries,
- $\bar{h}$ = average specific enthalpy of the volume.

The mass conservation equation states that the change in the total mass is equal to the difference between the inlet and outlet flow rates at the boundaries:

$$\frac{\partial M}{\partial t} = \dot{m}_{in} - \dot{m}_{out}$$

(2-2)

Since the only flowing material in the core is lead, the assumption of incompressible flow can be made. For incompressible flow, the inlet flow is always equal to the outlet flow such that the mass flow rate and mass constant according to the Equation (2-2):

$$\dot{m}_{in} = \dot{m}_{out} \Rightarrow M = \text{const}$$

(2-3)

Equation (2-3) yields a constant mass in a volume. In reality, the coolant mass would be changing due to changes in the coolant temperature. To account for this fact, the coolant mass is recalculated at the beginning of each time step, but still is assumed constant during the time step in accordance with Equation (2-3).
Figure 2-1. Reactor Structure, Geometry, and Nodes for the Plant Dynamics Code.
2.2. Coolant Temperatures

Figure 2-2 shows the core structure for the dynamics code.

Equations (2-1)-(2-3) applied to the coolant flow could be written as:

\[ M \frac{\partial \bar{h}}{\partial t} = \dot{m}[h_{\text{in}} - h_{\text{out}}] + Q_{\text{cl, Pb}} \]  \hspace{1cm} (2-4)

where \( Q_{\text{cl, Pb}} \) = heat flux from cladding to coolant.

For region \( i \) (Figure 2-3), Equation (2-4) becomes:

\[ M_{\text{Pb, } i} \frac{\partial \bar{h}}{\partial t} = \dot{m}_{\text{Pb, } i}[h_i - h_{i+1}] + Q_{\text{cl, Pb, } i} \]  \hspace{1cm} (2-5)
The total heat flux between cladding and coolant can be expressed in terms of the temperature difference between the coolant bulk temperature and average cladding temperature divided by the thermal resistance of coolant and cladding:

\[
 Q_{cl,\text{Ph}} = \frac{T_{cl} - \bar{T}_{pb}}{1 + \frac{1}{HTC_i A_i} + \frac{1}{2k_{cl} A_i}} \ln \left( \frac{r_{cl,\text{out}}}{r_{cl,\text{in}}} \right) r_{cl,\text{out}}
\]  

(2-6)

where \( T_{cl} \) = average cladding temperature,

\[
\bar{T}_{pb} = \frac{T_{pb_1} + T_{pb_{i+1}}}{2}
\]

= average coolant bulk temperature,

\[
HTC_i = \frac{k_{pb} \cdot Nu_i}{D_h}
\]

= coolant heat transfer coefficient,

\[
A_i = \pi d_{rod} \Delta x_i
\]

= heat transfer area,

\( d_{rod} \) = fuel rod (cladding) outer diameter,

\( N_{rod} \) = number of fuel rods in core,

\( k_{cl} \) = cladding thermal conductivity,

\( r_{cl,\text{in}}, r_{cl,\text{out}} \) = inner and outer cladding radii, respectively (\( 2r_{cl,\text{out}} = d_{rod} \)).

Note that Equation (2-6) assumes that the average cladding temperature occurs midway through the cladding thickness, such that half of the total cladding thermal resistance is added to the coolant thermal resistance.

Introducing the following notation for the coolant and cladding thermal resistances and the resistance between the coolant bulk temperature and average cladding temperature, the heat flux between the coolant and cladding could be re-written as shown in Equation (2-10).
\[ \text{res}_{cl_i} = \frac{\ln \left( \frac{r_{cl,\text{out}}}{r_{cl,\text{in}}} \right)}{2\pi k_{cl_i}} \quad (2-7) \]

\[ \text{res}_{\text{Pb}_i} = \frac{1}{\text{HTC} \cdot \pi d_{rod}} \quad (2-8) \]

\[ \text{res}_{\text{Pb},cl_i} = \text{res}_{\text{Pb}_i} + \frac{1}{2} \text{res}_{cl_i} \quad (2-9) \]

\[ Q_{cl,\text{Pb}_i} = \frac{\Delta x_i N_{\text{rod}}}{\text{res}_{\text{Pb},cl_i}} \left( T_{cl_i} - \frac{T_{\text{Pb}_i} + T_{\text{Pb}_{i+1}}}{2} \right) \quad (2-10) \]

In order to write Equation (2-5) in same variables (temperatures), the change in specific enthalpy can be expressed through the change in temperature:

\[ \frac{\partial h}{\partial t} = \frac{\partial h}{\partial T} \frac{\partial T}{\partial t} = c_p \frac{\partial T}{\partial t} \quad (2-11) \]

Also, in order to simplify Equation (2-5), two assumptions are made. First, it is assumed that the difference between the inlet and outlet enthalpies is equal to the difference in temperature times the specific heat calculated at the average temperature, i.e., the variation of the specific heat along the region is ignored:

\[ h_i - h_{i+1} = c_p \left( T_i - T_{i+1} \right) \quad (2-12) \]

Second, it is assumed that the change in the outlet temperature is equal to the change in average temperature:

\[ \frac{\partial T}{\partial t} = \frac{\partial T_{i+1}}{\partial t} \quad (2-13) \]

Using the assumptions, Equations (2-12)-(2-13), and the notations, Equations (2-6)-(2-10), the equation for the coolant temperature in the core becomes:

\[ \frac{\partial T_{\text{Pb}_{i+1}}}{\partial t} = \frac{1}{M_{\text{Pb}_i}} \dot{m}_{\text{Pb}_i} \left( T_{\text{Pb}_i} - T_{\text{Pb}_{i+1}} \right) + \frac{\Delta x_i N_{\text{rod}}}{M_{\text{Pb}_i} c_p \text{Pb}_i \text{res}_{\text{Pb},cl_i}} \left( T_{cl_i} - \frac{T_{\text{Pb}_i} + T_{\text{Pb}_{i+1}}}{2} \right) \quad (2-14) \]
Equation (2-14) is the equation for coolant temperature in the core as it is programmed in the code\textsuperscript{1}. The coefficients for each term are calculated at the beginning of the time step at the average coolant and cladding temperatures. The coefficients are assumed to remain constant during the time step, while the flow rate and temperatures are changing.

2.3. Core Structure Temperatures

Equation (2-1) applied to stationary core structures has the following forms for cladding, bond, and fuel, respectively:

\[
M_{cl} c_{p,cl} \frac{\partial T_{cl}}{\partial t} = Q_{bo,cl} - Q_{cl, Pb_i} \tag{2-15}
\]

\[
M_{bo} c_{p,bo} \frac{\partial T_{bo}}{\partial t} = Q_{f,bo} - Q_{bo,cl_i} \tag{2-16}
\]

\[
M_{f} c_{p,f} \frac{\partial T_{f_i}}{\partial t} = Q_{fis} - Q_{f,bo_i} \tag{2-17}
\]

The heat flows between the materials in Equations (2-15)-(2-17) are calculated using average temperatures and the thermal resistances similar to Equations (2-6)-(2-10):

\[
Q_{bo,cl_i} = \frac{\Delta x_i N_{rad}}{res_{cl,bo_i}} \left( T_{bo_i} - T_{cl_i} \right) \tag{2-18}
\]

\[
res_{cl,bo_i} = \frac{1}{2} res_{cl} + \frac{1}{2} res_{bo_i} \tag{2-19}
\]

\[
res_{bo_i} = \frac{\ln \left( \frac{r_{cl, in}}{r_{f, out}} \right)}{2\pi k_{bo_i}} \tag{2-20}
\]

\[
Q_{bo,f_i} = \frac{\Delta x_i N_{rad}}{res_{bo,f_i}} \left( T_{f_i} - T_{bo_i} \right) \tag{2-21}
\]

\[
res_{bo,f_i} = \frac{1}{2} res_{bo_i} + res_{f_i} \tag{2-22}
\]

\[
res_{f_i} = \frac{1}{4\pi k_{f_i}} \tag{2-23}
\]

The fission and decay heat power, $Q_{fis}$, in Equation (2-17) is calculated using the current value of the reactor power (see Section 4) assuming that the axial power distribution is the same as at steady-state (i.e., only the magnitude changes):

\textsuperscript{1} Highlighted equations in this document are the equations in the form programmed in the code
\[ Q_{\text{in}} = q'^* \pi d_{\text{rod}} \Delta x_i N_{\text{rod}} \]  

(2-24)

where \( q'^* = \frac{1}{\Delta x_i} \int q'^*(x)dx \) = average heat flux for the region \( \Delta x_i \),

\( q'^*(x) = \overline{q} f(x) \) = axial heat flux distribution,

\( \overline{q} = \frac{Q_{\text{Rx}}}{\pi d_{\text{rod}} N_{\text{rod}}} \) = core-average heat flux,

\( f(x) = \) given function (steady-state axial power distribution).

Using the Definitions (2-18)-(2-24), Equations (2-15)-(2-17) could be written as differential equations for the core structures:

\[
\frac{\partial T_{\text{cl}}}{\partial t} = \frac{\Delta x_i N_{\text{rod}}}{M_{\text{cl}} c_{p,\text{cl}} \rho_{\text{res}} \rho_{\text{cl},\text{bo}}} \left( T_{\text{bo}} - T_{\text{cl}} \right) - \frac{\Delta x_i N_{\text{rod}}}{M_{\text{cl}} c_{p,\text{cl}} \rho_{\text{res}} \rho_{\text{cl},\text{bo}}} \left( T_{\text{cl}} - \frac{T_{\text{ph}} + T_{\text{ph,cl}}}{2} \right) \quad (2-25)
\]

\[
\frac{\partial T_{\text{bo}}}{\partial t} = \frac{\Delta x_i N_{\text{rod}}}{M_{\text{bo}} c_{p,\text{bo}} \rho_{\text{res}} \rho_{\text{bo},f_i}} \left( T_{f_i} - T_{\text{bo}} \right) - \frac{\Delta x_i N_{\text{rod}}}{M_{\text{bo}} c_{p,\text{bo}} \rho_{\text{res}} \rho_{\text{bo},f_i}} \left( T_{\text{bo}} - T_{\text{cl}} \right) \quad (2-26)
\]

\[
\frac{\partial T_{f_i}}{\partial t} = \frac{\pi d_{\text{rod}} \Delta x_i N_{\text{rod}}}{M_{f_i} c_{p,f_i}} q'^* - \frac{\Delta x_i N_{\text{rod}}}{M_{f_i} c_{p,f_i} \rho_{\text{res}} \rho_{\text{bo},f_i}} \left( T_{f_i} - T_{\text{bo}} \right) \quad (2-27)
\]

Again, the coefficients in Equations (2-25)-(2-27) are calculated at the beginning of the time step and are assumed to be constant during the time step.

### 2.4. Hot Channel Temperatures

The equations for the core temperatures derived above represent an average channel in the core. In order to calculate the peak core temperatures, a hot channel is introduced into the system in parallel to the average channel. The equations for the hot channel are the same as Equations (2-14), (2-25)-(2-27) but with the axial power distribution, \( f(x) \), (Equation (2-24)), being the power distribution for the hot channel. This function is also given for steady-state calculations. Again, it is assumed that only the magnitude of the reactor power is changing while the power distribution and the peaking factors stay the same. As boundary conditions for the hot channel, the core inlet temperature is the same as for the average channel. It is expected that the peak temperatures for cladding or fuel, etc., could occur at the very top of the core. Therefore, an additional very small region (\( \Delta x = 1 \text{ mm} \)) is introduced at the top of the core for the hot channel. The average structure temperatures for this region accurately represent the local temperatures at the top of the core. The peak cladding, bond, and fuel temperatures are calculated as a maximum of the hot channel temperatures at every time step.
2.5. **Temperatures Outside the Core**

Equations for the other regions in the reactor are similar to the coolant and structural temperature equations derived for the core with adjustment for the corresponding structures. Also, there is no fission and decay heat source outside the core.

The reactor structures as well as the selected regions and nodes on the reactor part are shown in Figure 2-1. Figure 2-1 also shows which borders are assumed to be insulated, i.e. where the heat transfer through the borders is neglected. The core equations are described above. In the fission gas plenum (FGP), the only structure is the cladding (thermal inertia of the fission gas is neglected). There is no structure in the riser. In the heat exchanger, heat is transferred from the coolant to the HX tubes and then to the secondary fluid. Also, part of the heat is transferred from the coolant to the air outside the guard vessel through the HX shell wall, stagnant Pb outside the HX, reactor vessel (RV), gap between the vessels, and the guard vessel (GV). The treatment of the vessels and the air channels is described below. There is no heat transfer in the heat exchanger bypass (HXBP). In the downcomer, heat is transferred from the coolant to the reactor vessel and then to the air through the guard vessel. In the lower plenum (LP) region, heat is transferred from the coolant to the reactor vessel, flow distributor plate, and core support plate. In the lower axial reflector (LAR), the only structure is the fuel rods. Flow and heat transfer in the radial reflector (RR) are ignored.

The coolant outlet temperature for the last node in each region serves as the inlet temperature for first node in the next region. For example, the core-outlet temperature \( T_{\text{Pb,N+1}} \) is the FGP-inlet temperature \( T_{\text{Pb,1}} \). The mixing of flows (HX and HX bypass) is assumed to be instantaneous and the downcomer (DC)-inlet temperature is calculated at the beginning of each time step based on the energy (i.e., enthalpy) balance.

2.6. **Coolant Flow Rate**

The equation for natural circulation flow was previously obtained [2,3,4] from Bernoulli’s equation:

\[
\bar{\rho} A_c \int \frac{Y d\zeta}{A} \frac{dU_c}{dt} + 2 \bar{\rho} \int d\zeta \left( \frac{A_c}{A} \right)^2 \frac{Y^2 U_c^2 f}{D_h} + \sum_i \frac{1}{2} \rho_i U_i^2 K_i = \bar{\rho} \beta g \left( \int d\zeta T_{\text{Upwards}} - \int d\zeta T_{\text{Downwards}} \right) \tag{2-28}
\]

where

- \( \bar{\rho} = \) average coolant density,
- \( A_c = \) coolant flow area through core,
- \( Y = \) fraction of the total flow,
- \( A = \) flow area,
- \( U_c = \) average coolant flow speed in core,
- \( f = \) friction factor,
- \( D_h = \) hydraulic diameter,
- \( \beta = \) specific heat capacity of coolant.

\( \int \) denotes the integral along the path in the reactor.
Equation (2-28) is transformed into an equation for the coolant flow rate using the relationship between the flow rate and the flow speed in the core:

\[
\dot{m}_{pb} = \bar{\rho} A_c U_c = \frac{\rho A U}{Y}
\]

(2-29)

\[
\int \frac{Y \, dz \, dm_{pb}}{A} \frac{d \dot{m}_{pb}}{dt} = -2 \int \frac{d z}{A^2 \bar{\rho} D_h} \frac{Y^2 m_{pb}^2 f}{A^2 \bar{\rho} D_h} - \frac{1}{2} \sum \frac{m_{pb}^2 Y_i^2}{\rho_i A_i^2} K_i + (\bar{\rho} \beta g) \left( \int_{\text{Upwards}} d z T - \int_{\text{Downwards}} d z T \right)
\]

(2-30)

Introducing the following notations, Equation (2-30) can be re-written in the form:

\[
I_1 = \int \frac{Y \, dz}{A}
\]

(2-31)

\[
I_2 = \int d z \frac{Y^2 f}{A^2 D_h}
\]

(2-32)

\[
S_1 = \sum \frac{Y_i^2}{\rho_i A_i^2} K_i
\]

(2-33)

\[
I_1 \frac{d \dot{m}_{pb}}{dt} = \left( -2 \frac{I_2}{\bar{\rho}} - \frac{1}{2} S_1 \right) m_{pb}^2 + (\bar{\rho} \beta g) \left( \int_{\text{Upwards}} d z T - \int_{\text{Downwards}} d z T \right)
\]

(2-34)

To calculate the integrals in Equations (2-31)-(2-34,) they are replaced with the finite sums:

\[
\int f(x)dx \approx \sum_{i=1}^{N} \frac{f(x_i) + f(x_{i+1})}{2} \Delta x_i
\]

(2-35)

Integral \(I_1\) represents the sum over all regions along the coolant flow path according to Figure 2-1:

\[
I_1 = I_{1\text{LAR}} + I_{1\text{Core}} + I_{1\text{FGP}} + I_{1\text{RS}} + I_{1\text{HX}} + I_{1\text{DC}} + I_{1\text{LP}}
\]

(2-36)

The flow fraction parameter, \(Y\), is equal to unity for all regions except the heat exchanger region. In the heat exchanger it is assumed that the flow fraction is the same as
for steady-state. Since the flow area is constant inside each region, the integral for each region can be easily calculated. For example, in the core, its value is:

$$I_{1}^{Core} = \frac{1}{A_c} \sum_{i} \Delta x_i = \frac{L_c}{A_c}$$  \hspace{1cm} (2-37)

For the lower plenum, it is assumed that the flow area is constant throughout the plenum and equal to the flow area at the inlet, i.e. the flow area in the downcomer (Figure 2-1). The effective length is calculated as the total coolant volume in the plenum divided by the flow area. So, in the lower plenum the value of the integral is:

$$I_{1}^{LP} = \frac{L_{LP}^{eff}}{A_{LP}^{eff}} = \frac{V_{LP}}{A_{LP}^{eff}} = \frac{M_{Ph}^{LP}}{\rho_{Ph}^{LP} A_{DC}^2} 1$$  \hspace{1cm} (2-38)

Integral $I_2$ is calculated in a similar manner. However, as in the steady-state model, the friction losses in the rod array (LAR, core, and FGP) and HX only are accounted for; friction in all other regions is neglected.

$$I_2 = I_2^{LAR} + I_2^{Core} + I_2^{FGP} + I_2^{HX}$$  \hspace{1cm} (2-39)

$$I_{2}^{Core} = \frac{1}{A_c^2 D_{core}^2} \sum_{i} \bar{f}_i \Delta x_i$$  \hspace{1cm} (2-40)

where $\bar{f}_i = \text{average friction factor for region } i$.

Values for other regions are calculated similarly with an addition of a multiplier, $Y_{HX}^2$, in the HX region.

The temperature integrals in Equation (2-34) are calculated for the upward and downward coolant flows according to the steady-state flow direction shown in Figure 2-1. The upward integral includes the LAR, core, FGP, and riser regions; the downward integral includes the HX and downcomer regions. In the lower plenum regions, the flow is assumed to be approximately horizontal, so that it will not affect the driving force for natural circulation. The integrals are calculated using the Approximation (2-35), using the temperatures at the node borders (Figure 2-2). For example, the core part of the upward integral is:

$$\int_{Core} d\bar{z} T = \sum_{i=1}^{N_{core}} \frac{T_{Ph_i} + T_{Ph_{i+1}}}{2} \Delta x_i  \hspace{1cm} (2-41)$$

Since the temperatures are the changing variables, coefficients for the calculation of the temperature integrals defined by Equation (2-41) are introduced for use in Equation (2-34):
\[ C_{m_{LAR}}^{i} = \frac{\bar{\rho} \beta g \Delta x_{LAR}^{i}}{I_1} \frac{\Delta}{2} \]  
\[ C_{m_{core}}^{i} = \frac{\bar{\rho} \beta g \Delta x_{core}^{i}}{I_1} \frac{\Delta}{2} \]  
\[ C_{m_{FGP}}^{i} = \frac{\bar{\rho} \beta g \Delta x_{FGP}^{i}}{I_1} \frac{\Delta}{2} \]  
\[ C_{m_{RS}}^{i} = \frac{\bar{\rho} \beta g \Delta x_{RS}^{i}}{I_1} \frac{\Delta}{2} \]  
\[ C_{m_{HX}}^{i} = -\frac{\bar{\rho} \beta g \Delta x_{HX}^{i}}{I_1} \frac{\Delta}{2} \]  
\[ C_{m_{DC}}^{i} = -\frac{\bar{\rho} \beta g \Delta x_{DC}^{i}}{I_1} \frac{\Delta}{2} \]  

Note that the coefficients for the HX and DC have minus signs reflecting downward flow.

Overall, Equation (2-34) has the following form which is used in the code:

\[
\frac{dm_{Pb}}{dt} = -\frac{2 I_{\frac{\Delta}{2}} - \frac{1}{2} S_1}{\bar{\rho}} \frac{\Delta}{2} m_{Pb}^2 + \sum_{j = \{LAR, Core, FGP\}} \sum_{i} C_{m_{j}}^{i} (T_{Pb}^{i} + T_{Pb,i}^{i}) \]  

Table 2-1 summarizes the parameters used for each region. It also shows which regions are included in different integrals and sums in the flow Equation (2-30).

<table>
<thead>
<tr>
<th>Table 2-1. Summary of the Flow Equation Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Core</strong></td>
</tr>
<tr>
<td>( A_i )</td>
</tr>
<tr>
<td>( Y_i )</td>
</tr>
<tr>
<td>( D_h )</td>
</tr>
<tr>
<td><strong>K</strong></td>
</tr>
<tr>
<td>Friction</td>
</tr>
<tr>
<td>( I_1 )</td>
</tr>
<tr>
<td>( I_2 )</td>
</tr>
<tr>
<td>( I_{up} )</td>
</tr>
<tr>
<td>( I_{dn} )</td>
</tr>
<tr>
<td>( S_1 )</td>
</tr>
</tbody>
</table>
2.6.1. Treatment of Reversed Flow

Solution of Equation (2-34) may result in negative flow when the downwards integral exceeds the upwards integral for a sufficient amount of time. This negative value would mean that the flow is reversed in the system. The hydraulic resistance (first term in the right hand side of Equation (2-34)) should always tend to decrease the absolute value of the flow. In the case of negative flow, an increase in absolute value means a positive derivative. Therefore, the resistance term should be positive for reversed flow. To account for this, \( \dot{m}^2 \) is replaced with \( \dot{m} \dot{m} \) in Equation (2-48). For positive flow \( (\dot{m} > 0, \dot{m} \dot{m} = \dot{m}^2) \), the equation will have the same form as Equation (2-48). For negative flow \( (\dot{m} < 0, \dot{m} \dot{m} = -\dot{m}^2) \), the first term in Equation (2-48) will be multiplied by -1 resulting in a positive value. Note that a similar term, \( |u|u \), appears in Bernoulli’s equation, but the assumption of positive flow is applied in [2,3,4] to obtain Equation (2-28).

Also, the outlet and inlet nodes in Equation (2-14) for the coolant temperature will be interchanged for reversed flow. Therefore, Equation (2-14) will give the solution for \( T_{p_p} \) (not \( T_{p_p, o} \)). In the code, the index of the outlet node is determined at the beginning of a time step depending on the flow direction. Then, Equation (2-14) is used to calculate the coolant temperatures assuming that the flow direction does not change during the time step. The absolute value of the coolant flow rate is used in Equation (2-14).

It should be noted here that although the code allows for calculations with reversed flow, the transition to that regime has to go through stopped flow \( (u=0) \). It is expected that in this regime, the cross flows would have a significant effect on the flow. Meanwhile, the cross flows are neglected under the assumption of one-dimensional modeling. Therefore, it is to be expected that the transition to reversed flow could be inaccurately calculated with the current model.
3. Reactor Vessel Auxiliary Cooling System (RVACS) Equations

The calculation of the temperatures in the RVACS (Figure 3-1) is similar to the other structures in the reactor (Equations (2-14) and (2-25)). The only significant differences are: \textit{i}) the addition of radiative heat transfer between the vessels; and \textit{ii}) air temperatures and flow rate.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure3-1.png}
\caption{Reactor Vessel Auxiliary Cooling System.}
\end{figure}

3.1. Radiative Heat Transfer

The radiative heat transfer between the reactor vessel and the guard vessel is formulated as:

\[ q_R^*(z) = \sigma \varepsilon \left( T_{RVo}^4(z) - T_{GVi}^4(z) \right) \]  \hspace{1cm} (3-1)

where

- \( q_R^*(z) \) = radiative heat flux (W/m²),
- \( z \) = height along RV wall,
- \( \sigma = 5.6697 \times 10^{-8} \) W/m²K⁴ = Stefan-Boltzmann constant,
\[ \varepsilon = \frac{1}{\varepsilon_R + \varepsilon_G} \] = emittance, 
\[ \frac{1}{\varepsilon_R} + \frac{1}{\varepsilon_G} - 1 \] 
\[ \varepsilon_R, \varepsilon_G = \text{RV and GV emissivities, respectively,} \]
\[ T_{RVo} = \text{RV outer surface temperature (K),} \]
\[ T_{GVi} = \text{GV inner surface temperature (K).} \]

In the Plant Dynamics model, only average RV and GV temperatures are calculated. In order to calculate the surface temperatures, it is assumed that the heat flux from the left of the surface is equal to that from the right (Figure 3-2):

\[ q_{RV}^{-} = q_{RV}^{+} \] (3-2)

\[ q_{RV}^{-} = \frac{T_{RV} - T_{RVo}}{\text{res}_{RV}} \] (3-3)

\[ q_{RV}^{-} = \frac{T_{RVo} - T_{CG}}{\text{res}_{CG}} + q'_{R} \] (3-4)

where
\[ q'_{R} = \sigma \varepsilon \pi D_{RV} \left( T_{RVo}^{4} - T_{GVi}^{4} \right) \]
Substituting the heat fluxes defined in Equations (3-3)-(3-4) into Equation (3-2), the RV surface temperature can be calculated:

\[
\frac{T_{RV} - T_{RVo}}{2} = \frac{T_{RV0} - T_{CG}}{2} + \frac{q'_R}{2}
\]

\[
T_{RV} \frac{res_{CG}}{2} - \frac{T_{RVo} \ res_{CG}}{2} = T_{RVo} \ \frac{res_{RV}}{2} - \frac{T_{CG} \ res_{RV}}{2} + \frac{q'_R \ res_{CG} \ res_{RV}}{2}
\]

\[
T_{RV} \ \frac{res_{CG}}{2} + T_{CG} \ res_{RV} - \frac{q'_R \ res_{CG} \ res_{RV}}{2} = T_{RVo} \ (res_{RV} + res_{CG})
\]

\[
T_{RVo} = \frac{T_{RV} \ res_{CG} + T_{CG} \ res_{RV} - \frac{q'_R \ res_{CG} \ res_{RV}}{2}}{res_{RV} + res_{CG}} \quad (3-5)
\]

Similarly, the GV inner surface temperature is calculated:

\[
\frac{T_{CG} - T_{GVi}}{2} = \frac{T_{GVi} - T_{GV}}{2} + \frac{q'_R}{2}
\]

\[
T_{GVi} \ \frac{res_{CG}}{2} - \frac{T_{GV} \ res_{CG}}{2} = T_{CG} \ \frac{res_{GV}}{2} - \frac{T_{GVi} \ res_{GV}}{2} + \frac{q'_R \ res_{CG} \ res_{GV}}{2}
\]

\[
T_{GVi} \ (res_{GV} + res_{CG}) = T_{GV} \ res_{CG} + T_{CG} \ res_{GV} + \frac{q'_R \ res_{CG} \ res_{GV}}{2}
\]

\[
T_{GVi} = \frac{T_{GV} \ res_{CG} + T_{CG} \ res_{GV} + \frac{q'_R \ res_{CG} \ res_{GV}}{2}}{res_{GV} + res_{CG}} \quad (3-6)
\]

The surface temperatures are defined through the radiative heat flux which is itself a function of the surface temperatures. Therefore, iterations are applied to the solution of Equations (3-1), (3-5), and (3-6). It is assumed in the code that the vessel temperatures change slowly enough, due to large thermal inertia of the vessels, that the surface temperatures and the radiative heat flux are constant during the time step. Under this assumption, the radiative heat transfer is calculated at the beginning of the time step as described above and the heat transfer due to radiation is added to the RV and guard vessel equations as a constant term:
\[ M_{RV} c_{p,RV} \frac{\partial T_{RV}}{\partial t} = Q_{Pb,RV} - Q_{RV,CG} - Q_R \]  
\[ M_{GV} c_{p,GV} \frac{\partial T_{GV}}{\partial t} = Q_{CG,GV} - Q_{GV,air} + Q_R \]

where \[ Q_R = \sigma \varepsilon \pi D_{RV} (T_{RV,i}^4 - T_{GV,i}^4) \Delta x_i \]

### 3.2. Air Temperatures and Flow Rate

Since it is expected that the guard vessel temperature would not change significantly during a time step, and that the timescale for change in the air flow is shorter than that for the vessel temperature, steady-state calculations are applied to the air temperatures and flow rate. At the beginning of a time step, guard vessel temperatures are known for each region. The temperatures of the air are calculated as follows. First, it is assumed that the air flow rate is known. The air temperature at the inlet (bottom-most node in the DC region) is given and fixed during a transient. The air temperature rise in each region is defined by the heat flux from the GV to air (the heat transfer between the air channels is ignored):

\[ q_i' = \frac{T_{GV} - \bar{T}_{air}}{\Delta x} \]  
\[ q_i' \Delta x_i = m_{air} c_{p,air} \Delta T_i \]  

where \[ \bar{T}_{air} = \frac{T_{air,i} + T_{air,i+1}}{2} \] is the average air temperature in region \( i \),

\[ \Delta T_i = T_{air,i+1} - T_{air,i} \] is the air temperature rise in region \( i \)

Average air temperature could be expressed through the inlet temperature and temperature rise:

\[ \bar{T}_{air} = T_{air,i} + \frac{\Delta T_i}{2} \]

Substituting this value into Equation (3-9) and combining it with Equation (3-10), the equation for the air temperature rise is obtained:
\[
\frac{\dot{m}_{\text{air}} c_{p,\text{air}} \Delta T_i}{\Delta x_i} = T_{GV_i} - \left( T_{\text{air}_i} + \frac{\Delta T_i}{2} \right) \tag{3-11}
\]

\[
\Delta T_i \left( \frac{\dot{m}_{\text{air}} c_{p,\text{air}} \rho_{\text{GV,air},i}}{\Delta x_i} + \frac{1}{2} \right) = T_{GV_i} - T_{\text{air}_i}
\]

Then, the air temperature on the next border is calculated.

\[
T_{\text{air}_{i+1}} = T_{\text{air}_i} + \Delta T_i \tag{3-12}
\]

The calculations start with the bottom-most region of the downcomer and are repeated for each region up to the top of the heat exchanger region.

The air flow rate is calculated through the air speed obtained under the steady-state assumption by applying Bernoulli’s equation for natural circulation air flow [2,3,4].

\[
\int_{L_u} \int \rho g dx = \int_{L_d} \int \rho g dx \quad \left\{ \begin{array}{l}
\frac{1}{2} \rho_u u_u^2 - \frac{1}{2} \rho_d u_d^2 + \frac{2 \rho_u f_u L_u u_u^2}{D_u} + \frac{2 \rho_d f_d L_d u_d^2}{D_d} + \frac{1}{2} \rho_u K_u u_u^2 + \frac{1}{2} \rho_d K_d u_d^2
\end{array} \right. \tag{3-13}
\]

where

- \( u \) = air speed,
- \( \rho \) = air density,
- \( f \) = friction factor,
- \( D \) = hydraulic diameter,
- \( L \) = channel length,
- \( K \) = form loss coefficient,
- \( g \) = gravitational acceleration,
- subscripts \( u \) and \( d \) denote upward and downward channels, respectively.

The air channel consists of the channel outside the guard vessel plus a chimney above the vessel. The heat is added at the guard vessel upward channel only; the temperatures in the other channels remain constant. The density integrals can be simplified using constant temperatures and average density of the heated region:

\[
\int_{L_u} \rho g dx = \rho_d g L_d = \rho_d g (L_c + L_{GV})
\]
\[
\int \rho g dx = \int_{L_u}^{L_{GV}} \rho g dx + \rho_u g L_c = \bar{\rho} g L_{GV} + \rho_u g L_c
\]

where

\[
L_{GV} = \text{length of the air channel along GV wall},
\]

\[
L_c = \text{chimney length above GV},
\]

\[
\rho_d, \rho_u, \text{and } \bar{\rho} = \text{air densities in downward channel, chimney, and average density along GV wall, respectively.}
\]

The average density along the GV wall is calculated as the density at the temperature which is an average between the inlet and outlet air temperatures.

In order to solve Equation (3-13), it is assumed that the channel geometry above the GV is the same as in the GV region, and that the losses in the upward channels can be calculated based on the outlet density. The continuity equation relates the flow speeds in downward and upward channels with the densities and flow areas:

\[
\rho_d u_d A_u = m_{\text{air}} = \rho_d u_d A_d
\]  

(3-14)

\[
u_d = u_u \frac{A_u}{\rho_d A_d}
\]  

(3-15)

Using the above relationship and the expressions for the density integrals, Equation (3-13) can be used to solve for the air speed:

\[
u_u^2 \left[ \frac{1}{2} \rho_d - \frac{1}{2} \rho_u \left( \frac{\rho_u}{\rho_d} \right)^2 \left( \frac{A_u}{A_d} \right)^2 \right] + 2 \rho_d f_d \left( L_{GV} + L_c \right) \left( \frac{\rho_u}{\rho_d} \right) \left( \frac{A_u}{A_d} \right)^2 + 2 \rho_d f_u \left( L_{GV} + L_c \right)
\]

\[
+ \frac{1}{2} \rho_u K_d \left( \frac{\rho_u}{\rho_d} \right) \left( \frac{A_u}{A_d} \right)^2 + \frac{1}{2} \rho_u K_u \right] = \rho_d g \left( L_c + L_{GV} \right) - \bar{\rho} g L_{GV} - \rho_u g L_c
\]

or

\[
u_u^2 = \frac{\rho_d \left( L_c + L_{GV} \right) - \bar{\rho} L_{GV} - L_c}{Loss}
\]  

(3-16)

where

\[
Loss = \left( \frac{1}{2} + \frac{2 f_u \left( L_{GV} + L_c \right)}{D_u} + \frac{1}{2} K_u \right) + \rho_u \left( \frac{A_u}{A_d} \right)^2 \left( -\frac{1}{2} + \frac{2 f_d \left( L_{GV} + L_c \right)}{D_d} + \frac{1}{2} K_d \right)
\]

In the code, the form losses in air channels are neglected. The air flow rate is calculated by Equation (3-14). Once the flow rate is re-calculated, Equations (3-11) and (3-12) are repeated for each region to re-calculate the air temperatures. The iterations continue until the desired convergence (on air flow rate) is achieved.
The heat transfer from the guard vessel to air, $Q_{GV,air}$, is calculated by Equation (3-9) for each region and is assumed constant during the time step for the dynamic calculation of the guard vessel temperatures using Equation (3-8).
4. Reactor Power

The SAS4A/SASSYS-1 point kinetics subroutines [1] are incorporated into the code to calculate the reactor core power. Two modifications are made to the subroutines. First, the reactivity feedback calculations are modified to be based on the average coolant temperature in the core (rather than core inlet and outlet temperature). This modification is necessary since the reactivity feedback coefficients, obtained from the neutronics calculations, are provided for average coolant temperature in the core reflecting the fuel rod spacing and core restraint approaches for STAR-LM. In particular, spacing between the fuel pins in the open-lattice core is maintained by grip spacers similar to the approach followed in Light Water Reactor fuel assemblies. Thus, the radial core expansion is determined by the grid spacer temperature that is approximately equal to the coolant temperature. The mean core radial expansion is therefore assumed to follow the mean coolant temperature in the core. Second, a lower limit is introduced to the fission power to avoid unrealistically low numbers during transients that involve operation with negative reactivity for a long time. The limit is set to $10^{-10}$ of the full power to simulate the effect of spontaneous fission which is always present.

The SAS4A/SASSYS-1 subroutines predict the core power during the current time step based on the current reactivity coefficients and the reactor power history. The prediction of the reactor power has the following form:

$$Q(t) = Q_{nom} e^{C_1 + C_2 (t + \Delta t) + C_3 (t + \Delta t)^2}$$  \hspace{1cm} (4-1)

where

- $Q$ = current reactor power,
- $Q_{nom}$ = nominal (steady-state) reactor power,
- $C_1$, $C_2$, and $C_3$ = coefficients calculated by the subroutines,
- $t$ = time ($t=0$ at the beginning of the time step),
- $\Delta t_1$ = previous time step.

Typically, a transient calculation involves a steady-state initialization phase followed by the actual transient initiator and its consequences. The reactor power subroutines are used only during the transients. For the initial calculation (to obtain steady-state conditions before the transient), the nominal reactor power is used. At the last time step before the transient, the initialization subroutine is called to set up the parameters for the reactor power calculation during the transient.

The average coolant and fuel temperatures in the core are calculated at each time step. These two parameters are provided to the reactor power subroutine. The subroutine returns the coefficients for the reactor power prediction according to Equation (4-1). The reactivity is calculated based on the deviation of coolant and fuel temperatures from their steady-state values plus an external reactivity input:

$$\rho = (\alpha_d + \alpha_A) (\bar{T}_f - \bar{T}_{f,0}) + (\alpha_c + \alpha_R) (\bar{T}_c - \bar{T}_{c,0}) + \rho_{ext}$$  \hspace{1cm} (4-2)
where

\[ \rho = \text{reactivity}, \]
\[ \alpha_D = \text{Doppler reactivity feedback coefficient}, \]
\[ \alpha_A = \text{core axial expansion reactivity feedback coefficient}, \]
\[ \alpha_C = \text{coolant density reactivity feedback coefficient}, \]
\[ \alpha_R = \text{core radial expansion reactivity feedback coefficient}, \]
\[ \bar{T}_f = \text{average fuel temperature (} \bar{T}_{f,0} = \text{steady-state value}), \]
\[ \bar{T}_c = \text{average coolant temperature (} \bar{T}_{c,0} = \text{steady-state value}), \]
\[ \rho_{\text{ext}} = \text{given external reactivity source (e.g., from the control rod movement)}. \]
5. *Supercritical Carbon Dioxide Brayton Cycle Energy Converter Model*

The dynamics equations derived for the lead flow cannot be applied to the CO\textsubscript{2} flow because they are developed under conditions of incompressible flow; i.e. the lead flow rate is assumed to be uniform throughout the loop. The CO\textsubscript{2}, however, is a compressible fluid; therefore, the development of compressible flow dynamical equations is needed to correctly model the CO\textsubscript{2} flow and Brayton cycle.

Consider a flow of a compressible fluid in a heated channel (Figure 5-1).

![Figure 5-1. Flow in a Channel.](image)

The dynamical behavior of a compressible flow in a channel is described by mass (5-1), momentum (5-2), and energy (5-3) conservation equations [5]:

\[
\frac{\partial}{\partial t} \int_V \rho \, dV + \int_S \rho \vec{u} \cdot \vec{n} \, dS = 0 
\]  

(5-1)

\[
\frac{\partial}{\partial t} \int_V \rho \vec{u} \, dV + \int_S \rho \vec{u} \ddot{u} \cdot \vec{n} \, dS = \int_V \rho \vec{g} \, dV + \int_S \vec{n} \cdot \vec{\tau} \, dS 
\]  

(5-2)

\[
\frac{\partial}{\partial t} \int_V \left( \rho \left( e + \frac{1}{2} \rho (\vec{u})^2 \right) \vec{u} \ddot{u} \cdot \vec{x} \right) \, dV + \int_S \left( \rho \left( e + \frac{1}{2} \rho (\vec{u})^2 \right) - \rho \vec{g} \cdot \vec{x} \right) \vec{u} \cdot \vec{n} \, dS 
\]

\[= -\int_S \vec{q} \cdot \vec{n} \, dS + \int_S \vec{n} \cdot (\vec{\tau} \cdot \vec{u}) \, dS 
\]  

(5-3)

where \( S \) = boundary of the volume \( V \),
\( \vec{n} \) = normal to the surface \( S \),
\( \rho \) = density,
\( \vec{u} \) = velocity,
\( \vec{g} \) = gravitational acceleration,
\( \vec{\tau} \) = stress tensor,
\[ e = \text{internal energy}, \]
\[ \bar{q} = \text{heat flux}. \]

The following assumptions are made to simplify the dynamics equations.

- Ignore gravitational effects.
- Stress is represented by the friction at the channel wall only. This assumption means that the form losses are neglected compared to the frictional losses, which is expected to be true for the S-CO\(_2\) Brayton cycle. Also the equations are to be applied to a channel of constant cross-sectional area \((A=\text{const})\). The treatment of the flow through a valve – where form losses are important – is described separately below.
- Ignore acceleration pressure drop.
- Ignore energy loss due to friction.
- The total energy is represented by the specific enthalpy. This assumption means ignoring the kinetic energy of the flow. Since the kinetic energy is important in turbomachinery, this model is to be applied to the flow in heat exchangers and piping; a separate model is used for turbomachinery.
- The flow in the volume is characterized by mean properties and flow rates.

Under these assumptions, the conservation equations can be expressed in the following forms, respectively [6]:

\[
\frac{\partial \rho}{\partial t} = \frac{\dot{m}_{in} - \dot{m}_{out}}{A \Delta x} \tag{5-4}
\]

\[
\frac{\partial \bar{m}}{\partial t} = \left( p_{in} - p_{out} \right) \frac{A}{\Delta x} - 2 \bar{f} \left( \frac{\bar{m}^2}{A \bar{p} D_h} \right) \tag{5-5}
\]

\[
\frac{\partial h}{\partial t} = \frac{1}{\Delta x A \bar{p}} \left[ \dot{m}_{in} \left( h_{in} - \bar{h} \right) - \dot{m}_{out} \left( h_{out} - \bar{h} \right) + q' \Delta x \right] \tag{5-6}
\]

To apply Equations (5-4) through (5-6) in a computer code, they should be expressed in three variables only – the flow rate and two variables that define the fluid state. Since the CO\(_2\) properties are originally defined in terms of the temperature and density, selecting these variables would simplify the CO\(_2\) properties calculations and, therefore, increase the computational speed of the dynamics code. The other parameters (enthalpy and pressure) are expressed in terms of the temperature and density as following:

\[
\frac{\partial h}{\partial t} = \left( \frac{\partial h}{\partial T} \right)_\rho \frac{\partial T}{\partial t} + \left( \frac{\partial h}{\partial \rho} \right)_T \frac{\partial \rho}{\partial t} = h_t \frac{\partial T}{\partial t} + h_\rho \frac{\partial \rho}{\partial t} \tag{5-7}
\]

\[
\frac{\partial p}{\partial t} = \left( \frac{\partial p}{\partial T} \right)_\rho \frac{\partial T}{\partial t} + \left( \frac{\partial p}{\partial \rho} \right)_T \frac{\partial \rho}{\partial t} = p_t \frac{\partial T}{\partial t} + p_\rho \frac{\partial \rho}{\partial t} \tag{5-8}
\]
The partial derivatives in Equations (5-7) and (5-8) are derived analytically from the known functions, \( p = p(T, \rho) \), \( h = h(T, \rho) \).

The CO\(_2\) conditions (temperature and density) are calculated at the region boundaries (inlet and outlet); the flow rate is calculated between the boundaries (Figure 5-1). In order to calculate the density at the boundary, Equation (5-4) is applied to the region consisting of two halves of the temperature regions from each side of the boundary (Figure 5-2).

![Figure 5-2. Regions for Temperature, Density, and Flow Rate Calculations.](image)

\[
\frac{\partial \rho_i}{\partial t} = \frac{1}{A \left( \frac{\Delta x_i}{2} + \frac{\Delta x_{i-1}}{2} \right)} (\dot{m}_i - \dot{m}_{i-1}) \quad (5-9)
\]

The momentum Equation (5-5) applied to the region in Figure 5-2 gives

\[
\frac{\partial \dot{m}_i}{\partial t} = \frac{A}{\Delta x_i} (p_i - p_{i+1}) - \frac{2 f \Delta x_i}{M_i D_h} \dot{m}_i^2 \quad (5-10)
\]

where

\[ M_i = A \bar{\rho} \Delta x_i = \text{CO}_2 \text{ mass in region } i. \]

To apply the energy Equation (5-6) to the temperature region in Figure 5-2, an assumption of perfect mixing is made. Under this assumption, the enthalpy of the flow leaving the region is the same as the average enthalpy in the region \( h_{\text{out}} \approx \bar{h} \). The heat flux from the wall is defined by the temperature difference between the wall and average...
CO\textsubscript{2} temperature and the thermal resistance between the wall and CO\textsubscript{2} flow similar to Equation (2-10). The inlet flow rate is calculated as an average between the flow rates adjacent to the inlet boundary.

\[
\frac{\partial h_{i+1}}{\partial t} = \frac{1}{M_i} \left( \frac{\dot{m}_i + \dot{m}_{i-1}}{2} (h_i - h_{i+1}) + \frac{\Delta x_i N_i}{M_i} \left( T_{w,i} - \frac{T_i + T_{i+1}}{2} \right) \right) \tag{5-11}
\]

where

\( N_i \) = total number of channels (tubes) for heat transfer,

\( res_{w,CO_2} \) = thermal resistance between the wall and CO\textsubscript{2} (calculated similar to Equations (2-7)-(2-9)).

All coefficients in Equations (5-9)-(5-11) are assumed to be constant during the time step. To calculate the other properties derivatives, Relationships (5-7) and (5-8) are used:

\[
\frac{\partial T_{i+1}}{\partial t} = \frac{1}{h_{\rho,i}} \left( \frac{\partial h_{i+1}}{\partial t} - h_{\rho,i} \frac{\partial \rho_{i+1}}{\partial t} \right) \tag{5-12}
\]

\[
\frac{\partial p_{i+1}}{\partial t} = p_{\rho,i} \frac{\partial T_{i+1}}{\partial t} + p_{\rho,i} \frac{\partial \rho_{i+1}}{\partial t} \tag{5-13}
\]

5.1. Heat Exchangers

Equations (5-9)-(5-13) are readily applied to the flow in the heat exchangers. For the boundary nodes (HX inlet or outlet), the parameters of the connecting pipes (\( \Delta x \) and \( \dot{m} \)) are used in Equations (5-9) and (5-11), if the index is beyond the heat exchanger (e.g. \( \dot{m}_{i-1} \) is equal to the flow rate in the inlet pipe for region \( i=1 \)).

The region indexing is selected such that the border number increases in the direction of the hot flow\(^1\) (\( i=1 \) for the hot flow inlet). For the cold flow (CO\textsubscript{2} in IRHX, high pressure flows in the HTR and LTR), the indexing direction is opposite to the flow (\( i=1 \) at the outlet).

Although the same approach as for the other heat exchangers could also be applied to the cooler, a simplified model is used for the following reasons. First, it is noticed that the time step required for the momentum Equation (5-10) decreases as CO\textsubscript{2} conditions approach the critical point (due to increasing compressibility). So, implementing the same model for the cooler, even with a small number of regions, would result in significant increase in the computational time. Moreover, due to the strong properties variation near the critical point, a large number of the regions is required to accurately represent the conditions inside the cooler (In steady-state mode, 50 regions are used for a cooler while only 10-20 regions are used for the recuperators). This would

\(^1\) Hot and cold flows are defined based on steady-state conditions.
further slow down the calculations. Therefore, it was decided to implement a simplified model for the cooler. Again, the same model as for the other heat exchangers could be easily applied to the cooler in the future, if the computational time is not an issue. The simplified cooler model assumes that the minimum (cooler-outlet) temperature in the cycle is always maintained at the steady-state value. Thus, the temperature at the cooler outlet is fixed. To calculate the densities at the cooler inlet and outlet, a one-region approach is applied to Equations (5-9) and (5-10) to calculate an average flow rate in the cooler. The cooler-average friction factor is calculated for steady-state conditions and is assumed to be constant during the transient.

5.2. Pipes

The equations derived above are applied to the flows in the pipes in the Brayton cycle with the following simplifications. First, it is assumed that the pipes are insulated so that there is no heat transfer term in Equation (5-11). Also, the frictional pressure loss inside of the pipes (Equation (5-10)) is neglected.

The indexing of the regions for the pipe is selected such that the temperature and density are calculated at each important point of the cycle, such as the HX, turbine and compressor inlets and outlets, flow merge and split points. The flow rates are calculated between these points. Figure 5-3 shows the selected nodes for whole system including the reactor and Brayton cycle. The squares show the points for the temperature and density; the circles show the flow rate positions. Some additional points for the flow rates are added for a convenience. For example, the flow rate at the HTR inlet, $\dot{m}_5$, is used to calculate the density, $\rho_5$, according to Equation (5-9) but is actually calculated as a flow rate in the first region in the HTR.

Equation (5-10) assumes that there is no pressure loss due to the form losses. The corrections to the flow rate through the pipes with valves are described below.

5.3. Flow Split and Merge Points and Reversed Flow

At the flow merge points (such as Point 2 in Figure 5-3) there is more than a single inlet flow. Therefore, Equation (5-9) should have more than one inlet flow term. Also, Equation (5-11) will have more than one term for the heat addition due to inlet flows. Similarly, Equation (5-9) should be modified to include more than one outlet flow for the flow split points (such as Point 23). Also, the volumes for Equation (5-9) would include three terms for the merge and split point. Therefore, Equations (5-9) and (5-11) are modified as follows.

Before the dynamics calculations, the volumes for each density point are calculated. These volumes would include two adjustment regions halves for the regular point and three halves for flow split/merge points. Then, in the dynamic calculations, the derivative of the density is calculated as a sum of all of the inlet flow rates minus the sum of all outlet flow rates divided by the volume:
Subscript $\rho$ is used for the volume to distinguish it from the volumes for the enthalpy equations.

\[
\frac{\partial \rho_i}{\partial t} = \frac{1}{V_{\rho,i}} \left[ \sum_{i=\text{in}} \dot{m}_i - \sum_{i=\text{out}} \dot{m}_i \right] 
\]  
(5-14)

The same approach is used for Equation (5-11) to account for several inlet flows. This approach is combined with the treatment of negative flows.

A solution of the flow equation may result in a reversed flow, i.e. negative values of flow rates. The reversed flow would exchange the inlet and outlet points in Equations (5-9)-(5-11). To account for reversed flows and their effect on the equations, the following approach is used. Before the dynamics calculations, two arrays are generated which define inlet and outlet nodes for each flow rate in the system (including flows in the heat exchangers) under steady-state (positive flow) conditions. If, during a transient the flow is positive, then the outlet node for this flow as the same as at steady-state; if the flow is negative, then the outlet node is the steady-state inlet node. At the beginning of a time step, the density and enthalpy derivatives for each point are set to zero. Then, for each flow rate, the value of the density derivative is increased at the outlet node and decreased at the inlet node. Also, the value of the enthalpy derivative is increased for the outlet node.

**Figure 5-3. Plant Dynamics Model Nodes.**
\[
\frac{\partial \rho_{i-out}}{\partial t} = \frac{\partial \rho_{i-out}}{\partial t} + \frac{1}{V_{\rho,i-out}} \dot{m}_i \tag{5-15}
\]
\[
\frac{\partial \rho_{i-in}}{\partial t} = \frac{\partial \rho_{i-in}}{\partial t} - \frac{1}{V_{\rho,i-in}} \dot{m}_i \tag{5-16}
\]
\[
\frac{\partial h_{i-out}}{\partial t} = \frac{\partial h_{i-out}}{\partial t} + \frac{1}{M_i} \dot{m}_i (h_{i-in} - h_{i-out}) + \frac{\Delta x_i N_i}{M_i res_{w,CO}} \left( T_{w,i} - \frac{T_i + T_{i+1}}{2} \right) \tag{5-17}
\]

where

\( i\text{-in} \) and \( i\text{-out} \) = indices for the inlet and outlet nodes for flow, \( \dot{m}_i \),

the heat transfer term is omitted for the pipes.

Similar to the momentum equation for the reactor coolant, the flow equation is modified to account for negative flow:

\[
\frac{\partial \dot{m}_i}{\partial t} = \frac{A}{\Delta x_i} (p_i - p_{i+1}) - \frac{2 f \Delta x_i}{M_i D_h} \dot{m}_i |\dot{m}_i| \tag{5-18}
\]

5.4. Valves

Valves are used in the Brayton cycle for control purposes. There are six valves programmed in the code (with the corresponding flow rates in Figure 5-3 given in parentheses): IRHX bypass valve (28), turbine throttle valve (2), turbine bypass valve (29), Compressor No. 2 throttle valve (25), and inventory tank inlet (30) and outlet (31) valves. The valves are used to regulate flow by adding a resistance to the flow. In order to model the hydraulic resistance presented by a valve, a momentum Equation (5-18) is modified to account for the pressure losses in the valve. The valve is simulated as a sudden flow area contraction followed by a sudden flow area expansion. The total pressure loss is represented by the sum of the form losses from contraction and expansion:

\[
\Delta p_v = \frac{1}{2} K_{contr} \rho u_{small}^2 + \frac{1}{2} K_{exp} \rho u_{small}^2 \tag{5-19}
\]

where

\[
K_{contr} = \frac{1}{2} \left( 1 - \frac{A_{open}}{A_{total}} \right) = \text{contraction loss coefficient},
\]

\[
K_{exp} = \left( 1 - \frac{A_{open}}{A_{total}} \right)^2 = \text{expansion loss coefficient},
\]

\( A_{open} \) = flow area open to the flow (flow area inside a valve),

\( A_{total} \) = total flow area (flow area outside a valve),

33
\( \rho = \text{density,} \)
\( u_{\text{small}} = \text{flow speed at the opening.} \)

The following variables are introduced to simplify Equation (5-19).

- The ratio of the open area to the total area is defined as the valve open area fraction. The fraction is either user-defined or a result of the control calculations.

\[
f_{\text{open}} = \frac{A_{\text{open}}}{A_{\text{total}}} \quad (5-20)
\]

- Two loss coefficients are combined to form a valve loss coefficient:

\[
K_v = K_{\text{contr}} + K_{\exp} = \frac{1}{2}(1 - f_{\text{open}}) + (1 - f_{\text{open}})^2 = \frac{1}{2} - \frac{f_{\text{open}}}{2} + 1 - 2f_{\text{open}} + f_{\text{open}}^2
\]
\[
= \frac{1}{2} \left( 1 - \frac{f_{\text{open}}}{2} - f_{\text{open}} + f_{\text{open}}^2 \right) = \frac{1}{2} \left( 1 - f_{\text{open}} \right)^2 - f_{\text{open}} \left( 1 - f_{\text{open}} \right)
\]
\[
= \left( 1.5 - f_{\text{open}} \right) \left( 1 - f_{\text{open}} \right) \quad (5-21)
\]

- The flow speed is calculated based on the continuity equation:

\[
u_{\text{small}} = \frac{\dot{m}}{\rho A_{\text{open}}} \quad (5-22)
\]

- The density is calculated as the average density between inlet and outlet conditions:

\[
\rho = \frac{\rho_{\text{in}} + \rho_{\text{out}}}{2} \quad (5-23)
\]

Using the above definitions, the pressure drop across the valve becomes:

\[
\Delta p_v = \frac{1}{2} K_v \frac{1}{\rho A_{\text{open}}^3} \dot{m}^2 = \frac{1}{2} K_v \frac{1}{\rho f_{\text{open}}^2 A_{\text{total}}^3} \dot{m}^2 \quad (5-24)
\]

The open area fraction in the denominator is added to the valve loss coefficient to collect valve openings in one function:

\[
K'_v = \frac{K_v}{f_{\text{open}}^2} = \frac{(1.5 - f_{\text{open}})(1 - f_{\text{open}})}{f_{\text{open}}^2} \quad (5-25)
\]
After the derivatives for all flow rates in the cycle are calculated using Equation (5-18), the derivatives of the flow rates through valves are corrected to add the valve resistance (Equation (5-24) is corrected to allow for a negative flow):

$$\frac{\partial \dot{m}_{i,v}}{\partial t} = \frac{\partial \dot{m}_{i,v}}{\partial t} - \frac{A_{i,v}}{\Delta x_{i,v}} \frac{K'_r}{2 \rho A_{i,v}} \dot{m}_{i,v} \left| \dot{m}_{i,v} \right|$$  (5-26)

where

$$i-v = \{2, 25, 28, 29, 30, 31\} = \text{indices for the valves locations.}$$

### 5.5. Critical Flow Limitation

According to the momentum Equation (5-18), the flow between two points is determined by the pressures at those points. Pressure waves travel through a fluid at the sound speed. If the flow rate is high enough that the local speed reaches sonic speed, the conditions downstream of this point could not be communicated upstream any more. Therefore, the flow rate cannot increase even if outlet pressure decreases. This condition is called critical flow.

Critical flow occurs when the local speed reaches the speed of sound. According to the continuity equation, speed is reciprocal to the flow area, so that the maximum speed is achieved at the minimum flow area. In the cycle, the minimum flow areas occur at a valve opening. Therefore, it is expected that the flow in a valve could reach critical flow. The flow could not be increased beyond the critical flow no matter how low the outlet pressure is, even if Equation (5-18) allows for that. So, the critical flow rate should be calculated for the conditions of each valve and the limitation of the critical flow should be applied to the flow.

Consider flow through a valve (Figure 5-4). The conditions of the flow before the valve (Point 1) are given. At critical flow, the flow speed in the opening is equal to the sound speed defined for local properties.

$$u_0 = V_{s,0} \equiv V_{s}(h_0, s_0)$$  (5-27)

where $V_s = \text{sound speed.}$

![Figure 5-4. Flow Through a Valve Opening.](image-url)
The critical flow rate is defined from the continuity equation.

\[ \dot{m}_c = \rho_0 u_0 A_0 = \rho_0 u_0 A_1 f_{\text{open}} \]  
(5-28)

where

\[ f_{\text{open}} = \frac{A_0}{A_1} = \text{valve open area fraction}. \]

To find the conditions at the opening and the local sound speed an iterative process is applied. It is assumed that the flow through the valve is isentropic, i.e. \( s_0 = s_1 \). Suppose that the density and enthalpy at the opening are known (for the first approximation, the inlet parameters could be taken). Then the continuity equation defines the flow speed before the valve:

\[ u_1 = \frac{\dot{m}_c}{\rho_1 A_1} \]  
(5-29)

The enthalpy at the opening is found from the total energy conservation equation:

\[ h_1 + \frac{u_1^2}{2} = h_0 + \frac{u_0^2}{2} \]  
(5-30)

\[ h_0 = h_1 + \frac{u_0^2}{2} - \frac{u_1^2}{2} = h_1 + \frac{1}{2} \left( \frac{\dot{m}_c}{\rho_1 A_1} \right)^2 - \frac{V_{s,0}^2}{2} = h_1 + \frac{1}{2} \left( \frac{\rho_0 A_0 V_{s,0}}{\rho_1 A_1} \right)^2 - \frac{V_{s,0}^2}{2} \]

\[ h_0 = h_1 + \frac{V_{s,0}^2}{2} \left( \frac{\rho_0}{\rho_1} \right) f_{\text{open}}^2 - 1 \]  
(5-31)

When the enthalpy and entropy at the opening are known, the density and the sound speed are recalculated based on the CO\(_2\) properties.

\[ \rho_0 = \rho(s_0, h_0) \]  
(5-32)

\[ V_{s,0} = V_s(s_0, h_0) \]  
(5-33)

The iterations through Equations (5-28)-(2-33) are continued until convergence on the flow rate is achieved. Then the critical flow limits are applied to the flow rate calculated by Equations (5-18) and (5-26); i.e., the flow rate is not allowed to exceed the critical flow rate.

Note that the critical flow rate does not depend on the conditions downstream of the valve as it should be, since, at critical flow, the downstream conditions are not communicated back to the inlet point.
Although the critical flow algorithm is developed for the flow through a valve, it could also be applied to every flow in the Brayton cycle. Indeed, if a flow doesn’t go through a valve, then \( f_{\text{open}} = 1 \), and conditions at point 0 are the same as those at point 1. Therefore, the iterative process will automatically converge after just one iteration. So, the critical flow is calculated and limitations are applied to every flow in the Brayton cycle.

Because of the possibility of reversed flow, the critical flow rate is calculated for both directions. The critical flow in the reversed direction, \( \dot{m}_{\text{c}}^{-} \), is defined by the conditions which are outlet conditions for normal flow (Point 2 in Figure 5-4). The critical flow in the reversed direction is negative and the flow rate should always be between those two critical flows:

\[
\dot{m}_{\text{c}}^{-} \leq \dot{m} \leq \dot{m}_{\text{c}}^+
\]  

(5-34)

According to Equations (5-32) and (5-33) the iterative scheme for the finding critical flow rate requires calculating density and sound speed as functions of enthalpy and entropy. Since the \( \text{CO}_2 \) properties are defined as functions of temperature and density, using Equations (5-32) and (5-33) requires multiple iterations on the \( \text{CO}_2 \) properties. Because of the complicated properties calculations, these iterations are too slow to be used in the dynamics code. Therefore, the following approach has been taken to eliminate iterations on \( \text{CO}_2 \) properties. Tables of \( \text{CO}_2 \) properties have been generated to provide pressure, temperature, density, and sound speed as a function of enthalpy and entropy. The tables cover the entire possible range of operating conditions for the cycle (temperatures from 230 K to 1100 K, pressures from 0.05 MPa to 50 MPa). The tables give the properties at fixed \((h_i, s_j, i,j=1..500)\) points; linear interpolation is used between the points. The tables cover supercritical, liquid, gas, and two-phase regions. In all regions, except the two-phase region, the properties are defined by the properties subroutines. In two-phase region, Henry’s formula [7] is used to calculate the sound speed:

\[
V_s = \sqrt{\frac{[(1-x)\rho^* + x\rho]^2 + x(1-x)(\rho^*-\rho)^2}{x(\rho)^2 + (1-x)(\rho^*)^2}}
\]  

(5-35)

\[
h = x h^* + (1-x) h'
\]  

(5-36)

\[
s = x s^* + (1-x)s'
\]  

(5-37)

\[
\frac{1}{\rho} = \frac{x}{\rho^*} + \frac{1-x}{\rho'} \implies x = \frac{1}{\rho} - \frac{1}{\rho'}
\]  

(5-38)
where \( y' \) and \( y'' \) denote liquid and gas properties, respectively.

The tables are based on the properties only and need to be generated only once. They are stored in a file and are read before the calculations start. The tables have also proved to be useful as a first guess for iterations in turbomachinery calculations which frequently employ calculations of the properties based on the specific enthalpy and entropy.

The tables are only used in the critical flow calculations. Other properties calculations, such as for Equations (5-15)-(5-18), are based on the exact properties subroutines and do not use the tables. Although using the tables instead of the exact calculations will result in some error, that error is not expected to be significant for the system performance for the following reasons. Critical flow is expected to occur in valves. The valves themselves are regulated by the Brayton cycle control. The control mechanisms monitor the system parameters and adjust the valve openings accordingly. However, valve opening itself does not affect the system performance – it is flow rate through the valve that is important. So, the control mechanisms effectively adjust the flow rate through the valves. Therefore, if there is an error in the critical flow rate calculations, the flow rate will be slightly different from the actual value, but the control system will still be adjusting the flow rate to a required value so that the resulting valve opening calculated by the control system may differ slightly from the actual value. Thus, the reported valve opening may not be exact due to utilization of the properties tables, but the flow rate still will be accurate such that the system performance is not affected.

Even with the properties tables, calculation of critical flow requires iterations on Equations (5-28)-(5-33). So, the critical flow rate is calculated only at the beginning of the time step. However, it is found that smooth variation of a valve open area is beneficial to the stability of the control system action. Therefore, the critical flow rate should also change smoothly during the time step. It is assumed that during the time step, the parameters of the flow in the opening are constant and the critical flow rate is proportional to the valve open area fraction according to Equation (5-28). The flow parameters and the critical flow rate are recalculated at the beginning of the next step.

Implementing Equations (5-25)-(5-26) will result in a numerical singularity when a valve approaches fully closed position (\( f_{open} \to 0 \Rightarrow K' \to \infty \)). To avoid this problem a threshold value for a valve opening is introduced. If the opening is below this threshold value then the flow is assumed to be critical, and calculations of Equations (5-25) and (5-26) are skipped. A 1% open area is assumed for this threshold in the code.

### 5.6. Turbomachinery Models

Although the turbomachinery design models are not directly used in the dynamics code (they are part of the steady-state code), the off-design performance models are based on the design models. The performance models, in turn, are used to generate the turbomachinery maps which are used in the dynamics code. For these reasons, a detailed description of the design and performance models is presented here.

Turbine and compressor design and off-design prediction models were developed using two-dimensional (rather than complicated three-dimensional) models based on
empirical cascade tests parameters [8] for simplicity and calculation speed. The main ideas and equations of these models are described below.

5.6.1. Compressor Design

The compressor design model calculates steady-state design parameters for compressors for given cycle conditions. The input parameters are:

- Working fluid (CO₂)
- Inlet pressure and temperature
- Outlet pressure
- Flow rate
- Rotational speed
- Blade material density
- Maximum allowable blade stress
- Number of stages.

Note that the number of stages is also an optimization parameter and usually several compressor designs with different numbers of stages are calculated before the final selection is made.

The design parameters which need to be calculated include:

- Fluid property (pressure and temperature) distributions throughout the compressor
- Flow angles and speeds
- Blade parameters for each stage:
  - Hub and tip radii
  - Blade height
  - Blade chord
  - Spacing
  - Number of blades
  - Blade angles
- Stresses in the blades
- Compressor efficiency.

An attempt is always made to simplify the design procedure. In that regard, assumptions are made for the values for several parameters which in general need to be optimized based upon specific recommendations and engineering judgment. The parameters have to be changed only if the design or off-design characteristics turn out to be unsatisfying. It is understood that this approach will not produce the best optimal design, but it does produce reasonable design that is good enough to perform the cycle analysis which is the real goal of this work. This approach is used for the following parameters.

The axial component of the flow speed is assumed to be constant in the compressor. A repeating stage design is selected meaning that the flow angles are the same at each stage inlet.
The stage reaction is selected to be 50% which allows simplification of the design procedure. For example, for 50% reaction, it is possible to derive an analytical correlation between the inlet guiding vane (IGV) angle and blade deflection angle independent of other parameters to satisfy the de Haller criteria:

\[
\frac{W_2}{W_1} \leq 0.72 \tag{5-39}
\]

The components of the flow speed are shown in Figure 5-5.

Figure 5-5. Compressor Stage Velocity Triangles.

Under the assumption of constant axial speed

\[
\frac{W_2}{W_1} = \frac{C_z}{\cos \beta_2} = \frac{\cos \beta_1}{\cos \beta_2} \tag{5-40}
\]

For the 50%-reaction stage, the velocity triangles are symmetric meaning that \( \beta_2 = \alpha_1 \) which for the first stage is equal to the IGV angle, \( \alpha_{IGV} \), and remains the same for all stages under the repeating stage assumption. The deflection angle is defined as \( \beta_1 - \beta_2 \) and it limiting value can be found from Equations (5-39) and (5-40) as a function of the IGV angle:

\[
defl_{max} = \beta_1^{max} - \beta_2 = \arccos(0.72 \cos \alpha_{IGV}) - \alpha_{IGV} \tag{5-41}
\]

Table 5-1 shows the relationship between IGV angle and maximum deflection angle.
Table 5-1. Relationship between IGV Angle and Maximum Deflection for 50%-Reaction Stage

<table>
<thead>
<tr>
<th>IGV angle, deg</th>
<th>Max. deflection, deg</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>43.9</td>
</tr>
<tr>
<td>5</td>
<td>39.2</td>
</tr>
<tr>
<td>10</td>
<td>34.8</td>
</tr>
<tr>
<td>15</td>
<td>30.9</td>
</tr>
<tr>
<td>20</td>
<td>27.4</td>
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<tr>
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<td>21.4</td>
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<td>40</td>
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</tr>
<tr>
<td>50</td>
<td>12.4</td>
</tr>
<tr>
<td>60</td>
<td>8.9</td>
</tr>
</tbody>
</table>

It is always desirable for higher efficiency to select the deflection angle as high as possible. At the same time, Table 5-1 shows that large deflection angle requires a small IGV angle increasing the Mach number at the first stage inlet. As a trade-off, an IGV angle of 15 degrees and deflection of 30 degrees are selected for the design. Again, the maximum Mach number and compressor efficiency should be checked after the design is complete to see if this selection is satisfactory.

The blade row solidity (spacing-to-chord ratio) was selected following the recommendations for optimal turbine blade row spacing described in [9]. The NACA 65-series blade profiles were used for profile coefficients.

The design procedure starts by determining the pressure distribution between the stages. It is recommended for compressor design [9] that the enthalpy change for each stage should be approximately the same. Assuming that the stages are of equal efficiencies, the condition of equal enthalpy change in an isentropic process is used to determine the pressure and enthalpy at each stage inlet and outlet.

The known change in enthalpy in an isentropic process defines the axial component of the flow speed as follows. As mentioned above, under the repeating stage assumption, the flow inlet angle for every stage is equal to the IGV angle, and the flow velocity at each stage outlet is equal to that at the inlet. If the average deflection angle is specified, then one can calculate the average flow angle at the rotor outlet. For 50% reaction, this relation is explicit:

$$defl = \beta_1 - \beta_2 = \alpha_2 - \alpha_1$$  \hspace{1cm} (5-42)

For reactions other than 50%, the Relation (5-42) becomes implicit and more complicated but it has been shown that this relationship still exists.

Since the flow velocities at each stage outlet and inlet are equal, the change in enthalpy in the stage is equal to the change in total enthalpy which is defined by Euler’s equation:
\[ \Delta h_0 = \Delta h = u(\theta_2 - \theta_1) \quad (5-43) \]

Blade speed, \( u \), can be derived from the reaction definition:

\[ R = 1 - \frac{C_{2\theta} + C_{1\theta}}{2u} \quad (5-44) \]

\[ u = \frac{C_{2\theta} + C_{1\theta}}{2(1 - R)} \quad (5-45) \]

Assuming that the compressor efficiency is known (iterations are required), the average change in enthalpy can be related to the change in enthalpy in an isentropic process. Using the definition of efficiency and combining Equations (5-43)-(5-45), the following relationship is obtained:

\[ \frac{\Delta h}{\varepsilon} = \Delta h = \frac{C_{2\theta} + C_{1\theta}}{2(1 - R)} \left( \tan^2 \alpha_2 - \tan^2 \alpha_1 \right) \quad (5-46) \]

from which the axial component of flow speed is calculated:

\[ C_z = \frac{\sqrt{\frac{\Delta h}{\varepsilon} 2(1 - R)}}{\tan^2 \alpha_2 - \tan^2 \alpha_1} \quad (5-47) \]

The axial speed component is known, and the calculations can proceed for every stage. As mentioned above, the flow angle at a stage inlet is equal to the IGV angle, such that the remaining flow speed components can be found (Figure 5-5). The known inlet conditions specify the density at the inlet, \( \rho_1 \). The continuity equation is used to calculate the required flow area measured perpendicular to the compressor axis:

\[ A_\perp = \frac{A}{\cos \alpha_1} = \frac{\dot{m}}{\rho_1 C \cos \alpha_1} = \frac{\dot{m}}{\rho_1 C_z} \quad (5-48) \]

At the same time, the flow area is defined by the tip and hub radii of the blades

\[ A_\perp = \pi \left( r_t^2 - r_h^2 \right) \quad (5-49) \]

The design code is organized in such a manner that the hub radius is selected such that the calculated pressure at the stage outlet matches the required pressure from the distribution obtained earlier. Equation (5-49) is used to calculate the tip radius and then the medium radius, blade height, and blade speed:

\[ r_t = \sqrt{\frac{A_\perp}{\pi} - r_h^2} \quad (5-50) \]
\[ r_m = \frac{r_i + r_h}{2} \quad (5-51) \]

\[ H_b = r_i - r_h \quad (5-52) \]

\[ u = 2\pi n, r_m \quad (5-53) \]

The relative-to-the-blade speeds and angles are calculated based on Figure 5-5.

\[ W_{1z} = C_z \quad (5-54) \]

\[ W_{1\theta} = C_{1\theta} - u \quad (5-55) \]

\[ W_1 = \sqrt{W_{1\theta}^2 + W_{1z}^2} \quad (5-56) \]

\[ \sin \beta_1 = \frac{W_{1\theta}}{W_1} \quad (5-57) \]

The flow speed components at the rotor outlet are calculated using the Definition (5-44) of the stage reaction and velocity triangles on Figure 5-5.

\[ C_{2\theta} = 2u(1 - R) - C_{1\theta} \quad (5-58) \]

\[ C_{2z} = C_z \quad (5-59) \]

\[ C_2 = \sqrt{C_{2\theta}^2 + C_{2z}^2} \quad (5-60) \]

\[ \sin \alpha_2 = \frac{C_{2\theta}}{C_2} \quad (5-61) \]

\[ W_{2z} = C_z \quad (5-62) \]

\[ W_{2\theta} = C_{2\theta} - u \quad (5-63) \]

\[ W_2 = \sqrt{W_{2\theta}^2 + W_{2z}^2} \quad (5-64) \]

\[ \sin \beta_2 = \frac{W_{2\theta}}{W_2} \quad (5-65) \]

The optimum solidity [9] for the rotor blades is:
\[
\sigma = 2.5\left(\tan|\beta_2| + \tan|\beta_1|\right)\cos^2 \beta_1 \tag{5-66}
\]

Blade profile angles can now be calculated using the methodology described in [8] (Figure 5-6).

**Figure 5-6. Blade Profile Angles.**

Design incidence angle (i.e. incidence angle at design conditions):

\[
i^* = K_{i,i} K_{t,i} \left(\frac{\beta^*}{\sigma}\right)_{10} + n \theta \tag{5-67}
\]

where

\[
K_{i,i} = (10 t_b/c)^q,
\]
\[
q = 0.28/(0.1 + (t_b/c)^{0.3}),
\]
\[
\left(\frac{\beta^*}{\sigma}\right)_{10} = \frac{\beta^*}{5 + 46\exp(-2.3\sigma)} - 0.1\sigma^3 \exp\left[\frac{\beta_1 - 70}{4}\right],
\]
\[
x = 0.914 + \sigma^3/160,
\]
\[
n = 0.025\sigma - 0.06 - \frac{(\beta_1/90)^{(1+1.2\sigma)}}{1.5 + 0.43\sigma},
\]
\[
t_b/c = \text{blade maximum thickness-to-chord ratio (0.1 for NACA-65 series blades)}.
\]

Design deviation angle:

\[
\delta^* = K_{i,i} K_{t,\delta} \left(\delta^*_{0}\right)_{10} + m \theta \tag{5-68}
\]

where

\[
K_{t,\delta} = 6.25(t_b/c) + 37.5(t_b/c)^2,
\]
\[
\left(\delta^*_{0}\right)_{10} = 0.01\sigma\beta_1 + \left[0.7\sigma^{1.9} + 3\sigma\right]_\beta(\beta_1/90)^{(1.67+1.09\sigma)},
\]
\[
m = m_{1.0}/\sigma^b,
\]
\[
m_{1.0} = 0.17 - 0.0333x + 0.333x^2.
\]
\[
x = \beta i / 100, \\
b = 0.9625 - 0.17x - 0.85x^3,
\]

The rest of the blade profile angles are defined in Figure 5-6. Note that the camber angle, \( \theta \), needed for Equations (5-67) and (5-68), is defined through blade angles which are functions of the incident and deviation angles. Therefore, iterations are required on the camber angle.

When the blade profile parameters are calculated, it is possible to calculate flow conditions in the blade row. The total relative enthalpy\(^1\) at the rotor outlet is equal to the total relative enthalpy at rotor inlet:

\[
h'_{02} = h'_{01} = h_1 + \frac{W_1^2}{2} \tag{5-69}
\]

The total relative pressure at the rotor inlet is the pressure corresponding to the total relative enthalpy and inlet entropy:

\[
p'_{01} = p(h = h'_{01}, s = s_1) \tag{5-70}
\]

The total pressure at the rotor outlet is equal to the total relative pressure at inlet minus losses

\[
p'_{02} = p'_{01} - \bar{\omega}(p'_{01} - p_1) - \Delta p_{0c} \tag{5-71}
\]

where
\[
\bar{\omega} = \text{loss coefficient}, \\
\Delta p_{0c} = \text{tip clearance gap total pressure loss}.
\]

(The equations for the loss coefficient and clearance loss are provided below.)

Total relative pressure and enthalpy at the rotor outlet define the entropy at this point

\[
s_2 = s'_{02} = s(p = p'_{02}, h = h'_{02}) \tag{5-72}
\]

Enthalpy and other conditions at the rotor outlet can now be found:

\[
h_2 = h'_{02} - \frac{W_2^2}{2} \tag{5-73}
\]

\[
p_2 = p(h = h_2, s = s_2), \quad T_2 = T(h = h_2, s = s_2) \tag{5-74}
\]

\(^1\)Total relative enthalpy is defined as a sum of specific enthalpy and the relative (to the blades) kinetic energy which is calculated based on the relative velocity.
When the inlet and outlet conditions are known, the centrifugal, bending, vibrational, and total stresses can be calculated using Equations (5-75)-(5-78), respectively [10].

\[ \sigma_c = 2 \pi \rho_B n_r^2 A \]  \hspace{1cm} (5-75)

\[ \sigma_b = \frac{x}{I_{yy}} [M_\theta \cos \theta - M_a \sin \theta] - \frac{y}{I_{xx}} [M_a \cos \phi + M_\theta \sin \phi] \] \hspace{1cm} (5-76)

\[ \sigma_v = \alpha \sigma_b \] \hspace{1cm} (5-77)

\[ \sigma_T = \sigma_c + \sigma_b + \sigma_v \] \hspace{1cm} (5-78)

where

\( \rho_B \) = blade material density,
\( n_r \) = rotational speed,
\( x,y \) = coordinate system based on the principal axes of inertia,
\( I \) = polar moment of inertia,
\( \phi \) = moments of inertia angle,
\( \alpha \) = vibrational stress constant (\( \alpha = 0.75 \))

The blade chord is then adjusted such that the total stress is below the allowable limit and the calculations are repeated, if necessary.

The loss coefficient and tip leakage loss calculations follow the recommendations of [8].

Design loss coefficient

\[ \omega^* = \frac{2\sigma}{\cos \beta_2^*} \left( \frac{W_{\text{max}}}{W_1} \right)^2 K_1 \left[ K_2 + 3.1 \left( D_{\text{eq}}^* - 1 \right)^2 + 0.4 \left( D_{\text{eq}}^* - 1 \right)^3 \right] \] \hspace{1cm} (5-79)

where

\[ D_{\text{eq}} = \frac{W_{\text{max}}}{W_2} \],
\[ W_{\text{max}} = W_1 \left[ 1.12 + 0.61 \frac{\cos^2 \beta_1}{\sigma} (\tan \beta_1 - \tan \beta_2) + \alpha (i - i^*)^{1.43} \right] \],
\( \alpha = 0.0117 \),
\( K_1 = 0.0073 \),
\( K_2 = 1 + \left( \frac{s}{H_b} \right) \cos \beta_2 + 0.004 K_{Re} / K_1 \),
\( K_{Re} = \begin{cases} \sqrt{2.5 \times 10^5 / \text{Re}_c} - 1, & \text{Re}_c < 2.5 \times 10^5 \\ \left[ \log(2.5 \times 10^5) / \log(\text{Re}_c) \right]^{0.58} - 1, & \text{Re}_c > 2.5 \times 10^5 \end{cases} \)
The Correlation (5-79) includes the secondary flow effect \((K_1)\), Re number correction \((K_{Re})\), and end-wall losses \((s/H_b)\). The Mach number effect is taken into account as follows:

\[
\bar{\omega} = \bar{\omega}_m \left(1 + \xi^2\right) \tag{5-80}
\]

where

\[
\bar{\omega}_m = \bar{\omega}_s \left[1 + \left(\frac{i_m - i_s^*}{R_s^2}\right)\right],
\]

\[
\xi = \begin{cases} 
(i - i_m)/(i_m - i_s^*), & i < i_m \\
(i - i_m)/(i_s - i_m), & i \geq i_m 
\end{cases}
\]

\[
i_m = i_c + (i_s - i_c)\frac{R_c}{R_c + R_s},
\]

\[
i_s = i_s^* + \frac{R_s}{1 + 0.5(M_{1s}^2)}\xi^2,
\]

\[
i_c = i_c^* - \frac{R_c}{1 + 0.5M_{1c}^2}\xi^2,
\]

\[R_s = \alpha_s - \alpha^*,\]

\[\alpha_s = \alpha^* + 10.3 + \left[2.92 - \frac{\beta_{ls}}{15.6}\right] \frac{\theta}{8.2},\]

\[\beta_{ls} = \alpha_s + \gamma,\]

\[R_c = \alpha^* - \alpha_c,\]

\[\alpha_c = \alpha^* - 9 + \left[1 - \left(\frac{30}{\beta_{lc}}\right)^{0.48}\right] \frac{\theta}{4.176},\]

\[\beta_{lc} = \alpha_c + \gamma.\]

The total pressure loss due to tip clearance is:

\[
\Delta p_0 = \Delta p \frac{\dot{m}_c}{\dot{m}} \tag{5-81}
\]

where

\[
\dot{m}_c = \bar{\rho} U_c Z \delta \cos \gamma,
\]

\[U_c = \frac{0.816}{N_{row}^{0.2}} \sqrt{\frac{2\Delta p}{\bar{\rho}}},\]

The technique described above is now applied for the stator row to calculate the flow conditions and geometry of one stage. For the stator, the absolute (not relative) parameters (speed, angles, total pressure, enthalpy, and so on) are used. As described above, the hub radius is adjusted if needed to match the specified stage-outlet pressure. The process is then repeated for every stage.
To complete the design calculations, the compressor efficiency is found as the ratio of the enthalpy change in an isentropic process from inlet conditions to outlet pressure to the real enthalpy change from inlet to outlet conditions.

\[
\varepsilon = \frac{h_{s,\text{out}} - h_{i\text{n}}}{h_{\text{out}} - h_{i\text{n}}}
\]  

(5-82)

Note that since a repeating stage design was chosen, the flow speed at the compressor outlet is equal to that at the inlet, so the change in kinetic energy is zero and the change in total enthalpy is equal to the change in enthalpy. This means that the efficiency calculated by Equation (5-82) is in reality a total-to-total efficiency.

To calculate the compressor length, the blade row axial dimension is calculated as the chord, \(c\), multiplied by the cosine of the stagger angle, \(\gamma\) (Figure 5-6). The compressor length is equal to the sum of all rotor and nozzle blade axial dimensions.

The selection of the number of stages for each compressor in the S-CO\(_2\) Brayton cycle is based on engineering judgment using comparison of the main parameters presented in Figure 5-7 and Figure 5-8. A design with eight stages is selected for Compressor No. 1 and ten stages for Compressor No. 2. The design parameters are shown in Table 5-2 and Table 5-3.
Figure 5-7. Selection of Number of Stages for Compressor No. 1.
Figure 5-8. Selection of Number of Stages for Compressor No. 2.
### Table 5-2. Compressor No. 1 Design Parameters

**Properties distribution by stage (inlet point)**

<table>
<thead>
<tr>
<th>St#</th>
<th>p, MPa</th>
<th>T, C</th>
<th>C, m/s</th>
<th>rho, kg/m³</th>
<th>s, kJ/kg-K</th>
<th>h, kJ/kg</th>
<th>Vsd, m/s</th>
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Axial flow speed = 61.868 m/s

**Stage geometry**

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<th>r_m, cm</th>
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Out 61.87  16.58  64.05  0.192  15.00

**Total compressor length = 0.436 m**

**Compressor isentropic total-to-total efficiency = 91.578 %**

**Compressor isentropic total-to-static efficiency = 85.413 %**
### Table 5-3. Compressor No. 2 Design Parameters

**Properties distribution by stage (inlet point)**

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Axial flow speed = 84.246 m/s

**Stage geometry**

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<th>i/o</th>
<th>x, cm</th>
<th>r_h, cm</th>
<th>r_m, cm</th>
<th>r_t, cm</th>
<th>H_b, cm</th>
<th>A, cm²</th>
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</thead>
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<tr>
<td>0</td>
<td>IGV</td>
<td>in</td>
<td>0.00</td>
<td>26.43</td>
<td>28.22</td>
<td>30.00</td>
<td>3.57</td>
<td>632.59</td>
</tr>
<tr>
<td>0</td>
<td>IGV</td>
<td>out</td>
<td>2.29</td>
<td>26.43</td>
<td>28.22</td>
<td>30.00</td>
<td>3.57</td>
<td>632.59</td>
</tr>
<tr>
<td>1</td>
<td>Rot</td>
<td>in</td>
<td>2.54</td>
<td>26.43</td>
<td>28.22</td>
<td>30.00</td>
<td>3.57</td>
<td>632.59</td>
</tr>
<tr>
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<td>Rot</td>
<td>out</td>
<td>4.83</td>
<td>26.50</td>
<td>28.22</td>
<td>29.93</td>
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<tr>
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<td>5.08</td>
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<td>606.68</td>
</tr>
<tr>
<td>1</td>
<td>Sta</td>
<td>out</td>
<td>6.17</td>
<td>26.60</td>
<td>28.24</td>
<td>29.88</td>
<td>3.28</td>
<td>582.12</td>
</tr>
<tr>
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<td>3.28</td>
<td>582.12</td>
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<td>Chord,cm</td>
<td>AR</td>
<td>Spac,cm</td>
<td>C/S</td>
<td>s_c,MPa</td>
<td>s_b,MPa</td>
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<td>1.59</td>
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Blade other dimensions and stresses
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</tr>
<tr>
<td>1 Sta 44.85 41.71 3.14 19.20 25.65 32.13 15.00 9.58 5.42 29.85</td>
</tr>
<tr>
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</tr>
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</tr>
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<td>3 Rot 44.91 41.79 3.12 19.24 25.68 32.23 15.00 9.56 5.44 29.91</td>
</tr>
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</tr>
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<td>4 Rot 44.95 41.83 3.12 19.26 25.69 32.28 15.00 9.55 5.45 29.95</td>
</tr>
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</tr>
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</tr>
<tr>
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</tr>
<tr>
<td>8 Rot 45.09 42.00 3.09 19.34 25.75 32.50 15.00 9.50 5.50 30.09</td>
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</tr>
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</tr>
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<table>
<thead>
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<th>Velocity triangles (m/s, deg)</th>
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</thead>
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</tr>
<tr>
<td>-2 84.25 83.79 118.82 0.448 44.85 106.37 84.25 -22.57 87.22 0.329 -15.00</td>
</tr>
<tr>
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</tr>
<tr>
<td>-2 84.25 83.88 118.89 0.440 44.88 106.46 84.25 -22.57 87.22 0.323 -15.00</td>
</tr>
<tr>
<td>3 84.25 22.57 87.22 0.319 15.00 106.56 84.25 -83.99 118.96 0.436 -44.91</td>
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<td>8</td>
</tr>
<tr>
<td>9</td>
</tr>
<tr>
<td>10</td>
</tr>
</tbody>
</table>

**Total compressor length =** 0.318 m

**Compressor isentropic total-to-total efficiency =** 91.021 %

**Compressor isentropic total-to-static efficiency =** 86.053 %
5.6.2. **Compressor Off-Design Performance Model**

An off-design performance prediction model was developed and used to generate the compressor performance maps for the dynamics code.

For the off-design model the following parameters are given:

- Inlet conditions
- Flow rate
- Stage geometry
- Rotational speed
- Blade profile and angles.

The goal of the performance model is to calculate the flow conditions inside the compressor and at the compressor outlet. The flow velocities and angles at each row are also calculated by the model.

The calculations start from the inlet. Given the flow conditions and flow rate, together with the design parameters of the IGV (flow area and IGV angle), that information specifies the flow velocity and direction at the IGV outlet or first stage inlet. Using Equations (5-54)-(5-57), the relative to the blade flow velocity and angles are calculated at the stage inlet. The blade angle and flow angle define the incidence angle, \( i \), according to Figure 5-6. To determine the flow conditions at the outlet, iterations are required. It is assumed that the row-outlet density is known. The axial speed component at this point can then be found from the continuity Equation (5-48) with specified flow area by the design calculations.

The flow angle and the loss coefficient are to be found using the recommendations in [8]. The deviation angle is:

\[
\delta = \delta^* + \left( \frac{\partial \delta}{\partial i} \right)^* (i - i^*) + 10 \left( 1 - \frac{W_{z2}}{W_{z1}} \right)
\]

(5-83)

where

\[
\left[ \frac{\partial \delta}{\partial i} \right]^* = \frac{1 + \left( \sigma + 0.25\sigma^4 \left( \beta_i / 53 \right) \right)^{2.5}}{\exp(3.1\sigma)}
\]

is defined at design point.

The outlet flow angle is defined by Figure 5-6. The flow angle and axial speed component give the relative flow speed. The absolute speed, its components, and angles can be found using a relationship similar to Equations (5-62)-(5-65).

The loss coefficient is expressed as a function of the design loss coefficient and incidence angle:

\[
\bar{\omega} = \bar{\omega}_m \begin{cases} 
1 + \xi^2, & -2 \leq \xi < 1 \\
5 - 4(\xi + 2), & \xi < -2 \\
2 + 2(\xi - 1), & \xi > 1
\end{cases}
\]

(5-84)
where
\[ \xi = \begin{cases} \frac{(i - i_m)}{(i_m - i_c)}, & i < i_m \\ \frac{(i - i_m)}{(i_s - i_m)}, & i \geq i_m \end{cases} \]

The flow conditions at the rotor outlet are defined by Equations (5-69)-(5-74) with current values for the loss coefficient and flow speeds. The iterations on the rotor-outlet density are repeated, if necessary.

The same procedure is repeated for the stator and then for every stage until the outlet conditions are found.

Based on the inlet conditions and the outlet pressure, the outlet enthalpy in an isentropic process is calculated:
\[ h_{s,\text{out}} = h(p = p_{\text{out}}, s = s_{in}) \]

(5-85)

The total-to-total compressor efficiency is:
\[ \epsilon = \frac{h_{s,\text{out}} + \frac{C_{\text{out}}^2}{2} - h_{\text{in}} - \frac{C_{\text{in}}^2}{2}}{h_{\text{out}} + \frac{C_{\text{out}}^2}{2} - h_{\text{in}} - \frac{C_{\text{in}}^2}{2}} \]

(5-86)

To calculate the stall conditions, the stall criterion recommended by [8] is used:
\[ W_{RE} < W_{RE}^{\text{min}} \]

(5-87)

where
\[ W_{RE} = \sqrt{\frac{p_{02} - p_2}{p_{01} - p_1}} \]

= equivalent relative velocity ratio across the blade,
\[ W_{RE}^{\text{min}} = k \frac{(0.15 + 11t_b/c)/(0.25 + 10t_b/c)}{1 + 0.4[\theta\sigma/(2\sin(\theta/2)\cos \gamma)]^{0.65}} \]

= stall velocity ratio,

with condition \( \theta\sigma/(2\sin(\theta/2)\cos \gamma) \geq 1.1 \) applied,
\[ k = \begin{cases} 1, & D_{eq} \leq 2.2 \\ \left(2.2/D_{eq}\right)^{0.6}, & D_{eq} > 2.2 \end{cases} \]

In order to have a quantitative equivalent of Criterion (5-87), the stall coefficient is introduced:
\[ f_{\text{stall}} = \frac{W_{RE}}{W_{RE}^{\text{min}}} \]

(5-88)
The stall criterion would then be $f_{\text{stall}} < 1$, and $f_{\text{stall}}$ will show how far the compressor is from the stall conditions.

Choke conditions occur when the calculated Mach number (i.e., ratio of the local speed to the speed of sound) at any row inlet or outlet reaches unity. Therefore, the maximum Mach number in the compressor is used to measure how far the compressor is from the choking condition.

To demonstrate the application of the performance model, the example of the performance maps with fixed inlet conditions are calculated for each compressor. Figure 5-9 and Figure 5-10 show such performance maps for Compressors No. 1 and 2, respectively.
Figure 5-9. Example of Compressor No. 1 Performance Map.

Figure 5-10. Example of Compressor No. 2 Performance Map.
5.6.3. **Turbine Models**

The turbine models (both design and off-design) are very similar to the compressor model described above. The main differences include the row order (nozzles are followed by rotors in a turbine), absence of the inlet guiding vane, and the blade loss model.

The loss model for the turbine blades is implemented following the recommendations of [11]. This model uses the loss correlations presented by [12]. The advantage of the model is that it includes effects such as:

- Blade profile losses,
- Mach number correction,
- Shock losses,
- Tip clearance leakage,
- Reynolds number correction,
- Secondary flow losses, and
- Trailing edge losses.

The total loss coefficient is calculated as a sum of loss coefficients from profile, secondary, and tip leakage losses:

\[ Y = Y_p + Y_s + Y_{tip} \] (5-89)

The loss coefficient has the same definition as a compressor loss coefficient, \( \bar{\omega} \), in Equation (5-71), except that the tip leakage loss is included into the loss coefficient and does not appear in Equation (5-71) written for the turbine blades.

The profile loss coefficient [12] is calculated as the combination of the profile loss coefficient derived in [13] and the effects of Mach number and Reynolds number. The incident loss coefficient is added to the profile loss coefficient following the recommendations of [13] as described below.

\[ Y_p = 0.914 \left( \frac{2}{3} Y_{p[i=0]} K_p K_p + Y_{shock} \right) f_{Re} \] (5-90)

where

- \( Y_{p[i=0]} \) = profile loss coefficient at zero incidence,
- \( K_p = \frac{Y_p}{Y_{p[i=0]}} \) = incidence loss coefficient, i.e., ratio of profile loss at any incidence to profile loss at zero incidence,
- \( K_p \) = correction coefficient from exit Mach number and channel acceleration,
- \( Y_{shock} \) = subsonic shock loss coefficient,
- \( f_{Re} \) = Reynolds number correction.

The profile loss coefficient at zero incidence is defined [13] as
\[ Y_{p\{i=0\}} = \left\{ Y_{p,n} + \frac{K_1}{\alpha_2} \left[ \frac{K_1}{\alpha_2} \left( Y_{p,i} - Y_{p,n,i} \right) \right] \left( \frac{t_b}{c} \right)^{\frac{\alpha_1}{\alpha_2}} \right\} \] (5-91)

where

\[ Y_{p,n} = \text{profile loss coefficient for nozzle (}\kappa_1=0\text{) blades,} \]
\[ Y_{p,i} = \text{profile loss coefficient for impulse (}\kappa_1=-\alpha_2\text{) blades,} \]
\[ \kappa_1 = \text{blade angle at the inlet,} \]
\[ \alpha_2 = \text{flow angle at the outlet,} \]
\[ t_b = \text{maximum blade thickness,} \]
\[ c = \text{blade chord.} \]

The indices, 1 and 2, denote blade inlet and outlet, respectively.

The profile loss coefficients for nozzle and impulse blades in Equation (5-91) are given in [13] as plots versus blades spacing-to-chord ratio, \( s/c \), for different flow outlet angles, \( \alpha_2 \). Those plots were fitted with the TableCurve 3D software (demo version) to find the fitting function which gives the best approximation.

\[ Y_{p,n} = f_1(s/c, \alpha_2) \] (5-92)
\[ Y_{p,i} = f_2(s/c, \alpha_2) \] (5-93)

The incidence loss coefficient is reported in [13] as a plot versus the ratio of incidence angle, \( i \), to stalling incidence angle, \( i_s \). The stalling incidence, \( i_s \), is derived in [13] through a series of plots versus spacing-to-chord ratio and flow outlet angle. Again, those plots, as well as the plot for incidence loss coefficient, were fitted with functions using the TableCurve software.

\[ K_{ip} = f_3(i/i_s) \] (5-94)
\[ i_s = i_s(s/c = 0.75) + \Delta i_s \] (5-95)
\[ i_s(s/c = 0.75) = f_4 \left( \frac{\kappa_1}{\alpha_2(s/c = 0.75)}, \alpha_2(s/c = 0.75) \right) \] (5-96)
\[ \frac{\alpha_2}{\alpha_2(s/c = 0.75)} = f_5(s/c) \] (5-97)
\[ \Delta i_s = f_6(s/c, \alpha_2) \] (5-98)

As examples of plot fitting, Figure 5-11 shows the 3D fitting function (for the stalling incidence angle for a blade spacing-to-chord ratio = 0.75) and Figure 5-12 shows the 2D fitting of the incidence loss coefficient.
Stalling Incidence Angle (s/c=0.75)

Rank 3  Eqn 1138  \( z = \frac{(a + b \ln x + c y + d y^2)}{(1 + e \ln x + f y + g y^2)} \)

\( r^2 = 0.97725726 \)  DF Adj \( r^2 = 0.97451245 \)  FitStdErr = 1.6392854  Fstat = 422.53915

\( a = 3.6653366 \)  \( b = -0.33031158 \)  \( c = 1.3547229 \)  \( d = -0.18000932 \)

\( e = -0.22448634 \)  \( f = 0.023396452 \)  \( g = 0.055922252 \)

Figure 5-11. Stalling Incidence Plot Fitting.

Incidence Loss Correction

Rank 6  Eqn 7004  \( y = \frac{(a + c x + e x^2)}{(1 + b x + d x^2 + f x^3)} \)

\( r^2 = 0.99973383 \)  DF Adj \( r^2 = 0.99961975 \)  FitStdErr = 0.032441058  Fstat = 11267.792

\( a = 0.97953746 \)  \( b = -0.41749684 \)  \( c = -0.49682766 \)

\( d = 0.020476472 \)  \( e = 0.78361624 \)  \( f = 0.0073812324 \)

Figure 5-12. Incidence Loss Coefficient Plot Fitting.
The correction coefficient from exit Mach number, channel acceleration, and their combined effect for Equation (5-90) are defined in [12], respectively:

\[
K_1 = \begin{cases} 
1, & \text{if } M_2 \leq 0.2 \\
1 - 1.25(M_2 - 0.2), & \text{if } M_2 > 0.2 
\end{cases} \quad (5-99)
\]

\[
K_2 = \left( \frac{M_1}{M_2} \right)^2 \quad (5-100)
\]

\[
K_p = 1 - K_2(1 - K_1) \quad (5-101)
\]

where

\[M = \text{Mach number.}\]

The subsonic shock loss coefficient in Equation (5-90) is defined in [12].

\[
Y_{\text{shock}} = 0.75(M_1 - 0.4)^{0.75} \frac{r_h}{r_t} \frac{p_1}{p_2} \frac{1 - \left(1 + \frac{\gamma - 1}{2} M_1^2 \right)^{\frac{\gamma}{\gamma - 1}}}{1 - \left(1 + \frac{\gamma - 1}{2} M_2^2 \right)^{\frac{\gamma}{\gamma - 1}}} \quad (5-102)
\]

where

\[r_h, r_t = \text{hub and tip radii, respectively,} \]

\[p = \text{pressure,} \]

\[\gamma = \frac{c_p}{c_v} = \text{ratio of specific heats.} \]

The correction for the Reynolds number is defined [12] as:

\[
f_{Re} = \begin{cases} 
\left( \frac{Re}{2 \cdot 10^5} \right)^{-0.4}, & \text{if } Re \leq 2 \cdot 10^5 \\
1, & \text{if } 2 \cdot 10^5 < Re \leq 10^6 \\
\left( \frac{Re}{10^6} \right)^{-0.2}, & \text{if } Re > 10^6 
\end{cases} \quad (5-103)
\]

where

\[Re = \frac{\rho_2 V^2 c}{\mu_2} = \text{Reynolds number based on blade chord and outlet conditions.} \]

The secondary loss coefficient in Equation (5-89) is defined in [12] as the original secondary loss coefficient from [13] corrected for the subsonic Mach number:
\[ Y_s = 1.2 Y_{s,i} K_s \]  

(5-104)

\[ Y_{s,1} = 0.0334 f_{AR} \left( \frac{\cos \alpha_2}{\cos \kappa_1} \right) \left( \frac{C_L}{s/c} \right)^2 \frac{\cos^2 \alpha_2}{\cos^3 \alpha_m} \]  

(5-105)

where

\[ \frac{C_L}{s/c} = 2 (\tan \alpha_i + \tan \alpha_t) \cos \alpha_m, \]

\[ \alpha_m = \tan^{-1} \left( \frac{\tan \alpha_i + \tan \alpha_t}{2} \right), \]

\[ f_{AR} = \begin{cases} 1 - 0.25 \sqrt{2 - h/c}, & \text{if } h/c \leq 2 \\ \frac{h/c}{h/c}, & \text{if } h/c > 2 \end{cases} \]

\( h/c \) = blade height-to-chord ratio.

\[ K_s = 1 - K_3 (1 - K_p) \]  

(5-106)

where

\[ K_3 = \left( \frac{c_x}{h} \right)^2, \]

\( c_x \) = axial projection of the blade chord.

The tip leakage loss coefficient in Equation (5-89) is defined for shrouded blades only [12].

\[ Y_{tip} = 0.37 \frac{c}{h} \left( \frac{t}{c} \right)^{0.78} \left( \frac{C_L}{s/c} \right)^2 \frac{\cos^2 \alpha_2}{\cos^3 \alpha_m} \]  

(5-107)

where

\[ t' = \frac{t}{N_{seal}^{0.42}}, \]

\( t \) = tip clearance,

\( N_{seal} \) = number of seals.

For unshrouded blades, the tip leakage coefficient is set to be zero, but the overall blade row efficiency is corrected for the tip leakage losses:

\[ \frac{\Delta \eta}{\eta_0} = 0.93 \frac{t}{h \cos \alpha_2} \frac{r_i}{r_m} \]  

(5-108)

where

\( \Delta \eta \) = correction to the blade efficiency,

\( \eta_0 \) = blade efficiency without tip leakage losses,
\[ r_m = \text{blade mean radius}. \]

Introduction of the Mach number correction had been found to give rise to some unexpected results. Turbine designs with a small number of stages (where the velocities are higher) might have higher efficiency than designs with more stages as Figure 5-13 shows. This is due to the decrease in profile and secondary loss coefficients for higher Mach number. A similar behavior is reported in [11]. However, operating at higher Mach number at the design point will reduce the margin to turbine choke. For this reason and from comparison of turbine length, diameter, and blade dimensions, a four-stage design is still considered to be optimal.

The same loss model is used in the off-design performance model. The turbine map generated for fixed inlet conditions is presented in Figure 5-14.
Figure 5-13. Selection of the Number of Stages for Turbine.
Figure 5-14. Example of Turbine Performance Map.
5.6.4. Turbomachinery in the Plant Dynamics Model

The common approach in turbomachinery modeling is an instantaneous response assumption \[6\]. Under this assumption, the flow rate and outlet parameters react instantaneously to changes in the inlet and outlet pressure and/or rotational speed. The steady-state off-design performance model is used to calculate the turbomachinery behavior under this approach. However, those off-design models usually involve multiple layers of iterations on fluid properties, speeds, and losses and, therefore, are much too slow to be directly used in dynamic calculations. The iterations could be very slow for CO\textsubscript{2} due to rapid properties variations.

Instead, performance maps are generated for each turbomachinery component before the dynamic calculations are performed. These maps are then used in the dynamics model. The maps should cover the whole possible range of parameter variations to avoid extrapolation. The parameters which define the turbomachinery state are:

- Inlet temperature;
- Inlet pressure;
- Outlet pressure; and
- Rotational speed.

Based on these parameters, the steady-state off-design performance model calculates all other parameters and conditions. Of the calculated parameters, the mass flow rate though the turbomachinery and outlet temperature (or specific enthalpy) are used in the dynamics model. So, the goal of the performance maps is to generate a behavior law for flow rate and outlet temperature for variations in each of the four parameters listed above. The traditional approach for ideal gas cycles has been to reduce the number of varying parameters to two (usually, pressure ratio and temperature-corrected rotational speed) using the fact that the fluid follows the ideal gas law. Therefore, the turbine and compressor maps are usually generated as functions of two variables in traditional approaches.

It is known, however, that carbon dioxide does not follow the ideal gas law, especially near the critical point where Compressor No. 1 is operated. For example, a very small change in inlet pressure, say 7.5 MPa vs. 7.4 MPa at design, causes a very significant change in inlet density (594 kg/m\textsuperscript{3} vs. 369 kg/m\textsuperscript{3}, for this example and design temperature). This change in density will disturb the fluid velocities resulting in significant differences in outlet pressure (30.6 MPa vs. 20.0 MPa) for the same flow rate through the compressor and the same rotational speed. Therefore, the flow rate could not be defined as a function of rotational speed and pressure ratio only (the pressure ratios are 4.08 and 2.70, in this example). The inlet temperature has a similar effect for CO\textsubscript{2}. Although it could theoretically be possible to find the non-dimensional parameters which will allow a reduction of the number of variables even for CO\textsubscript{2}, finding these parameters could be very difficult, if at all successful. Instead, an approach is adopted that involves generation of performance maps for individual variations of each of the four parameters involving four-dimensional arrays. Fortunately, modern computers can generate, store,
and work with million-element arrays (e.g., 40 points for each parameter requires $40^4 = 2.56 \times 10^6$ elements).

Four-dimensional performance maps are generated for the turbine and two compressors. These maps provide the flow rate, outlet temperature, maximum Mach number (for checking choke conditions), and stall parameter (for checking compressor stall conditions) for each value of rotational speed, inlet temperature, and inlet and outlet pressures. The parameters between the points are obtained in the dynamics model by linear interpolation between the map points.

Figure 5-15 shows the example of the performance map for Compressor No. 1. It shows the flow rate (relative to nominal) for each point of inlet and outlet pressure with the design values for inlet temperature and rotational speed. The points in inlet pressure represent values of inlet pressure with the design value in the middle. The points for outlet pressure are obtained by dividing the range between stall and choke conditions into a fixed number of intervals such that the actual values for outlet pressure at each point are different for different inlet conditions (these values are stored in a separate array). Similar maps are constructed for each point in inlet temperature and rotational speed to compose the complete performance map for the flow rate through Compressor No. 1. Similarly, the maps for outlet temperature, maximum Mach number, and stall criteria (for compressors) are obtained for the turbine and each compressor.

While a significant amount of computational time is required to generate the four-dimensional maps, the maps only need to be generated once. The utilization of the relative to steady-state, rather than absolute, parameters allows use of the same maps even if design conditions are different\(^1\). The usage of the four-dimensional arrays has proven to be fast enough for the dynamics code.

---

\(^1\)The turbomachinery design is integrated into the steady-state code which is based on several layers of iterations with finite convergence criteria. Therefore, the steady-state results, including the turbomachinery design conditions, are slightly different at each start of the dynamics calculations.
5.7. **Turbomachinery Shaft Dynamics**

The turbomachinery rotor dynamics equation defines the change in the turbomachinery rotational speed [6]:

\[
\frac{\partial \omega}{\partial t} = \frac{W_{\text{gen}} - W_{\text{grid}}}{I \omega}
\]  

(5-109)

where

- \( \omega = 2\pi n_r \) = rotational speed,
- \( W_{\text{gen}} \) = generator power (see below),
- \( W_{\text{grid}} \) = demand from the grid,
- \( I \) = total moment of inertia.

The generator power is defined as the difference between the power produced by the turbine \( (W_T) \) and power consumed by the two compressors \( (W_{C1}, W_{C2}) \) with
accounting for the mechanical losses ($W_l$), generator efficiency ($\varepsilon_{\text{gen}}$), and power input for water pump for the cooling water provided to the cooler:

$$W_{\text{gen}} = (W_l - W_{C1} - W_{C2} - W_i) \varepsilon_{\text{gen}} - W_{\text{water}}$$  \hspace{1cm} (5-110)

The component power is calculated as the flow rate through the component times the change in specific enthalpy across the component. The default mechanical losses are assumed to be 1 % of the full generator power and proportional to the square of the rotational speed. The default generator efficiency is assumed to be 98.5 %. The grid demand is given as a function of time. The total moment of inertia is a sum of the moments for the turbine, compressors, and generator rotors. The moment of inertia is defined as [14]:

$$I = \int r^2 dm$$  \hspace{1cm} (5-111)

For a solid cylinder of radius $r$, length $L$, mass $m$, and density $\rho$, it is equal to:

$$I_{\text{cyl}} = \frac{1}{2} mr^2 = \frac{1}{2} \pi \rho L r^4$$  \hspace{1cm} (5-112)

Equation (5-112) is used to obtain the moment of inertia for each stage of the turbine and compressor. For example, the moment of inertia of a compressor stage is a sum of moments for each rotating part as shown in Figure 5-16.

$$I_{\text{comp}} = \sum_{i=1}^{N} \left( I_{\text{shaft}_i}^{\text{rot}} + I_{\text{hub}_i}^{\text{rot}} + I_{\text{blades}_i}^{\text{rot}} + I_{\text{shaft}_i}^{\text{stat}} \right)$$  \hspace{1cm} (5-113)

$$I_{\text{shaft}_i}^{\text{rot}} = \frac{1}{2} \pi \rho_{\text{shaft}} \text{Chord}_i^{\text{rot}} r_{\text{shaft}_i}^4$$  \hspace{1cm} (5-114)

$$I_{\text{shaft}_i}^{\text{stat}} = \frac{1}{2} \pi \rho_{\text{shaft}} \text{Chord}_i^{\text{stat}} r_{\text{shaft}_i}^4$$  \hspace{1cm} (5-115)

$$I_{\text{hub}_i}^{\text{rot}} = \frac{1}{2} \pi \rho_{\text{hub}} \text{Chord}_i^{\text{rot}} \left( r_{\text{hub}_i}^4 - r_{\text{shaft}_i}^4 \right)$$  \hspace{1cm} (5-116)

$$I_{\text{blades}_i}^{\text{rot}} = \frac{1}{2} \pi \rho_{\text{blade}} K_{\text{blade}} \text{Chord}_i^{\text{rot}} \left( r_{\text{tip}_i}^4 - r_{\text{hub}_i}^4 \right)$$  \hspace{1cm} (5-117)

where

1 At this stage of the turbomachinery design, it is not clear whether to include the stator row hub into rotating parts or not. In any case, as it will be shown below, the moments of inertia of turbine and compressors are small compared to generator’s moment of inertia. So the effect of the stator hub is not significant for the total moment of inertia.
\[ K_{blade} = \frac{\sigma t_b}{2c} \frac{\theta}{2\sin\frac{\theta}{2}} \]  

where (see figure to the right)

\[ \sigma = \frac{c}{s} \]  
blade solidity,
\[ c = \text{blade chord}, \]
\[ s = \text{blade spacing}, \]
\[ t_b = \text{maximum blade thickness}, \]
\[ \theta = \text{blade angle}. \]

The blade row “porosity” is a ratio of the volume occupied by the blades to the total volume between the hub and tip. The blade’s total volume is calculated under the assumption that the blades are arcs with an average thickness equal to half of the maximum thickness.

Figure 5-16. Compressor Stage for Moment of Inertia Calculations.

The moment of inertia for the generator rotor is calculated using Equation (5-42) by interpolating data for existing generators. Table 5-4 shows the calculated moments of inertia for the turbine, compressors and generator.
Table 5-4. Moments of Inertia
(kg-m²)

<table>
<thead>
<tr>
<th>Generator</th>
<th>Turbine</th>
<th>Compressor #1</th>
<th>Compressor #2</th>
<th>TOTAL</th>
</tr>
</thead>
<tbody>
<tr>
<td>4706</td>
<td>223.451</td>
<td>4.931</td>
<td>14.456</td>
<td>4948.8</td>
</tr>
</tbody>
</table>
6. Plant Control System

The goal of the automatic control system is to follow the load; that is, to adjust the plant characteristics in such a way that the plant produces electricity in an amount exactly equal to the electric grid demand at all times without violating limits on the Brayton cycle or reactor conditions. Usually, the plant control system consists of a reactor control system and the power conversion plant control system. However, the STAR-LM reactor is designed as an autonomous reactor such that in many instances no control action is needed on the reactor side as shown below.

6.1. Autonomous Reactor Control

The strong negative reactivity feedbacks of the STAR-LM fast reactor allow for self-adjustment of the reactor power. The reactor power adjusts itself through the reactivity feedbacks as shown on Figure 6-1 to match the heat removal rate by the balance-of-plant. This self-adjustment of the reactor power is called autonomous reactor control. An example is autonomous load following whereby the reactor power adjusts itself to the changes in the heat removal by the S-CO$_2$ Brayton cycle system to match routine changes in the demand from the electric grid.

Even though the process in Figure 6-1 is applicable to any reactor with negative reactivity feedbacks, the actual temperature increase is defined by the magnitude of the reactivity feedbacks. Only if the feedbacks are strong enough (as in the STAR-LM reactor) will the temperature changes be small enough to not exceed their limits. In this case, no control action from control rods is needed to control the reactor power. The large reactivity feedbacks are an inherent feature of the fast neutron spectrum core with lead coolant and nitride fuel.

The STAR-LM reactor operates on natural circulation, so there are no main coolant pumps on the reactor side. Since no intermediate loop is utilized in the design, there are no intermediate coolant pumps as well. Therefore, the need for control action for coolant pump control is also eliminated by design.
6.2. Brayton Cycle Automatic Control

The Brayton cycle automatic control system adjusts the cycle in such a way that the plant produces electricity in an amount exactly equal to the grid demand. The Brayton cycle control system for STAR-LM incorporates several control mechanisms as shown in Figure 6-2.
One of the motivations for STAR-LM development was to design a reactor which could supply electricity to customers who are not connected to a major electrical grid. Therefore, it is assumed in the analysis that the STAR-LM turbogenerator rotational speed dictates the grid frequency such that the generator works in an asynchronous mode at all power levels. Thus, the rotational speed of the generator (and the turbine and compressors, since they are all on one shaft) has to be actively controlled all of the time. The turbogenerator rotational speed can be controlled by the power supplied by the turbine to the generator. If the generator power exceeds the demand from the grid, then the excess of energy is stored as the kinetic energy of the rotating components and the shaft starts to rotate faster. Therefore, the rotational speed can be controlled by adjusting the net power produced by the turbine (turbine work minus the work demand of the compressors).

6.2.1. Turbine Bypass Control

The common practice for gas Brayton cycles is to use turbine bypass for turbomachinery speed control. The same approach is used for this cycle. If the generator power exceeds the grid demand, the turbine bypass valve opens reducing the flow rate through the turbine and, therefore, decreasing the turbine work. The turbine bypass automatic control monitors the rotational speed of the turbogenerator. It calculates the difference between the actual rotational speed and its specified value. It is assumed that it
also calculates the derivative and integral value of this difference. The bypass valve action, valve opening or closing, is calculated based on the three parameters representing proportional, differential, and integral control as defined by Equation (6-1).

\[
\int_{0}^{t} E_{TBP}^\prime(t) dt
\]

where \( f_{TBP}^\prime \) = rate of change of open area fraction, \( f_{\text{open}}^\prime \), or speed at which the valve is opening or closing,

\[
E_{TBP} = \frac{n_r(t) - n_r^\text{spec}}{n_r^\text{spec}} = \text{deviation of the rotational speed, } n_r, \text{ from its specified value,}
\]

\( k_d, k_p, k_i = \text{user-defined coefficients of control.} \)

The coefficients for the proportional, differential, and integral parts of the control are defined by the user in the input file. They are selected to achieve a satisfactory response from the control system.

The derivative of deviation is generally calculated as the difference between the current value and the value at the previous time step divided by the time step.

\[
E' = \frac{E(t) - E(t - \Delta t)}{\Delta t}
\]

However, the derivative for the turbine bypass deviation is known at every time step since the derivative of the shaft rotational speed is calculated by the shaft dynamics Equation (5-109).

\[
E_{TBP}^\prime = n_r^\prime
\]

The deviation integral is calculated as a sum of error values multiplied by the time step for all time steps from the beginning of a transient.

\[
\int_{0}^{t} E_{TBP}^\prime(t) dt = \sum_{i=0}^{t} E_{TBP}^\prime(t) \Delta t
\]

The valve open fraction at the end of the time step is calculated based on the value at the beginning of the time step and the opening or closing rate.

\[
f_{\text{open}}^\prime = \left(f_{\text{open}}^\prime\right)_{t=0} + f_{TBP}^\prime \Delta t
\]

The limitations on the open area fraction (between 0 and 1) and valve opening and closing rates (defined by the user) are applied to the results obtained from Equation (6-1).

It should be noticed here that opening the bypass valve reduces the amount of useful work performed by the turbine. Therefore, the valve should be closed during
operation at full power. However, it is found to be beneficial from the stability point of view to maintain a small bypass flow (valve open fraction is much less than 1%). This small bypass flow does not show any significant effect on the generator output while allowing usage of turbine bypass control for very small changes (uncertainties) in the generator output. Although inventory control could be used for small power increases, that control mechanism is shown not to be fast enough for turbogenerator speed control.

If the calculated value of the turbine bypass valve open area fraction is less than zero then the valve is closed \( f = 0 \) and the remaining value (i.e., \(-f\)) is added to the inventory control action (Equation (6-6) below) to simulate shaft speed control by inventory addition. This would be the case, for example, when the plant operates above 100% load as described below.

6.2.2. Inventory Control

Quasi-static calculations [15] showed that inventory control is the preferable control mechanism, since it provides the highest cycle efficiency at reduced loads. At the same time, during the development of the dynamic control system, it was realized that inventory control could not be fast enough to regulate the turbomachinery speed. Rapid removal of the CO\(_2\) inventory from the cycle introduces large disturbances in the compressor conditions causing stalling or choking in one of the compressors. Thus, turbine bypass control is utilized for small changes in loads as a means of shaft speed control. In order to eliminate dual action from both control mechanisms, the control system logic is specified such that turbine bypass controls the cycle from 100% to 90% load and inventory control regulates the cycle below 90%. Turbine bypass control still regulates the shaft speed at any power level. In order to implement an automatic control on the CO\(_2\) inventory in the cycle, an inventory control table is generated. The inventory control table specifies how much CO\(_2\) mass needs to be removed from the cycle for each power level. The table is generated based on quasi-static calculations. Any inaccuracy in the table will result in a mismatch between the generator output and grid demand. This imbalance will be reflected in the shaft rotational speed, and, therefore, will be regulated by the turbine bypass automatic control.

The automatic inventory control system monitors the CO\(_2\) mass in the inventory tanks and opens and closes inlet and outlet valves by an approach similar to Equation (6-1), in order to match the required mass obtained from the control table. Since opening an inlet or outlet valve adds or removes CO\(_2\) inventory from the tank (acting as derivative of the inventory), the inventory control system calculates valve openings (not their derivatives).

\[
\dot{f}_{INV} = k_p^{INV} E_{INV} + k_d^{INV} E_{INV}' + k_i^{INV} \int_0^t E_{INV}(t) dt + |f_{open}^{TBP}|
\]  

(6-6)

where

\( \dot{f}_{INV} \) = total action from inlet and outlet valves (see below),
\[ E_{INV} = \frac{\Delta M_{spec} - \Delta M_t(t)}{M_{t,t=0}} \]

= deviation of the mass addition to the inventory tank, \( \Delta M_t \) from its specified value, \( M_{t,t=0} \)

\[ \int f_{open}^{TBP} = \text{remaining action from turbine bypass control (used here only if calculated } f_{open}^{TBP} \text{ is less than zero).} \]

The deviation derivative and integral are calculated as shown in Equations (6-2) and (6-4).

If the inventory control action calculated by Equation (6-6) is positive (\( \Delta M_t \) is less than a required value) then the inlet valve opens; otherwise the outlet valve opens:

\[
\begin{align*}
    f_{open}^{INV} &= f_{INV}, \quad \text{if } f_{INV} > 0 \\
    f_{open}^{INV_0} &= -f_{INV}, \quad \text{if } f_{INV} < 0
\end{align*}
\]

(6-7)

It is shown [15] that the inventory control system has limited range mostly due to inventory tank volume limitations – inventory control is to be used from 90% to 50% load. The control table is generated to cover this range; if the load falls below 50%, the control table will still provide the 50% load value such that inventory control will not take any action. Instead, other control mechanisms will be initiated as described below.

It is also shown that inventory control is capable of increasing the plant power above 100% nominal, if CO\(_2\) inventory is added to the cycle. However, the results demonstrated that this power increase results in increase of the core structure temperatures beyond the safety limits for steady-state operation. Still, the capability of short-term power increase from the cycle is demonstrated, mostly as an exercise. The inventory control table is extended up to 110% for these purposes.

6.2.3. Turbine Throttle Valve Control

For loads below 50% of nominal full power, turbine throttle valve control is initiated. Again, based on quasi-static calculations, a control table is generated showing the required pressure drop across the throttle valve as a function of the grid demand. Similar to the inventory control table, any inaccuracy in the throttle valve control table will be compensated by turbine bypass control. The turbine throttle valve automatic control system calculates the difference between the actual pressure drop across the valve and the required value, and adjusts the valve opening.

\[
f_{TIN}' = k_{p}^{TIN} E_{TIN} + k_{s}^{TIN} E_{TIN}' + k_{i}^{TIN} \int_{0}^{f} E_{TIN}(t) dt
\]

(6-8)

where

\[ f_{TIN}' = \text{rate of change of open area fraction, } f_{open}^{TIN}, \text{ or speed at which the valve is opening or closing,} \]
\[ E_{TIN} = \frac{\Delta p_{TIN}^{spec} - \Delta p_{TIN}(t)}{P_{TIN}} = \text{deviation of the pressure drop across the turbine inlet valve, } \Delta p_{TIN}, \text{ from its specified value.} \]

The deviation derivative and integral are calculated as shown in Equations (6-2) and (6-4). The valve open area is calculated similar to Equation (6-5).

### 6.2.4. Flow Split Control

In parallel to the turbine throttle valve, a flow split control is activated. This control monitors the conditions in the compressors. If an approach to stall or choke is detected in any of the compressors, the flow split between the compressors is adjusted. To regulate the flow split, a throttle valve before Compressor No. 2 is utilized. The valve action is calculated as a sum of the actions from each limiting condition (stall or choke). For example, if the Compressor No. 1 stall parameter, \( f_{stall} \), is below 1.1 (\( f_{stall} = 1 \) means stall), the control system will start closing the valve before Compressor No. 2 to allow for more flow through Compressor No. 1. The closing rate will be increasing as the stall parameter approaches unity.

\[ (f_{C2IN}^{'})_{C1stall} = -\frac{1.1 - f_{C1stall}}{1.1 - 1} f_{c2v}^{el.\ lim} \tag{6-9} \]

where

\( (f_{C2IN}^{'})_{C1stall} \) = part of the Compressor No. 2 inlet valve opening or closing rate due to stall conditions at Compressor No. 1,

\( f_{c2v}^{el.\ lim} \) = maximum closing rate for the valve.

It is shown [15] that the compressor operating ranges are the main limiting parameters for every control mechanism (except inventory control) range. Since flow split control is used to prevent limiting conditions in the compressors, its utilization extends the range of the turbine bypass and turbine inlet valve controls.

The turbine throttle valve (with flow split control) controls the cycle between 50% and 20% load. For smaller loads, turbine bypass control will automatically regulate the generator output by controlling the shaft speed.

### 6.2.5. IRHX Bypass Control

It is show that the control mechanisms described above are sufficient to control the plant over the full load range (between 0% and 100% loads). Therefore, an automatic control for IRHX bypass is not implemented in the code. It could be done similarly to turbine inlet valve control, if a need for automatic IRHX bypass control arises in the future. At this time, a “manual” control action is simulated, i.e., the user can specify the
valve open area fraction as a function of time (similar tables for other controls are also specified in the input file).

6.3. **Plant Operation Limits**

The plant control system has to allow plant operation without exceeding the safety and operational limits. These limits include:

- Peak temperatures in the core, such as the coolant boiling temperature and cladding temperature limit,
- Coolant freezing,
- Compressor stall and choking conditions,
- Turbine choking,
- Excessive CO$_2$ pressures and temperatures, and
- Excessive shaft rotational speeds.
7. Heat Transfer and Pressure Drop Correlations

7.1. Heat Transfer Correlations

Coolant (Lead) Flow in Core (Ref. 16)

\[
Nu = Nu_{\text{lam}} + \frac{0.041}{x^2} \left[ 1 - \frac{1}{x^{30} - \frac{1}{6} + (1.15 + 1.24 \varepsilon_b)^{1/2}} \right] Pe^2
\]

where

\[
Nu_{\text{lam}} = 7.55x - \frac{6.3}{x^b} \left[ 1 - \frac{3.6x}{x^{20}(1 + 2.5 \varepsilon_6^{0.86} + 3.2)} \right]
\]

\[
x = \frac{p}{d}
\]

\[
\varepsilon_6 = \frac{k_{cl}}{k_{bo}} \frac{X_{cl}(m_{bo} - X_{bo})m_{cl} + X_{bo}(1 - X_{bo}m_{bo})}{k_{bo} X_{bo}(m_{bo} + X_{bo})m_{cl} + X_{bo}(1 + X_{bo}m_{bo})}
\]

\[
X_{cl} = \left( \frac{D_{c,i}}{D_{rod}} \right)^{12}, \quad X_{bo} = \left( \frac{D_{f,o}}{D_{rod}} \right)^{12}
\]

\[
m_{cl} = \frac{k_{cl} - k_f}{k_{cl} + k_f}, \quad m_{bo} = \frac{k_{bo} - k_f}{k_{bo} + k_f}
\]

\[
Pe = Re Pr
\]

\[
a = 0.56 + 0.19x - 0.1/x^{80}
\]

\[
b = 17x(x - 0.81)
\]

\[p = \text{fuel pin pitch},\]
\[d = D_{rod} = \text{fuel pin diameter},\]
\[k = \text{thermal conductivity},\]
\[D_{c,i} = \text{cladding inner diameter},\]
\[D_{f,o} = \text{fuel outer diameter},\]
\[\text{subscripts cl, bo, and f denote cladding, bond, and fuel, respectively.}\]

Lead Flow around Rods – LAR and FGP Regions (Ref. 17)

\[
Nu = 24.15 \log_{10} \left[ -8.21 + 12.76 \frac{p}{d} - 3.65 \left( \frac{p}{d} \right)^2 \right] + x
\]

where
\[
x = \begin{cases} 
0.0174 \left[ 1 - e^{-6.0 \left( \frac{P}{d} - 1 \right)} \right] (\text{Re} \cdot \text{Pr} - 200)^{0.9}, & \text{if } \text{Re} \cdot \text{Pr} \geq 200 \\
0, & \text{if } \text{Re} \cdot \text{Pr} < 200 
\end{cases}
\]

Lead Flow in IRHX (Ref. 18)

\[
\text{Nu} = 7.55 \frac{P}{d} - 14 \left( \frac{P}{d} \right)^5 + 0.007 \text{Pe}^{0.64 + 0.246 \frac{P}{d}}
\]

Lead Flow along a Cylindrical Wall – IRHX wall and Downcomer (Ref. 19)

\[
\text{Nu} = \alpha + \beta (\psi \cdot \text{Re} \cdot \text{Pr}) \gamma
\]

where

<table>
<thead>
<tr>
<th></th>
<th>Outer wall</th>
<th>Inner wall</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\alpha) =</td>
<td>(5.54 - 0.023y)</td>
<td>(4.82 + 0.697y)</td>
</tr>
<tr>
<td>(\beta = )</td>
<td>(0.0189 + 0.00316y = 0.0000867y^2)</td>
<td>0.0222</td>
</tr>
<tr>
<td>(\gamma = )</td>
<td>(0.758 \frac{y}{\sqrt{0.0204}})</td>
<td>(0.758 y^{0.053})</td>
</tr>
</tbody>
</table>

\(y = \frac{R_o}{R_i}\) = radius ratio,

\(\psi = 1 - \frac{1.82}{\text{Pr} \left( \frac{E_m}{v} \right)^{1.4}_{\text{max}}} \),

\(\left( \frac{E_m}{v} \right)^{1.4}_{\text{max}} = 7.833 \cdot 10^{-4} \text{Re}^{1.2}\)
Air Flow along Guard Vessel Wall (Ref. 20)

\[
Nu = F_{enh} 0.0217 \text{Re}^{0.8} \text{Pr}^{0.4} \left( \frac{T_{air}}{T_w} \right)^{0.2}
\]

where

- \( F_{enh} \) = heat transfer enhancement factor,
- \( T_{air}, T_w \) = air bulk and wall surface temperatures, respectively.

CO₂ Flows

The user can select in the input files which correlation to use for each component. The code supports two correlations:

- Dittus-Boelter [21]

\[
Nu = 0.023 \text{Re}^{0.8} \text{Pr}^n
\]

where

- \( n = 0.4 \) for heated flow and 0.3 for cooled flow.

- Petukhov-Gnielinski [22]

\[
Nu = \frac{f(\text{Re} - 1000)\text{Pr}}{1 + 12.7 \left( \frac{f}{2} \right)^{1/2} \left( \frac{f}{4L} \right)^{1/4} \left( \frac{\text{Pr}^{2/3} - 1}{\text{Pr}^{1/3}} \right)}
\]

where

- \( f \) = friction factor, found from \( 1/\sqrt{f} = 4.0 \log_{10} \left( \text{Re} \cdot \sqrt{f} \right) - 0.4 \)
- \( D \) = hydraulic diameter,
- \( L \) = channel length.

Both correlations above are applicable to turbulent flow (\( \text{Re} > 2300 \)) only. In laminar flow the following correlation is used [23].

\[ Nu_{lum} = 4.364 \]
7.2. Pressure Drop Correlations

Turbulent Flow (Re>2300, Except Air, Ref. 24)

\[ f = \frac{1}{4X^2} \]

where

\[ X = \text{hydraulic resistance found from} \]

\[ X = 1.74 - 2\log_{10}\left(\frac{2k_s}{D_h} + \frac{18.7X}{Re}\right) \]

\( k_s = \text{wall roughness (10} \, \mu\text{m is assumed),} \)
\( D_h = \text{hydraulic diameter.} \)

Laminar Flow (Re<2300, Ref. 7)

\[ f = \frac{16}{Re} \]

Air Flow (Ref. 21)

\[ f = \frac{0.0791}{Re^{0.25}} \]
8. Solution Scheme

8.1. Numerical Solution of Differential Equations

The equations described above for time-dependent temperatures, densities, and flow rates are combined into a system of differential equations. The total number of equations required to be solved to characterize the whole plant is about 400\(^1\). The resulting system has three major characteristics important for the solution scheme.

First, the system is highly-coupled meaning that the system could not be divided into independent sub-systems. For example, the equations for the reactor side depend on the coolant temperatures. The coolant temperatures depend on its flow rate. The coolant flow rate in natural circulation is defined by coolant temperatures everywhere in the reactor. So, all of the equations in the reactor are inter-dependent on the coolant temperatures and flow rate. Moreover, the coolant temperatures in the IRHXs are defined by the wall and CO\(_2\) temperatures and CO\(_2\) flow rates. The CO\(_2\) flow rates are defined by CO\(_2\) conditions everywhere in the cycle. The CO\(_2\) temperatures in the IRHXs are defined by its flow rate and lead temperatures. So, both the reactor and Brayton cycle parts of the plant are inter-dependent on the lead and CO\(_2\) temperatures and flow rates. Therefore, the whole system of the differential equations has to be solved simultaneously.

Second, the majority of the differential equations are non-linear. For example, Equation (2-14) for coolant temperatures has the term, \(\dot{m}_{Pb} T_{Pb}\), where both variables are found as a solution of separate differential equations. Therefore, linear solution schemes cannot be applied to this system.

Third, it is found that the system is a stiff system of differential equations. The stiff system means that the time constants of the equations in the system differ significantly. For example, time constants for the CO\(_2\) flow rate equations are very small, especially near the critical point. The time constant for some structures in the reactor, such as the reactor vessel, are very large. Traditional methods (such as Euler or Runge-Kutta) could be applied to the stiff system; however, the time step would be selected by the smallest time scale of the system resulting in significant amount of computation.

In order to address these specific features of the system, a special solution technique has been developed for this code. The numerical scheme is based on Taylor series methods with accounting for different time scales and a dynamically-controlled time step.

It is noticed that all of the equations can be described by the general form:

\[
y_i' = \frac{\partial y_i}{\partial t} = A y_i + B y_j + C y_k + D,
\]

\(\text{(8-1)}\)

\(^1\) The number of equations depends on the number of regions selected for each heat exchanger and other zones, such as core. For the selected nodalization (see the input files at the end of the report), the total number of equations is 367 (see Appendix A).
where $i$ could be equal to $j$ (to produce $y_j^2$) and some of the coefficients, $A$, $B$, $C$, or $D$, could be zero.

The coefficients in Equation (8-1) include properties of the materials and structures (such as specific heat, thermal resistance, pressure derivatives with respect to temperature and so on). Assuming that the time step is small enough that those coefficients could be considered constant during the time step, the higher order derivatives can be calculated:

$$\frac{\partial}{\partial t} y_j^{(n)} = A y_j^{(n)} + B (y_i y_j)^{(n)} + C y_k^{(n)}$$

$n=1,2,3,\ldots$ (8-2)

The derivative of the product is calculated using the Leibniz identity [25]:

$$(y_i y_j)^{(n)} = \sum_{k=0}^{n} nCk y_i^{(k)} y_j^{(n-k)}$$

(8-3)

where $nCk \equiv \frac{n!}{(n-k)!k!} = \text{binomial coefficient}$.

Note that in equations of the form of Equation (8-1), the derivatives are expressed in terms of parameter values such as temperature, density, and flow rate and they are known at the beginning of the time step. The derivatives in Equation (8-2) are expressed in terms of lower-order derivatives only, which have been calculated before. So the derivatives of any order can be calculated for the developed system using Equations (8-1)-(8-3).

This fact is used to apply the Taylor series method to find the solution for the dynamics equations. The method is based on the fact that the value of a function at the end of a time step can be represented using the Taylor series [26]:

$$y(t+h) = y(t) + y'(t) \frac{h}{1!} + y''(t) \frac{h^2}{2!} + y'''(t) \frac{h^3}{3!} + \ldots$$

(8-4)

The representation is exact, if an infinite number of terms is used. In practice, the required number of terms is included until the next one is less than a pre-specified error:

$$\left| y^{(n+1)}(t) \frac{h^{n+1}}{(n+1)!} \right| < \varepsilon$$

(8-5)

As it follows from Equation (8-5), the accuracy of the Expansion (8-4) can be controlled by both the number of terms, $n$, and the time-step, $h$. This fact is used in the code. The minimum number of terms for the Expansion (8-4) is specified by the user in the input file (“Order” parameter). This number is used for the most slowly-changing variables, such as the reactor vessel temperatures. For the faster-changing variables, more terms are used. Experience has shown that the smallest time scale in the system is for the
CO₂ flow rate equations. So, the largest number of terms is used for the CO₂ flow rates. At the same time, the \( n \)-th derivative of the flow rate requires knowledge of the \((n-1)\) derivatives of CO₂ temperatures and densities. Thus, the CO₂ properties require one-order fewer calculations. Consequently, in the case of the IRHX, for example, the HX structure temperatures require two-order fewer derivatives and all the parameters on the reactor side require three-order fewer derivatives than that of the CO₂ flow rate. If, for example, just one term is used for the equations on the reactor side (representing Euler’s method), then two terms are used for the HX structure temperatures, three terms are used for CO₂ temperatures and densities, and four terms are used for the CO₂ flow rate. This approach allows saving of computational effort by not calculating unnecessary derivatives. The accuracy is controlled by the time step as described below.

Equation (8-2) can be readily applied to most of the differential equations described previously. Derivatives of temperatures and pressures in Equations (5-17) and (5-18) are calculated using Relationships (5-12) and (5-13). The derivatives for the reactor power are calculated based on the power interpolation form specified in Equation (4-1):

\[
Q'(t) = Q_{\text{nom}} e^{C_1+C_2(t+\Delta t)+C_3(t+\Delta t)^2} [C_2 + 2C_3(t + \Delta t)] = Q(t)[C_2 + 2C_3(t + \Delta t)]
\]

\[
Q''(t) = Q'(t)[C_2 + 2C_3(t + \Delta t)] + Q(t) \cdot 2C_3
\]

\[
Q^{(n)}(t) = Q^{(n-1)}(t)[C_2 + 2C_3(t + \Delta t)] + (n-1) \cdot 2C_3 Q^{(n-2)}(t), \quad n = 2, 3, \ldots \quad (8-6)
\]

8.2. Convergence Criteria and Accuracy Control

The following scheme is used to solve the system of differential equations with the required accuracy. The system parameters (flow rates, temperatures, and densities) are known and the coefficients for each differential equation are calculated at the beginning of a time step. The differential equations, as derived above, are used to calculate first derivatives of each parameter. The higher derivatives are calculated at the beginning of a time step using Equation (8-2). Equation (8-4) is then used to calculate the values of each parameter at the end of the time step. These values are stored in a common array, \( Y \), according to the structure shown in Appendix A. The time step is then divided into two time sub-steps. Equation (8-4) is used to calculate the parameters at the end of first sub-step. Based on new values of the system parameters at the end of a sub-step, the derivatives are updated in the same way as described above. These derivatives are used to calculate the system parameters at the end of the second sub-step, i.e., at the end of the original time-step. Again, all parameters are stored in array, \( Y \), and compared with values obtained with a single time step. The calculations stop if the difference between the parameters at the end of the time step does not exceed the specified accuracy for each parameter, i.e., the following condition is satisfied:
\[
\max \left\{ \frac{Y_i - Y_{i_{\text{iter}}}}{Y_i} \right\}_{i = 1 \ldots N} \leq \varepsilon \tag{8-7}
\]

where

\( Y \) = array of parameters obtained with current sub-step,
\( Y_{i_{\text{iter}}} \) = \( Y \) on previous iteration (with larger sub-step),
\( N \) = total number of system parameters (equations),
\( \varepsilon \) = convergence criteria (user-defined in an input file).

If Condition (8-7) is not satisfied, then the current sub-step is divided in half and the process is repeated to calculate the values at the end of the original time step. The iterations continue until the Condition (8-7) is satisfied.

At each convergence check, the index for the variable with the largest difference from the previous iteration step is stored. This index is then reported in an output file to allow the user to determine which variables have the slowest convergence. This approach has been used to determine that the \( \text{CO}_2 \) flow rates are the fastest-changing variables in the system.

Again, it is assumed in the process described here that the coefficients in the differential equations (such as mass or specific heat) are constant during the time step. Meanwhile, this solution method will converge for any time step (when the number of sub-steps is large and the sub-step is small). If the time step is large, then the properties could change significantly. In this case, the assumption of constant coefficients would not hold. Therefore, special arrangements are required to avoid this situation. This is done through a dynamic time step control, as described below.

The number of the sub steps required to achieve a given accuracy is stored in a public variable. On the next time step, the calculations start with this number divided by 4. Division by 4 is introduced to check if convergence is improved and the number of the sub steps could be decreased. At least two iterations are required to check the accuracy according to Equation (8-7). The number of the sub steps on the second iteration step is twice that on the first iteration step. Therefore, convergence is first checked with the number of the time steps equal to twice what was selected for the first iteration step. Thus, if for the first iteration step the number of the sub steps is selected to be four times smaller than that on the previous time step, the convergence will be checked with a number of the sub steps two times smaller than what was needed on the previous time step. So, if convergence has improved, the number of the sub steps will be reduced. At the same time, this approach avoids unnecessary calculations by starting with approximately the same time sub step as before.

### 8.3. Dynamic Time Step Control

As described above, the main assumption of the implemented solution scheme is that the coefficients in the differential equations are constant during a time step. For slow transients (when conditions change slowly), the coefficients could be considered approximately constant during a relatively long time. In this case, a large time step is beneficial to reduce the computational effort. For fast transients, however, large time
steps cannot be used since the conditions could vary significantly over a large time step. The following approach is implemented in the code to determine how fast the transient is and which time step to use.

As described above, convergence is achieved by increasing the number of sub-steps. High and low limits are imposed on the number of sub-steps. If the number of sub-steps exceeds its high limit and convergence is still not achieved, then the transient is considered to be too fast for the current time step. The time step is decreased by a factor of 10 in this case and the solution is repeated for the new time step. (Note that the coefficients in the differential equations are not recalculated, since they are determined at the beginning of the time step; however, the time interval over which these coefficients are considered to be constant is reduced). If convergence is achieved with a small number of steps (less than the lower limit), then the time step is increased by a factor of 10 for the next time-step calculations. Eight and 256 sub-steps are somewhat arbitrarily selected as lower and upper limits on the number of sub-steps.

This dynamic control of the time step enables the calculation of slow transients (or part of a transient) with large time steps increasing the computational speed while fast transients (or the fast part of a transient) are calculated with smaller time steps for greater accuracy.
9. Input Files

Below are the input files with the parameters used to generate the results reported in Chapters 11 and 12\(^1\). Since the dynamics code calculations start with steady-state calculations, the steady-state input files are used in the code and are included in the list. The following input files (Appendix B) are included:

- `Reactor.dat.txt` – data for steady-state reactor calculations,
- `RVACS.dat.txt` – data for steady-state RVACS calculations,
- `Cycle.dat.txt` – data for steady-state Brayton cycle calculations,
- `Maps.dat.txt` – parameters for turbomachinery maps generation,
- `Dynamic.dat.txt` – parameters for the dynamics model,
- `BCcontrol.dat.txt` – parameters for Brayton cycle control system.

Note that the input files are changed for each simulation, so the files presented here show the example of the parameters used for one particular run. For example, the parameter “Grid load table” in `Dynamic.dat.txt` sets the linear grid demand reduction to 0% in first 2000 seconds and then operation on this level for another 2000 seconds.

\(^1\) This report was written some time after the results were obtained. During that time, work on improvement of the Brayton cycle models has continued, so some models have been subsequently updated. Therefore, the files presented here may not be exactly those used to generate the results shown on the Figures. For example, turbomachinery input parameters in `Cycle.dat.txt` are those for the updated models, while the results were obtained using the previous models.
Reactor_dat.txt

****************** Input data for Reactor calculations ******************

Reactor power, MWt
400.0
Core inlet coolant temperature, C
438.0
Maximum cladding temperature, C
650.0
Accuracy on dT in core
1.D-6

------------------------ RV ----------------------------------------
RV outer diameter, m
5.5
RV thickness, cm
3.0 5.08
RV height, m
16.9
Lower plenum height-to-RV radius ratio
1.0
Fraction of core power removed by RVACS
0.001

------------------------------------------ Core ---------------------
Hot channel outlet power factor
0.42913
Hot channel factor
1.19863D00
Power peaking factor
1.62085D0
Core outer diameter, m
2.459
Radial refletor thickness, m
0.297
Active core height, m
2.0
Core height extrapolation coefficient (Le/L)
1.20401
Fission gas plenum height-to-core height ratio
0.25
Lower axial reflector, m
0.25
Fuel rod (cladding) outer diameter, cm
1.3
Cladding thickness, cm
0.1
Cladding material
HT9
Pitch-to-diameter ratio
1.536
Fuel smear density
0.78
Fuel porosity
0.05
Number of flow distributors below the core
2
Array of fractions of open flow areas in distributors
0.6 0.6
Array of the flow distribution plate thicknesses, cm
1.27 10.16
Array of the flow distribution plate flow channel diameters, cm
1.27 1.27
Array of the flow distribution plate materials
SS316 SS316
Number of spacer grids
3
Fraction of flow area blocked by the grid
0.441
Total number of regions in core
10
------------------------------------------ HX -----------------------
Number of heat exchangers
4
HX height, m (0 - adjust height for max claddding temperature)
6.0
Core shroud thickness, cm
2.54
Gap between core shroud and HX wall, cm
1.27
HX wall thickness, cm
1.27
HX wall material
SS316
Gap between RV wall and HX outer wall, cm
6.985
Plenum above HX top, m
1.6524
Gap between HXs, cm
15.24
Accuracy of CO2 outlet temperature
1D-6
Fraction of flow that bypass HX
0.01
HX type:
   ( 1 - Staked U-Tubes   )
   ( 2 - U-tubes         )
   ( 3 - Concentric Tubes )
   ( 4 - Straight Tubes  )
   ( 5 - Straight Annuli )
   ( 6 - Helical Coil    )
   ( 7 - PCHE            )
4
HX lattice layout (3 - triangular, 4 - square)
3
HX tube outer diameter, cm (outer tube for HX types 3&5)
0.9
HX tube inner diameter, cm (outer tube for HX types 3&5)
0.5
Pitch-to-diameter ratio (0 - adjust for max claddding temperature)
0.0
Tube material
SS316
Grooves height, relative to diameter, (e/d). 0 means no grooves. [0<=e/d<=0.1] 0.0
Grooves pitch, relative to diameter, (t/d). [0.25<=p/d<=10.0] 0.25
Grooves angle to the tube axis, (a/90). [0.2<=a/90<=1.0] 0.2
Enhancement correlation (Z - Zukauskas, B - Bergles) B
Heat transfer correlation on CO2 side DB
Number of regions for temperature calculations 5
HX inner tube outer diameter, cm (for HX types 3&5 only) 2.4
HX inner tube inner diameter, cm (for HX types 3&5 only) 2.0
Number of turns per tube (Type 6 only) 3.0
************** Input data for RVACS calculations ******************

RV material
SS316
RV emissivity
0.4
Conduction gap thickness, cm
12.7
Conduction gap material
PbBi
Guard vessel thickness, cm
5.08
Guard vessel material
SS316
GV emissivity
0.4
GV surface heat transfer enhancement coefficient
2.0
Air channel length above reactor vessel, m
15
Air upward channel thickness, cm
15
Air wall thickness, cm
1.0
Air downward channel thickness, cm
60.96
Air ambient temperature, C
36
Number of regions
10
Cycle_dat.txt

****** Input data file for CO2 cycle efficiency calculations *******
Maximum pressure in cycle (MPa)
20.0D0
Minimum pressure in cycle (MPa)
7.4D0
Minimum temperature in cycle (C)
31.250D0
Turbine efficiency, %
91.0
Compressor 1 efficiency, %
91.6
Compressor 2 efficiency, %
91.0
Generator efficiency, %
98.5
Mechanical losses, %
1.0
Fraction of flow sent to LT recuperator
0.66
Maximum number of iterations
40
Array of pipe lengths, m (10 points)
20.0 5.0 2.0 7.0 2.0 2.0 1.0 2.0 1.0 20.0
Array of pipe diameters, m (10 points)
1.0 0.5 0.5 0.5 0.5 0.5 0.5 0.5 0.5 1.0
***** High Temperature Recuperator parameters ***********************
Recuperator type (0 - Ideal, 1 - Shell-and-tube, 2 - PCHE)
2
------------------ Shell-and-tube HX data -----------------------------
Recuperator outer diameter, m
6.0
Recuperator length, m
10.
Inner and outer tube diameters, m
0.01 0.014
Recuperator pitch-to-diameter ratio
1.33
Tube material (5 characters)
SS316
Number of points for temperature calculations
11
Required accuracy (in secondary outlet temperature), C
0.01
Tube side (1 - Primary, 2 - Secondary)
2
Heat transfer correlation (DB - Dittus-Boelter, PG - Petukhov-Gnielinski)
DB
Number of fins on inner surface per tube
12
Width of fins on inner surface, m
0.001
Length of fins on inner surface, m
0.0015
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of fins on outer surface per tube</td>
<td>12</td>
</tr>
<tr>
<td>Width of fins on outer surface, m</td>
<td>0.001</td>
</tr>
<tr>
<td>Length of fins on outer surface, m</td>
<td>0.0025</td>
</tr>
<tr>
<td>Recuperator length, m</td>
<td>2.5</td>
</tr>
<tr>
<td>Recuperator width, m</td>
<td>6.0</td>
</tr>
<tr>
<td>Recuperator height, m</td>
<td>6.0</td>
</tr>
<tr>
<td>Semi-spherical channel diameter, mm</td>
<td>1.0</td>
</tr>
<tr>
<td>Pitch-to-diameter ratio</td>
<td>1.2</td>
</tr>
<tr>
<td>Layer thickness, mm</td>
<td>0.8</td>
</tr>
<tr>
<td>Material</td>
<td>SS316</td>
</tr>
<tr>
<td>Number of points for temperature calculations</td>
<td>21</td>
</tr>
<tr>
<td>Required accuracy (in secondary outlet temparature), C</td>
<td>1.E-4 0.001</td>
</tr>
</tbody>
</table>

Heat transfer correlation (DB - Dittus-Boelter, PG - Petukhov-Gnielinski)

**Low Temperature Recuperator parameters***************

| Recuperator type (0 - Ideal, 1 - Shell-and-tube, 2 - PCHE) | 2 |

** Shell-and-tube HX data ****************************

| Recuperator outer diameter, m | 6.0 |
| Recuperator length, m         | 10.0 |
| Inner and outer tube diameters, m | 0.01 0.014 |
| Recuperator pitch-to-diameter ratio | 1.33 |
| Tube material (5 characters)  | SS316 |
| Number of points for temperature calculations | 11 |
| Required accuracy (in secondary outlet/inlet temparature), C | 0.01 |
| Tube side (1 - Primary, 2 - Secondary) | 2 |

Heat transfer correlation (DB - Dittus-Boelter, PG - Petukhov-Gnielinski)

** PCHE HX data ****************************

| Recuperator length, m | 2.5 |
| Recuperator width, m  | 6.0 |
| Recuperator height, m | 6.0 |
| Semi-spherical channel diameter, mm | 1.0 |
| Pitch-to-diameter ratio | 1.2 |
| Layer thickness, mm   | 0.8 |
| Material              | SS316 |
| Number of points for temperature calculations | 21 |
| Required accuracy (in secondary outlet temparature), C | 1.0E-4 0.001 |

Heat transfer correlation (DB - Dittus-Boelter, PG - Petukhov-Gnielinski)
Cycle_dat.txt

Width of fins on inner surface, m
0.001
Length of fins on inner surface, m
0.0015
Number of fins on outer surface per tube
12
Width of fins on outer surface, m
0.001
Length of fins on outer surface, m
0.0025

Recuperator length, m
5.0
Recuperator width, m
6.0
Recuperator height, m
6.0
Semi-spherical channel diameter, mm
1.0
Pitch-to-diameter ratio
1.2
Layer thickness, mm
0.8
Material
SS316
Number of points for temperature calculations
21
Required accuracy (in secondary outlet temperature), C
1D-4 0.001
Heat transfer correlation (DB - Dittus-Boelter, PG - Petukhov-Gnielinski)
DB

Cooling fluid (3 characters)
H2O
Inlet temperature of cooling fluid, C
30
Outlet pressure of cooling fluid, Pa
1.01325D5 (1 atm)
Cooling fluid flow rate, kg/s
10000
Cooling fluid inlet and outlet pipe diameters, m
1.0 1.0
Cooling fluid pump efficiency, %
90
Accuracy on outlet temperature
1.D-5
Cooler type (1 - Shell-and-tube, 2 - PCHE)
2

Shell-and-tube HX data

Cooler outer diameter, m
6.0
Inner and outer tube diameters, m
0.01 0.014
Pitch-to-diameter ratio
1.3
Tube material (5 characters)
SS316
Number of points
51
Tube side (1 - Primary, 2 - Secondary)
1
Heat transfer correlation (DB - Dittus-Boetler, PG - Petukhov-Gnielinski)
DB
Number of fins on inner surface per tube
0
Width of fins on inner surface, m
0.001
Length of fins on inner surface, m
0.0015
Number of fins on outer surface per tube
0
Width of fins on outer surface, m
0.001
Length of fins on outer surface, m
0.0025
------------------ PCHE HX data -------------------------------
Cooler width, m
6.0
Cooler height, m
6.0
Semi-spherical channel diameter, mm
1.0
Pitch-to-diameter ratio
1.2
Layer thickness, mm
0.8
Material
SS316
Number of points for temperature calculations
51
Heat transfer correlation (DB - Dittus-Boetler, PG - Petukhov-Gnielinski)
DB
****************************** Turbine ******************************
Shaft revolution speed (rev/s)
60
Minimum hub radius (cm)
10
Degree of reaction
0.5
Design incidence angle, deg
0
Design deviation angle, deg
0
Blade maximum thickness-to-chord ratio
0.2
Tip clearance-to-radius ratio
0.002
Number of tip seals for rotor (0 - unshrouded)
0
Number of tip seals for nozzle (0 - unshrouded)
0
Trailing edge thickness-throat opening ratio for rotor
0.1
Trailing edge thickness-throat opening ratio for nozzle
0.1
Blade material density (kg/m3)
8300
Blade maximum total stress (MPa)
300
Vibrational stress factor
0.75
Number of stages
7
Inlet nozzle efficiency, %
90
Outlet diffuser efficiency, %
90
Pressure recovery coefficient
0.5
Accuracy on exit pressure
1.D-8
----------------- Blade profile coefficients -----------------
Coefficient for Ixx
1.165D-3
Coefficient for Iyy
1.0381D-2
Coefficient for x coordinate of center of gravity
7.434D-2
Coefficient for y coordinate of center of gravity
2.738D-2
Coefficient for x coordinate of trailing edge
4.372D-1
Coefficient for y coordinate of trailing edge
-4.656D-1
Principal axes angle, degrees
60.33
Average blade angle, degrees
67
************************ Compressor #1 ************************
Shaft revolution speed (rev/s)
60
Minimum hub radius (cm)
1
Degree of reaction
0.85
Blade profile coefficient
1.0
Design incidence angle thickness correction factor
1.0
Blade maximum thickness-to-chord ratio
0.1
Tip clearance-to-radius ratio
0.002
Blade material density (kg/m³)
8300
Blade maximum total stress (MPa)
300
Vibrational stress factor
0.75
Number of stages
20
Inlet nozzle efficiency, %
90
Outlet diffuser efficiency, %
90
Pressure recovery coefficient
0.5
Accuracy on exit pressure
1.D-8

--------------- Blade profile coefficients ----------------
Coefficient for Ixx
1.165D-3
Coefficient for Iyy
1.0381D-2
Coefficient for x coordinate of center of gravity
7.434D-2
Coefficient for y coordinate of center of gravity
2.738D-2
Coefficient for x coordinate of trailing edge
4.372D-1
Coefficient for y coordinate of trailing edge
-4.656D-1
Principal axes angle, degrees
60.33
de Haller coefficient (W2/W1)
0.75
Inlet GV angle, degrees
0

*************************** Compressor #2 ************************
Shaft revolution speed (rev/s)
60
Minimum hub radius (cm)
5
Degree of reaction
0.85
Blade profile coefficient
1.0
Design incidence angle thickness correction factor
1.0
Blade maximum thickness-to-chord ratio
0.1
Tip clearance-to-radius ratio
0.002
Blade material density (kg/m³)
8300
Blade maximum total stress (MPa)
300
Vibrational stress factor
0.75
Number of stages
20
Inlet nozzle efficiency, %
90
Outlet diffuser efficiency, %
90
Pressure recovery coefficient
0.5
Accuracy on exit pressure
1.D-8

------------------ Blade profile coefficients -------------------
Coefficient for Ixx
1.165D-3
Coefficient for Iyy
1.0381D-2
Coefficient for x coordinate of center of gravity
7.434D-2
Coefficient for y coordinate of center of gravity
2.738D-2
Coefficient for x coordinate of trailing edge
4.372D-1
Coefficient for y coordinate of trailing edge
-4.656D-1
Principal axes angle, degrees
60.33
de Haller coefficient (W2/W1)
0.75
Inlet GV angle, degrees
0 15
Maps.dat.txt

********* Input data for turbo=machinery maps generation *********
------------------- Turbine ----------------------------------------
Generate (1) or use existing (0) map? 0
Max and min values for relative shaft speed
0.1 2.0
Number of points for relative shaft speed 20
Max and min values for relative inlet temperature
0.35 1.3
Number of points for relative inlet temperature 20
Max and min values for relative inlet pressure
0.5 1.25
Number of points for relative outlet pressure 61
Minimum pressure ratio 1.0
Number of points for relative outlet density 61
Min (start) value for relative flow rate 0.01
Initial and minimum steps for relative flow rate 0.01 1D-6
---------------- Compressor #1 -------------------------------------
Generate (1) or use existing (0) map? 0
Max and min values for relative shaft speed
0.25 2.0
Number of points for relative shaft speed 36
Max and min values for relative inlet temperature
0.992 1.008
Number of points for relative inlet temperature 3
Max and min values for relative inlet pressure
0.5 1.5
Number of points for relative inlet pressure 61
Lower limit for stall factor 0.95
Number of points for relative outlet density 61
Min (start) value for relative flow rate 0.01
Initial and minimum steps for relative flow rate 0.01 1D-6
--------------- Compressor #2 ---------------------------------------
Generate (1) or use existing (0) map? 0
Max and min values for relative shaft speed
0.25 2.0
Number of points for relative shaft speed 19
Maps.dat.txt

Max and min values for relative inlet temperature
0.5 2.0
Number of points for relative inlet temperature
7
Max and min values for relative inlet pressure
0.5 1.5
Number of points for relative inlet pressure
61
Lower limit for stall factor
0.95
Number of points for relative density
61
Min (start) value for relative flow rate
0.01
Initial and minimum steps for relative flow rate
0.01 1D-6
************ Input data for dynamic calculations ***************

Simulation time, s
4000

Time to attain SS (not included into simulation time), s
500

Time step (initial), s
0.01

Report every N calculations
200

Number of regions in FGP
3

Number of regions in riser
10

Number of regions in HX bypass
3

Number of regions in LAR
3

Fuel Doppler reactivity coefficient, cents/C
-0.12 -0.1

Fuel Axial Expansion reactivity coefficient, cents/C
-0.06851285

Core Radial Expansion reactivity coefficient, cents/C
-0.14007261

Coolant Density reactivity coefficient, cents/C
0.150686901

Effective delayed neutron fraction
0.0035

Prompt neutron generation time, mks
0.534

Required convergence
1.D-5

Solution method
TS

Order
1

Reactor power calculation method (1 - PRKE, 2 - Quasi-Static)
1

Volumes at turbine and compressors (1 and 2) exhausts, m3
1.0 1.0 1.0

Generator's moment of inertia, kg-m2
4706

Number of points in grid load table (1 - no action)
3

Grid load table (Time, s; % nominal)
0 2000 4000
100 0 0

Equivalent pipe break diameter, cm
0

Location of the break (dynamic-code node)
5
****** Input parameters for the Brayton cycle control *******
------------------------- Automatic control -------------------------
-------- Rotational speed control --------
Number of points in the rotational speed control table
1
Rotational speed control table (Time; Nr, % nominal)
0   20  500
100 100 60
------- TBP -------
Coefficients for turbine bypass control (P D I)
5 4.5 0.1
Opening and closing rate limits, %/s
5 10
------- INV -------
Number of points in the inventory control table
11
Inventory control table (Load,%; dM,kg)
50 51.786 58.38 66.706 74.524 78.397 82.304 86.223 90 99.99 101
13583 13204.4 11802.5 10073.1 8108.8 6856.7 5133.3 2533.2 0 0 -1000
Coefficients for inventory control (P D I)
5 5 0
Inlet valve opening and closing rate limits, %/s
10 10
Outlet valve opening and closing rate limits, %/s
10 10
Coefficient for inventory compensation shaft control
10000
------- TIN -------
Number of points in the valve control table
8
Valve control table (Load,%; dp_valve,MPa)
20 22.767 28.195 32.918 37.584 42.169 46.254 50
22 3.0 2.5 2.0 1.5 1.0 0.5 0
Coefficients for valve control (P D I)
5 5 0.1
Valve opening and closing rate limits, %/s
10 0.25
Stall margins (stop-closing and start-opening)
0.05 0.025
------- C2IN -------
Valve opening and closing rate limits, %/s
0.5 0.5
--------------------------- HX Bypass -----------------------------
Length of the HX bypass pipe, m
1
Diameter of the HX bypass pipe, m
0.2
Number of points in the HX Bypass control table (1 - no action)
1
HX Bypass valve control table (Time; f_open, %)
50 150
0 99.5
### Turbine Inlet Valve

Number of points in the Turbine inlet valve control table (1 - no action)

<table>
<thead>
<tr>
<th>Time</th>
<th>f_open, %</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>100</td>
</tr>
<tr>
<td>100</td>
<td>11.5</td>
</tr>
</tbody>
</table>

### Turbine Bypass

Length of the turbine bypass pipe, m

1

Diameter of the turbine bypass pipe, m

0.2

Number of points in the turbine bypass control table (1 - no action)

<table>
<thead>
<tr>
<th>Time</th>
<th>f_open, %</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>100</td>
</tr>
<tr>
<td>0</td>
<td>40</td>
</tr>
</tbody>
</table>

### Inventory Control

Length of the high pressure pipe, m

1

Diameter of the high pressure pipe, m

0.2

Length of the low pressure pipe, m

1

Diameter of the low pressure pipe, m

0.2

Tank volume, m³

120

CO₂ initial temperature in tank, °C

85

CO₂ initial pressure in tank, MPa

8

Number of points in the inventory control table - inlet (1 - no action)

<table>
<thead>
<tr>
<th>Time</th>
<th>s; f_open, %</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1 199 200</td>
</tr>
<tr>
<td>0</td>
<td>5 5 0</td>
</tr>
</tbody>
</table>

Number of points in the inventory control table - outlet (1 - no action)

<table>
<thead>
<tr>
<th>Time</th>
<th>f_open, %</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1 99 100</td>
</tr>
<tr>
<td>0</td>
<td>5 5 0</td>
</tr>
</tbody>
</table>
10. Result Graphs Explanation

Chapters 11 and 12 present the results obtained with the dynamics model. The model solves a large number of equations to find the time-dependent values of parameters which characterize the system. Therefore, a large number of parameters is reported in the output graphs in order to understand the system behavior. To avoid confusion, different parameters are reported on different graphs. A total of 20 graphs is selected as necessary and more or less sufficient to characterize the system behavior in a transient. The graphs are presented as four on a page. Below are the brief descriptions of the parameters for the graphs grouped by page (in left-to-right, then down order).

Page 1:
- **Heat balance in reactor**
  - Reactor core power (Q_Rx)
  - Heat removal rate by CO₂ in IRHX (Q_RHX)
  - Heat removal rate by air in RVACS (Q_air)

- **Coolant speed in core**
  - Average coolant speed in core (u_Pb)

- **Coolant temperatures in core**
  - Average core-outlet temperature (T_out)
  - Average core-inlet temperature (T_in)

- **Coolant temperatures in HX**
  - IRHX-inlet temperature (Tin_HX)
  - IRHX-outlet temperature (Tout_HX)

Page 2:
- **Peak temperatures in core**
  - Peak fuel temperature (Tf_max)
  - Peak cladding temperature (Tcl_max)
  - Peak coolant temperature (TPb_max)

- **Peak RV temperatures**
  - Maximum reactor vessel inner surface temperature (TRVi_mx)
  - Maximum average reactor vessel temperature (TRV_mx)
  - Maximum reactor vessel outer surface temperature (TRVo_mx)

- **Average temperatures in core**
  - Core-average fuel temperature (Tf_av)
  - Core-average coolant temperature (TPb_av)

- **Reactivity contribution**
  - Coolant density component of reactivity (r_cool)
- **Core axial expansion component of reactivity (r_aex)**
- **Doppler component of reactivity (r_dop)**
- **Net reactivity (r_net)**
- **Core radial expansion component of reactivity (r_rex)**

**Page 3:**

- **Reactor power components**
  - Total reactor power (Q_Rx)
  - Decay heat power (Q_dc)
  - Fission power (Qfis)

- **Reactor power and its prediction**
  - Reactor power (Q_Rx)
  - Relative error of reactor power prediction by extrapolation by kinetics subroutines (E_ex)

- **Turbine and compressor work and generator output**
  - Turbine work (W_Turb)
  - Compressor No. 1 power input (W_Comp1)
  - Compressor No. 2 power input (W_Comp2)
  - Net generator power (W_gen)
  - Grid demand (W_grid)

- **CO₂ flow split**
  - Fraction of flow which goes through cooler and compressor No. 1 (FS)

**Page 4:**

- **Compressor stall check**
  - Stall parameter for Compressor No. 1 (Comp1)
  - Stall parameter for Compressor No. 2 (Comp2)

- **Turbine and compressor choke check**
  - Maximum Mach number in Compressor No. 1 (Comp1)
  - Maximum Mach number in Compressor No. 2 (Comp2)
  - Maximum Mach number in turbine (Turb)

- **Shaft speed**
  - Turbomachinery shaft rotational speed (N_r)

- **Valve control actions**
  - Open area fraction for turbine bypass valve (f_TBP)
  - Open area fraction for inventory tank inlet valve (f_INVi)
  - Open area fraction for inventory tank outlet valve (f_INVo)
  - Open area fraction for turbine throttle valve (f_TIN)
  - Open area fraction for Compressor No. 2 throttle valve (f_C2IN)
Page 5:
- Brayton cycle flow rates
  - CO₂ flow rates in the Brayton cycle according to Figure 5-3 nodes

- Brayton cycle temperatures
  - CO₂ temperatures in the Brayton cycle according to Figure 5-3 nodes

- Brayton cycle densities
  - CO₂ densities in the Brayton cycle according to Figure 5-3 nodes

- Brayton cycle pressures
  - CO₂ pressures in the Brayton cycle according to Figure 5-3 nodes
11. Simulation of Normal Operation

The dynamics code is used to simulate normal operational transients and postulated accident conditions. The results of normal operational transients are presented here; the results for accidents are presented in the next chapter.

The normal operational transients are selected to test the plant control system. The transient simulate the variation of the grid demand anticipated for normal plant operation. These variations include full power operation (no change), small instantaneous (step) changes, and smooth (linear) load changes.

The grid demand (load) is specified by user in the input file (see above) as a table of grid loads versus time at a finite number of points; a linear load change is assumed between those points. The load is equal to the first value for times before the first point and is equal to the last value for times after the last point.

Any transient calculations are preceded by initial system stabilization. During that time the same dynamics equations are solved; however, no transient initiator is introduced into the system and no control system action is initiated. The reactor power is equal to the steady state value and is not calculated by the reactor kinetics subroutines. The shaft rotational speed is also constant at the steady-state value and is not calculated by the shaft dynamics equation. It is found that 300-500 s are enough to achieve stabilized conditions. It is assumed that the transient (system disturbance) starts at $t = 0$ s; the initial system stabilization is reported for negative times. For the positive times, all dynamics equations are solved, including calculation of the reactor power by the kinetics subroutines and shaft rotational speed by the shaft dynamics equation, and the control systems are functional.

11.1. Full Power Operation – Steady-State and Stability Test

Before any disturbances were introduced into the system, the code was checked for stability and prediction of the steady-state parameters. For this purpose, a transient run was initiated with the grid load set to 100% for all times. The plant control system has to maintain stable operation at full power; the system parameters have to be constant close to the steady-state values. Some differences from steady-state values as well as initial stabilization of the parameters are expected, since the assumptions made for the dynamics model are not the same as for the steady-state model. These differences are acceptable as long as they are small.

Since no disturbance is introduced into the system, the only changes at the beginning of the transient are: i) switching to the reactor power calculations by the kinetics subroutines; ii) start of the shaft dynamics calculations; and iii) plant control system actions.

Figure 11-1 shows the results of system operation at the full power simulation. All calculated system parameters are close to the steady-state values (perhaps, the biggest difference is in reactor power which stabilizes at 399.4 MW vs. 400 MW at steady-state – about a 0.15% difference). Also, all parameters are stable for at least 1000 s (the total simulation time). The Brayton cycle control system adds some CO$_2$ inventory to the cycle.
and opens the turbine bypass valve slightly. Some bypass flow is required for stable operation of the control system (to avoid constant switching between turbine bypass and inventory controls). These changes are too small to affect the cycle, as shown in Figure 11-1, but are enough for stable operation.

11.2. Plant Response to Step Changes in Load

The sudden changes (occurring over 0.1 s) of the generator load have been introduced into the system to simulate step changes in grid load. Figure 11-2 and Figure 11-3 show the system response to 1% increase and decrease in grid load, respectively. Figure 11-4 shows the system response to a 10% step decrease in load. All responses are satisfactory; the control system can safely handle the changes and the system operates stably at the new load levels.

11.3. Plant Response to Linear Changes in Load

Linear changes in grid demand at 3%/min are simulated. Several runs are made to show that the system can safely accommodate linear changes in grid demand at this rate from full power down to any power level.

Figure 11-5 shows the system response to a linear change from full power to 50% load, operation at 50% load, and return to full power. The system response is satisfactory. However, there is a small amount of non-steady behavior at the point of return to full power when the inventory control has to be used in addition to the turbine bypass control for the shaft speed regulation. The non-steady-behavior is overcome in a short time and does not cause any long-term consequences.

Figure 11-6 shows the system response to a full-range (from 100% to 0%) load change. It demonstrates operation and interaction between all Brayton cycle control mechanisms. It also demonstrates that the plant can be controlled over the full range of grid changes without any active control on the reactor side (no external reactivity control is programmed in the code). Thus, autonomous reactor control is demonstrated.
Figure 11-1. Operation at Full Power.
Figure 11-1. Operation at Full Power (2).
Figure 11-1. Operation at Full Power (3).
Figure 11-1. Operation at Full Power (4).
Figure 11-1. Operation at Full Power (5).
(End of Figure 11-1)
HEAT BALANCE IN REACTOR

COOLANT SPEED IN CORE

COOLANT TEMPERATURES IN CORE

COOLANT TEMPERATURES IN HX

Figure 11-2. 1% Step Decrease in Load.
Figure 11-2. 1% Step Decrease in Load (2).
Figure 11-2. 1% Step Decrease in Load (3).
Figure 11-2. 1% Step Decrease in Load (4).
Figure 11-2. 1\% Step Decrease in Load (5).
Figure 11-3. 1% Step Increase in Load.
Figure 11-3. 1% Step Increase in Load (2).
Figure 11-3. 1% Step Increase in Load (3).
Figure 11-3. 1% Step Increase in Load (4).
Figure 11-3. 1% Step Increase in Load (5).
(End of Figure 11-3)
Figure 11-4. 10% Step Decrease in Load.
Figure 11-4. 10% Step Decrease in Load (2).
Figure 11-4. 10% Step Decrease in Load (3).
Figure 11-4. 10% Step Decrease in Load (4).
Figure 11-4. 10% Step Decrease in Load (5).
(End of Figure 11-4)
Figure 11-5. 50% Linear Decrease in Load with Return to Full Load.
Figure 11-5. 50% Linear Decrease in Load with Return to Full Load (2).
Figure 11-5. 50% Linear Decrease in Load with Return to Full Load (3).
Figure 11-5. 50% Linear Decrease in Load with Return to Full Load (4).
Figure 11-5. 50% Linear Decrease in Load with Return to Full Load (5).
(End of Figure 11-5)
HEAT BALANCE IN REACTOR

COOLANT SPEED IN CORE

COOLANT TEMPERATURES IN CORE

COOLANT TEMPERATURES IN HX

Figure 11-6. 100% Linear Decrease in Load.
Figure 11-6. 100% Linear Decrease in Load (2).
Figure 11-6. 100% Linear Decrease in Load (3).
Figure 11-6. 100% Linear Decrease in Load (4).
Figure 11-6. 100% Linear Decrease in Load (5).
12. Accident Simulations

The goal of the accident calculations is to investigate the system response to all possible accident conditions. As a passive safety design approach, simplifications are incorporated in the STAR-LM concept to reduce the number of accident initiators.

12.1. Passive Safety Design of STAR-LM

A number of special features are incorporated into STAR-LM design for passive safety. The Pb primary coolant system operates on natural circulation eliminating the need for main coolant pumps. Natural circulation removes the core power at levels up to and exceeding 100% nominal. This feature follows from the low neutron capture in Pb which enables the coolant volume fraction to be increased enlarging the core hydraulic diameter and reducing the core pressure drop thereby enhancing natural circulation. The open-lattice core configuration that does not incorporate individual fuel assemblies with hexcans eliminates flow blockage accidents. Traditional loss-of-flow (LOF) accident initiators that have been considered for liquid metal-cooled fast reactors are eliminated.

The system vessel pool configuration with ambient pressure Pb coolant and a surrounding guard vessel reduces the potential for loss of primary coolant. The reactor vessel has no penetrations (such as pipes) and has no external welds under the coolant level; all penetrations for secondary system pipes, control, and instrumentation are installed at the vessel closure head. The simultaneous failure of both vessels is assumed to have an incredibly low frequency and is excluded from the spectrum of considered events.

The high boiling temperature of the Pb primary coolant ($T_{\text{boil}} = 1740 \, ^\circ \text{C}$) eliminates concerns about coolant loss due to flashing, and together with the high decomposition temperature of the transuranic nitride fuel ($> 1350 \, ^\circ \text{C}$) and natural circulation of the primary coolant results in enhanced passive safety whereby the core and heat exchangers remain covered by ambient pressure single-phase primary Pb coolant and single-phase natural circulation removes the core power under all operational and postulated accident conditions at higher temperatures than traditional liquid metal-cooled reactors. The high thermal conductivity of the nitride fuel combined with the molten Pb bond filling the gap between the nitride fuel pellets and cladding reduces the temperature difference between the fuel and coolant reducing the thermal energy stored in the fuel and the positive Doppler reactivity contribution upon fuel cooldown during transients and accidents.

The Pb coolant does not react chemically with the CO$_2$ working fluid above $\sim 250 \, ^\circ \text{C}$ which is below the Pb melting temperature ($327 \, ^\circ \text{C}$). The Pb coolant also does not react vigorously with air or water/steam. The consequences of a tube rupture inside of an in-vessel Pb-to-CO$_2$ heat exchanger have been analyzed and found to be benign with CO$_2$ bubbles/void rising to the Pb free surface without undergoing transport to the core region [27]. Overpressure protection including a rupture disc is provided on the reactor system.

Relying on autonomous reactor control eliminates the need for rapid reactivity compensation using control rods. The absence of fast-moving power control rods distinct
from rods used for startup and shutdown eliminates a source of rapid reactivity insertion eliminating transient over power (TOP) accidents. The core incorporates control rods to compensate for the low burnup reactivity swing during the core lifetime, but those rods can be designed to move very slowly. The removal of a control rod at such a low rate would provide sufficient time for the plant protection system and/or operator to detect and scram the reactor either automatically or manually before any significant amount of positive reactivity were introduced.

For emergency heat removal, the outside of the guard vessel is cooled by natural circulation of air; the Reactor Vessel Auxiliary Cooling System (RVACS) guard vessel air cooling system is always in effect and does not require human activation. Multiple separated chimneys for inflow and outflow of air provide protection against the effects of aircraft impact or sabotage.

12.2. Identification of Accident Scenarios

As a result of the passive safety features and simplifications of STAR-LM, all traditional accident initiators for liquid metal-cooled reactors on the reactor side are eliminated; only initiators from the balance-of-plant (BOP) are considered. From the set of S-CO\textsubscript{2} Brayton Cycle initiators, only those which have a potential to influence the reactor itself are of a safety concern and are considered in this work. The most severe conditions on the BOP side, in terms of impacting the reactor, involve the complete loss of BOP heat removal capability; that is, loss-of-heat-sink (LOHS) events. The other accident scenario initiators on the S-CO\textsubscript{2} Brayton Cycle side are sudden disconnection of the generator from the electric grid (i.e., the loss-of-load (LOL) event), and a pipe break with loss of the CO\textsubscript{2} working fluid.

In order to investigate the passive response of the reactor, it assumed that the reactor protection system (scram system) fails to detect and/or to respond to the accident conditions. Therefore, only unprotected (unscrammed) accidents are considered here. Unscrammed accidents scenarios are considered to be extremely unlikely because they involve both an accident initiator and failure of the plant protection system to scram the reactor. The S-CO\textsubscript{2} Brayton Cycle automatic control system is considered to act normally; in case of its failure, any scenario would most likely transition into a LOHS event. The malfunction of the BOP control system, which could potentially worsen the accident situation, falls into a triple-fault scenario and is considered to be beyond the scope of this work.

Relying on negative temperature feedbacks for autonomous power control presents the possibility of increasing the power level by “overcooling” the Pb primary coolant. Overcooling might result from either an increase in the heat removal rate by the BOP or decreased heat production rate by the reactor itself. The former may result from an increased flow rate of the secondary fluid through the heat exchanger; for example, in the case of a pipe break downstream of the Pb-to-CO\textsubscript{2} heat exchangers (HXs). The latter may be caused by an accidental reactor scram. The major concern here is possibility to overcool and freeze the primary coolant, if the heat removal rate from the BOP is significantly higher than the reactor power after shut down. Both of these accident
scenarios are considered in the analysis as well as the unprotected accidents described above.

The current evaluation does not consider Pb-to-CO\textsubscript{2} heat exchanger tube rupture. An analysis of the in-vessel phenomena following the rupture of a HX tube in STAR-LM has been presented in [27].

\textbf{12.3. Loss of Heat Sink (LOHS) without SCRAM}

In this accident scenario, it is assumed that at time, \( t=0 \), the heat transfer in the heat exchangers from the structure (HX tubes) to the secondary fluid instantaneously drops to zero. In reality, there would always be some heat transfer, even if only for a short time. The scenario therefore represents a bounding case of mismatch between heat production in the reactor and heat removal by the S-CO\textsubscript{2} Brayton Cycle. For these reasons, the accident scenario is identified for safety evaluation. As mentioned above, the scenario also incorporates the additional assumption of no protective action (scram) by the plant protection system. The reactor power changes due to the inherent reactivity feedbacks only.

Figure 12-1 shows the short term behavior of the system parameters at the beginning of the transient. Due to imbalance between heat production and heat removal, the system temperatures start to rise. This temperature rise introduces negative reactivity to the reactor core through the combination of core radial expansion (i.e., expansion of the grid spacers separating the fuel pins), coolant density, fuel Doppler, and fuel axial expansion reactivity feedbacks shutting down the fission power and decreasing the peak temperatures in the core. Since the BOP does not affect the reactor in this scenario, its parameters are not calculated and are not shown in the figure.

In the long term (Figure 12-2), the heat generation rate is decreasing with time due to decay of the radioactive elements. Heat removal from the reactor is directly provided by the RVACS. The system temperatures continue to rise until the decay heat drops below the heat removal rate by the RVACS. During the temperature rise, more negative reactivity is inserted into the reactor, such that the fission power is temporarily neutronically shut down. The fission power decreases to the spontaneous fission level and temporarily stays at this level. After the heat removal by the RVACS exceeds the decay heat, the system starts to cool down. As the system temperatures drop, the inherent reactivity feedbacks cause a positive reactivity change in the core and the fission power begins to rise. For some time, however, the fission power is significantly lower than the decay heat such that it does not affect the system temperatures which continue to drop thereby further increasing the reactivity. By the time that the fission power reaches the level at which it affects the system temperatures, the reactivity is sufficiently positive causing the power to rise fast and exponentially – faster than the system temperatures can respond. This results in a peak in the reactor power to a level of about 1.75 \% nominal around 35 hours. Subsequently, the temperatures start to rise in response to the power increase introducing negative reactivity and shutting down the fission power again. This gives rise to the oscillatory spikes in the power and oscillations in the temperatures. However, the actual power level remains low at a value representative of decay heat.
levels. The peaks in fission power are repeated with reducing magnitude until the temperatures and the power balance reach a new equilibrium state at zero reactivity.

The equilibrium power level of 0.5% nominal, representative of decay heat after 8 hours, and the equilibrium temperatures are calculated to subsequently remain unvarying. Fission power maintains this unvarying power level as the decay heat continues to decrease. Because it is assumed that the reactor protection system fails, human intervention would be required to neutronically shut down the reactor.

The results in Figure 12-1 and Figure 12-2 demonstrate that the temperatures return to values comparable with nominal steady-state operating temperatures. For example, the peak cladding temperature (PCT) equalizes at about 605 oC, which is below the nominal operational value of 650 oC. However, the structural temperatures could temporarily attain higher values during the accident scenario. The PCT remains above 650 oC for about 17 hours and above 700 oC for about 7 minutes. A structural analysis is necessary to determine if the temperatures pose a hazard to the structures and whether it could be expected that the reactor would be permitted to restart after the accident. It is expected that the structural temperatures are low enough to withstand the transient.

12.4. Loss of Generator Load (LOL) without SCRAM

In the LOL scenario, it is assumed that the generator is instantly separated from the grid at t=0 (the separation is modeled with the grid load going from 100% to zero in 0.1 s). Since the turbine energy is no longer consumed by the grid, it tends to be transferred to the kinetic energy of the turbomachinery; i.e., as an increase in the rotational speed. The S-CO$_2$ Brayton Cycle control system is assumed to work and attempts to regulate the turbine power to keep the rotational speed at the normal value. The protection system on the reactor is assumed to fail to simulate the passive response.

This scenario is considered to be one of the most severe for the S-CO$_2$ Brayton Cycle side. The turbine blades could separate from the rotor if their rotational speed becomes too high. Although the exact limit on rotational speed needs to be calculated by stress analysis of the blades and is not known at this time, it is believed that the blades should maintain their integrity, if the rotational speed is remains below 120-130% of the nominal value.

Figure 12-3 shows the first 100 seconds of the loss-of-load event. In response to the initial increase in the shaft S-CO$_2$ Brayton Cycle rotational speed, the control system opens the turbine bypass valve. It is found that a very fast acting turbine bypass valve is required in order to keep the turbomachinery rotating speed under control. In the model, it is assumed that the valve can move from the fully closed position to the fully open position in 0.5 s. This rapid action is initiated if the rotational speed exceeds 105% nominal. In reality, a separate fast-acting bypass valve would be installed in parallel with the regular bypass valve used for the cycle control.

Figure 12-4 shows the plant response to the LOL accident over the entire simulation time. Opening the turbine bypass valve reduces the CO$_2$ flow rate through the turbine as well as through the heat exchangers. As a result of reduced cooling, the Pb temperature exiting the heat exchangers starts to increase followed by the core coolant temperature and other temperatures in the core. This temperature increase introduces
negative reactivity, reducing the core power level. The reactor peak temperatures following the small initial increase drop below the nominal values. On the S-CO\textsubscript{2} Brayton Cycle side, the automatic control system adjusts the system parameters such that the turbine power is equal to the compressional work representing zero generator output, and the rotational speed is kept at the nominal value. The compressors work far away from design points and are calculated to approach surge/stall conditions (stall occurs when $f_{\text{stall}}=1$ on Figure 12-4). Compressor surge, however, is avoided ($f_{\text{stall}}>1$ on Figure 12-4) by the control action so that the compressors continue to operate all of the time.

Overall, the system response to the LOL event is benign from a safety point of view. The reactor system temperatures do not exceed values that would raise concerns. On the S-CO\textsubscript{2} Brayton Cycle side, additional structural analysis is required to investigate the effect of the initial increase in the shaft rotational speed on the blades as well as the effect of compressor surge. These questions are important in terms of protection of the energy conversion components (i.e., protection of plant investment) but do not represent concerns about the conditions of the reactor system.

### 12.5. CO\textsubscript{2} Pipe Break Accidents

The S-CO\textsubscript{2} Brayton Cycle operates at high pressure (up to 20 MPa) such that CO\textsubscript{2} pipe breaks must be considered as accident initiators. The location of the break in this analysis is selected to include the accident initiator that is expected to influence the reactor side in the most severe way; that is, a pipe break in the line between the heat exchanger exit and the inlet to the turbine. This is also expected to be the most probable break location, since the working fluid is at the highest pressure and temperature. In addition, a break location located downstream of the turbine between the turbine exit and the high temperature recuperator is selected for investigation for reasons discussed below.

The main effect of a pipe break on the S-CO\textsubscript{2} Brayton Cycle side from the safety point of view is the reduced or complete loss of heat removal capability of the S-CO\textsubscript{2} Brayton Cycle. As discussed above, the limiting case for these conditions, a total loss-of-heat sink, has already been analyzed and it has been shown that the reactor can withstand that accident. It is not expected that the consequences from the partial loss of heat removal would exceed those from the total loss. Therefore, loss of the heat sink (total or partial) is not a concern in this accident. However, it is possible to increase rather than decrease the heat removal rate in the heat exchanger as a result of the CO\textsubscript{2} pipe break. The reactor power could rise due to the reactivity feedbacks, if colder coolant is introduced into the core with the possibility to subsequently increase the coolant and structural temperatures. It is important to determine the magnitude of the overcooling of the Pb coolant. If the Pb coolant could be frozen upon the outside of heat exchanger tubes partially blocking the Pb flow path, the frictional pressure drop through the HXs would increase causing a reduction in the natural circulation flowrate.

By analyzing the S-CO\textsubscript{2} Brayton Cycle layout (Figure 5-3), two possibilities for overcooling of the primary coolant in the heat exchangers are found. First, the flow rate of the CO\textsubscript{2} in heat exchangers could increase if the pipe break occurs downstream the HXs. Second, colder than normal CO\textsubscript{2} could be introduced to the HXs. One of the possibilities for such conditions is a pipe break downstream of the turbine. In this case,
the hot CO₂ flow to the high temperature recuperator (HTR) would be reduced such that the cold stream would not be heated as effectively and colder CO₂ would enter the HXs. The two pipe break locations, upstream and downstream of the turbine, are simulated with the plant dynamics analysis code.

In order for overcooling of the primary side to be significant, the changes in the conditions of the secondary flow (flow rate and/or temperature) must be sufficiently large and must last for a sufficient time. The extent to which the pipe break affects heat removal in the HXs, the conditions in the S-CO₂ Brayton Cycle, as well as the time scale of the transient are dependent upon both the break size and location. For a large break, the change in CO₂ conditions would be significant, but the time interval before the S-CO₂ Brayton Cycle depressurizes is short. Thus, the accident rapidly transitions to a loss-of-heat sink accident and no significant overcooling is expected. In contrast, a small break, even if it is expected to last longer, could be accommodated by the BOP control system and could be similar to action of the inventory control system (when CO₂ inventory is slowly being removed from the cycle). Therefore, an intermediate break, which has a potential to have an impact on the primary side for a significant amount of time, is expected to provide more of a potential safety concern than the large or small breaks.

12.5.1. Turbine Inlet Pipe Break Accident

When a small area main CO₂ line break (1 cm or smaller effective diameter) is simulated, the calculations show that the S-CO₂ Brayton Cycle control system is able to maintain the system pressures by means of inventory compensation from the inventory control tanks with very little overcooling. Figure 12-5 shows the response to a 1 cm turbine inlet pipe break with virtually no change in coolant temperature and minor change in reactor power. The inventory control system is involved in both inventory compensation for the leak and shaft speed regulation control. Since it is prohibited for inlet and outlet inventory valves to be opened simultaneously, the inventory control can fulfill only one function at a time, resulting in repeating pattern of opening and closing the valves.

When a larger 5 cm equivalent diameter break is simulated (Figure 12-6), the S-CO₂ Brayton Cycle automatic control system is able to maintain the system pressures for some time by opening the outlet valve from the inventory tank to compensate for the loss in CO₂ inventory. Immediately after the break, the CO₂ flow rate through the in-reactor HXs increases such that the heat removal by CO₂ exceeds heat generation in the reactor. However, the loss of CO₂ inventory quickly decreases the CO₂ flow rates such that the heat removal from the reactor decreases below the nominal level following the initial increase. As a result, the overcooling of the primary coolant is small and the coolant temperature starts to increase after about 30 seconds. The cycle depressurization causes the turbine work to decrease and the turbomachinery which all share a common shaft with the generator begin to slow down; it is assumed that the grid consumes a constant power from the generator. In about 60 seconds, the turbine power drops below the power level consumed by the compressors such that the turbomachinery and CO₂ circulation stop. Due to the significant changes in the conditions of the S-CO₂ Brayton Cycle, the compressors are expected to stall at about 40 seconds following the break. At about 60
seconds, the flow has stopped and the S-CO$_2$ Brayton Cycle does not operate anymore. The accident evolves into a loss-of-heat sink accident (Figure 12-7) in which the coolant temperatures start to rise. No significant overcooling of the Pb primary coolant is calculated for this case.

The system response to 10 cm break (Figure 12-8) is similar to that of 5 cm with increased rate of depressurization. The CO$_2$ turbine and compressors stop at about 40 seconds. Again, no significant coolant overcooling is calculated.

Figure 12-9 shows the result of a larger break of 25 cm equivalent diameter of the turbine inlet pipe. The cycle depressurization is calculated to occur very rapidly such that the turbine work rapidly falls and the turbomachinery starts to slow down. In about 6 seconds, the turbomachinery and CO$_2$ circulation stop. Also, the change in the conditions of the S-CO$_2$ Brayton Cycle is so dramatic that the compressors are expected to stall at about 3 seconds after the break. The fast depressurization of the volume downstream of the HXs causes the CO$_2$ flow rate through the HXs to rapidly increase thereby increasing the heat removal rate by the CO$_2$. However, the Pb primary coolant temperature in the HX drops by only a few degrees Celsius during this time, and the coolant temperatures in the core do not have time to react at all over this short timescale. The enhancement of Pb natural circulation due to the decrease of the cold pool temperature reduces heat transfer in the HXs and the coolant temperature starts to rise again. At about 6 seconds, the S-CO$_2$ Brayton Cycle does not operate anymore. The transient evolves into a loss-of-heat sink accident where the coolant temperatures start to rise. Overall, no significant overcooling of the Pb primary coolant is observed under these conditions.

12.5.2. Turbine Outlet Pipe Break Accident

For the case of a turbine outlet pipe break, overcooling is caused by the reduced CO$_2$ temperature at the HX inlet due to decreased preheating of CO$_2$ in the high temperature recuperator. The response to the small break (Figure 12-10) is calculated to be very similar to that of the turbine inlet pipe breaks; namely, the control system maintains the CO$_2$ pressure for significant time. Figure 12-11 shows the results for a 5 cm turbine-outlet pipe break. The results show that initially the CO$_2$ temperature at HTR outlet (HX inlet) is maintained by the thermal inertia of the HTR structure. By the time that this temperature starts to react to the effects of the change in flow conditions of the hot CO$_2$ stream at about 60 seconds, the depressurization causes a decrease in CO$_2$ circulation. Even though some overcooling occurs in the HX at this time, it is too small to cause a significant reduction in Pb coolant temperature. The results for other break diameters show no significant overcooling as well. For large diameters (Figure 12-9 shows results for 25 cm break), cycle depressurization occurs before the temperatures react to the break. For breaks smaller than 5 cm equivalent diameter, the effect of depressurization is too small to cause any overcooling concerns.

In general, the plant dynamics code calculations reveal that overcooling of the Pb primary coolant cannot be practically achieved following a CO$_2$ pipe break in the S-CO$_2$ Brayton Cycle Energy Converter of the STAR-LM plant. Overcooling is limited either by the duration of the cycle depressurization, the extent of increase of the CO$_2$ flow rate
through the Pb-to-CO$_2$ HXs (for turbine inlet line breaks), the extent of decrease in the CO$_2$ temperature inlet to the reactor (for turbine outlet line breaks), or both.

12.6. Transient Overpower (TOP) without SCRAM

As discussed above, fast TOP accidents are eliminated in the STAR-LM concept. The only source of excess reactivity is the burnup compensation rods which are designed to move very slowly or only a small amount per adjustment. It is expected that the reactivity insertion rate from run out of those rods would be very small (cents per hour or less). As a conservative approach, a reactivity insertion of 15 cents at the rate of 1.0 cent per second is assumed in this accident. Since the reactivity insertion rates are significantly higher than expected from the reactivity compensation rods, the safety limits on the reactor side are of lesser concern in this simulation. The main goal of this accident simulation is to look at the response of the S-CO$_2$ Brayton Cycle; in particular, to see if the BOP control system can handle the increased heat addition to the BOP, and the imbalance between heat addition and grid demand.

Figure 12-13 shows the system response to the reactivity insertion of 15 cents over 15 seconds. The reactivity insertion causes the reactor power to initially increase raising the system temperatures. The negative reactivity feedback from temperature increases compensates the reactivity insertion from the postulated rod withdrawal and the reactor returns to a zero reactivity state with almost the same power level and elevated temperatures. The BOP control system reacts to the increased heat addition by opening the turbine bypass valve to keep the generator output equal to the grid demand.

12.7. Reactor SCRAM

The lead coolant has a high melting temperature (327 °C). Therefore, the possibility of overcooling and freezing the coolant has to be considered with regard to plant operations. During normal operation, the lead temperature is maintained by the balance between heat generation in the reactor and heat removal by the BOP. It is shown above that this balance could not be disturbed by an event on the BOP side enough to cause freezing of the Pb. However, there is a possibility of introducing a significant mismatch between heat generation and removal by scrambling the reactor. If significant negative reactivity is introduced to the reactor over a short period of time, the reactor power will drop very rapidly, while the heat removal could stay at the nominal level for a longer time. These conditions will lead to overcooling of the lead in the heat exchangers. Note that the reactivity feedback from the decrease in system temperatures would not be able to raise the reactor power, if the initial reactivity insertion is large. The other concern in this situation is the natural circulation behavior. Like the temperatures, the coolant flow rate depends on the balance between heat production and heat removal. When heat production drops so does the driving force for natural circulation leading to a situation in which the coolant circulation may stop or reverse.
The Plant Dynamics Analysis Code is applied to simulate the system response to a reactor SCRAM. Figure 12-14, Figure 12-15, and Figure 12-16 show the system behavior in response to the introduction of -5 δ in 1 second for short, middle, and long terms, respectively. Introduction of the negative reactivity shuts down the fission power and the coolant temperature at the core outlet starts to drop. With some delay, the coolant temperature in heat exchanger also begins to decrease. Since the cold side temperature always lags behind the hot side temperature, the temperature difference between the hot and cold sides is decreasing causing a decrease in the natural circulation flow. The decrease in lead temperature in the HX leads to a decrease in the CO\textsubscript{2} temperature at the HX outlet, or turbine inlet, such that the turbine work starts to fall. The S-CO\textsubscript{2} Brayton Cycle control system tries to adjust for these conditions, but at about 20 seconds (Figure 12-14) the turbine work is so small that the rotational speed of the turbomachinery cannot be maintained at the design value. When the compressors stop (at around 60 seconds), the heat removal by the S-CO\textsubscript{2} Brayton Cycle is discontinued and the lead temperatures start to rise. Figure 12-15 shows that during the transient, the lead temperature in the HX drops to about 365 °C (only about 40 °C higher than the freezing temperature); the core inlet temperature drops to about 405 °C. The natural circulation stops and even reverses at about 250 seconds but is subsequently reestablished. After the BOP ceases to operate, the plant undergoes a transition to a regime similar to a loss-of-heat sink event, but since the system temperatures at the beginning of LOHS conditions are low, the temperatures rise to a level which is lower than that during the LOHS scenario discussed above (Figure 12-16). The natural circulation flow stabilizes at about one-tenth of the nominal flow.

Although coolant bulk freezing is not calculated, the initial drop in the Pb coolant temperature and the predicted loss of natural circulation (even for a short time) could cause concerns and one could question applicability of the current strictly one-dimensional model. Also, as it will be shown below, the assumed response of the S-CO\textsubscript{2} Brayton Cycle might not represent a worst case behavior.

It has been shown that reactor scram can cause a significant drop in the Pb coolant temperature. This temperature drop is shown to be reversed when the BOP cannot operate anymore at a reduced core power level. However, the simulation assumed that the generator stays connected to the grid causing the turbomachinery to slow down and stop if the turbine power is not great enough to drive the generator and compressors. However, the BOP control strategy or operator action might include disconnecting the generator from the grid following generation of the reactor scram signal. In this case, the turbine could continue to operate and drive the compressors for a longer time causing a further decrease in coolant temperatures.

Figure 12-17 shows the system response to this scenario that includes discontinuing the generator from the grid. Again, -5 δ of reactivity is introduced into the core in 1.0 second. At 20 seconds, however, the generator is separated from the grid and the S-CO\textsubscript{2} Brayton Cycle control system adjusts the Brayton cycle to keep the turbomachinery rotational speed at the nominal level. It can be seen that the turbomachinery could operate for at least 1000 seconds in this case. During that time, the CO\textsubscript{2} flow through the heat exchanger is maintained and the Pb is being cooled down. A specific feature of the Brayton cycle is that because of the recuperative cycle configuration, a decrease in the turbine outlet temperature will result in a drop in the HX inlet temperature, leading to a further decrease in the temperature at the turbine inlet and
outlet. The decrease in CO₂ temperature in the HX causes further decrease in the lead temperature. It is calculated that the coolant bulk temperature will reach the Pb freezing temperature (327 °C) in the heat exchanger in about 450 seconds. The results obtained beyond this point assume that the coolant can still circulate as liquid, even if the bulk temperature is below the freezing temperature.

The actual S-CO₂ Brayton Cycle would also include a separate circuit with a pathway for normal shutdown heat removal involving a motor-driven compressor and a CO₂-to-water heat exchanger. That separate heat transfer pathway and the transition to its operation are not modeled in the calculations. A transition from the BOP circuit for normal operation to the circuit for normal shutdown heat removal would decrease the heat removal from the reactor relative to that obtained in the calculations presented above. Thus, the calculations may be viewed as belonging to the scenarios that arise when the transition to the shutdown heat removal path fails to be established.
Figure 12-1. Loss of Heat Sink Accident - Short Term.
Figure 12-1. Loss of Heat Sink Accident - Short Term (2).
Figure 12-1. Loss of Heat Sink Accident - Short Term (3).

------------------------------------------- (End of Figure 12-1) -------------------------------------------
Figure 12-2. Loss of Heat Sink Accident - Long Term.
Figure 12-2. Loss of Heat Sink Accident - Long Term (2).
Figure 12-2. Loss of Heat Sink Accident - Long Term (3).

------------------------------------------- (End of Figure 12-2) -------------------------------------------
Figure 12-3. Loss of Load Accident - Short Term.
Figure 12-3. Loss of Load Accident - Short Term (2).
HEAT BALANCE IN REACTOR

COOLANT SPEED IN CORE

COOLANT TEMPERATURES IN CORE

COOLANT TEMPERATURES IN HX

Figure 12-4. Loss of Load Accident - Long Term.
Figure 12-4. Loss of Load Accident - Long Term (2).
Figure 12-4. Loss of Load Accident - Long Term (3).
Figure 12-4. Loss of Load Accident - Long Term (4).
Figure 12-4. Loss of Load Accident - Long Term (5).
Figure 12-5. Turbine-Inlet Pipe Break Accident – 1 cm Break.
Figure 12-5. Turbine-Inlet Pipe Break Accident – 1 cm Break (2).
Figure 12-5. Turbine-Inlet Pipe Break Accident – 1 cm Break (3).
Figure 12-5. Turbine-Inlet Pipe Break Accident – 1 cm Break (4).
Figure 12-5. Turbine-Inlet Pipe Break Accident – 1 cm Break (5).
Figure 12-6. Turbine-Inlet Pipe Break Accident – 5 cm Break, Short Term.
Figure 12-6. Turbine-Inlet Pipe Break Accident – 5 cm Break, Short Term (2).
Figure 12-6. Turbine-Inlet Pipe Break Accident – 5 cm Break, Short Term (3).
Figure 12-6. Turbine-Inlet Pipe Break Accident – 5 cm Break, Short Term (4).
Figure 12-6. Turbine-Inlet Pipe Break Accident – 5 cm Break, Short Term (5).
(End of Figure 12-6)
Figure 12-7. Turbine-Inlet Pipe Break Accident – 5 cm Break, Long Term.
Figure 12-7. Turbine-Inlet Pipe Break Accident – 5 cm Break, Long Term (2).
Figure 12-7. Turbine-Inlet Pipe Break Accident – 5 cm Break, Long Term (3).
Figure 12-8. Turbine-Inlet Pipe Break Accident – 10 cm Break.
Figure 12-8. Turbine-Inlet Pipe Break Accident – 10 cm Break (2).
Figure 12-8. Turbine-Inlet Pipe Break Accident – 10 cm Break (3).
Figure 12-8. Turbine-Inlet Pipe Break Accident – 10 cm Break (4).
Figure 12-8. Turbine-Inlet Pipe Break Accident – 10 cm Break (5).
Figure 12-9. Turbine-Inlet Pipe Break Accident – 25 cm Break.
Figure 12-9. Turbine-Inlet Pipe Break Accident – 25 cm Break (2).
Figure 12-9. Turbine-Inlet Pipe Break Accident – 25 cm Break (3).
Figure 12-9. Turbine-Inlet Pipe Break Accident – 25 cm Break (4).
Figure 12-9. Turbine-Inlet Pipe Break Accident – 25 cm Break (5).
Figure 12-10. Turbine-Outlet Pipe Break Accident – 1 cm Break.
Figure 12-10. Turbine-Outlet Pipe Break Accident – 1 cm Break (2).
Figure 12-10. Turbine-Outlet Pipe Break Accident – 1 cm Break (3).
Figure 12-10. Turbine-Outlet Pipe Break Accident – 1 cm Break (4).
Figure 12-10. Turbine-Outlet Pipe Break Accident – 1 cm Break (5).
Figure 12-11. Turbine-Outlet Pipe Break Accident – 5 cm Break.
Figure 12-11. Turbine-Outlet Pipe Break Accident – 5 cm Break (2).
Figure 12-11. Turbine-Outlet Pipe Break Accident – 5 cm Break (3).
Figure 12-11. Turbine-Outlet Pipe Break Accident – 5 cm Break (4).
Figure 12-11. Turbine-Outlet Pipe Break Accident – 5 cm Break (5).
Figure 12-12. Turbine-Outlet Pipe Break Accident – 25 cm Break.
Figure 12-12. Turbine-Outlet Pipe Break Accident – 25 cm Break (2).
Figure 12-12. Turbine-Outlet Pipe Break Accident – 25 cm Break (3).
Figure 12-12. Turbine-Outlet Pipe Break Accident – 25 cm Break (4).
Figure 12-12. Turbine-Outlet Pipe Break Accident – 25 cm Break (5).
Figure 12-13. Transient-Overpower without Scram Accident.
Figure 12-13. Transient-Overpower without Scram Accident (2).
Figure 12-13. Transient-Overpower without Scram Accident (3).
Figure 12-13. Transient-Overpower without Scram Accident (4).
Figure 12-13. Transient-Overpower without Scram Accident (5).
Figure 12-14. Reactor SCRAM with Full Grid Load – Short Term.
Figure 12-14. Reactor SCRAM with Full Grid Load – Short Term (2).
Figure 12-14. Reactor SCRAM with Full Grid Load – Short Term (3).
Figure 12-14. Reactor SCRAM with Full Grid Load – Short Term (4).
Figure 12-14. Reactor SCRAM with Full Grid Load – Short Term (5).
(End of Figure 12-14)
Figure 12-15. Reactor SCRAM with Full Grid Load – Middle Term.
Figure 12-15. Reactor SCRAM with Full Grid Load – Middle Term (2).
Figure 12-16. Reactor SCRAM with Full Grid Load – Long Term.
Figure 12-16. Reactor SCRAM with Full Grid Load – Long Term (2).
Figure 12-17. Reactor SCRAM with Separation from the Grid.
Figure 12-17. Reactor SCRAM with Separation from the Grid (2).
Figure 12-17. Reactor SCRAM with Separation from the Grid (3).
Figure 12-17. Reactor SCRAM with Separation from the Grid (4).
13. Summary and Proposed Future Developments

13.1. Dynamics Code

13.1.1. Code Development Summary

The dynamics code for a STAR-LM lead-cooled fast reactor coupled to a supercritical carbon dioxide Brayton cycle has been developed. The code addresses the specific features of the system, such as natural circulation of the reactor coolant, autonomous reactor control, non-ideal behavior of the secondary fluid, and recompression cycle configuration. The code is based on the first-principal conservation equations. The special solution technique has been developed and implemented to the code which allows solving the highly coupled stiff system of differential equations effectively. The code is tested against steady-state conditions and showed stability and good agreement with steady-state calculations. The code is applied to the normal operation transient and accident conditions. It is shown that the plant can be controlled from 0% to 100% power without exceeding limitations on either S-CO$_2$ Brayton cycle or reactor conditions. The accident conditions simulations confirm the passive safety performance of the natural circulation LFR.

Since some assumptions are made during the implementation of the first-principles equations in the code, further improvements to the code could be made when any of these assumptions are revised or eliminated. Possible improvements to the code are discussed below.

13.1.2. Updates to Reactor Model

The reactor model could be updated to eliminate as many assumptions of the current model as possible. For example, a realistic IRHX bypass flow could be calculated. Also, the heat losses through the reactor closure head and in the lower plenum could be added to the model. Another possible update would relate to the core radial expansion coefficient for the core support structure (such as the core support plate) dependent upon the support plate temperatures rather than coolant temperature.

13.1.3. Turbomachinery Update

The S-CO$_2$ Brayton cycle turbomachinery design and performance models have been updated since the results reported in this document were obtained. The improved turbomachinery models have to be incorporated into the code to generate new performance maps. Important for dynamics code improvement in the turbomachinery models has been incorporation of the turbine and compressor inlet nozzles into the model. This change leads to fact that the minimum temperature in the cycle is now calculated
between the inlet nozzle and first stage of Compressor No. 1; i.e., inside the compressor. Since it is important to control this temperature to stay above the critical CO$_2$ temperature, control of a temperature inside the compressor should be introduced to the code.

The other option to improve the turbomachinery treatment in the dynamics code would be introduction of dynamic equations describing flow inside the turbomachinery (rather than the current instantaneous-response approach based on performance maps).

13.1.4. PCHE Model

Like the turbomachinery model, the model for Printed Circuit Heat Exchangers (PCHE) has been recently updated. The model updates include better treatment of the geometry as well as heat transfer and pressure drop correlations. These updates would be very easily incorporated into the dynamics code since they do not involve revisions of the model and equations themselves.

13.1.5. Cooler Model

A major simplifying assumption of the current model is ideal control of the minimum cycle (i.e., cooler-outlet) temperature. This assumption allows use of a simplified cooler model. A more realistic cooler model could be introduced into the code together with realistic temperature control. The major challenge here is that due to the significant properties variation, many axial nodes would be required to accurately calculate the heat transfer and pressure drop, significantly increasing total number of equations to be solved. Moreover, the CO$_2$ compressibility near critical point is expected to result in a decreased time scale of the momentum equations inside the cooler. Incorporating the accurate cooler model would require either improving the solution technique or modifying the code to run on multi-processor computers.

13.1.6. Normal Decay Heat Removal Path

The preliminary analysis showed that the CO$_2$ Brayton cycle may not be suitable for operation under decay heat removal after reactor shut down (i.e., scram). Therefore, a separate dedicated system for decay heat removal may be added to the balance of plant. Modeling of this system as well as the transition to its operation would be required.

13.1.7. Working Fluid Conditions

The model, at the current stage of development, is not capable of estimating the consequences of coolant freezing. The code does not model the formation of frozen crusts of Pb on the outside of tube surfaces inside the HX. It also does not model bulk freezing of the coolant should the Pb fluid temperature be calculated to drop below the freezing
point. The presence of a partial or complete flow blockage would change the flow patterns in the remaining liquid region. The model could be extended to include those effects.

Also, the treatment of two-phase flow on CO₂ side should be added to the model if the simulation of the heat sink temperature variation would indicate that such a two-phase regime could be achieved in the cycle during particular transients or accidents.

13.2. Normal Operational Transients

13.2.1. Summary of Normal Operation Analysis

The analysis of the normal operational transients performed so far has shown an acceptable system response during those transients - no major operational issues have been identified. However, further improvements in the plant control system logic could be made to eliminate minor issues, such as the transition from one control action to another. The results also showed that under some conditions, the Brayton cycle compressors could operate in an unstable (surge) regime. However, updated compressor designs (which have not yet been implemented in the dynamics code) have shown an increase in margin to stall/surge conditions which should improve the stability of the compressors and the whole cycle.

The other possibility for control system modeling improvement would be incorporation of a more realistic control system including the control electronics and all of the delays associated with it.

13.2.2. Startup and Shutdown, Decay Heat Removal

The normal operating conditions should be expanded to include normal (non-emergency) plant start up and shutdown. Simulating these conditions, however, would require modeling of the processes which are beyond of the current model capabilities, such as turbine start-up and the transition to the dedicated decay heat removal system. Therefore, significant model updates would be required.

13.2.3. Heat Sink Conditions

Incorporating the realistic cooler model would allow adding to the normal operating transients set of conditions associated with the heat sink, such as heat sink temperature and flow rate. It would also facilitate calculations to answer concerns about controllability of the CO₂ conditions near critical point. (For code change requirements for the cooler model, see Section 13.1.5 above).
13.3. Accident Simulations

13.3.1. Summary of Accidents Simulation

The plant dynamics analysis code for the STAR-LM reactor coupled to the S-CO₂ Brayton cycle has been used to simulate postulated accidents. To demonstrate the passive safety of STAR-LM, unprotected accidents are emphasized; i.e., it is assumed that the reactor protection system fails to respond to upset conditions. It is discussed how some traditional accident scenarios (LOCA and LOF) are eliminated by specific features of the STAR-LM concept.

Among the remaining unprotected accidents, the most severe one is a LOHS accident. It is demonstrated that the reactor can tolerate this accident without exceeding the safety limits. It is demonstrated that events and accidents on the BOP side do not give rise to safety concerns about the reactor. In particular, it is demonstrated that there is very limited overcooling in case of a CO₂ pipe break, either due to the timescale or the magnitude of the overcooling, to cause any significant decrease in Pb primary coolant temperature. The BOP control system can safely accommodate the disconnecting of the generation from the grid without the need for the reactor protection system to scram the reactor.

Calculations indicate that a combination of postulated events is required to calculate bulk freezing of the primary coolant in the heat exchangers. In particular, disconnect from the grid and reactor scram need to be accompanied by failure to change from the normal at power heat removal circuit to the normal shutdown heat removal circuit. (See Section 13.1.7 for the requirements for code modification for coolant freezing). Since freezing of the Pb coolant could potentially be damaging to certain structures, measures should be taken to eliminate or significantly reduce the possibility of conditions that can lead to its occurrence. One use of the plant dynamics code is thus to delineate situations in which freezing might occur or in which the margin in Pb temperatures above the freezing temperature is reduced.

13.3.2. Additional Accident Scenarios

Other accident scenarios, such as pipe breaks at locations not considered in this report, could be simulated with the existing code. The protected class of accidents, where the plant protection system responds to accident conditions, could be simulated. This would require development of a plant protection system model similar to the plant control system model described in this report. Significant code modifications may be required to model accidents which involve significant deviations from operating conditions, such as coolant leakage in case of the reactor vessel failure or turbomachinery shaft rupture.
14. Acknowledgements

Argonne National Laboratory’s work was supported by the U. S. Department of Energy 2002 Nuclear Energy Research Initiative Program under Project No. 02-065, “Coupling of High Temperature, Lead-Cooled, Closed Fuel Cycle Fast Reactors to Advanced Energy Converters,” under contract W-31-109-Eng-38. The authors are grateful to Dr. Paul S. Pickard (Sandia National Laboratories), the Generation IV National Technical Director for Energy Conversion, and Dr. Carl Sink (U.S. DOE) as well as Dr. Eric Loewen (Idaho National Laboratory), the Generation IV System Integration Manager for the LFR, and Dr. Rob Versluis (U.S. DOE) for their continued support for development and validation of the plant dynamics code for S-CO2 cycle applications. The authors are also indebted to many individuals for their helpful advice during the course of this work including Drs. James E. Cahalan, David C. Wade, George Klopp, and Roald A. Wigeland (ANL/NE), as well as Kenneth Nichols and Robert Fuller (Barber-Nichols, Inc.).
15. References


Appendix A.

Plant Dynamics Code Variables.

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Plant Dynamics Code Project Structure.

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Nuclear Engineering Division
Argonne National Laboratory
9700 South Cass Avenue, Bldg. 208
Argonne, IL 60439-4842

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