Improving LER Coupling and Increasing PEP-II Luminosity
with Model-Independent Analysis

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Abstract

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The PEP-II storage ring at SLAC houses electrons (in the High-Energy Ring, or HER) and positrons (in the Low-Energy Ring, or LER) for collision. The goal of this project was to improve the linear optics of the LER in order to decrease coupling, thereby decreasing emittance and increasing luminosity. To do this, we first took turn by turn BPM (Beam Position Monitor) data of a single positron bunch at two betatron resonance excitations, extracted orbits from this data using Model-Independent Analysis, and from these orbits formed a virtual model of the accelerator. We then took this virtual model and found an accelerator configuration which we predicted would, by creating vertical symmetric sextupole bumps and adjusting the strengths of several key quadrupole magnets, improve the coupling and decrease the emittance in the LER. We dialed this configuration into the LER and observed the coupling, emittance, and luminosity. Coupling immediately improved, as predicted, and the y emittance dropped by a dramatic 40%. After the HER was adjusted to match the LER at the Interaction Point (IP), we saw a 10% increase in luminosity, from $10.2 \times 10^{33} cm^{-2} sec^{-1}$ to $11.2 \times 10^{33} cm^{-2} sec^{-1}$, and achieved a record peak specific luminosity.
Introduction

The PEP-II storage ring at the Stanford Linear Accelerator Center is designed to store high-energy electrons and positrons for collision. The electrons circulate in the HER (High-Energy Ring) at 9 GeV and the positrons circulate in the LER (Low-Energy Ring) at 3.1 GeV. In order to maximize the likelihood of collisions between electrons and positrons at the Interaction Point (IP), PEP-II must have high luminosity. Luminosity is defined as the interaction rate per unit cross-section [6], and is proportional to the product of the intensities of the electron and positron beams over the cross-sectional area of the interaction region of the beams. Luminosity can be improved by increasing beam currents, improving LER-HER alignment, or decreasing emittance, the constant area in x or y phase space which the beam occupies. The latter was the goal of this project, and was accomplished by improving the linear optics (no higher-order corrections included) of the LER.

The linear optics of a beam can be fully characterized with ten parameters [6]. Four of these parameters have to do with coupling between x and y, or the degree to which the two transverse directions of beam motion are dependent. As a result of the fact that x emittance is typically much higher than y emittance, coupling causes y emittance to increase. The goal of this project was to decrease coupling in order to decrease emittance in the LER, and thus increase luminosity. Model-Independent Analysis (MIA) has previously been used to study the linear optics of the PEP-II beams [4], and we used MIA again here. From our MIA-derived orbits we derived Green’s Functions and Phase Advances for use in fitting a virtual machine [5], which we would use to improve the linear optics of the LER. Usually, attempts to improve the linear optics of either of the PEP-II beams focus solely on adjusting the quadrupole magnets. This time, we attempted to also adjust the orbit corrector magnets in order to create symmetric Y sextupole bumps, which are pairs of nonlinear magnets used to produce linear effects by being separated from the beamline by a vertical symmetric ‘bump’ created by the orbit correctors.

The project consisted of four phases: obtaining beam data from the PEP-II LER and extracting four independent orbits with MIA, creating a virtual accelerator model which
fit the data, creating a solution which adjusted the strengths of several LER quadrupole magnets and orbit correctors in order to improve coupling in the LER, and then implementing the solution in the actual accelerator.

Materials and Methods

All steps in the procedure except for the dialing in of the final solution were done in MATLAB. The solution was dialed into PEP-II with a physical knob in the control room.

0.1 Taking BPM Data and Extracting Orbits

First, we took BPM data for a single LER positron bunch in order to measure the ten parameters which characterize the linear optics of the positron beam. In the PEP-II LER there are 319 BPM sites, some single-view in Y (measure displacement only along the Y axis), some single-view in X, and some double-view. We used MIA, which is capable of reducing noise by a factor of $\sqrt{N}$ or $\sqrt{M}$, whichever is larger, for M turns and N BPM measurements, to extract betatron oscillation modes from the BPM data. Unlike in a linac, where the beam jitter produces a signal which is extractable with MIA, in a storage ring the betatron motion is damped by synchrotron radiation and thus is difficult to measure. To aid in measurement, we excited the beam at the horizontal and then the vertical betatron eigentunes. For each excitation mode we collected a set of BPM data for 1024 consecutive turns. This produced four data matrices: a set of X data and a set of Y data for both the horizontal and vertical excitation modes. Each matrix was arranged such that a column contains the measurement data for a single BPM and a row contains the data for a single turn.

Since the position data at each BPM is strongly related to the data of its neighbors, we can use statistical methods to remove BPM data which is likely to be incorrect. We applied both strong and weak criterion to our data in order to find and discard unusable BPM columns.
Once we had isolated the good BPM data, we performed a Discrete Fourier Transform on each data set. The $0^{th}$ Fourier mode represents the self-consistent closed orbit; this is the path where the beam comes back to the same position after each turn. The other dominating mode is at the frequency of the resonance excitation. From this mode we can and did extract two independent orbits: a cosine-like orbit, represented by the real part of the mode coefficient, and a sine-like orbit, represented by the imaginary part. Since we could do this for both the horizontal and vertical excitation modes, we extracted four independent orbits $(x_1, y_1), (x_2, y_2), (x_3, y_3)$ and $(x_4, y_4)$ from the BPM data. These four orbits completely determine the transfer matrix, a matrix which operates on a phase space vector at one location to give the phase space vector at another location, between any two BPM positions.

0.2 Calculating Green’s Functions and Phase Advances

In order to fit a virtual machine model to the measured data, we need measurement-derived quantities which we can use as fitting parameters in creating the virtual machine. One such quantity we can use are the betatron phase advances at each BPM location [5]. We extracted these phase advances by taking the arctangent of the ratio of the imaginary to the real part of the Fourier Transform mode at each BPM. Since we are taking a ratio, any BPM gains will cancel.

In addition to phase advances, we can use Green’s functions to assist in fitting a virtual machine model to the data [5]. This is done by deriving relationships between machine invariants, our four calculated orbits, and calculable transfer matrices, of which the Green’s functions are a part. From the four orbits we have already calculated, we can form, for each BPM ‘a’, a phase space matrix $Z^a$:

$$Z^a = \begin{pmatrix} x_1^a & x_2^a & x_3^a & x_4^a \\ x_1^a & x_2^a & x_3^a & x_4^a \\ y_1^a & y_2^a & y_3^a & y_4^a \\ y_1^a & y_2^a & y_3^a & y_4^a \end{pmatrix}$$
For any two adjacent BPMs there exists a symplectic transfer matrix $\mathcal{R}^{ab}$ such that $Z^b = \mathcal{R}^{ab} Z^a$. The symplectic condition on $\mathcal{R}$, $\mathcal{R}^T J \mathcal{R} = J$, implies that there exists an invariant antisymmetric matrix $Q$ such that $Q = Z^a T J Z^b = Z^{bT} J Z^a$. While in general $Q$ (being antisymmetric) would have six scalar invariants, the fact that the beam is excited along the eigentunes means that the coupling parameters are zero ($Q_{13} = Q_{14} = Q_{23} = Q_{24} = 0$) and thus that there are only two scalar invariants. While, due to individual BPM gains, we cannot directly calculate $Q_{12}$ and $Q_{34}$, we can calculate their ratio. Taking the ratio of $Q_{12}$ to $Q_{34}$ cancels the BPM gain, and thus is an invariant.

Now, we can use the four orbits and the invariant ratio $Q_{12} / Q_{34}$ to determine the linear Green’s functions between each pair of BPMs. Applying the symplectic condition and the relationship between phase space coordinates gives the following relationships between the four Green’s functions, $\mathcal{R}_{12}$, $\mathcal{R}_{14}$, $\mathcal{R}_{32}$, $\mathcal{R}_{34}$; the four orbits; and $Q_{12}$ and $Q_{34}$:

\[
\begin{align*}
(x_1^a x_2^b - x_2^a x_1^b)/Q_{12} + (x_3^a x_4^b - x_4^a x_3^b)/Q_{34} &= \mathcal{R}_{12}^{ab} \\
(x_1^a y_2^b - x_2^a y_1^b)/Q_{12} + (x_3^a y_4^b - x_4^a y_3^b)/Q_{34} &= \mathcal{R}_{32}^{ab} \\
(y_1^a x_2^b - y_2^a x_1^b)/Q_{12} + (y_3^a x_4^b - y_4^a x_3^b)/Q_{34} &= \mathcal{R}_{14}^{ab} \\
(y_1^a y_2^b - y_2^a y_1^b)/Q_{12} + (y_3^a y_4^b - y_4^a y_3^b)/Q_{34} &= \mathcal{R}_{34}^{ab}
\end{align*}
\]

Since the BPMs are not perfect, we must take measurement error into account when we fit a virtual machine model to the measurement data. Thus we express the Green’s functions in the BPM measurement frame, given by Eqs. 1-4, in terms of the Green’s functions in the virtual machine frame, $R_{ij}$, the individual BPM gains, $g_x$ and $g_y$, and the individual BPM cross-couplings, $\theta_{xy}$ and $\theta_{yx}$:

\[
\begin{align*}
\mathcal{R}_{12}^{ab} &= g_x^b R_{12}^{ab} g_x^a + g_y^b R_{12}^{ab} g_y^a + \theta_{xy}^b R_{32}^{ab} g_x^a + \theta_{xy}^b R_{34}^{ab} g_y^a \\
\mathcal{R}_{32}^{ab} &= g_x^b R_{32}^{ab} g_x^a + g_y^b R_{32}^{ab} g_y^a + \theta_{yx}^b R_{12}^{ab} g_x^a + \theta_{yx}^b R_{14}^{ab} g_y^a \\
\mathcal{R}_{14}^{ab} &= g_x^b R_{14}^{ab} g_x^a + g_y^b R_{14}^{ab} g_y^a + \theta_{xy}^b R_{34}^{ab} g_x^a + \theta_{yx}^b R_{32}^{ab} g_y^a \\
\mathcal{R}_{34}^{ab} &= g_x^b R_{34}^{ab} g_x^a + g_y^b R_{34}^{ab} g_y^a + \theta_{yx}^b R_{14}^{ab} g_x^a + \theta_{yx}^b R_{12}^{ab} g_y^a
\end{align*}
\]

With Eqs. 1-4 we calculated the Green’s Functions in the BPM measurement frame, $\mathcal{R}_{ij}$. Then, using these results and Eqs. 5-8, we went on to fit a virtual model.
0.3 Fitting a Virtual Model to the Green’s Functions and Phase Advances

To create a virtual machine model from the measured BPM data, we performed an SVD-enhanced least-squares fitting of the machine variables to the Green’s functions and phase advances calculated from the BPM data. In order to do such a fitting, the measurement derived quantities, which are the Green’s functions and phase advances, can be expressed as a vector, $\vec{Y}_m$. The fitted machine variables are then expressed as a vector, $\vec{Y}$, which is a function of the fitting variables $\vec{X}$, such as the BPM gains and cross-couplings.

To perform the fitting, we initialize $\vec{X}$ at a reasonable guess, such as a prior machine configuration, and call that $\vec{x}_0$. Then we try to find $\vec{x}$ such that:

$$\vec{Y}(\vec{x}_0 + \vec{x}) = \vec{Y}(\vec{x}_0) + M\vec{x} + \vartheta(\vec{x}^2) = \vec{Y}_m$$

(9)

Or, equivalently:

$$M\vec{x} = \vec{Y}_m - \vec{Y}(\vec{x}_0) = \vec{b}$$

(10)

where $M = \frac{\partial \vec{Y}}{\partial \vec{X}}|_{\vec{x}_0}$ and $\vartheta(\vec{x}^2)$ represents terms of second order or higher, which we disregard. The standard least-squares fitting solution is $\vec{x} = (M^T M)^{-1} M^T \vec{b}$. However, there may exist degeneracies which cause this solution to diverge, and so instead of using a standard least-squares fitting, we use an SVD-enhanced least-squares fitting [3]. The SVD-enhanced fitting solves eqn. 10 by using dominant SVD modes of $M$. After each iteration, we set $\vec{x}_0 = \vec{x}_0 + \vec{x}$, update the derivative matrix $M$, and begin again. After many iterations, the calculated Green’s functions and phase advances, $\vec{Y}$, converge to the measured values, $\vec{Y}_m$, and we have obtained our virtual accelerator model. We used this process to fit a virtual accelerator to our four orbits.

0.4 Finding and Implementing a Wanted Model

Once we obtained a virtual accelerator model, we found a Wanted Model, or a solution which should improve the optics of the accelerator. To do this, we first selected a small number of magnets to use as variables. The magnets which we chose were the trombone quadrupoles, the global and local skew quadrupoles, and the symmetric Y sextupole
bumps. We then figured out, with MATLAB, a way to best vary these magnet strengths in order to improve certain linear optics parameters, which we weight according to their current importance to us. In this particular project, the goal was to improve the coupling in the LER, and so we heavily weighted the coupling parameters. We also assigned heavy weight to the tunes and beta functions at the interaction point, as we did not want to disturb the LER’s current interaction point configuration. Disturbing this configuration would lead to different interaction with the HER, and so even if the LER had improved linear optics, it would not interact with the HER properly and thus we would not achieve high luminosity.

After we found the Wanted Model, we programmed our magnet strength adjustments into a physical knob which would dial the adjustments slowly into the real accelerator. We dialed the adjustments in and then measured the coupling, emittance, and luminosity. After dialing in our solution, the HER required some adjustment in order to match the new LER configuration at the IP, as some IP changes in the LER are unavoidable. We kept measuring luminosity as the HER was adjusted.

**Results**

After taking data from the PEP-II LER, we successfully created a virtual accelerator which fit the BPM gains and cross-coupling, as well as the betatron phase advances and even the coupling parameters, very well [See Figure 1 and Figure 3]. Surprisingly, we discovered a strange pattern in the BPM gains. A section of about 60 BPMs had a gain significantly lower than 1, as low as .6 and with an average of about .75, meaning that a very large section of BPMs was measuring with an average error of 25% [See Figure 2].

After finding the virtual machine and discerning that it was a good fit, we went on in search of a solution which would improve the coupling of the LER. We came up with three solutions which each had their various good and bad points. We compared the solutions based upon their predicted emittance, coupling, tune, and other factors [See Figure 4 for a comparison of emittance]. We wanted the solution to improve coupling
while not changing IP parameters. We picked our first solution, which improved the coupling greatly and also decreased emittance, while not changing any other parameters by too much [See Figure 5].

When we dialed this solution into the machine, the coupling decreased almost exactly as expected. This was a great success [See Figure 6]. Emittance also decreased, as predicted, by 40%. After adjusting the HER to match the LER and restoring its correct energy settings, luminosity increased by 10%, from $10.2 \times 10^{33} \text{cm}^{-2} \text{sec}^{-1}$ to $11.2 \times 10^{33} \text{cm}^{-2} \text{sec}^{-1}$. We also achieved a record peak specific luminosity, or luminosity divided by current [See Figure 7].

**Discussion and Conclusion**

One concern that arose during the modeling process was the section of BPMs with abnormally low gain. To discover whether these BPMs were actually measuring badly or whether our model was simply a bad fit, we tried removing this section of BPMs and even fitting with up to 300 Green’s functions, and in both situations the virtual machine still fit very well [See Figure 1]. Also, since the coupling parameters of our virtual model fit the coupling parameters of the data well [See Figure 3], and coupling was not a fitting variable we used but a check of the model correctness, we had strong reason to believe that our virtual model was a good fit, and thus that the strange BPM gain patterns were real. When we measured the accelerator again after dialing in our solution, we found the same patterns, further supporting the claim that the BPM gains were not an effect of our virtual model.

When dialing our solution into the machine, we noticed that there was a large amount of improvement in the machine when only 50% of the planned magnet strength adjustment had been made. We did some quick calculation, and discovered that 50% of our solution still decreased emittance by a large amount. Staying at this 50% was also less risky, as it involved less magnet strength change. Based upon this, we decided to stop at 50% of the original solution. This may have sacrificed a small bit of emittance decrease, but with
much less risk of causing unpredicted changes in the machine.

At the end of this project, the decrease in emittance and large increase in luminosity showed that our procedure for improving coupling was a good one, and that we effectively altered the strengths of the symmetric Y sextupole bumps to achieve our goal, which had not been done before. After this success, a similar procedure will be done to improve the linear optics of the HER and possibly further increase luminosity.

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**References**


Figure 1: A plot showing linear optics parameters as a function of ring circumference in meters (unless otherwise indicated) for the LER data (green), the Virtual Machine created from the data (red), and the ideal lattice configuration (blue). The Virtual Machine is a good fit to the data but is far from the ideal lattice. Note especially the coupling, in the fifth row and first column, which is the target parameter to be fixed in this project.
Figure 2: Top: BPM gain, for x and y measurements, as a function of BPM number. These gains are fitted from the BPM data. Bottom: BPM cross-coupling, for x and y measurements, as a function of BPM number. These are also fitted from the BPM data.

Figure 3: Coupling in our calculated Virtual Machine (blue) compared to the raw BPM data (red). The coupling fits very well, though we did not use coupling as a fitting parameter. This shows that our model is a good one.
Figure 4: A comparison of the integrated Y emittance of the original virtual machine (red) and our three derived solutions (chosen solution in dark blue). X emittance, which is less important as it is generally much larger, is not shown and was similar for all three solutions.

Figure 5: The coupling, as a function of distance from the Interaction Point, of the original virtual machine (red) and our chosen solution (blue). This predicts that coupling will improve with our solution, as the goal is for coupling to be smaller in magnitude.
Figure 6: The coupling, as a function of distance from the IP, of the original virtual machine (red), as predicted for our solution (blue), and the machine after we implemented our solution (green). This shows that we succeeded in improving coupling, though we were not completely successful in some areas.

Figure 7: The Luminosity as a function of current. In red is the luminosity after our solution was applied. This shows a new record peak specific luminosity.