Domain Wall Fermions at Ten Years

March 15-17, 2007

Organizers:

Thomas Blum and Amarjit Soni

RIKEN BNL Research Center

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Preface to the Series

The RIKEN BNL Research Center (RBRC) was established in April 1997 at Brookhaven National Laboratory. It is funded by the "Rikagaku Kenkyusho" (RIKEN, The Institute of Physical and Chemical Research) of Japan. The Center is dedicated to the study of strong interactions, including spin physics, lattice QCD, and RHIC physics through the nurturing of a new generation of young physicists.

The RBRC has both a theory and experimental component. The RBRC Theory Group and the RBRC Experimental Group consist of a total of 25-30 researchers. Positions include the following: full time RBRC Fellow, half-time RHIC Physics Fellow, and full-time, post-doctoral Research Associate. The RHIC Physics Fellows hold joint appointments with RBRC and other institutions and have tenure track positions at their respective universities or BNL. To date, RBRC has ~50 graduates of which 14 theorists and 6 experimenters have attained tenure positions at major institutions worldwide.

Beginning in 2001 a new RIKEN Spin Program (RSP) category was implemented at RBRC. These appointments are joint positions of RBRC and RIKEN and include the following positions in theory and experiment: RSP Researchers, RSP Research Associates, and Young Researchers, who are mentored by senior RBRC Scientists. A number of RIKEN Jr. Research Associates and Visiting Scientists also contribute to the physics program at the Center.

RBRC has an active workshop program on strong interaction physics with each workshop focused on a specific physics problem. Each workshop speaker is encouraged to select a few of the most important transparencies from his or her presentation, accompanied by a page of explanation. This material is collected at the end of the workshop by the organizer to form proceedings, which can therefore be available within a short time. To date there are eighty-three proceeding volumes available.

A 10 teraflops RBRC QCDOC computer funded by RIKEN, Japan, was unveiled at a dedication ceremony at BNL on May 26, 2005. This supercomputer was designed and built by individuals from Columbia University, IBM, BNL, RBRC, and the University of Edinburgh, with the U.S. D.O.E. Office of Science providing infrastructure support at BNL. Physics results were reported at the RBRC QCDOC Symposium following the dedication. QCDSP, a 0.6 teraflops parallel processor, dedicated to lattice QCD, was begun at the Center on February 19, 1998, was completed on August 28, 1998, and was decommissioned in 2006. It was awarded the Gordon Bell Prize for price performance in 1998.

N. P. Samios, Director
March 2007
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INTRODUCTION

Thomas Blum and Amarjit Soni

The workshop was held to mark the 10th anniversary of the first numerical simulations of QCD using domain wall fermions initiated at BNL. It is very gratifying that in the intervening decade widespread use of domain wall and overlap fermions is being made. It therefore seemed appropriate at this stage for some "communal introspection" of the progress that has been made, hurdles that need to be overcome, and physics that can and should be done with chiral fermions.

The meeting was very well attended, drawing about 60 registered participants primarily from Europe, Japan and the US. It was quite remarkable that pioneers David Kaplan, Herbert Neuberger, Rajamani Narayanan, Yigal Shamir, Sinya Aoki, and Pavlos Vranas all attended the workshop. Comparisons between domain wall and overlap formulations, with their respective advantages and limitations, were discussed at length, and a broad physics program including pion and kaon physics, the epsilon regime, nucleon structure, and topology, among others, emerged.

New machines and improved algorithms have played a key role in realizing realistic dynamical fermion lattice simulations (small quark mass, large volume, and so on), so much in fact that measurements are now as costly. Consequently, ways to make the measurements more efficient were also discussed.

We were very pleased to see the keen and ever growing interest in chiral fermions in our community and the significant strides our colleagues have made in bringing chiral fermions to the fore of lattice QCD calculations. Their contributions made the workshop a success, and we thank them deeply for sharing their time and ideas.

Finally, we must especially acknowledge Norman Christ and Bob Mawhinney for their early and continued collaboration without which the success of domain wall fermions would not have been possible.
David Kaplan
Institute for Nuclear Theory
Seattle

The Old Fart Talk:

- Tedious Reminiscences
- Narrow Survey of the Current Situation
- Pretentious Prognostications about the Future
- Ideas of the Past Week
- Closing platitude

Anomalies, and their role in the genesis of DWF

Chiral fermions as they appear in supersymmetric theories

The future challenge: systems with many fermions

An quick alternative to sewing many propagators together
I. Tedious Reminiscences: Anomalies and DWF

Why was the Callan-Harvey paper so exciting?

Because it was an example of a theory with accidental low energy chiral symmetry:

- No relevant operator could spoil the chiral symmetry, since LH and RH modes were physically separated by P violating bulk mass term
- The UV theory possessed no chiral symmetry at all
- Anomalies were explicitly accounted for.

Applicable to lattice QCD? No interest in 1988.
Revived DWF as a theory for chiral gauge theories (1992). Found that CH scenario works perfectly on the lattice, with chiral symmetry violating Wilson operators in the bulk.

Problem: on finite lattice need bizarre gauge dynamics or explicit gauge symmetry violation so that gauge fields only see chiral mode instead of vector.

Lattice exposes CH error: Anomalous current flow from one side (with Golterman, Jansen (1992))
One of the challenges for lattice SUSY is the fact that SUSY theories typically possess chiral symmetries.

- \( N=1 \) SYM: Gauge + gaugino fields, classical \( U(1) \) (broken by anomalies)
- \( N=4 \) SYM: Gauge + 4 gauginos + 6 scalrs, \( SU(4) \) R-symmetry

Need accidental SUSY + accidental chiral symmetry...compatible??
• Latticized Dirac-Kahler fermions are equivalent to staggered fermions.

• Staggered fermions have a well defined geometric significance

• Point group of the lattice lies in a nontrivial subgroup of (Lorentz x Flavor)

• Key to SUSY lattices: staggered scalars, stagger gauge bosons. Nontrivial lattice symmetry repps can become quite different continuum reps
III. Pretentious Prognostications and My Thoughts of Last Week

A fundamental obstacle for more nucleons?

\[ O(x, y) = u(x_1) \cdots u(x_m) d(y_1) \cdots d(y_n) \]

Correlator \( O(z_i)O^\dagger(z_f) \) requires \((m! \times n!)
propagator contractions?!

Triton: 5! x 4! = 2,880

\(^4\text{He}: 6! \times 6! = 518,400

\(^{238}\text{U} \sim 10^{1516}

No! Calculating \( \langle \det[D^{-1}(x_i,x_j)] \rangle \) for N
fermions...difficulty scales as \( N^3 \), not \( N! \)

Can this observation help us with other aspects of
dense fermions?
2+1 flavor DWF QCD:
Zero and finite temperature ensembles and $\Delta S = 1$ physics

Robert D. Mawhinney
For the RBC and UKQCD Collaborations\textsuperscript{1}

During the last 10 years, studies of domain wall fermion QCD have progressed from quenched simulations, to 2 flavor simulations, and now to 2+1 flavor simulations. The residual mass ($m_{\text{res}}$) in quenched simulations was markedly decreased by moving from the Wilson gauge action to the Iwasaki and DBW2 gauge actions. These actions smoothed the gauge fields at the lattice scale and reduced the density of small dislocations, $\rho(0)$, seen by the four-dimensional Wilson operator, that govern the domain wall fermion modes that are not localized in the fifth dimension. The form for the residual mass is seen to be

$$m_{\text{res}} \sim \frac{\rho(0)}{L} + a \exp(-aL)/L$$

Adding fermions and keeping the lattice spacing fixed requires simulations at smaller $\beta$ values, which leads to rougher gauge fields and a large value for $\rho(0)$. Even with the DBW2 gauge action, for 2+1 flavors the coarser gauge fields lead to values for $m_{\text{res}}$ that are similar to those obtained in the quenched case with the Wilson gauge action.

The RBC and UKQCD collaborations have found that the Iwasaki gauge action provides a reasonable balance between a small value for $m_{\text{res}}$ and the rate of tunneling of global topological charge. Using the QCDOC computers, these collaborations have generated the ensembles shown on the next page, except for the first ones which were generated on QCDSP. Many talks about the physics of these ensembles will be presented at this workshop.

The earliest 2 flavor DWF QCD simulations were at finite temperature. Along with Pavlos Vranas of IBM, we are now doing simulations of 2 and 2+1 flavor DWF QCD at finite temperature on $N_t = 8$ lattices. As we saw for earlier simulations with $N_t = 6$, the $N_t = 8$ evolutions show noticeable fluctuations in $\langle \bar{\psi} \psi \rangle$ (second page). We are currently scanning $\beta$ for 2+1 flavor DWF QCD to determine the location of the finite temperature phase transition on $16^3 \times 8 \times 32$ lattices.

We are measuring the matrix elements needed for a determination of $K \rightarrow \pi \pi$ decays, using lowest order chiral perturbation theory, on our $24^3 \times 64 \times 16$ 2+1 flavor ensembles. The third page shows our measurements. Page four shows the long plateaus we determine on our current lattices for $\Delta I = 3/2$ amplitude and compares them with earlier results on quenched $16^3 \times 32 \times 16$ lattices. Page five shows our first results for the subtracted matrix element $\langle \pi^+ | Q_{\text{sub}}^{(1/2)} | K^+ \rangle$. This matrix element requires the subtraction of a power divergent quantity and we see relatively small statistical errors from the data already accumulated.

### Zero Temperature Ensembles

<table>
<thead>
<tr>
<th>Volume</th>
<th>$a^{-1}$ (GeV)</th>
<th>$(m_l, m_s)$</th>
<th>$m_{res}$</th>
<th>MD time units</th>
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<tr>
<td>$16^3 \times 32 \times 12$</td>
<td>1.69(5)</td>
<td>$(0.02, \infty)$</td>
<td>0.00137(5)</td>
<td>2680.5</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$(0.03, \infty)$</td>
<td></td>
<td>3097.5</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$(0.04, \infty)$</td>
<td></td>
<td>3252.5</td>
</tr>
<tr>
<td>$16^3 \times 32 \times 8$</td>
<td>1.8(1)</td>
<td>$(0.02, 0.04)$</td>
<td>0.0107(1)</td>
<td>1797.5</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$(0.04, 0.04)$</td>
<td></td>
<td>1797.5</td>
</tr>
<tr>
<td>$16^3 \times 32 \times 16$</td>
<td>1.62(4)</td>
<td>$(0.01, 0.04)$</td>
<td>0.00308(4)</td>
<td>4015</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$(0.02, 0.04)$</td>
<td></td>
<td>4045</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$(0.03, 0.04)$</td>
<td></td>
<td>4020+3580</td>
</tr>
<tr>
<td>$24^3 \times 64 \times 16$</td>
<td>1.6-1.7</td>
<td>$(0.005, 0.04)$</td>
<td>0.0031</td>
<td>4500</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$(0.01, 0.04)$</td>
<td></td>
<td>3785</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$(0.02, 0.04)$</td>
<td></td>
<td>2850</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$(0.03, 0.04)$</td>
<td></td>
<td>2813</td>
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<tr>
<td>$32^3 \times 64 \times 16$</td>
<td>2.1-2.2</td>
<td>$(0.004, 0.03)$</td>
<td>$\approx 0.0005$</td>
<td>500</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$(0.006, 0.03)$</td>
<td></td>
<td>892</td>
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First row is with DBW2 gauge action, all others use the Iwasaki action.
$N_t = 8$ DWF Thermodynamics

- Preliminary simulations underway on BGLs at Yorktown Heights, Livermore and on QCDOC at CU.

- Considerable coding effort by Jung, Vranas and Cheng.

- Vranas has scanned the transition for 2 flavors and sees features in $\langle \bar{\psi} \psi \rangle$ just above $T_c$.

- 2+1 flavor parameters estimated from $T=0$ results - simulations underway by Cheng and Renfrew.
$\Delta S = 1$ Measurements on 2+1 flavor, $24^3$ ensembles

- Valence masses 0.001, 0.005, 0.01, 0.02, 0.03, 0.04
- Concentrating on 0.005/0.04 and 0.01/0.04 ensembles
- Large contributions by Tom Blum, Saul Cohen, Sam Li.
- 0.005/0.04 ensemble: 40 configurations separated by 80 MD time units
  0.01/0.04 ensemble: 30 configurations separated by 80 MD time units
- Concentrating on lighter quark masses where NLO chiral perturbation theory should be reasonable.
- Coulomb gauge fixed wall sources at $t = 5$ and 59
- Random noise source of length 40 for pupil calculations
$\Delta I = 3/2$ Plateau Comparisons

Previous Quenched

New 2+1 Flavor
Subtracted $Q_6$ Comparison

$$\langle \pi^+ | Q_{i,\text{lat}}^{(1/2)} | K^+ \rangle + \eta_{1,i} (m_s + m_d) \langle \pi^+ | (\bar{s}d)_{\text{lat}} | K^+ \rangle$$

Previous Quenched  

New 2+1 Flavor

![Graphs comparing previous and new results]
Dynamical QCD with exact chiral symmetry — from the $p$-regime to the $\varepsilon$-regime

Shoji Hashimoto (KEK)

@ “Domain Wall Fermions at Ten Years”, BNL, Mar 15, 2007
Panel discussion on chiral extrapolation (chaired by S. Sharpe)

- Chiral log not seen.
- Sea quark masses too large for controlled extrapolation.
  - Introduce a model dependence.
  - Extraction of LECs unreliable.

- Maybe, the standard $\chi$PT not applied for Wilson fermion (nor staggered)

JLQCD (2002)
Nf=2 improved Wilson fermion

Motivation for
- much lighter sea quarks
- exact chiral symmetry
Plan of this talk

1. Overlap-Dirac operator
   - Spectrum of $H_W$
   - Topology conserving action
   - Topology issues
   - Locality

2. Dynamical overlap
   - Implementation/ improvement

3. Pions and Kaons
   - Low-Mode
     Preconditioning
   - Low-Mode Averaging
   - Chiral log
   - Other targets

4. $\epsilon$-regime
   - Simulations
   - Eigenvalue spectrum
   - prospects

An overview: Details will be covered by other members.
Near-zero mode suppression

Obvious solution = introduce unphysical (heavy negative mass) Wilson fermions (Vranas, JLQCD, 2006)

\[ \det H_W (-m_0)^2 \]

- Our choice

\[ \det \left[ \frac{H_W (-m_0)^2}{H_W (-m_0)^2 + \mu^2} \right] \]

to minimize the effect in UV region.

Plaquette gauge,
\[ \beta=5.83, \mu=0; \beta=5.70, \mu=0.2 \]

Completely wash-out the near-zero modes. Overlap is much faster.
Parameters

- $\beta=2.30$, $\alpha=0.12$ fm, $16^3 \times 32$
- 6 sea quark masses $m_q = 0.015 \ldots 0.100$, covering $m_s/6 \sim m_s$
- 10,000 HMC traj.
  - $\sim 4,000$ with 4D solver
  - $\sim 6,000$ with 5D solver
- $Q=0$ sector only, except $Q=-2, -4$ runs at $m_q = 0.050$

$\Rightarrow$ Kaneko's talk
Chiral log?

Data points now extended down to $m_s/6$ with exact chiral symmetry.

Is the chiral log seen?

\[
\frac{f_\pi}{f} = 1 - \frac{N_f}{2} \frac{m_\pi^2}{(4\pi f)^2} \ln \frac{m_\pi}{\Lambda^2} + \ldots
\]

Testing with

- $m_\pi^2/m_q$
- $f_\pi$

$\Rightarrow$ Noaki’s talk

further tests
Comparison with $\chi$RMT

$\chi$RMT

- Equivalent to $\chi$PT at the LO in the $\varepsilon$-expansion.
- Predicts eigenvalues of $D$ in unit of $\lambda\Sigma V$.
  - $\langle \lambda_1 \rangle \Sigma V = 4.30$
  - $\langle \lambda_2 \rangle \Sigma V = 7.62$
  - ...
  for $N_f=2$, $Q=0$.
- $\Sigma$ may be extracted from average eigenvalues.
QCD Simulations at Realistic Quark Masses: Probing the Chiral Limit

G. Schierholz
Deutsches Elektronen-Synchrotron DESY

Abstract – To better constrain the extrapolation of the lattice results to the chiral limit, and to catch the physics of the pion cloud surrounding hadrons, simulations at realistic quark masses are necessary. In this talk I report on recent simulations by the QCDSF collaboration of two-flavor lattice QCD at small quark masses. Results are presented for a variety of observables. To guide the extrapolation, contact is made with chiral perturbation theory, after correcting for finite size effects. Some of the pertinent low-energy constants are computed and compared with phenomenology. For the first time the rho mass is extracted from data below the two-pion threshold.
\[ r_0 f_0 = 0.179(2), \quad r_0 \Lambda_3 = 1.82(7) \]
Pion Decay Constant

\[ f_{PS} = f_0 \left[ 1 + x \hat{l}_4 + O(x^2) \right] \]

\[ \frac{f_{PS} - f_{PS}(L)}{f_{PS}} = \sum_{|n| \neq 0} \frac{x}{\lambda} \left[ I_{fPS}^{(2)}(\lambda) + x I_{fPS}^{(4)}(\lambda) \right] \]

\[ r_0 f_0 = 0.179(2) \quad r_0 \Lambda_4 = 3.32(6) \]

\[ f_{PS} \leftarrow \text{NPRen} \]
Determination of Scale

\[ Z_A \text{ cancels, FS effects & leading log's largely cancel} \]

\[ r_0 = 0.45(1) \text{ fm} \]
Rho Mass

\[
\begin{align*}
\text{Raw data}
\end{align*}
\]
Effective range formula: 

\[ \frac{k^3}{W} \cot \delta_{11} = \frac{4k_\rho^5}{m_\rho^2 \Gamma_\rho} \left( 1 - \frac{k^2}{k_\rho^2} \right), \quad \Gamma_\rho = \frac{g_{\rho \pi \pi}^2 k_\rho^3}{6\pi m_\rho^2} \Rightarrow m_\rho \]
Chiral phase transition in large N QCD
with overlap fermions

R. Narayanan

Department of Physics, Florida International University, Miami, FL 33199.

H. Neuberger

Department of Physics and Astronomy, Rutgers University, Piscataway, NJ 08855.

Overlap fermions and domain wall fermions both use the hermitian Wilson-Dirac operator, $H_w$, with negative mass as the kernel. Wilson fermions feel gauge field topology and therefore are also sensitive to topology changing configurations. One finds arbitrarily small eigenvalues of $H_w$ and consequently, one needs to go to large $N_c$ in domain wall fermions. There is also the need to use projection operator for the correct evaluation of $\epsilon(H_w)$.

There is a bulk transition on the lattice in the large N limit at some lattice coupling $b_B = \frac{1}{3}g_B$ where the eigenvalue distribution of the Polyakov loop opens a gap. The space of lattice gauge fields comes in disconnected pieces for $b > b_B$ and $H_w$ does not have small eigenvalues. This along with the fact that fermions in the fundamental representation are naturally quenched in the large N limit makes it attractive to numerically study the chiral limit of large N QCD using overlap or domain wall fermions.

Continuum large N gauge theory on a torus of size $l$ can occur in several phases labeled as $Xc$ with $X = 0, 1, 2, 3, 4$. There is a physical torus size associated with each one of these transitions. Chiral symmetry is broken in the 0c phase in the large N limit of QCD. The theory in the 1c phase behaves like finite temperature large N QCD in the deconfined phase. The first order deconfinement transition drives a first order chiral transition.

Simple minded random matrix models do not work in the chirally symmetric phase in contrast to the broken phase. One has to augment the random matrix models with two more fluctuating random variables that are correlated with the standard random matrix variables.

The above results are described in detail in the following papers:

1. J. Kiskis, R. Narayanan and H. Neuberger, hep-lat/0203005; hep-lat/0308033.

2. R. Narayanan and H. Neuberger, hep-lat/0405025; hep-th/0605173; hep-lat/0612006.

3. J. Kiskis, hep-lat/0507003.
1 Domain Wall Fermions and Overlap Formalism 1992-1998

- Chiral fermions on the lattice using a mass defect: D. Kaplan: hep-lat/9206013
- Infinite number of Wilson fermions can be used to realize one chiral fermion: R. Narayanan and H. Neuberger, hep-lat/9212019
- Integrating out infinite number of fermions – Overlap formalism: R. Narayanan and H. Neuberger, hep-lat/9307006
- Schwinger model – Test of the overlap formalism: R. Narayanan, H. Neuberger and P. Vranas, hep-lat/9503013
- Can the results of the Schwinger model be reproduced using domain wall fermions (finite number of Wilson fermions)?: P. Vranas, hep-lat/9608078; hep-lat/9705023
- Domain wall fermions with ten Wilson fermions (length of the extra direction) are enough to get the expected behavior in the chiral limit: T. Blum and A. Soni: hep-lat/9611030
- There is an explicit fermion operator for vector-like theories in the overlap formalism: H. Neuberger: hep-lat/9707022

2 A lattice phase transition at large N

- Let \( P \in SU(N) \) denote the parallel transporter around a single plaquette.
The eigenvalues $e^{i\theta_k}$, $k = 1, \ldots, N$ of $P$ are gauge invariant and independent of the point where the loop is opened.

Consider the quantity $\rho(\theta) d\theta$ which is the probability of finding an eigenvalue $e^{i\theta_k}$ in the range $\theta < \theta_k < \theta + d\theta$ for some $k$.

$\rho(\theta)$ has no gap at lattice strong coupling and develops a gap around $\theta = \pi$ as the coupling gets weaker on the lattice in the large $N$ limit.

This is the bulk transition on the lattice and is a cross-over for finite $N$.

It is the third order Gross-Witten transition in QCD$_2$ D. Gross and E. Witten, Phys. Rev. D21 (1980) 446.

This transition continues to be third order in $d = 3$ (F. Bursa and M. teper, hep-th/0511081) and it is first order in $d = 4$ (J. Kiskis, R. Narayanan and H. Neuberger, hep-lat/0203005).

The weak coupling side of this transition separates lattice gauge fields into disconnected pieces. This prohibits topology changing configurations in a typical local update algorithm. Global changes in the gauge field are needed to change topology. Wilson fermions will not have small eigenvalues unlike the case in finite $N$ (J. Kiskis, R. Narayanan and H. Neuberger, hep-lat/0203005).

3 Phases of large $N$ QCD:

- R. Narayanan and H. Neuberger, hep-lat/0303023; J. Kiskis, R. Narayanan and H. Neuberger, hep-lat/0308033

Large $N$ gauge theory on a continuum $t^4$ torus has several phases denoted by $X_c$. $X$ ranges from 0 to 4 and corresponds to the number of directions along which Polyakov loops are broken.

There exists a critical size $l_c$ that separates the 0c phase ($l > l_c$) from the 1c phase ($l < l_c$).

Continuum reduction holds in the 0c phase and the theory does not depend on $l$ if $l > l_c$. $l_c = 1/T_c$ and physics is independent of the
temperature $T$ for $T < T_c$. Chiral symmetry is broken in the $0c$ phase
(R. Narayanan and H. Neuberger, hep-lat/0405025).

- The theory in the $1c$ phase behaves like finite temperature large $N$
  QCD in the deconfined phase. The deconfinement transition drives
  the chiral transition and chiral symmetry is restored in the $1c$ phase
  (R. Narayanan and H. Neuberger, hep-th/0605173) Numerical evidence
  indicates that the phase transition from $0c$ to $1c$ is first order (J. Kiskis,
  hep-lat/0507033).

4 Numerical computation of the chiral condensate

- Pick some $L$ and choose $b < b_c(L)$ such that the theory is in the $0c$
  (confined) phase

- No finite volume effects.

- Keep $b$ close to $b_c(L)$ to minimize finite spacing effects.

- Use the overlap Dirac operator that respects exact chiral symmetry on
  the lattice. Let $A(\mu)$ denote the massive overlap Dirac operator with
  $\mu$ being the bare mass on the lattice.

\[
\Sigma = \lim_{\mu \to 0} \lim_{N \to \infty} \frac{1}{L^4 N} \langle \text{Tr} A^{-1}(\mu) \rangle_{N,L}
\]

- We are working at a fixed lattice volume.

- The first step is to take the large $N$ limit.

- The second step is to take the massless limit.

- Absence of finite volume effects means that $\Sigma$ does not depend on $L$ at
  the given gauge coupling.

- Continuum limit is obtained by increasing $b$ and suitably changing $L$
  such that one is always in the $0c$ phase.

- Let $\pm i \lambda_i$ with $\lambda_1 \leq \lambda_2 \leq \cdots \leq \lambda_K$ be the eigenvalues of $A(0)$.
• Consider the scales variables $z_k = \lambda_k \Sigma N L^4$.

• Extensive work in the area of chiral RMT has shown that the probability distributions, $p(z_k)$, are universal functions as $L \rightarrow \infty$ at fixed $N$.

• Compute the two lowest eigenvalues $\lambda_1$ and $\lambda_2$.

• Check if the ratio $r = \frac{\lambda_1}{\lambda_2}$ obeys the universal function dictated by chiral RMT.

• Find the common $\Sigma$ that converts $\lambda_1$ and $\lambda_2$ into $z_1$ and $z_2$.

• If we use $T_c \approx 0.6\sqrt{\sigma} \approx 264\text{MeV}$ (B. Lucini, M. Teper and U. Wenger, hep-lat/0307017), and if we use perturbative tadpole improved estimates for $\Sigma$ in the $\overline{MS}$ scheme, we get $\frac{1}{N^2}(\overline{\psi}\psi)_{\overline{MS}}(2\text{GeV}) \approx (174\text{MeV})^3$

5 Restoration of chiral symmetry in the 1c phase

• Fermions do matter in the 1c phase even in the 't Hooft limit.

• Fermion determinant will depend on the “momentum” that is force-fed in the broken direction.

• In other words, boundary conditions in the temperature direction matters.

• Let $\theta$ be the phase associated with the $U(1)$ that defines the boundary condition with respect to the phase of the Polyakov loop in the broken direction. Let $\theta = 0$ define anti-periodic boundary conditions.

• The fermion determinant will depend on $\theta$ and one hopes that fermions will pick $\theta = 0$.

• Consider the lowest eigenvalue of the overlap Dirac operator as a measure of the fermion determinant and look at this as a function of $\theta$.

• The data shows a gap in the spectrum for all $\theta$ as long as $T > T_c$. This shows strong interaction in the color space.
• The gap is the biggest for $\theta = 0$.

• We do not have a chiral Langrangian to motivate us toward a Random Matrix Model in the chirally symmetric phase.

• The lattice data does not agree with the prediction of standard random matrix model.

• In order to model the lattice data in the deconfined phase, we propose the following relation:
  \[ \lambda_j = \alpha' \xi_j + \beta' \]
  where $\alpha'$ and $\beta'$ are two additional fluctuating variables. Let $\mu_j = \lambda_j - \lambda_1$ and $\eta_j = \xi_1 - \xi_j$.

• We have
  \[ \ln \mu_j \leftrightarrow \ln \eta_j + \ln(-\alpha') \]
  and this can be tested by looking for $j$ independence of the LHS. The lattice data is in agreement.

• In order to look for fluctuations, we consider
  \[ \Delta_j = \ln \mu_j - \langle \ln \mu_j \rangle, \delta_j = \ln \eta_j - \langle \ln \eta_j \rangle, \delta = \ln(-\alpha') - \langle \ln(-\alpha') \rangle \]
  and obtain
  \[ \langle \Delta_j \Delta_k \rangle \approx \langle \delta_j \delta_k \rangle + \langle \delta_j \rangle + \langle \delta_k \rangle \]
  The first term on the LHS is obtained from the data and the second term on the LHS from the simplest, unextended, RMM.

• If $\alpha'$ were a fixed parameter, $\delta \equiv 0$ and the LHS of the above equation should be zero. The data is not consistent with this scenario.

• If $\delta$ and $\delta_j$ are not correlated, the LHS should come out independent of $j$ and $k$. The data is not consistent with this scenario.

• $\alpha'$ and $\beta'$ are fluctuating random variables that are correlated to the rest of the random matrix model variables.
• The gap is the biggest for $\theta = 0$.

• We do not have a chiral Langrangian to motivate us toward a Random Matrix Model in the chirally symmetric phase.

• The lattice data does not agree with the prediction of standard random matrix model.

• In order to model the lattice data in the deconfined phase, we propose the following relation:

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• We have

$$\ln \mu_j \leftrightarrow \ln \eta_j + \ln(-\alpha')$$

and this can be tested by looking for $j$ independence of the LHS. The lattice data is in agreement.

• In order to look for fluctuations, we consider

$$\Delta_j = \ln \mu_j - <\ln \mu_j>, \delta_j = \ln \eta_j - <\ln \eta_j>, \delta = \ln(-\alpha') - <\ln(-\alpha')>$$

and obtain

$$<\Delta_j \Delta_k> = <\delta_j \delta_k> + <\delta_j + \delta_k>$$

The first term on the LHS is obtained from the data and the second term on the LHS from the simplest, unextended, RMM.

• If $\alpha'$ were a fixed parameter, $\delta \equiv 0$ and the LHS of the above equation should be zero. The data is not consistent with this scenario.

• If $\delta$ and $\delta_j$ are not correlated, the LHS should come out independent of $j$ and $k$. The data is not consistent with this scenario.

• $\alpha'$ and $\beta'$ are fluctuating random variables that are correlated to the rest of the random matrix model variables.
Domain Wall Fermions and other 5D Algorithms

A D Kennedy
University of Edinburgh

All the algorithms we have for chiral lattice fermions can be formulated as a rational approximation to Neuberger's operator, and for a large class of such methods including DWF this approximation is the Schur complement of a five dimensional linear system. We compare the merits of implementing such five dimensional systems using four or five dimensional pseudofermion fields.
Neuberger’s Operator

\[ D_N(\mu, H) = \frac{1}{2} \left[ 1 + \mu + (1 - \mu) \gamma_5 \text{sgn}(H) \right] \]

- All the methods that are used to put chiral fermions on the lattice are rational approximations to Neuberger’s operator
  - They are not just analogous, there is a well-defined mapping between them
Disadvantages of 5D pseudofermions

- Introduce extra noise into the 4D world
  - Letting the 5D pseudofermions cancel stochastically with their Pauli-Villars partners ("pseudo-pseudo-fermions") is a very bad idea
  - Cancelling them explicitly is better, but one is still has $L_s^{-1}$ unnecessary noisy estimators of 1
  - These increase the maximum force on the 4D gauge fields and force the MD integration step-size to be smaller
Disadvantages of 5D pseudofermions

- Extent of the fifth dimension is fixed
  - At least over an entire HMC trajectory
  - With 4D pseudofermions one can adjust the degree of the rational approximation at each MD step to cover the spectrum of the kernel with fixed maximum error
Disadvantages of 4D Pseudofermions

- Cannot evaluate roots of Neuberger operator
  - We cannot use the multishift solver techniques used with 5D pseudofermions because we would need constant shifts of the 4D operator and not the 5D one
  - Not obvious how to implement odd number of flavours efficiently with 4D pseudofermions
  - Not obvious how to use $n^{th}$ root RHMC acceleration trick
    - But we can still use Hasenbusch's technique
  - These techniques can be implemented using nested 4D CG solver with multishift on both inner and outer solvers
Desiderata & Conclusions

- It would be nice to make progress in the following areas
  - Express $n^{th}$ roots of Neuberger operator as a 5D system
  - Work out how to systematically improve the condition number of 5D systems
    - The Schur complement of a 5D system is uniquely defined, but there are many 5D matrices with the same Schur complement
    - With each class of such 5D matrices it is often possible to greatly change the condition number by fairly simple transformations
    - It would be nice to know how to do this systematically
    - The evidence at present indicates that better approximations (e.g., Zolotarev rather than tanh) are not intrinsically worse conditioned
Speeding up Domain Wall Fermion Algorithms using QCDLAB

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Abstract

Simulating lattice QCD with chiral fermions and indeed using Domain Wall Fermions continues to be challenging project however large are concurrent computers. One obvious bottleneck is the slow pace of prototyping using the low level coding which prevails in most, if not all, lattice projects. Recently, we came up with a new proposal, namely QCDLAB, a high level language interface, which we believe will boost our endeavours to rapidly code lattice prototype applications in lattice QCD using MATLAB/OCTAVE language and environment. The first version of the software, QCDLAB 1.0 offers the general framework on how to achieve this goal by simulating set of the lattice Schwinger model http://phys.fsnh.edu.al/qcdlab.html. In this talk we introduce QCDLAB 1.1, which extends QCDLAB 1.0 capabilities for real world lattice computations with Wilson and Domain Wall fermions.

*Invited talk given at the 'Domain Wall Fermions at Ten Years', Brookhaven National Laboratory, 15-17 March 2007
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1 The challenge of Domain Wall Fermions computations &nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&n...
3 QCDLAB 1.0

QCDLAB is a high level language tool a collection of MATLAB functions for the simulation of lattice Schwinger model. It can be used as a small laboratory to test and validate algorithms. In particular, QCDLAB 1.0 serves as an illustration of the minimal prototyping code concept.

QCDLAB 1.0 can also be used for newcomers in the field. They can learn and practice lattice projects which are based on short codes and run times. This offers a “learning by doing” method, perhaps a quickest route into answers of many unknown practical questions concerning lattice QCD simulations.

The next two sections describe basic algorithms for simulation of lattice QCD and foundations of Krylov subspace methods. Then, we present the QCDLAB 1.0 functions followed by examples of simple computing projects. The last section outlines the future plans of the QCDLAB project for lattice QCD computations. It is based on the MATLAB and OCTAVE language and environment. While MATLAB is a product of The MathWorks, OCTAVE is its clone, a free software under the terms of the GNU General Public License.

MATLAB/OCTAVE is a technical computing environment integrating numerical computation and graphics in one place, where problems and solutions look very similar and sometimes almost the same as they are written mathematically. Main features of MATLAB/OCTAVE are:

- Vast build-in mathematical and linear algebra functions.
- Many functions form Blas, Lapack, Minpack, etc. libraries.
- State-of-the-art algorithms.
- Interpreted language.
- Dynamically loaded modules from other languages like C/C++, FORTRAN.

QCDLAB 1.0 is

- **General functions:**
  - Solvers: BiCg5, BiCgstab, CG, CGNE, POM, GMRES, Lanczos, SCG, SUMR
  - Data processing: Autocorel, Binning.

- **Specialised functions for the Schwinger model:**
  - Operators: Dirac.KS, Dirac.r, Dirac.W, cdot5
  - Measurements: wloop

a collection of functions for the simulation of lattice Schwinger model. It can be used as a small laboratory to test and validate algorithms. In particular, QCDLAB 1.0 serves as an illustration of the minimal prototyping code concept.

QCDLAB 1.0 can also be used for newcomers in the field. They can learn and practice lattice projects which are based on short codes and run times. This offers a “learning by doing” method, perhaps a quickest route into answers of many unknown practical questions concerning lattice QCD simulations. It contains the following MATLAB/OCTAVE functions:

<table>
<thead>
<tr>
<th>Autocorel</th>
<th>BiCg5</th>
<th>BiCgstab</th>
<th>Binning</th>
<th>cdot5</th>
</tr>
</thead>
<tbody>
<tr>
<td>CG</td>
<td>CGNE</td>
<td>Dirac.KS</td>
<td>Dirac.r</td>
<td>Dirac.W</td>
</tr>
<tr>
<td>POM</td>
<td>Force.KS</td>
<td>Force.W</td>
<td>GMRES</td>
<td>HMC.KS</td>
</tr>
<tr>
<td>HMC.W</td>
<td>Lanczos</td>
<td>SCG</td>
<td>SUMR</td>
<td>wloop</td>
</tr>
</tbody>
</table>

The functions can be grouped as in the following:

- **General functions:**
  - Solvers: BiCg5, BiCgstab, CG, CGNE, POM, GMRES, Lanczos, SCG, SUMR
  - Data processing: Autocorel, Binning.

- **Specialised functions for the Schwinger model:**
In order to get started with QCDLAB 1.0 one can run the following three projects:

- For Matlab: MProject1, MProject2, MProject3
- For Octave: OProject1, OProject2, OProject3

The user is required to download these m-files to the working directory of MATLAB/OCTAVE and then type the corresponding project names in the MATLAB/OCTAVE environment. The first project is a simulation project, the second one is a linear system and eigenvalue solver project, whereas the third one is a linear system and eigenvalue solver project for chiral fermions.

For more information on QCDLAB 1.0 the reader is referred to the complete documentation available at the project web page http://phys.fshn.edu.al/qcdlab.html.

4 QCDLAB 1.1

As stated in the first section, the goal of the QCDLAB project is to create an algorithmic prototyping environment for lattice QCD computations. This goal has no ending station, but rather it is a process which will enhance the computing capabilities as the time goes on. The first step in this direction is the 1.1 version. It offers new functionality for 4D and 5D computations at the linear system and eigenvalue solver level. The new functions are:

\[
\begin{align*}
\cdot5 & \quad \text{Dirac4} & \quad \text{InitialiseDiracW} & \quad \text{InversePower} \\
\text{Mult.DiracW} & \quad \text{Mult.DiracW.H} & \quad \text{Mult.DWF} & \quad \text{Mult.DWF.H} \\
\text{Multi.gamma5} & \quad \text{P5minus} & \quad \text{P5plus} & \quad \text{PowerMethod}
\end{align*}
\]

There are two ways to implement the Wilson operator:

- Creation of a sparse matrix using Dirac4

In the first case one needs to initialise the Wilson operator using the InitialiseDiracW function and to hack inversion solvers of QCDLAB 1.0 in the place where the multiplication with A takes place, e.g.

\[A \cdot x \rightarrow \text{Mult.DiracW}(x)\]

The same comment is for the Domain Wall Fermion matrix-vector functions Mult.DWF, Mult.DWF.W. Note that the 1.0 version of the \cdot5 function is now redefined for the 4D fermion theory and is included in the 1.1 version. Other useful functions are multiplication of a 4D-vector by \gamma_5: Multi.gamma5, the chiral projection functions applied to 4D-vectors: P5minus, P5plus.

Gauge field data structure

Both Dirac4 and InitialiseDiracW need the gauge field, which must be supplied as a set of four matrices: u1, u2, u3, u4. Each gauge field component is a \(9N \times 2\) matrix, where \(N\) is the total number of lattice sites. The first column is the real part and the second the imaginary part of the particular SU(3) matrix element. Note that, if reshaped, the most inner dimensions of gauge field component are 3 x 3 matrices, i.e.

\[\text{reshape(u1, 3, 3, N, 2)}\]

The user should care to organise the gauge field data in this way for the QCDLAB functions to work as required.

Sparse Wilson matrices

In case one want to create a sparse Wilson matrix one uses the Dirac4 function:
function A=Dirac4(u1,u2,u3,u4);
% Constructs Wilson-Dirac operator
mass=0; N1=8; N2=8; N3=8; N4=16; N=N1*N2*N3*N4;
% Gamma matrices
gamma1=[0,0,0,-i; 0,0,i,0; i,0,0,0; 0,0,0,0];
gamma2=[0,0,-1,0; 0,0,0,i; i,0,0,0; 0,0,0,0];
gamma3=[0,0,0,-i; 0,0,i,0; i,0,0,0; 0,0,0,0];
gamma4=[0,0,0,-i; 0,0,i,0; i,0,0,0; 0,0,0,0];
% Projection operators
P1_plus=eye(4)+gamma1; P1_minus=eye(4)-gamma1;
P2_plus=eye(4)+gamma2; P2_minus=eye(4)-gamma2;
P3_plus=eye(4)+gamma3; P3_minus=eye(4)-gamma3;
P4_plus=eye(4)+gamma4; P4_minus=eye(4)-gamma4;
% Shift operators
I1=speye(N1); I2=speye(N2); I3=speye(N3); I4=speye(N4);
T1=[I1(: ,pl); T2=[I2(: ,p2)]; T3=[I3(: ,p3)]; T4=[I4(: ,p4)];
E1=spkron(I4,spkron(I3,spkron(I2,spkron(T1,spkron(T2,spkron(T3,spkron(T4,spkron(I1,speye(3)))))))));
E2=spkron(I4,spkron(I3,spkron(I2,spkron(I1,spkron(T1,spkron(T2,spkron(T3,spkron(T4,spkron(I1,speye(3)))))))));
E3=spkron(I4,spkron(I3,spkron(I2,spkron(I1,spkron(T1,spkron(T2,spkron(T3,spkron(T4,spkron(I1,speye(3)))))))));
E4=spkron(I4,spkron(I3,spkron(I2,spkron(I1,spkron(T1,spkron(T2,spkron(T3,spkron(T4,spkron(I1,speye(3)))))))));
% Gauge Field configuration \{u_1, u_2, u_3, u_4\}: 9*N by 2 matrices
I_N=speye(N);
[I, J]=spfind(spkron(I_N,ones(3)));
U1=sparse(I, J, u1(: ,1)+i*u1(: ,2),3*N, 3*N);
U2=sparse(I, J, u2(: ,1)+i*u2(: ,2),3*N, 3*N);
U3=sparse(I, J, u3(: ,1)+i*u3(: ,2),3*N, 3*N);
U4=sparse(I, J, u4(: ,1)+i*u4(: ,2),3*N, 3*N);
% Upper triangular
U=spkron(P1_minus, U1*E1)+spkron(P2_minus, U2*E2)+spkron(P3_minus, U3*E3)+spkron(P4_minus, U4*E4);
% Lower triangular
L=spkron(P1_plus, U1*E1)+spkron(P2_plus, U2*E2)+spkron(P3_plus, U3*E3)+spkron(P4_plus, U4*E4);
%M==U+L';
A=(mass+4)*speye(12*N)-0.5*(U+L');
% Eigenvalue solvers
The 1.1 version comes with two eigenvalue solvers: PowerMethod, InversePower, which are implementations of the methods with the same name. They can be used for the Hermitian eigenvalue problems. For example, if one would like to compute the smallest eigenvalue of the Hermitian Wilson operator one can use the InversePower function:
function [v, lambda, rr]=InversePower(b,x0, tol, nmax);
% Inverse power method for the Hermitian Wilson operator
v=b/norm(b);
while 1,
  u=bicg5(v,x0,1e-6,1000);
  u=multi_gamma5(u);
  lambda=v'*u;
  r=v-u/lambda;
  rnorm=norm(r); rr=[rr;rnorm];
  if rnorm<tol, break, end
  v=u/norm(u);
end
% Copyright (C) 2006-2007 Artan Borici.
% This program is a free software licensed under the terms of the GNU General Public License.
Domain Wall Fermion operator

The `Mult_DWF` implements the Domain Wall Fermion operator

\[
M = \begin{pmatrix}
1 - D_W & P_+ & -mP_- \\
P_- & 1 - D_W & \\
-mP_+ & P_- & 1 - D_W
\end{pmatrix}
\]

applied to a vector:

```matlab
function y=Mult_DWF(x,N5);
% Multiplies a vector by the Domain Wall Fermion matrix
global N mass_dwf
x=reshape (x, 12*N,N5);

% y(:,1)=x(:,1)-Mult_Dirac_W(x(:,1))+P5plus(x(:,1))+P5minus(x(:,N5));
for j5=2:N5-1;
    y(:,j5)=x(:,j5)-Mult_Dirac_W(x(:,j5))+P5plus(x(:,j5)+1)+P5minus(x(:,j5-1));
end
y(:,N5)=x(:,N5)-Mult_Dirac_W(x(:,N5))+P5plus(x(:,1))+P5minus(x(:,N5-1));

x=reshape (x,12*N*N5,1);

y=reshape (y,12*N*N5,1);
```

% Copyright (C) 2006-2007 Artan Borici.
% This program is a free software licensed under the terms of the GNU General Public License

Acknowledgements

The author wishes to thank Tom Blum and Amarjit Soni for the invitation and the kind hospitality at BNL as well as Stefan Sint for useful discussion on possible extensions of QCDLAB’s Dirac operators with non-trivial boundary conditions.

References


Moebius DW Fermions
&
Ward-Takahashi identities*

Richard C. Brower


*RCB, Hartmut Neff and Kostas Orginos
hep-lat/0409118 & hep-lat/0703XXX
Moebius Generalization

\[
D_{Moebius}(M_5) = \frac{(b_5 + c_5)D_w(M_5)}{2 + (b_5 - c_5)D_w(M_5)} = \alpha \frac{a_5 D_w(M_5)}{2 + a_5 D_w(M_5)} = \alpha D_{Shamir}(M_5)
\]

Since \( \epsilon [\alpha x] = \epsilon [x] \)

Moebius is an new (scaled) polar algorithm

Parameters: \( M_5 \), \( a_5 = b_5 - c_5 \) and scale: \( a = b_5 + c_5 \)
$16^3 \times 32$ Gauge Lattice $@ \; \beta = 6 \; \& \; m = 0.44$

Shamir, $M_5=1.8, a_s=1.0$
- $\alpha = 2.0$
- $\alpha = 2.5$
- $\alpha = 3.0$
- $\alpha = 3.5$
- $\alpha = 4.0$
- $\alpha = 4.5$

$m_{res}$ vs. Number of Dirac applications
DW/Overlap Equivalence:

\[ \langle \mathcal{O}(q, \bar{q}) \rangle_{DW} = \langle \mathcal{O}(\psi, \bar{\psi}) \rangle_{ov} \]

where \( q = [\mathcal{P}^\dagger \psi]_1 \), \( \bar{q} = [\bar{\psi} D_{DW}(1) \mathcal{P}]_1 \)

\[ \Rightarrow \langle q_y \bar{q}_x \rangle \equiv \ldots \equiv D_{ov}^{-1}(m)_{xy} \equiv \langle \psi_y \bar{\psi}_x \rangle \]

\[ \mathcal{P}^\dagger \frac{1}{D_{DW}(m)} D_{DW}(1) \mathcal{P} = \]

\[ = \begin{bmatrix}
D_{ov}^{-1}(m) & 0 & 0 & \cdots & 0 \\
X_2 D_{ov}^{-1}(m) & 1 & 0 & \cdots & 0 \\
X_3 D_{ov}^{-1}(m) & 0 & 1 & \cdots & 0 \\
& \cdots & \cdots & \cdots & \cdots \\
X_L D_{ov}^{-1}(m) & 0 & 0 & \cdots & 1
\end{bmatrix} \]

**note:** **Standard**

\[ \bar{q} = [\bar{\psi}(-D_{-}^{(1)}) \mathcal{P}]_1 \]

\[ \Rightarrow \langle q \bar{q} \rangle = \frac{1}{1-m} [D_{ov}^{-1}(m) - 1] \]
Application of DW/overlap equ to currents

\[ \langle J_{\mu}^{ov}(x) \psi_y \bar{\psi}_z \rangle_{ov} = \langle J_{\mu}^{DW}(x) q_y \bar{q}_z \rangle_{DW} \]
Model for $m_{res}$ dependence on $\& L_s$

$$m_{res} = \sum_{\lambda} \rho(\lambda) \Delta_L(\lambda) \quad \rho(\lambda) = \frac{\langle \lambda | G_{ov} G_{ov}^\dagger | \lambda \rangle}{\sum_{\lambda} \langle \lambda | G_{ov} G_{ov}^\dagger | \lambda \rangle} \geq 0$$

( ) has negligible dependence on $\& L_s$

$$m_{res} \simeq \int d\eta(\lambda) \rho(\lambda) \Delta_L(\alpha \lambda)$$

$$\Delta_L(\lambda) = \langle \lambda | \Delta_L(H) | \lambda \rangle = \frac{4}{2 + \left[ \frac{1+\lambda}{1-\lambda} \right]^{-L} + \left[ \frac{1+\lambda}{1-\lambda} \right]^L} \geq 0$$

$$\rightarrow e^{-L \log[(1+\lambda)/(1-\lambda)]} \quad \text{for } O(L^{-1}) < \lambda < O(L)$$
Summary: I present the gap domain wall fermion (GDWF) method. I show that GDWF induce a substantial gap in the transfer matrix Hamiltonian along the fifth dimension. As a result they significantly improve the chiral properties of domain wall fermions in the large to intermediate lattice spacing regime of QCD, 1 to 2 GeV. Furthermore, I argue that this method should also improve the chiral properties of related lattice fermions [P.M. Vranas, hep-lat/0606014]
Gap Domain Wall Fermions

- Improve DWF in the region $1 \text{ GeV} < a^{-1} < 2 \text{ GeV}$.

- Since the problem occurs when $H(m_0)$ is small
  multiply the Botzeman weight with: $\det[H(m_0)] = \det[D(-m_0)]$

- This is the same as inserting Wilson fermions with heavy mass in the supercritical
  region (for example $m_0 = 1.9$). I will use 2 flavors.

- This will forbid zero crossings at $m_0$ and therefore enlarge the gap and reduce the
  residual mass.

- It will suppress instantons with size near the lattice spacing which are a lattice
  artifact (dislocations).

- Must check that the added Wilson fermions:
  - have hadron spectrum above the cutoff and are therefore irrelevant.
  - do not break parity (Aoki phase).
  - allow crossings due to instantons/anti-instantons with sizes $> a$ (active topology).
Quenched DWF, GDWF scale matching

- DWF data (diamonds) are from [RBC, PRD 69 (2004) 074502].
- Matching is better than 5%.
- Use the rho to set the scale.
$\beta = 5.7$

$\beta = 5.85$

$\beta = 6.0$

$\beta = 4.8$

$\beta = 4.4$

$\beta = 4.8$

$a^{-1} = 1 \text{ GeV}$

$a^{-1} = 1.4 \text{ GeV}$

$a^{-1} = 2 \text{ GeV}$

$\lambda$

$m_0$ (0 flavors)

$m_0$ (2 flavors)
The residual mass

\[ \bar{\beta} = \frac{1}{\beta} \]

- \( a^{-1} = 1.0 \text{ GeV} \)
- \( a^{-1} = 1.4 \text{ GeV} \)
- \( a^{-1} = 2.0 \text{ GeV} \)

DWF

GDWF

\[ m_f = 0.02 \text{ and } m_0 = 1.9 \]
Net topology change

- GDWF may reduce the net-topology sampling of the traditional HMC because it forbids smooth deformations of topological objects. The eigenvalue flow cannot cross $m_0$.
- This is only an algorithmic issue. We need algorithms that can tunnel between topological sectors.
- The same problem occurs in QCD anyway with or without GDWF. The QCD topological sectors are separated by energy barriers that become infinitely high as we approach the continuum. We have not been to small enough coupling yet in QCD to see the phenomenon.
- GDWF resemble continuous QCD in this way even more.
- In many cases net-topology change is not important provided one uses a large enough volume (see H. Leutwyler, A. Smilga, Phys. Rev D 46 (1992) 5607).
- It is important to see crossings in the larger-instanton regime since they confirm a topologically active vacuum. The net index may be fixed but the appearance/disappearance of instantons/anti-instantons is a property of the QCD vacuum and has to be there. Obviously then for large enough volumes cluster decomposition ensures correct physics.
Approaching the Chiral Limit with Dynamical Overlap Fermions

T. Kaneko for the JLQCD collaboration

1High Energy Accelerator Research Organization (KEK)
2Graduate University for Advanced Studies

"Domain Wall Fermions at Ten Years", March 15–17, 2007
0.1 basic properties of HMC

area preserving

\[ \Delta H \text{ at } m_{sea} = 0.025 \]

- a few spikes per \( O(10,000) \) trajectories: \( P_{\text{spike}} \lesssim 0.03 \% \)
- \( \exp[-\Delta H] = 1 \) in all runs
- does not need "replay" trick

reversibility

\[ \Delta U \text{ vs } \epsilon \]

\[ \Delta U = \sqrt{\sum [U(r+1)-U(r)]^2 / N_{\text{eff}}} \]

- \( \Delta U \lesssim 10^{-8} \): comparable to previous simulations

\[ \epsilon : \text{stop. cond. for MS/overlap solver} \]
0.2. Effects of low modes of $D_{OV}$

- as approaching to $\epsilon$-regime, cost is governed by $\lambda_{ov,\, min}$ rather than $m_{sea}$

- too small volume?
  $M_{PS} \, L \gtrsim 2.7$, $\exp[-M_{PS} \, L] \Rightarrow \lesssim 1-2\%$ effects on $M_{PS}$, larger $L$ for $m_{sea} \ll 0.015$

T. Kanelko - Approaching the chiral limit with dynamical overlap fermions
0.2 timing

- mild $m_{\text{sea}}$ dep. of $N_{\text{inv},H}$ and $N_{\text{MD}}$
  
  $\Downarrow$
  
  CPU time $\propto 1/m_{\text{sea}}^{-\alpha}$, w/ $\alpha \sim 0.53$
  
  $\Downarrow$
  
  naive expectation: $N_{\text{inv}} \propto 1/m_{\text{sea}}$, $N_{\text{MD}} \propto 1/m_{\text{sea}}$

- BG/L $\times$ 10 racks $\times$ 1 month
  
  $\Rightarrow$ 4000 traj. at all $m_{\text{sea}}$

### CPU time [min] on BG/L $\times$ 10 racks

<table>
<thead>
<tr>
<th>$m_{\text{sea}}$</th>
<th>HMC-4D</th>
<th>HMC-5D</th>
</tr>
</thead>
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<tr>
<td></td>
<td>traj.</td>
<td>time</td>
</tr>
<tr>
<td>0.015</td>
<td>2800</td>
<td>6.1</td>
</tr>
<tr>
<td>0.025</td>
<td>5200</td>
<td>4.7</td>
</tr>
<tr>
<td>0.035</td>
<td>4600</td>
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</tr>
<tr>
<td>0.050</td>
<td>4800</td>
<td>2.6</td>
</tr>
<tr>
<td>0.070</td>
<td>4500</td>
<td>2.1</td>
</tr>
<tr>
<td>0.100</td>
<td>4600</td>
<td>2.0</td>
</tr>
</tbody>
</table>

T.Kaneko  Approaching the chiral limit with dynamical overlap fermions
0.3 autocorrelation

- plaquette: local
  ⇒ small $m_q$ dependence
- $N_{\text{inv,H}}$: long range
  ⇒ rapid increase as $m_q \to 0$
  ⇒ may need large statistics

Tasanello  Approaching the chiral limit with dynamical overlap fermions
1. Summary

- Algorithm for JLQCD's dynamical overlap simulations
  - Hasenbusch precond. + multiple time scale MD + ...
  - 5D solver
  - Extra-Wilson fermion to suppress (near-)zero modes
    ⇒ cheap approx. for \( \text{sgn}[F_W] \), ⇒ turn off reflection/refraction
- Effects due to fixed (global) topology (R.Brower et al., 2003)
  - Topological properties \( (\chi_t, \ldots) \) ⇒ talks by T.W.Chiu, T.Onogi
  - \( Q \)-dependence of observables ⇐ simulations w/ \( Q \neq 0 \)
  - Suitable for \( \epsilon \)-regime ⇒ talk by S.Hashimoto
- On-going/future plans
  - Spectrum/matrix elements ⇒ talks by J.Noaki, N.Yamada
  - Simulations of \( N_f = 3 \) QCD
  - Extend to larger volumes
Topological Susceptibility in the Trivial Sector with Dynamical Overlap

Ting-Wai Chiu
Physics Department, National Taiwan University

for JLQCD and TWQCD Collaborations
Lattice Setup (See Talks by Hashimoto, Kaneko, and Onogi)

- Lattice size: $16^3 \times 32$
- Gluons: Iwasaki gauge action at $\beta = 2.30$
- Quarks ($n_f = 2$): overlap Dirac operator with $m_0 = 1.6$
- Add extra Wilson fermions and pseudofermions

$$\det(H_{ov}^2) \rightarrow \det(H_{ov}^2) \frac{\det(H_w^2)}{\det(H_w^2 + \mu^2)}, \mu = 0.2$$

to forbid $Q_{top}$ crossing zero, thus $Q_{top}$ is invariant.

- Quark masses: $m_{sea} = 0.015, 0.025, 0.035, 0.050, 0.070, 0.100$, each of $\sim 1000$ confs with $Q_{top} = 0$.

- For each configuration, 50+50 low-lying eigenmodes of overlap Dirac operator are projected.
Preliminary Results (using 396 confs for each $m_{sea}$)

On the $16^3 \times 32$ lattice, measure the time-correlation function

$$C(t_1 - t_2) = \langle Q(t_1) Q(t_2) \rangle = \sum_{x_1, x_2} \langle \rho(x_1) \rho(x_2) \rangle$$

$16^3 \times 32$, $\beta = 2.30$, $m_{val} = m_{sea} = 0.015$

$$\rho_1(x) = m \text{tr}[\gamma_5 (D_0 + m)^{-1}]$$

no. of configurations = 396

no. of low-lying eigenmodes = 50+50
Topological Susceptibility

\[ a^4 \chi_{\text{top}} = \frac{32}{16^3} \langle Q(t_1) Q(t_2) \rangle, \quad |t_1 - t_2| = 16 \]

16³x32, \( \beta=2.30 \), \( m_{\text{val}}=m_{\text{sea}} \)

\[ \rho_1(x) = \text{tr} \{ \gamma_5 (D_0^i + m)^{-1} \} \]

no. of configurations = 396
no. of low-lying eigenmodes = 50+50

<table>
<thead>
<tr>
<th>a^4 \chi_{\text{top}}</th>
</tr>
</thead>
</table>
| 3.0e-4
| 2.0e-4
| 1.0e-4
| 0.0
| 0.00 | 0.02 | 0.04 | 0.06 | 0.08 |

AQC/TCO Collaboration: 50/50

T.W. Chiu, Topological Susceptibility – p.4
Realization of Leutwyler-Smilga relation

In the limit $m \to 0$, $\chi_{top} \to m\Sigma/n_f$, in agreement with the Leutwyler-Smilga relation!
Conclusion and Outlook

- For the topologically-trivial gauge configurations generated with \( n_f = 2 \) dynamical overlap quarks constrained by extra Wilson and pseudofermions, they possess topologically non-trivial excitations (e.g., instanton and anti-instanton pairs) in sub-volumes.

- These near-zero modes allow us to determine \( \chi_{top} \) and \( \Sigma \).

- In the chiral limit, the Leutwyler-Smilga relation is realized!

- Similar studies for \( Q_{top} = 2 \), and \( Q_{top} = 4 \) sectors are now in progress.
Baryon spectrum from 2+1 flavours of DWF QCD

RBC and UKQCD collaborations

Chris Maynard

Chris Allton, Tom Blum, Paul Cooney, Luigi del Debbio, Meifeng Lin, Aurora Trivini, Takeshi Yamazaki, James Zanotti
Nucleon FSE

- No obvious FSE for Nucleon
  - 2 and 3 fm box
  - \( m_p \sim 390 \text{ Mev} \)
  - \( m_p L \sim 3.9 \) and 5.8
  - \( m_p / m_V \sim 0.45 \)
- No FSE expected
  - \( m=0.005 \) \( m_p L \sim 4.6 \)
Edinburgh plot

- Displays all 2+1 flavour DWF data
- Apparently lies on universal curve
  - $L_S$, gauge coupling, volume
- Lightest data noisy due to vector correlator
Chiral extrapolation II

- NLO $\chi$PT term
  - $m_p^3$
- Radius of convergence?
  - Definitely not heaviest
- Maybe lightest datum
- Not realistic
Baryon spectrum

- Ultimately expect non-linear chiral behaviour
  - Can't get spectrum right with linear
- NLO $\chi$PT fit not realistic
  - goes in right direction
  - maybe just statistics
- Require lighter quarks and better statistics
Conclusions

- Preliminary results for Baryon spectrum
  - Broadly agrees with experiment
  - starting to reach interesting regime
  - $m_u/m_s < 1/5$, $m_p/m_V \sim 0.38$, $m_p \sim 310$ MeV
  - Measurements still running
- Complex chiral behaviour must be explored to get spectrum right
- Finer lattice spacing simulation under way
  - control lattice artefacts
Spectrum Study of Nf=2 QCD with Overlap Fermions

Jun Noaki
for JLQCD Collaboration

Abstract: We present JLQCD's ongoing project on the dynamical overlap fermions. Some important new techniques used in our calculation are explained. With our high precision data, we discuss the chiral log. Another focus is the renormalization factor of quark mass, whose preliminary result is estimated.
Improvements with low eigenmodes
DeGrand and Schaefer, 2004; Giusti et al., 2004

- Lowest 50 eigenmodes
  \[ D_{ov} u_i = \lambda_i u_i \]

- Low mode preconditioned quark propagator
  \[ S_q(x, y) = \sum_{i=1}^{50} \frac{u_i(x) u_i(y)}{\lambda_i + m_q} + S_q^{\text{Higher}}(x, y) \]

- Low mode averaged meson correlator
  \[ C(t) = C_{HH}^{LL}(t) + C_{HL}^{LL}(t) + C_{HL}^{HH}(t) + C_{LL}^{HH}(t) \]

Average over space
\[ f = 94.4(1.4) \text{ MeV} : 4\text{pts fit} \]
\[ \text{FSE: } \sim 3\% / \sim +5\% \text{ for the lightest mass} \]

Gasser and Leutwyler, 1987; Colangelo et al., 2005
Partially quenched data
Sharpe, 1997; Golterman and Leung, 1997

- Fit with common $f$ is impossible
- Independent fits are done
NPR for quark mass  Martinelli et al., 1995

- Renormalization condition

\[ Z_l^{-1} Z_l \Lambda_l^{(\text{latt})}(p) = \Lambda_l^{(\text{tree})} \]  
(Landau gauge)

- OPE + WTI

\[
\Lambda_P(p) = A \frac{\langle \bar{\psi} \gamma_\mu \psi \rangle}{m_q} + Z_\alpha Z_m + B_P \cdot m_q^2
\]

\[
\Lambda_S(p) = A \frac{\partial}{\partial m_q} \langle \bar{\psi} \gamma_\mu \gamma_5 \psi \rangle + Z_\alpha Z_m + B_S \cdot m_q^2
\]

\[
\langle \bar{\psi} \psi \rangle = \left( \frac{1}{V} \sum_{l=1}^{50} \right) m_q^2 \left( \frac{2}{m_q^2 + \Lambda_l^2} \right)
\]
Strange quark mass

- Dynamical limit

\[ \zeta_{\overline{\text{MS}}}(2\text{GeV}) = w_{\overline{\text{MS}}}(2\text{GeV}) \cdot Z_{\overline{\text{RGI}}} \]

\[ = 0.742(12) \]

- PCAC rel.

\[ m_K = 495 \text{ MeV} \quad \Rightarrow \quad a \cdot m_s^{(\text{bare})}/2 \approx 0.050 \]

\[ m_s^{\overline{\text{MS}}}(2\text{GeV}) = 117(2) \text{ (sys.)} \]
Isospin breaking Study with Nf=2 domain-wall QCD + Quenched QED Simulation

Takumi Doi (Univ. of Kentucky / RBRC)

In collaboration with T.Blum (Univ. of Connecticut, RBRC), M.Hayakawa (Nagoya Univ.), T.Izubuchi (Kanazawa Univ., RBRC), N.Yamada (KEK)

- We investigate the isospin breaking effect on hadron spectrum using Lattice QCD+QED simulation
- Determination of the LECs which appear in meson spectrum + experimental input \(\rightarrow\) quark mass
  \[
  \frac{m_d}{m_u} = 2.055(32)^{+0.3}_{-0.2} \\
  \frac{m_s}{m_{ud}} = 28.63(32)
  \]
  - Further refinement is underway for nondegerate quark mass X QED correction \(\alpha_{em} * m_s\)
- In QED effect determination, \(Q = +e, -e\) trick gives remarkable improvement, while the QED effect on baryons still need additional work
- The QCD (mu-md) effect on baryons obtained reasonably
Quark mass determination

- Offset of quark mass in DWF
  - Residual quark mass with QED on determined by PCAC
  - Fit to the quark mass dependence of neutron mesons and pion mass splittings
    - LECs are determined
- LECs obtained + experimental inputs
  - $M(\pi_0)^2 \leftrightarrow$ sensitive to (mu+md), insensitive to (mu-md)
    - $\Rightarrow$ determine (mu+md)
  - $M(K^+)^2 + M(K^0)^2 \leftrightarrow$ sensitive to ms, (mu+md) insensitive to (mu-md)
    - $\Rightarrow$ determine ms
  - $[M(K^0)^2 - M(K^+)^2] - [M(\pi_0)^2 - M(\pi^+)^2] \leftrightarrow$ sensitive to (mu-md), ms
    - $\Rightarrow$ determine (mu-md)
Quark masses and splittings

- **Masses**
  \[
  \frac{m_d}{m_u} = 2.055(32) \pm 0.3 \quad \text{MILC w/o QED}
  \]
  \[m_s/m_{ud} = 28.63(32)\]
  \[
  \frac{m_d}{m_u} = 2.33(0)(5)(43)
  \]
  \[m_s/m_{ud} = 27.4(1)(4)(1)\]
  \[
  m_{u_{\overline{\text{MS}}}}(2\text{GeV}) = 2.85(11)(29)\text{MeV}
  \]
  \[m_{d_{\overline{\text{MS}}}}(2\text{GeV}) = 5.85(24)(30)\text{MeV}\]
  \[m_{u_{\overline{\text{MS}}}}(2\text{GeV}) = 4.35(17)\text{MeV}\]
  \[m_{s_{\overline{\text{MS}}}}(2\text{GeV}) = 124.5(39)\text{MeV}\]

- **By employing RBC**
  nonperturbative \(1/Z_m = 0.62\)

- **Systematic error**
  - Neglection of nondegenate mass effect \(\alpha(m_1 - m_2)\)
  - Finite \(V\): estimation by Cottingham formula
  + Vector saturation model would be negligible

- **Splittings**
  \[m_{\pi^+} - m_{\pi^0}(QED) = 3.81(16)\text{MeV} \quad \text{vs.} \quad [4.59(\exp) - 0.17(3)]\text{MeV}\]
  Kaon suffer from large systematic error
Baryons: much larger noise:

The idea for the S/N improvement

- Q = +e, -e trick

- Physical observables are expected to
  \[ m(e) = m^0 + e^2 m' + e^4 m'' \ldots \]
  - (Perturbatively, only O(e^{2n}) appear)
  - \[ [ m(+e) + m(-e) ] \] kill the fluctuation of O(e)

- QED confs: \( \{ A_\mu(em) \} \rightarrow \{ +A_\mu(em), -A_\mu(em) \} \)

*We actually achieve Remarkable Improvement!*

*Same Boltzmann Weight!*
Proton neutron mass difference from the QED effect

The lattice result indicates 
$M(p) > M(n) \text{ (QED)}$ at each $m_{sea}$

c.f. Cottingham formula: 
$M(p)-M(n)(\text{QED}) = 0.76 \text{MeV}$

Need more statistics? 
Finite $V$?
The isospin breaking on baryons from QCD ([md-mu] ≠ 0 effect)

- \( p - n \rightarrow -2.55(18)(51) \text{ MeV} \)
- \( \Xi(-) - \Xi(0) \rightarrow +3.86(11)(77) \text{ MeV} \)
- \( \Sigma^+ - \Sigma(0) \rightarrow -3.32(12)(66) \text{ MeV} \)
- \( \Sigma^+ - \Sigma(-) \rightarrow +3.04(11)(61) \text{ MeV} \)
- \( \Sigma^+ - \Sigma^- \rightarrow -6.37(22)(127) \text{ MeV} \)

Inputs: \((md-mu))^{MS} = 3.0(6) \text{ MeV} \)

\( a_{(md-mu)^{bare}} = 0.0011(2) \) from meson spectrum

cf. S.R. Beane, K. Orginos, M.J. Savage hep-lat/0605014

\( p - n = -2.26(57)(42)(10) \text{ MeV} \)
Chiral Extrapolations for Domain Wall Fermions

Meifeng Lin
[RBC and UKQCD Collaborations]

Columbia University
mflin@phys.columbia.edu

March 15, 2007 @ BNL
Difficulty in chiral extrapolations

- Chiral perturbation theory fails if the masses are too heavy!

- In this talk......
  - lighter quark masses: down to \( m_s/5 \)
  - larger volume: \((3\text{fm})^3\)

⇒ Can the data be described by NLO ChPT then??
## Simulation Overview

**RBC and UKQCD 2+1 flavor DWF ensembles:**

<table>
<thead>
<tr>
<th>$\beta$</th>
<th>Volume</th>
<th>$L_s$</th>
<th>$m_l/m_s$</th>
<th># Time Units</th>
<th>$m_\pi$ (MeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.13</td>
<td>$16^3 \times 32$</td>
<td>16</td>
<td>0.01/0.04</td>
<td>4000</td>
<td>390</td>
</tr>
<tr>
<td>2.13</td>
<td>$16^3 \times 32$</td>
<td>16</td>
<td>0.02/0.04</td>
<td>4000</td>
<td>520</td>
</tr>
<tr>
<td>2.13</td>
<td>$16^3 \times 32$</td>
<td>16</td>
<td>0.03/0.04</td>
<td>4000</td>
<td>620</td>
</tr>
<tr>
<td>2.13</td>
<td>$24^3 \times 64$</td>
<td>16</td>
<td>0.005/0.04</td>
<td>4500</td>
<td>310</td>
</tr>
<tr>
<td>2.13</td>
<td>$24^3 \times 64$</td>
<td>16</td>
<td>0.01/0.04</td>
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<td>2.13</td>
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<td>16</td>
<td>0.03/0.04</td>
<td>2813</td>
<td>620</td>
</tr>
</tbody>
</table>

We are hoping to have better chiral fits with the lighter mass simulations!
The Residual Mass

- The ratio $R(t)$ is used to compute $m_{\text{res}}$ on the lattice

\[ R(t) = \frac{\langle \sum_{x} J_{5q}^{a}(x,t) \pi^{a}(0) \rangle}{\langle \sum_{x} J_{5q}^{a}(x,t) \pi^{a}(0) \rangle} \]

- Fit $R(t)$ where plateaux are reached to a constant to get $m_{\text{res}}$

- $O(m_{f}a)$ quark mass dependence

- Defined in the chiral limit:
  - $m_{f} \rightarrow 0$ (strictly speaking, $m_{f} = -m_{\text{res}}$, but not a big effect)

\[ am_{\text{res}} = 0.00314(2) \]
The simultaneous NLO fit of $M_{PS}^2$ and $f_{PS}$ fails. $\chi^2$/dof = 11

Pion mass range: $\sim 230 - 530$ MeV
Decreasing the range of pion masses in the fit helps. $\chi^2$/dof = 2

Using the quark masses from linear fit of $M^2_{PS}$,

\[ f_\pi = 124(3) \text{ MeV}, f_K = 153(3) \text{ MeV} \]

\[ f_K/f_\pi = 1.24(2) \text{ Preliminary!} \]
Spontaneous chiral symmetry breaking in QCD: a finite-size scaling study on the lattice

Silvia Necco (IFIC Valencia)

Workshop “Domain wall fermions at ten years”
Brookhaven National Laboratory
March 15-17 2007

Spontaneous chiral symmetry breaking in QCD with massless quarks at infinite volume can be seen in a finite box by studying the dependence of the chiral condensate from the volume and the quark mass. We performed a feasibility study of this program by computing the (quenched) quark condensate at small quark masses, using the Neuberger Dirac operator. We carried out simulations in various topological sectors, at several volumes, quark masses and lattice spacings and we focused on observables which are infrared stable and free from mass-dependent ultraviolet divergences. The numerical calculation has been performed with an exact variance-reduction technique, which is designed to be particularly efficient when spontaneous symmetry breaking is at work in generating a few very small low-lying eigenvalues of the Dirac operator. The finite-size scaling behaviour of the condensate in the topological sectors considered agrees, within our statistical accuracy, with the expectations of the chiral effective theory. Close to the chiral limit we observe a detailed agreement with the first Leutwyler-Smilga sum rule. By comparing the mass, the volume and the topology dependence of our results with the predictions of the chiral effective theory, we extract the corresponding low-energy constant.
Quark condensate with Ginsparg-Wilson fermions

- Quark condensate in the chiral limit:
  \[ \langle \bar{\psi} \psi \rangle = \lim_{a \to 0} Z_S \langle \bar{\psi} \psi \rangle, \quad \bar{\psi} = \left(1 - \frac{\bar{a}}{2} D\right) \psi \]

- At finite quark mass: UV divergences \((b_1 \propto 1/a^2, b_2 \propto \ln(a))\)
  \[ -\frac{Z_S \langle \bar{\psi} \psi \rangle}{N_f} = b_1 m + b_2 m^3 + \{\text{finite terms}\}. \]

- The condensate can be defined at fixed topological charge \(\nu = |Q|\): same UV divergences
  \[ -\frac{\langle \bar{\psi} \psi \rangle_\nu}{N_f} = \frac{\nu}{V m} + \chi_\nu; \quad \tilde{\chi}_\nu = Z_S \chi_\nu = b_1 m + b_2 m^3 + \{\text{finite terms}\} \]

1/m divergence: zero-modes contributions; 
\(Z_S, b_1, b_2\) topology independent; \(b_1, b_2\) volume independent and suppressed by \(1/V\) with respect to the finite terms

\(\Rightarrow (\tilde{\chi}_\nu_1 - \tilde{\chi}_\nu_2)\) is unambiguously defined at finite quark mass
\[ \rho(\lambda): \text{spectral density of massive Dirac operator} \]

\[ \tilde{\tau}_\nu(\lambda_{\text{min}}, \lambda_{\text{max}}) = 2\tilde{m} \int_{\lambda_{\text{min}}}^{\lambda_{\text{max}}} \frac{1}{\lambda^2 + \tilde{m}^2} \tilde{\rho}_\nu(\lambda) d\lambda; \quad \tilde{\chi}_\nu = \tilde{\tau}_\nu(0, \infty) \]

- \( \tilde{\tau}_\nu(\lambda_{\text{min}}, \lambda_{\text{max}}) \) has a well defined continuum limit, if \((\lambda_{\text{min}}, \lambda_{\text{max}})\) are kept fixed when \(a \to 0\)

- \( \tilde{\tau}_\nu(\lambda_{\text{min}}, \lambda_{\text{max}}) \) has a well defined continuum limit, if \((\lambda_{\text{min}}, \lambda_{\text{max}})\) are kept fixed when \(a \to 0\)

\[ \tilde{\tau}_\nu(\lambda_{\text{min}1}, \infty) - \tilde{\tau}_\nu(\lambda_{\text{min}2}, \infty) = \tilde{\chi}_{\nu_1} - \tilde{\chi}_{\nu_2} - [\tilde{\tau}_\nu(0, \lambda_{\text{min}1}) - \tilde{\tau}_\nu(0, \lambda_{\text{min}2})] \]

Strategy: this quantity is UV-finite

- can be computed using stable numerical estimators
  → low-mode averaging

- can be matched directly with chiral effective theory at NLO → extraction of the low-energy constant

L. Giusti, S.N. (hep-lat/0701023)

Finite-size scaling

latt. c1: $\beta = 5.8485$, $V = 12^4$, $L \approx 1.5$ fm

latt. c2: $\beta = 5.8485$, $V = 16^4$, $L \approx 2.0$ fm

$a^3(\chi_{\nu_1} - \chi_{\nu_2})$ as a function of $(mV)a^{-3}$

- at leading order, we expect $a^3(\chi_{\nu_1} - \chi_{\nu_2})$ to be a function of $(mV)$
- within our precision, we are not sensitive to NLO corrections
- $V_{c2}/V_{c1} \approx 3$ non-trivial verification of finite-size-scaling
- similar behaviour for higher topologies
Leutwyler-Smilga sum rules

\[
\frac{X_{\nu_1} - X_{\nu_2}}{X_{\nu_3} - X_{\nu_4}} \quad \leftrightarrow \quad \frac{\tilde{X}_{\nu_1}(\mu) - \tilde{X}_{\nu_2}(\mu)}{\tilde{X}_{\nu_3}(\mu) - \tilde{X}_{\nu_4}(\mu)}
\]

\[
(\nu_1 - \nu_2)\nu_3\nu_4
\]

\[
(\nu_3 - \nu_4)\nu_1\nu_2
\]

latt. c1: \( \beta = 5.8485, V = 12^4, L \sim 1.5 \text{ fm} \)
latt. c2: \( \beta = 5.8485, V = 16^4, L \sim 2.0 \text{ fm} \)
latt. c3: \( \beta = 6.0, V = 16^4, L \sim 1.5 \text{ fm} \)

- summary for the three lattices, at the lightest quark mass at our disposal
- the topology dependence in (quenched) QCD is well reproduced by chiral effective theory
Finite volume effects and lattice artefacts below statistical uncertainty

Conversion to $\overline{\text{MS}}$ scheme: $\overline{m}_{\overline{\text{MS}}}(2 \text{ GeV})/M = 0.72076$

\begin{equation}
(c3): \Sigma_{\overline{\text{MS}}}(2 \text{GeV}) = (290 \pm 11 \text{MeV})^3
\end{equation}

\begin{table}
\begin{tabular}{|c|c|c|}
\hline
latt. & $\nu_1 - \nu_2$ & $a^3 \Sigma_{\text{eff}}/Z_\beta$ \\
\hline
$\text{c1}$ & 0-1 & 0.0040(6) \\
 & 1-2 & 0.0039(3) \\
 & 2-3 & 0.0034(3) \\
\hline
$\text{c2}$ & 0-1 & 0.0035(8) \\
 & 1-2 & 0.0049(9) \\
 & 2-3 & 0.0040(5) \\
\hline
$\text{c3}$ & 0-1 & 0.0015(3) \\
 & 1-2 & 0.00178(18) \\
 & 2-3 & 0.00188(12) \\
\hline
\end{tabular}
\end{table}

RGI condensate: $\tilde{\Sigma}_{\text{eff}} = \tilde{Z}_\beta \Sigma_{\text{eff}}$

\begin{equation}
(c1): \tilde{\Sigma}_{\text{eff}}(L = 1.5 \text{ fm}) r_0^3 = 0.33(3)
\end{equation}

\begin{equation}
(c2): \tilde{\Sigma}_{\text{eff}}(L = 2.0 \text{ fm}) r_0^3 = 0.34(5)
\end{equation}

\begin{equation}
(c3): \tilde{\Sigma}_{\text{eff}}(L = 1.5 \text{ fm}) r_0^3 = 0.29(3)
\end{equation}

ALPHA collaboration (2000)
Domain wall filters.*

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ABSTRACT: We propose using the extra dimension separating the domain walls carrying lattice quarks of opposite handedness to gradually filter out the ultraviolet fluctuations of the gauge fields that are felt by the fermionic excitations living in the bulk. This generalization of the homogeneous domain wall construction has some theoretical features that seem nontrivial.

KEYWORDS: Chirality, Lattice Gauge Field Theories.

*Talk delivered by H. Neuberger at the workshop "Domain Wall Fermions at Ten Years", March 15-17, 2007, BNL.
1. Introduction.

To explain our basic idea it is best to start using continuum notation and language.

In Callan and Harvey's original domain wall construction one had fermions propagating in a full five dimensional gauge background. This situation was maintained in Kaplan's original proposal, but was early on discarded in favor of a five dimensional gauge field that is restricted to be independent of the fifth dimension and whose component in the fifth dimension is zero.

Here we focus only on the vector-like case, with two domain walls, separated by a fifth dimension of the topology of a circle. The domain walls are at diametrically opposed points on the circle. On the lattice, one can take the fermion mass to infinity on one of the semicircles, and effectively reduce the circle to a segment, with a domain wall at each end. The Weyl partners that make up a Dirac fermion live separately on the walls, with small leakage into the "bulk", the interior of the fifth dimension. For a finite separation there is an exponentially small direct interaction between the partners, which can be thought of as an effective vertex of the structure of an exponentially small mass term. Sometimes, in the lattice field theory literature, such mass terms are referred to as a "residual mass". Only at infinite separation does one get exactly massless Dirac particles, with the associated exact chiral symmetry. The latter holds since one can independently rotate the partners due to the infinite separation between the walls.

Here we shall re-introduce a dependence of the four components on the fifth dimension. For the time being, we maintain the fifth component at zero. However, the five dimensional gauge field is uniquely defined in terms of a four dimensional background; the dependence on the fifth dimension is chosen by hand, in a way designed to improve the speed of convergence to an infinite separation limit.

For our construction we need to introduce the concept of a "UV" filter. This is a term making its appearance in the lattice field theory literature, but its meaning varies slightly,
depending on context. To be specific, we first define the term in the continuum. A “UV”
filter \( \mathcal{F} \) will be an operation that gets as input some four dimensional Euclidean
gauge field \( A \) and produces a new four dimensional Euclidean gauge field \( \mathcal{F}(A) \). We require that
\( \mathcal{F} \) and the operation of a four dimensional gauge transformations of \( A \) commute. \( \mathcal{F}(A) \) is
smoother than \( A \) in that its the gauge invariant content fluctuates less than that of \( A \). As
an example, we define a filter using APE smearing:

\[
\mathcal{F}_\tau(A) = A(\tau_0), \quad \partial_\tau A_\mu(\tau) = \partial_\tau S(A)|_{A=A(\tau)}, \quad A(0) = A
\]  

(1.1)

Here \( S \) is some local four dimensional gauge invariant action, like the ordinary Euclidean
Yang Mills action possibly including also higher derivative terms. \( \tau_0 \) is a quantity of
dimensions length (for separations along the four physical dimension) squared. \( \sqrt{\tau_0} \) is a
relatively short distance on the scale of four dimensional physics. Linearizing the above
equation is required to produce, for \( \tau > 0 \) a suppression for higher momentum modes in
\( A(\tau) \) in Feynman gauge.

We want to use a UV filter on the gauge fields seen by most of the fermions (bulk) to
hasten the convergence to the infinite wall separation limit. However, we do not want the
UV filter to affect too much the physical fermions, which live on the walls, because, for
example, if we add a mass to make the physical quarks heavy, we still want the charmonium
spectrum to come out right, and this requires the Coulomb potential to be well represented
even at short distances, making UV filtering undesirable.

This leads us to introduce a profile in the five dimensional gauge field. We label the
segment connecting the domain walls by the variable \( s \), where \( 0 \leq s \leq S \) and introduce
the profile function \( \tau(s) \) which obeys \( \tau(s) = \tau(S - s) \) and \( \tau(0) = 0 \). \( \tau(s) \) increased from
0 gradually to some value \( \tau_0 \), stays there for a stretch of \( s \) given by \( I \), until it reaches
the midpoint \( s = S/2 \), after which it reverses its behavior in accordance with the above
mentioned constraint under reflection. This constraint is needed to produce an extra
symmetry, a parity operation connecting the fermions bound to the walls. When \( I = \infty \)
the walls are no longer coupled, and we have exact chiral symmetry.

As is well known, it is useful to view \( s \) as an Euclidean auxiliary time, with an associated
\( s \)-dependent Hamiltonian \( H(\tau(s)) \). Obviously, the \( s \)-independent case is easier to analyze.
What we gain here is that the gap, \( g \), separating negative and positive energies in \( H(s) \)
increases in the “flat” region of length \( I \). The approach to infinite \( I \) is governed essentially by
e^{-\frac{I}{\tau}} \), so it becomes obvious why we want \( g \) to be large. The impact of the UV fluctuations
contained in \( A(\tau) \) is such that \( |g| \) increases with \( \tau \).

There can be other choices of the filter and profile. Also, the mass term can have a
profile, like in the original constructions. What was described above is close to what we
actually implemented on the computer, in a lattice version of the above.

2. Definitions.

We shall label the four dimensional slices by \( s = 1, 2, ..., S \); the physical fermions live on
slices 1 and \( S \) and the associated five dimensional wave functions are localized in the fifth
coordinate \( s \), to the vicinities of these walls.
Following [1], we write the kernel for the five dimensional fermionic action in the following form:

\[
D = \begin{pmatrix}
B_1 & B_2 & B_3 & \cdots & 0 \\
B_2 & B_3 & 0 & \cdots & 0 \\
B_3 & 0 & B_1 & \cdots & 0 \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
0 & 0 & 0 & \cdots & \frac{C_i}{C_i^2}
\end{pmatrix}
\]

The matrix \(D\) is of size \(2k \times 2k\) where the entries are \(q \times q\) blocks and \(k = S\) and \(q = 2NL^4\), where the gauge group is taken as \(SU(N)\). The factor \(2N\) counts spinorial and gauge group indices.

The matrices \(B_i\) and \(C_i\) are dependent on the gauge background defined by the collection of link matrices \(U_{ij}^\mu(x)\). These matrices are of dimension \(N \times N\). \(\mu\) labels the positive 4 directions on a hypercubic lattice and \(U_{ij}^\mu(x)\) is the unitary matrix associated with a link that points from the site \(x\) in the \(\mu\)-direction on slice \(s\)

\[
\begin{align*}
(C_{\alpha\beta})_{xoa,\beta ij} &= \frac{1}{2} \sum_{\mu=1}^{4} \sigma_{\mu}(\delta_{\alpha,\mu} - \delta_{\beta,\mu}) \left( U_{ij}^\mu(x) \right)_{ij} \\
(B_{\alpha\beta})_{xoa,\beta ij} &= \frac{1}{2} \sum_{\mu=1}^{4} \delta_{\alpha,\mu} \left( U_{ij}^\mu(x) \right)_{ij} - \frac{1}{2} \left( M_{\alpha\beta} \delta_{\alpha,\mu} \right) \left( U_{ij}^\mu(x) \right)_{ij}
\end{align*}
\]

The effective four dimensional fermion action. Following the method of [1] we get

\[
\det D = \left( -\frac{1}{2} \right)^{k} \left( \prod_{i=1}^{S} \det B_i \right) \det \left[ \frac{1 - \Gamma_5}{2} - \frac{1 + \Gamma_5}{2} \right]
\]
Define

\[ T_s \equiv e^{-H_s} = \left( \begin{array}{ccc} B_s & 0 & 0 \\ 0 & C_s & 0 \\ 0 & 0 & C_s \end{array} \right) \tag{3.2} \]

For any \( s \), \( \det T_s = 1 \).

\( T_I \) is a symmetric product of \( T_{s-1} \) factors. By definition, gauge fields and mass parameters \( M_0 \) labeled by \( s \) and \( S-s \) are identical.

\[ T_I = R^l T_{r-1}^{-(l+1)} R \tag{3.3} \]

The \( R, R^l \) factors are complex matrices (neither unitary, nor hermitian) representing “ramps” and are given by:

\[ R = T_r^{-1} T_{r-1}^{-1} \cdots T_1^{-1}, \quad R^l = T_s^{-1} T_{s+1}^{-1} \cdots T_{s-l}^{-1} \tag{3.4} \]

In between the “ramps” we have a uniform “plateau” with identical transfer matrix factors. The total number of slices is \( k = l + 2r \)

\[ \det D = (-1)^k \prod_{s=1}^{S} \det B_s \det \left[ 1 + T_1^{-1} \right] \det \left[ \frac{1 + \Gamma_x \frac{1 - T_1}{1 + T_1}}{2} \right] \tag{3.5} \]

The same derivation for Pauli Villars (PV) fields should give a PV determinant made out of the first two factors in the formula above. Dividing the two expressions, leaves us with the last factor as representing the almost massless Dirac fermion.

Thus, the operator

\[ E_l = \frac{1 - T_1}{1 + T_1} \tag{3.6} \]

carries the \( l \) dependence and its spectrum governs the approach to the chiral limit \( l = \infty \).

Unlike in the homogeneous case, we are so far unable to prove that \( E_l \) indeed has a limit, \( E_\infty \), as \( l \) tends to infinity. If we assume that such a limit exists, the rate of convergence would be controlled by the eigenvalue of \( T_I \) that is closest to unity as \( l \) becomes very large.

Physically, using effective Lagrangian intuition, one would expect \( E_\infty \) to exist for most four dimensional gauge fields. Simulations with the five dimensional action could practically resolve this question.

4. How to test the method.

Before proceeding to a five dimensional implementation one would like to find by numerical methods the lowest or few lowest eigenvalues of the matrix \( T_I + T_I^{-1} \) but this is tough because the condition number of \( T_I \) rapidly becomes too large as the number of slices increases. A possible trick to get around this goes as follows:
It is easy to check that the spectrum of

\[ B_S = \begin{pmatrix} 0 & A_1 & 0 & \cdots & 0 \\ 0 & 0 & A_2 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \cdots & A_{S-1} \\ A_S & \vdots & \vdots & \cdots & 0 \end{pmatrix} \] (4.1)

is given by \( \lambda_{n,k} = \rho_n e^{ik} e^{\frac{2\pi i}{S}} \), with \( k = 0,1,2,\ldots, S - 1 \) where \( \lambda_n = \rho_n e^{\phi_n} \) are the eigenvalues of \( \Pi_S \equiv A_1,\ldots,A_S, \) In our application, \( \Pi_S \) is hermitian and positive, so \( \phi_n = 0. \) From the relation \( |\lambda_{n,k}| = \rho_n^{1/2} \) we see that we would get directly the quantity we expect to have a finite limit as \( l \to \infty. \) Writing \( \rho_n \equiv e^{\phi_n}, \) with a weak \( S \)-dependence in \( \kappa_n \) suppressed, we can find the dominating \( \kappa_n \) using a routine based on a restarted Arnoldi procedure looking for the eigenvalues of \( B_S - 1 \) of smallest magnitude \( [2]. \)

In some sense, it might be surprising that this relatively simple generalization of the homogeneous domain wall setup is substantially more difficult to analyze theoretically, with or without numerical tools. Since \( E_{\infty} \) would provide a new variant of an overlap operator, the issue might be of interest separately from the question of increased numerical efficiency of QCD domain wall simulations.

5. Further directions.

One could adapt to the lattice a definition of \( A(s) \) based on simple interpolation. This would allow for an adiabatic limit, with an almost homogeneous extra dimension where theoretical issues would be under better control, setting aside the question how far one can deviate from this limit.

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References

Abstract

The use of lattice regularizations with exact chiral symmetry has made possible to explore the $\epsilon$-regime of $\chi$PT. A number of simulations have been carried out in the quenched approximation in volumes up to 2 fm. I will describe some recent results.
Weak effective couplings in $SU(4)$ limit

$$\mathcal{H}_{\text{ChPT}}^{\text{ChPT}} = \frac{g_w^2}{4 M_W^2} (V_{us})^* V_{ud} \sum_{\sigma=\pm} g^\sigma [O^\sigma]$$

$$O^\pm = \frac{F^4}{4} \left[ (U \partial_\mu U^\dagger)_{us} (U \partial_\mu U^\dagger)_{du} \pm (U \partial_\mu U^\dagger)_{uu} (U \partial_\mu U^\dagger)_{ds} - (u \to c) \right]$$

In contrast with $SU(3)$, only two operators appear in $SU(4)$-ChPT at LO:

$$\frac{A_0}{A_2} = \frac{1}{\sqrt{2}} \left( \frac{1}{2} + \frac{3}{2} \frac{g^-}{g^+} \right) \quad [g^+]_{Nc} = [g^-]_{Nc} = 1$$
The Matching

We perform the matching by equating certain correlation functions in lattice QCD and in the chiral theory: three-point functions of the bare operators and two left currents

\[ R^\sigma(x_0, y_0) \equiv \frac{\sum_{x,y} \langle [J_{L0}(x)]_{du} O^\pm(0) [J_{L0}(y)]_{us} \rangle}{\sum_x \langle [J_{L0}(x)]_{\alpha\beta} [J_{L0}(0)]_{\beta\alpha} \rangle \sum_y \langle [J_{L0}(y)]_{\alpha\beta} [J_{L0}(0)]_{\beta\alpha} \rangle} \]

\[ g^\sigma \quad [R^\sigma(m, V, LEC'S)] = k^\sigma \left( \frac{M_W}{\Lambda} \right) \quad \frac{Z^\sigma(g_0)}{Z^2_A} \quad R^\sigma \]

\[ \downarrow \quad \chi_{PT} \quad \downarrow \quad \text{P.T.} - 2 \text{ loop} \quad \downarrow \quad \text{N.P.} \quad \text{Lattice} \]

In the $\epsilon$-regime at NLO: $R^\sigma(x_0, y_0)$ independent of $x_0, y_0, |\nu|$ and any other LEC different from $g^\pm$
Tested in the quenched approximation:

<table>
<thead>
<tr>
<th></th>
<th>$\beta$</th>
<th>$L/a$</th>
<th>$T/a$</th>
<th>$n_{\text{low}}$</th>
<th>$L_{\text{fm}}$</th>
<th>$m$</th>
<th># cfgs</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\epsilon$-regime</td>
<td>5.8485</td>
<td>16</td>
<td>32</td>
<td>20</td>
<td>2</td>
<td>$m_s/40, m_s/60$</td>
<td>$O(800)$</td>
</tr>
<tr>
<td>$p$-regime</td>
<td>5.8485</td>
<td>16</td>
<td>32</td>
<td>20</td>
<td>2</td>
<td>$m_2/2 - m_s/6$</td>
<td>$O(200)$</td>
</tr>
</tbody>
</table>

The expected features of the $R^\sigma(x_0, y_0)$ in the $\epsilon$-regime: independence on $x_0$, $y_0$, $m$ and $\nu$ are well reproduced by the data.
$g^\pm$ in $SU(4)$-limit

<table>
<thead>
<tr>
<th></th>
<th>$g^+$</th>
<th>$g^-$</th>
</tr>
</thead>
<tbody>
<tr>
<td>This work</td>
<td>0.51(3)(5)(6)</td>
<td>2.6(1)(3)(3)</td>
</tr>
<tr>
<td>&quot;Exp&quot;</td>
<td>$\sim 0.5$</td>
<td>$\sim 10.4$</td>
</tr>
<tr>
<td>Large $N_c$</td>
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<td>1</td>
</tr>
</tbody>
</table>
Fat links for dynamical fermion simulations

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We investigate a variant of hypercubic gauge link smearing where the SU(3) projection is replaced with a normalization to the corresponding unitary group. This smearing is differentiable and thus suitable for use in dynamical fermion simulations using molecular dynamics type algorithms. We show that this smearing is as efficient as projected hypercubic smearing in removing ultraviolet noise from the gauge fields. We test the normalized hypercubic smearing in dynamical improved (clover) Wilson and valence overlap simulations on $12^3 \times 24$ lattices with lattice spacing $a \approx 0.13$fm.

The clover Wilson simulations are stable and demonstrate that the non-analyticity in the projection is no problem in practice. The spectral gap is wide allowing for much smaller quark masses than the $m_{PS}/m_V = 0.6$ reached. The tests with the overlap operator show an improvement of a factor of three in cost over the overlap operator constructed from the thin link Wilson kernel. This comes together with improved locality.
Main building blocks

n-APE

\[ \tilde{U}_\mu(x) = \text{Proj}_{U(3)} \left[ (1 - \alpha)U_\mu(x) + \frac{\alpha}{6} V_\mu(x) \right] \]

\[ \text{Proj}_{U(3)} A = A(A^\dagger A)^{-1/2} \]

- differentiable everywhere if \( A \) non-singular
  \( \implies \) no problem in practice
- projection has been used in the past: Kentucky'93, FLIC, Narayanan&Neuberger'06
- force term can be computed exactly (à la stout)
- the projection costs about the same as stout smearing
HYP links for dynamical fermions

\[ V_{n,\mu} = \text{Proj}_{U(3)}[(1 - \alpha_1)U_{n,\mu} + \frac{\alpha_1}{6} \sum_{\pm \nu \neq \mu} \tilde{V}_{n,\nu;\mu} \tilde{V}_{n+\rho,\mu;\nu} \tilde{V}_{n+\rho,\nu;\mu}] \]

\[ \tilde{V}_{n,\mu;\nu} = \text{Proj}_{U(3)}[(1 - \alpha_2)U_{n,\mu} + \frac{\alpha_2}{4} \sum_{\pm \rho \neq \nu, \mu} \bar{V}_{n,\rho;\nu,\mu} \bar{V}_{n+\rho,\mu;\nu} \bar{V}_{n+\rho,\nu;\mu}] \]

\[ \bar{V}_{n,\mu;\nu,\rho} = \text{Proj}_{U(3)}[(1 - \alpha_3)U_{n,\mu} + \frac{\alpha_3}{2} \sum_{\pm \eta \neq \rho, \nu, \mu} U_{n,\eta} U_{n+\eta,\mu} U_{n+\rho,\eta}^\dagger] \]

- n-HYP: same as HYP with projection to U(3)
- virtually indistinguishable from standard HYP
- more efficient than if built from stout smearing

How does the projection perform in MD simulations?
Tests: Dynamical clover Wilson

- clover Wilson with $c_{SW} = 1$
- standard HYP parameters: $\alpha_1 = 0.75$, $\alpha_2 = 0.6$, $\alpha_3 = 0.3$
  no tuning necessary
- Lüschter-Weisz gauge action
- $12^3 \times 24$
- $a \approx 0.13\text{fm}$
- $m_{PS}/m_V \approx 0.6$
Dynamical clover Wilson

- large spectral gap $\Rightarrow$ smaller quark masses possible
- fat link cost: 11% of total budget
- gain on inversions
Overlap: Cost

\[ D_{ov} = R \left[ 1 + \gamma_5 \text{sign}(H_W(-R)) \right] \]

\[ \text{sign}\, H_W \approx \sum_{\lambda} \text{sign}\lambda \, P_\lambda + (1 - \sum_{\lambda} P_\lambda) \text{sign}_{\text{app}} H_W \]

- lower eigenmode density of \( H_W(-R) \)
  \( \Rightarrow \) easier to approximate sign function; lower cost
- \( n\)-HYP as good as \( 3 \times \) stout at \( 6\rho = 0.9 \)
Recent calculations of the structure of the nucleon by the LHPC collaboration are presented using domain wall fermions on a staggered sea. Matrix elements of twist-two operators are calculated using domain wall valence quark propagators on MILC lattices with lattice spacing $a = 0.125$ fm and pion masses down to 350 MeV on lattices of spatial size up to 3.5 fm. The axial charge, $g_A = \langle t \rangle_{u-d}$, is calculated with 6.8% error in agreement with experiment, and chiral extrapolations of $\langle x \rangle_{u+d}$, $\langle x \rangle_{u-d}$, and $\langle x^3 \rangle_{u-d}$ also agree with experiment. Electromagnetic form factors $F_1$, $F_3$, $G_A$, and $G_P$ all qualitatively approach the experimental results as the pion mass decreases, and chiral extrapolation of the rms charge radius is in quantitative agreement with experiment. The generalized form factors $A_{10}$, $A_{20}$, and $A_{30}$ provide clear evidence of strong dependence of the transverse size of the nucleon on the momentum fraction and agree with the phenomenological parameterization of Diehl et al. The generalized form factors $A_{30}$, $B_{30}$, and $C_{30}$ are calculated, chirally extrapolated, and used to determine the connected diagram contributions of the quark spin and orbital angular momentum to the nucleon spin. The success of these calculations strongly motivates taking the next step in controlling systematic errors by undertaking a fully consistent, unitary calculation with dynamical domain wall fermions with lattice spacings of $a = 0.123$ and 0.093 fm at masses of approximately 390, 310, and 260 MeV.
Nucleon axial charge $g_A$ and $\langle I \rangle_{\Delta q}^{u-d}$

![Graph showing the relationship between $g_A$ and $m_{\pi}^2$ (GeV$^2$)].

LHPC hep-lat/0510062
Chiral extrapolation of \( \langle x \rangle_q^{u-d} = A_{20}^{u-d}(t = 0) \)

Chiral extrapolation \( O(p^4) \) relativistic ChPT (Dorati, Hemmert, et al.)

\[
A_{20}^{u-d}(t, m_{\pi}) = A_{20}^{0,u-d}(f_A(m_{\pi}) + g_A(t, m_{\pi})) + \tilde{A}_{20}^{0,u-d} h_A(m_{\pi}) + A_{20}^{m_{\pi}^2} m_{\pi}^2 + A_{20}^t t
\]
$\langle r^2 \rangle_{u-d} = a_0 - \frac{(1 + 5y^2_A)}{(4\pi f_\pi)^2} \log \left( \frac{m^2}{m^2 + A^2} \right)$
Nucleon spin decomposition

\[ \Delta \Sigma^{u+d}/2 \]

\[ L^{u+d} \]

Contributions to nucleon spin

\[ m_{\pi}^2 \text{ [GeV}^2\text{]} \]
Chiral extrapolation of \( J_q = \frac{1}{2} \left( A_{20}^{u+d}(0) + B_{20}^{u+d}(0) \right) \)

ChPT including Delta (Chen and Ji)

\[
J_q(m_\pi, \Delta) = J_q(m_\pi) - \frac{1}{2} \left( \frac{9}{2} b g_N + 3 a g_N - \frac{15}{2} b g_{\Delta} \right) \frac{8 g^2 N A}{3 \sqrt{2} m_\pi} \left( m_\pi^2 - 2 \Delta^2 \right) \ln \left( \frac{m_\pi^2}{\Delta^2} \right) + 2 \Delta \sqrt{\Delta^2 - m_\pi^2} \ln \left( \frac{\Delta - \sqrt{\Delta^2 - m_\pi^2}}{\Delta + \sqrt{\Delta^2 - m_\pi^2}} \right)
\]
Status of the 2+1 Flavor Wilson-Clover Simulation of QCD by PACS-CS Collaboration

Y. Kuramashi (Univ. of Tsukuba) for PACS-CS collaboration

We report the current status of the PACS-CS project which aims at completing the Wilson-clover \( N_f = 2 + 1 \) program using the Lüscher’s domain-decomposed HMC algorithm. Some very preliminary results for the hadron effective masses and the pseudoscalar decay constants are presented.
§4. Simulation details

- Iwaseski gauge action + clover quarks with $c_{SW}^{NP}$
- $\beta = 1.9 \ (a = 0.1\text{fm}, \ a^{-1} = 2.0\text{GeV})$, lattice size $= 32^3 \times 64$
- $\kappa_s = 0.13640$, $N_{\text{poly}} = 180$ for PHMC
- ud quarks: block size $= 8^4$, $\delta_{\Lambda} = \tau/(N_1 N_2)$, $\delta_{\pi} = \tau/N_2$
- s quark: not domain-decomposed, $\delta_{\pi} = \tau/(N_1 N_2)$
- SAP+GCR for IR part, SSOR+GCR for UV part
- tolerance: $|\text{residual}|/|\text{source}| \leq \epsilon$, $\epsilon = 10^{-9}\text{(force), } 10^{-14}\text{(}\delta)$

<table>
<thead>
<tr>
<th>$\kappa_{ud}$</th>
<th>0.13727</th>
<th>0.13754</th>
<th>0.13770</th>
<th>0.13776</th>
<th>0.13781</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m_{ud}^{\text{AWI}}[\text{MeV}]$</td>
<td>45</td>
<td>24</td>
<td>12</td>
<td>9</td>
<td>6</td>
</tr>
<tr>
<td>$N_0, N_1, N_2$</td>
<td>4,4,14</td>
<td>4,4,20</td>
<td>4,4,16</td>
<td>4,4,26</td>
<td>4,4,12</td>
</tr>
<tr>
<td>$\tau$</td>
<td>0.5</td>
<td>0.5</td>
<td>0.25</td>
<td>0.25</td>
<td>0.075</td>
</tr>
<tr>
<td>$\text{trajs}$</td>
<td>$-$</td>
<td>4600</td>
<td>3300</td>
<td>$-$</td>
<td>1400</td>
</tr>
<tr>
<td>$\tau_{\text{int}}[P]$</td>
<td>$-$</td>
<td>12.5(4.2)</td>
<td>7.7(2.3)</td>
<td>$-$</td>
<td>$-$</td>
</tr>
</tbody>
</table>
§5. Hadron effective masses (preliminary)

\[ \kappa_{ud} = 0.13754 (m_{ud}^{AWI} = 24 \text{MeV}) \]
more than encouraging albeit possible $O((a\Lambda_{QCD})^2)$ errors
\[ f_K/f_\pi = 1.201(29) \]
\[ 1.207(12) \text{ (exp.)} \]

consistent with the experimental value
§7. Summary

target of the PACS-CS project
– three $\beta$ values, two $\kappa_s$, (3.0fm)$^3$
– aim at the physical point

next step and on-going project
– examine chiral logs
– investigate phase structure
– simulations at another $\kappa_s$
– nonperturbative $Z_A$, $Z_m$ with the SF method
– calculation of $\eta'$ meson mass
A steady stream of developments in Lattice QCD have made it possible today to begin to address the question of how nuclear physics emerges from the underlying theory of strong interactions. Central role in this understanding play both the effective field theory description of nuclear forces and the ability to perform accurate non-perturbative calculations in low energy QCD. Here I present some recent results that attempt to extract important low energy constants of the effective field theory of nuclear forces from lattice QCD.
Nuclear physics

- Connect Nuclear physics to QCD
- Yet a smaller scale
- Nuclear binding energy ~ MeV
- Does it look hopeless?
- Not really!
Hadronic Interactions

- Scattering processes from Lattice QCD are not straightforward.
- Miani-Testa no-go theorem ('90) [see C. Michael '89]
- Infinite Volume:
  - Euclidean → Minkowski
- Finite volume: discrete spectrum
- Avoids Miani-Testa no-go theorem [M. Luscher]

Luscher Formula

Energy level shift in finite volume:

\[ \Delta E_L = E_L - 2m - 2 \sqrt{p^2 + m^2} - 2m \]

**P_\infty** solutions of:

\[
p \cos \delta(p) = \frac{1}{\pi L} S \left( \frac{p^2 L^2}{4 \pi^2} \right) \]

\[
S(q) = \sum \frac{1}{q^2 - q} - 4m
\]

\[
p \cot \delta(p) = \frac{1}{a} + \cdots
\]

\[
\frac{1}{a} = \frac{1}{\pi L} S \left( \frac{p^2 L^2}{4 \pi^2} \right) + \cdots
\]

Expansion at \( p \to 0 \):

\[ \Delta E_0 = -\frac{4 \pi a}{m L^3} \left[ 1 + \frac{a}{L} + c_2 \left( \frac{a}{L} \right)^2 \right] + O \left( \frac{1}{L^4} \right) \]

\( c_1 \) and \( c_2 \) are universal constants

\( a \) is the scattering length
I=2 Pion Scattering

\[ m_N a_2 = \frac{m_N^2}{8 \pi f} \left( 1 + \frac{3m_N^2}{16\pi f^2} \left( \log \frac{m_N^2}{\mu^2} + 2 \pi (\alpha) \right) \right) \]

[Gasser-Leutwyler '84] [Colangelo et al. '01]

- \( m_N a_2 = -0.0422(3)(18) \)
- Experiment: \( m_N a_2 = -0.0454(31) \)
- SyPT has insignificant effect to the result [Chen et al. '05]

Kaon Pion Scattering Lengths

- Upcoming experiments on Kaon-pion molecules (DIRAC collab.)
- Continuum extrapolation still needed
Nucleon-Nucleon


BBSvK: Beane, Bedaque, Savage, van Kolck 02
W: Weinberg '90, Weinberg '91, Ordonez et al. '95
Fukugita et al. '95: Quenched heavy pions

Nucleon-Hyperon

NPLQCD: hep-lat/0612026
Hadronic interactions (Future)

- These calculations are the beginning of the beginning!
- Need lighter pion masses, multiple volume sizes, and lattice spacings
  - Determine if we see scattering states
- Meson baryon channels: (K-ν, Kξ...) ---- Neutron stars
- Hyperon-Hyperon and Hyperon-Nucleon channels [NPLQCD: hep-lat/0610286]
  - Hyper-nuclear physics and Neutron stars
- Need to make lattices designed for this project
- Higher statistics: (JLAB spectrum program – INCITE recent award)

Hadron Interactions: Projected errors

- Errors on scattering nucleon-nucleon scattering length as function of computational resources
- Only cost for correlation function calculation presented
Semileptonic Hyperon Decays in Full QCD

Huey-Wen Lin

Jefferson Lab
Thomas Jefferson National Accelerator Facility

in collaboration with Kostas Orginos

Huey-Wen Lin — DWF@10 Workshop
Action and Parameters

- This calculation:
  - Mixed action: DWF on staggered sea
  - Pion mass range: 360–700 MeV
  - Strange-strange Goldstone fixed at 763(2) MeV
  - Volume fixed at 2.6 fm
  - $a \approx 0.125$ fm, $L_s = 16$, $M_s = 1.7$
  - HYP-smeared gauge, box size of $20^3 \times 32$

- Octet spectrum along with experimental numbers

<table>
<thead>
<tr>
<th>Label</th>
<th>$m_\pi$ (MeV)</th>
<th>$m_K$ (MeV)</th>
<th>$\Sigma^- \rightarrow n$ conf.</th>
</tr>
</thead>
<tbody>
<tr>
<td>m010</td>
<td>358(2)</td>
<td>605(2)</td>
<td>600</td>
</tr>
<tr>
<td>m020</td>
<td>503(2)</td>
<td>653(2)</td>
<td>420</td>
</tr>
<tr>
<td>m030</td>
<td>599(1)</td>
<td>688(2)</td>
<td>561</td>
</tr>
<tr>
<td>m040</td>
<td>689(2)</td>
<td>730(2)</td>
<td>306</td>
</tr>
</tbody>
</table>

Huey-Wen Lin — DWF@10 Workshop
Momentum Extrapolation

\begin{align*}
\text{\(m_x=359\) MeV} & \quad & \text{\(m_x=503\) MeV} \\
\text{\(m_x=599\) MeV} & \quad & \text{\(m_x=689\) MeV}
\end{align*}

Huey-Wen Lin — DWF@10 Workshop
Mass Dependence

- Use $\delta = a^2(M_K^2 - M_\pi^2)$ to describe the SU(3) symmetry breaking

$f_1(0) = 0.90(7)$ (Preliminary)

Huey-Wen Lin — DWF@10 Workshop
Axial Coupling Constants: $g_{\Xi\Xi}$ and $g_{\Sigma\Sigma}$

- Cannot be determined by exp.
- Existing predictions from xPT and large $N_c$ calculations

$0.18 < -g_{\Xi\Xi} < 0.36$

$0.30 < g_{\Sigma\Sigma} < 0.55$

- Applications such as hyperon scattering, non-leptonic decays, etc.

Huey-Wen Lin — DWF@10 Workshop
Summary/Outlook

- First non-quenched calculation of hyperon semileptonic decays
- Lighter pion masses as low as 350 MeV
- Preliminary results show $|V_{us}|$ (from $\Sigma \rightarrow n$)
  - Consistent with the previous lattice measurement
  - Larger error due to lighter pion mass
  - More statistics needed for lightest point!

In the near future:
- Finish the semileptonic form factor analysis, including $\Xi \rightarrow \Sigma$ channel
- $\Sigma$ and $\Xi$ structure-function form factors
- Possibly take $\Lambda \rightarrow p$ data, if time allows

Huey-Wen Lin — DWF@10 Workshop
Nucleon structure from $N_f = 2 + 1$ DWF simulations

T. Yamazaki for RBC-UKQCD collaboration

We present our results of nucleon matrix elements, e.g. the nucleon axial charge, moments of quark distributions, and iso-vector form factors, calculated on $N_f = 2 + 1$ dynamical domain wall fermion configurations, recently generated by our collaboration where $a^{-1} \approx 1.6$ GeV. We employ four quark masses which give the pion mass from about 300 MeV to 700 MeV to extrapolate the results to the chiral limit. While the result are preliminary because the statistics is not enough at the lighter two quark masses, we found that the axial charge in the chiral limit is reasonably consistent with the experiment, and the moments are closer to the experiment than the results with quenched domain wall fermion.
1. Introduction
Motivation: understand nucleon structure from first principle

We calculate matrix elements related to nucleon structure on $N_f = 2 + 1$ DWF configuration.

- $g_A/g_V$
  Well determined experimentally: $g_A/g_V = 1.2673(35)$

- Moments of quark distributions
  Deep inelastic scattering; structure functions
  $\langle x \rangle_q \rightarrow$ Unpolarized: $F_1(x, Q^2), F_2(x, Q^2)$
  $\langle x \rangle_{\Delta q} \rightarrow$ Polarized: $g_1(x, Q^2), g_2(x, Q^2)$

- Form factors
  Elastic scattering
  \[
  F_1(q^2) = \frac{1}{(1 + q^2/M_V^2)^2}, \quad \langle r_{ch}^2 \rangle = 12/M_V^2,
  \]
  \[
  G_A(q^2) = \frac{g_A}{(1 + q^2/M_A^2)^2}, \quad \langle r_{ax}^2 \rangle = 12/M_A^2
  \]
3. Simulation parameters

- $N_f = 2 + 1$ Iwasaki gauge + Domain Wall fermion actions
- $\beta = 2.13 \ a^{-1} = 1.62 \ GeV \ M_5 = 1.8 \ m_{\text{res}} = 0.003$
- Lattice size $24^3 \times 64 \times 16 \ (L a \approx 3 \ fm)$
- $m_s = 0.04$ fixed (close to $m_s^{\text{phys}}$)
- quark masses and confs.

<table>
<thead>
<tr>
<th>$m_f$</th>
<th>$m_\pi [\text{MeV}]$</th>
<th># of confs. $\times N_{\text{meas}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.005</td>
<td>310</td>
<td>$52 \times 4$</td>
</tr>
<tr>
<td>0.01</td>
<td>390</td>
<td>$119 \times 4$</td>
</tr>
<tr>
<td>0.02</td>
<td>520</td>
<td>$49 \times 4$</td>
</tr>
<tr>
<td>0.03</td>
<td>690</td>
<td>$53 \times 4$</td>
</tr>
</tbody>
</table>

Results at two lighter masses do not have good accuracy, so that all results are preliminary.

- We focus only on iso-vector quantities. (no disconnected diagram)
4. Preliminary results

4.1. $g_A/g_V$

$m_\pi$ is lighter than $m_\pi$ in $N_f = 0$ case.

Results at two lighter $m_\pi$ has larger error and fluctuation.

There seems to be no dynamical effect.

Preliminary result

$$g_A/g_V = \begin{cases} 1.22(10) \text{ (lat.)} \\ 1.267(4) \text{ (exp.)} \end{cases}$$

We will confirm the result is reliable by improving statistics.
4.2. Moments of quark distributions (cont’d)

Each component in $N_f = 0$ is independent of $m_\pi$.

\[ \langle x \rangle_{u-d} \text{ and } \langle x \rangle_{\Delta u-\Delta d} \text{ are closer to experiment, and have some } m_\pi \text{ dependence.} \]

Perturbative $Z(2\text{GeV}) = 0.88(5)$ from PLB641,67

→ We will calculate $Z$ factor by non-perturbative method.

Preliminary result
4.3. Form factors (cont’d)

\[ F_1(q^2) / F_1(0) \]

\[ G_A(q^2) / G_A(0) \]

\[ F_1(q^2) = \frac{1}{(1 + q^2 / M_V^2)^2} \quad G_A(q^2) = \frac{g_A}{(1 + q^2 / M_A^2)^2} \]

\( F_1 \) is almost independent of \( m_\pi \) except for lightest mass.
\( G_A \) has \( m_\pi \) dependence.
Here we discuss the accuracy with which the matrix elements of an operator such as the strong penguin operator $O_6$ can be computed using the domain wall fermion formulation. The matrix element $\langle \pi | O_6 | K \rangle$ is one of the two most important parts of a calculation of $\epsilon'/\epsilon$ and is made difficult by the mixing of the dimension-6 operator $O_6$ with the lower dimension operators $\mathfrak{F}d$ and $\mathfrak{F} \gamma^5 d$. The very mild, explicit chiral symmetry breaking present in the domain wall fermion formulation makes it possible to treat this operator mixing with considerable precision. As discussed in our earlier RBC paper\(^1\), the leading order effect of the explicit chiral symmetry breaking on the matrix element $\langle \pi | O_6 | K \rangle$ is to add a small constant, proportional to the residual mass, $m_{\text{res}}$ to this matrix element. The physical matrix element vanishes linearly in the quark mass as the chiral limit is approached and the quantity of interest is the slope of this mass dependence. Thus, the slope of interest is not affected by this additive constant, $\propto m_{\text{res}}$. Here we consider next-leading corrections to this result, examining the extent to which unphysical terms are present which are proportional to products of the quark mass $m_f$ and the residual mass but enhanced by the $1/a^3$ factor associated with the power divergence of this lower-dimension operator mixing. We conclude that while such effects are present on the few percent level they can be explicitly subtracted leaving uncertainties well below the 1%.

Power Divergent Subtraction from $O_6$

- Matrix element $\langle \pi | O_6 | K \rangle$
  - Contains divergent, unphysical $\langle \pi | \bar{s}d | K \rangle$ contribution.
  - Removed by subtraction specified by chiral perturbation theory.
  - How large is the systematic error on this 5% difference?

(a) Two terms to be subtracted

(b) Result after subtraction
Recall Leading Order Argument

- $O_6$, $(8, 1)$ operator contains $(\bar{3}, 3)$ and $(3, \bar{3})$ counter terms.

\[
(\cdot O_6)_{\text{finite}} = \bar{s} \gamma^\mu (1 - \gamma^5)d \sum_{q = u, d, s} \bar{q} \gamma^\mu (1 + \gamma^5)q \\
+ \eta_0 \bar{s}(1 + \gamma^5)q_b (M_\ell)_{2,b} + \eta_0 \bar{q}_a (1 - \gamma^5)d (M^*_\ell)_{3,a} \\
+ \eta_1 \bar{s}(1 + \gamma^5)q_b (\Omega)_{2,b} + \eta_1 \bar{q}_a (1 - \gamma^5)d (\Omega^*)_{3,a}
\]

- $\eta_0$ and $\eta_1$ are $O(1/a^3)$.
- $M_\ell$ is the usual $(3, \bar{3})$ fermion mass matrix.
- $\Omega$ is a $(3, \bar{3})$ spinor matrix in the DWF $s$-derivative connecting the $s = L_s/2$ and $s = L_s/2 - 1$ slices.

\[
\bar{\Psi}_{x,L_s/2}(1 + \gamma^5)\Psi_{x,L_s/2-1} + \bar{\Psi}_{x,L_s/2-1}(1 - \gamma^5)\Psi_{x,L_s/2} \\
\rightarrow \bar{\Psi}_{x,L_s/2}(1 + \gamma^5)\Omega \Psi_{x,L_s/2-1} + \bar{\Psi}_{x,L_s/2-1}(1 - \gamma^5)\Omega^\dagger \Psi_{x,L_s/2}
\]

- We expect $\eta_1 \propto e^{-\alpha L_s}$. 
Mixing of Order $M_f \times \Omega$?

- A new counter term appears:
  \[ \delta O_6 = \eta_2 \bar{s}(1 + \gamma^5)q_{a'} (M_f^*)_{b,b'}(\Omega^*)_{c,c'} \epsilon^{2, b, c \epsilon a', b', c'} + \eta_2 \bar{q}_{a'}(1 - \gamma^5)d (M_f)_{b,b'}(\Omega)_{c,c'} \epsilon^{3, b, c \epsilon a', b', c'}. \]

- For conventional values for $M_f$ and $\Omega$:
  \[ \delta O_6 = \eta_2 (m_s - m_d) \bar{s}\gamma^5 d + \eta_2 (m_d + m_s + 2m_u) \bar{s}d. \]

- Trouble?
  - $\eta_2 \approx 0.003$ enhanced $20\times$ gives $6\%$ correction to physical slope!
  - Could discover and remove using $m_u$ dependence?

- Ruled out by perturbative $U_L(3) \times U_R(3)$ symmetry:
  Even powers of $\Omega$ required.
Mixing of Order $M_f \times \Omega \times \Omega$?

- If $\Omega^2$ term were of order $m_{\text{res}}^2$ we could drop it: $(0.003)^2$ enhanced $20 \times$ gives $0.02\%$ effect.

- Residual mass reflects two effects:
  - Eigenstates of $H_T$ above the mobility edge: $m_{\text{res}} \propto \frac{1}{L_s} e^{-\lambda_c L_s}$.
  
  \[ \Omega^2 \propto \left( \frac{1}{L_s} e^{-\lambda_c L_s} \right)^2 \]

  - Eigenstates of $H_T$ below the mobility edge: $m_{\text{res}} \propto \frac{\rho^{(0)}}{L_s}$.
  
  \[ \Omega^2 \propto \frac{\rho^{(0)}}{L_s} \]

- Thus, $\Omega^2$ term might $\mathcal{O}(m_{\text{res}})$!
Examine $M_f \times \Omega \times \Omega$ contribution to $O_6$

- Possible counter terms:
  
  $$\delta O_6 = \bar{s}(1 + \gamma^5)q_a (\Omega)_{2,a} \eta_3 \text{tr}(M_f^\dagger \Omega)$$
  
  $$\quad + \bar{q}_a (1 - \gamma^5)d (\Omega^*)_{2,a} \eta_3 \text{tr}(M_f \Omega^\dagger)$$

- Extract the allowed 2nd order polynomial in $\Omega_{ij}$:
  
  $$\Omega_{uu} \Omega_{dd} + \Omega_{uu} \Omega_{ss} + \Omega_{dd} \Omega_{ss}$$

- Substitute conventional values for $M_f$ and $\Omega$:

  $$\delta O_6 = \eta_3 (m_s - m_d) \bar{s} \gamma^5 d + \eta_3 (m_d + m_s + 2m_u) \bar{s} d.$$ 

- Should be subtracted!

  - Reversed sign implies standard subtraction fails.
  - Must fit to $m_u$ dependence and remove.
  - Size of term in question: $0.5 \cdot 0.003 \times 20 \approx 3\%$. 
$B_K$ with two flavors of dynamical overlap fermions

Norikazu Yamada (KEK)

for JLQCD Collaboration
Simulation parameters

- **Gauge: Iwasaki RG (\( \beta=2.30 \))**
  - extra Wilson fermions \((m_0=1+s=1.6)\)
    - to prevent topological charge, \(Q\), from changing
  - ghosts with twisted mass \(\mu=0.2\)
    - to suppress unwanted UV effects due to extra Wilson fermions

\[
\exp(-S_g^{Iwasaki}) \Rightarrow \det \left| \frac{H_W(m_0)^2}{H_W(m_0)^2 + \mu^2} \right| \exp(-S_g^{Iwasaki}) \quad \text{[JLQCD, PRD74 (2006)094505]}
\]

\(r_0=0.49 \text{ fm} \quad a=0.1184(12) \text{ fm} \quad (1/\alpha=1.667(17) \text{ GeV})\)

\((L/a)^3 \times (T/a) = 16^3 \times 32 \quad V \approx (1.9 \text{ fm})^3\)

Results are from configurations in \(Q=0\).
Simulation parameters

- Sea and valence quarks: Overlap fermion
  - 6 sea quark masses \((am_{\text{sea}}=0.015, 0.025, 0.035, 0.050, 0.070, 0.100)\)
  - \(1/6\, m_s < m_q < m_s\) \(0.34 < m_\pi/m_\rho < 0.67\)
  - our lightest pion \(m_\pi \approx 293\text{ MeV}, m_\pi L \approx 2.8\)

- 6 valence quark masses take the same values as the sea’s.

- LMP and LMA are implemented for all valence propagators.

- While all degenerate and non-degenerate mesons are measured,
  
  I focus on the degenerate mesons \((m_{\text{val1}}=m_{\text{val2}})\) in this talk.
Test with NLO ChPT

(ii) with free $f$

- While $\chi^2$/dof is reasonable for all fit ranges, $f$ largely depends on them.
- When the fit range is $[0, m_s/2]$ or smaller, $f$ takes a reasonable value.

Surely confident of that our three or four lightest quarks are well inside the NLO ChPT regime.
Introducing higher order terms

- naive: (4 or 5 free parameters)
- non-naive: (1 free parameters)

In both cases, the whole data are well interpolated.
Preliminary result for $B_K$

Interpolate to physical $m_K$ using the NNLO-like functions to obtain $B_K$ at $m_{\text{sea}}=m_{\text{ud}}$ and $m_{\text{val1}}=m_{\text{val2}}=m_s/2$ as

$$B_K^{\overline{\text{MS}}}(2\text{GeV}) = \begin{cases} 0.534(3) & \text{naive NNLO w/ free } f \\ 0.540(10) & \text{non-naive w/ free } f \end{cases}$$

(statistical error only)

$$B_K^{\overline{\text{MS}}}(2\text{GeV}) = 0.509(18) \ [\text{RBC (incl. systematic error)}]$$
Summary

• Calculation of $B_K$ is now in progress.
• Preliminary result looks promising.
• Our three or four lightest quarks are well inside the NLO ChPT regime.
• To do
  - include non-degenerate mesons,
  - study topological charge dependence numerically,
  - clarify finite volume effects using ChPT with FV,
  - estimate systematic errors.
The Kaon Bag Parameter
from 2+1-Flavors of Domain-Wall Fermion

Saul D. Cohen
sdcohen@phys.columbia.edu

RBC and UKQCD Collaborations
(Columbia University)
Calculations done using the QCDOC supercomputers
at RIKEN-BNL Research Center and the University of Edinburgh
The Solution

Operator Mixing
In the continuum, $B_K$ contains only the operator of the form $VV + AA$, which renormalizes multiplicatively:

$$O_{VV + AA}^{\Delta S=2} = (\bar{s}\gamma_\mu d)(\bar{s}\gamma_\mu d) + (\bar{s}\gamma_\mu \gamma_5 d)(\bar{s}\gamma_\mu \gamma_5 d)$$

Domain-Wall Fermions
The domain-wall formalism allows us to control chiral symmetry breaking; as long as our residual masses small, we avoid the unwanted terms which contribute at $O(m^2_{\text{res}})$. In this framework, we may also apply continuum partially quenched chiral perturbation theory and use non-perturbative renormalization.
$16^3 \times 32$ Results

Kaon Bag Parameter

$B_K$ is computed for 15 nondegenerate combinations of valence strange and light masses. Light mass runs along the $x$-axis; strange mass is denoted by color. We fit to the partially quenched 2+1 flavor NLO chiral perturbation theory form of Sharpe and Van de Water. $B_K^{\text{bare}} = 0.607(9)$
Nonperturbative Renormalization

Lattice-to-Continuum Matching

$Z_{VV+AA, VV+AA}$ can be divided into a constant $Z^{RGI}$ and a running $f(\mu)$. 

![Graph showing the relationship between $(ap)^2$ and a quantity representing $Z^{RGI}$ and $f(\mu)$.]
$16^3 \times 32$ Results

Comparisons
Here we compare our 2+1 flavor domain-wall fermion $B_K$ result with other chiral fermion and unquenched results.

With $Z_{B_K} = 0.917(07)(18)$ in $\overline{\text{MS}}$ at 2 GeV

$$B_K = 0.557(12)(16)$$
Three-Point Plateaux
The much longer length of the time-dimension allows us to average over more fluctuations in the gauge field, yielding smaller statistical errors.

\[ B \text{ Plateau on } 24^3, m_{\text{sea}} = 0.04, m_{\text{sea}} = 0.005 \]

\[ B \text{ Plateau on } 24^3, m_{\text{sea}} = 0.04, m_{\text{sea}} = 0.01 \]
Update on $B_K$ with a mixed action

Jack Laiho, Fermilab
Christopher Aubin
Ruth Van de Water

Domain Wall Fermions after ten years
RIKEN-BNL 2007
March 16
Mixed action simulations

Our simulations use MILC lattices with asq-tad staggered quarks in the sea sector and domain wall quarks in the valence sector.

Advantages

- A large number of ensembles with different volumes, sea quark masses and lattice spacings exist and are publicly available.
- The existing ensembles have 2+1 flavors of light sea quarks ($m_{strange}/8$ for the lightest quarks)
- The good chiral properties of the valence sector make things much simpler than the staggered case. There are only two additional parameters (over pure domain wall) that appear at one loop in the mixed action ChPT for $m_\pi$, $f_\pi$, and $B_K$. They can both be obtained from spectrum calculations.
- NPR can be carried through in the same way as in domain wall.
Values for $\Delta_{\text{mix}}$

(a = 0.125 fm)

- $m_{\text{sea}} = 0.007/0.05$
- $m_{\text{sea}} = 0.01/0.05$
- MILC splittings: $\Delta_A, \Delta_r, \Delta_V, \Delta_l$
- $\Delta_{\text{mix}}$ extrapolation
Parameters of the simulation

- Done on MILC lattices with improved staggered (asqtad) sea quarks

- Many MILC lattice ensembles exist. This work only makes use of the MILC coarse lattices \((a \approx 0.12 \text{ fm})\). We will add the fine \((a \approx 0.09)\) soon.

- The lightest quark masses have \(\approx m_{\text{strange}}/8\) and \(m_{\pi}L \geq 4\) for all data points.

- Following LHPC, we are using HYP smearing with the usual Hasenfratz parameters to reduce residual chiral symmetry breaking.

- We are using periodic+antiperiodic boundary conditions and periodic-antiperiodic boundary conditions to create forward and backward propagators, effectively doubling the time extent of the lattice.
$B_K$ on a single ensemble

$a = 0.125$ fm; $m_{\text{sea}} = 0.007/0.05$
Estimate of future error budget

- 3% chiral extrapolation error (similar to MILC's $f_K$ number)

- Finite Volume Effects: 1.5%

- NPR: Similar to RBC ≈ 3%

- Lattice spacing dependence: again, similar to RBC ≈ 4%. This will improve with another lattice spacing.

- Statistical: 2 – 4% now. This will improve also.

Added in quadrature, we estimate around 7% now, and 5% after another year of running.
K_{13} Form Factor with N_f=2+1 Domain Wall Fermions

James Zanotti
University of Edinburgh
UKQCD/RBC Collaboration

We present the latest results from the UKQCD/RBC collaborations for the K_{13} form factor with 2+1 flavours of dynamical domain wall quarks. Simulations are performed on $16^3 \times 32 \times 16$ and $24^3 \times 64 \times 16$ lattices with three values of the light quark mass, allowing for an extrapolation to the chiral limit.

After interpolating to zero momentum transfer, we obtain the preliminary result $f_+(0)=0.9609(51)$, which is in agreement with the result of Leutwyler & Roos.
Motivation

- $K \rightarrow \pi \ell \nu$ ($K_{l3}$) decay leads to determination of

$$\text{decay rate } \propto |V_{us}|^2 |f_+(q^2 = 0)|^2$$

- Require precise theoretical determination

- Current conservation $\quad \rightarrow \quad f_+(0) = 1|_{su(3) \text{ flavour limit}}$

- Ademollo-Gatto Theorem $\rightarrow$ second order SU(3) breaking effects in $f_+(0)$

$$f_+(0) = 1 + f_2 + f_4 + \cdots$$

$$\Rightarrow \Delta f = 1 + f_2 - f_+(0)$$

- [Leutwyler & Roos: $f_2 = -0.023$]
Parameters

- $N_f = 2 + 1$ flavours of dynamical domain wall fermions
- Iwasaki gauge action

$\beta = 2.13, \ L_s = 16, \ am_{\text{res}} \approx 0.03, \ a \approx 0.121 \text{ fm}, \ am_s = 0.04$

<table>
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<th>$am_q$</th>
<th>Volume</th>
<th>$m_\pi$ [MeV]</th>
<th>$m_K$ [MeV]</th>
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<td>0.668(2)</td>
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<tr>
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<td>0.567(1)</td>
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<td>0.03</td>
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<td>0.516(1)</td>
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<tr>
<td>0.01</td>
<td></td>
<td>0.385(1)</td>
<td>0.559(1)</td>
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</tbody>
</table>

For more details, see hep-lat/0701013
Pole: \[ f_0(q^2) = \frac{f_0(0)}{1 - q^2/M^2} \]
\[ |V_{us}| \]

\[ \Delta f = -0.0161(46)(15)(16) \Rightarrow f_+^{K\pi}(0) = 0.9609(51) \]

Using \[ |V_{us}f_+(0)| = 0.2169(9) \] from experimental decay rate:

\[ |V_{us}| = 0.2257(9)_{\text{exp}}(12)f_+(0) \]

\[ |V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1 - \delta, \quad \delta = 0.00076(62) \]

**PDG(2006)/LR:**

\[ |V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1 - \delta, \quad \delta = 0.0008(10) \]
Summary and Future Work

Preliminary $N_f = 2 + 1$ result for

* agrees well with L/R result
* no obvious finite size effects
* small statistical error
* progress towards controlling systematics

Further Improvements

* Lighter quark masses
* Another $\beta \rightarrow$ continuum limit
* Twisted boundary conditions $\rightarrow$ smaller $q^2$
Chiral Violations in Perturbative Domain Wall QCD

Stefano Capitani
Institut für Kernphysik
UNIVERSITÄT MAINZ

We calculate in lattice perturbation theory, using the exact propagators corresponding to the setting of a finite number of points in the fifth dimensions, the residual mass and other matrix elements which measure the breaking of chiral symmetry in domain-wall fermions. We present results for several choices of the domain-wall parameters.
Domain-wall simulations: finite number of points $N_s$ in the 5th dimension
$\Rightarrow$ violations of chiral symmetry

Only in the theoretical limit in which $N_s$ becomes infinite the chiral modes can fully decouple from each other, yielding an exact chiral symmetry

We investigate the chirality-violating effects using perturbative calculations

The focus is at small $N_s$, where the simulations are currently performed

We have computed three quantities:
- the residual mass
- the difference $\Delta = Z_V - Z_A$
- a chirally-forbidden mixing (nonzero at finite $N_s$) of an operator which describes the lowest moment of the $g_2$ structure function

For all this, we had to derive the required propagator functions

We have computed the same quantities with the plaquette action as well as with improved gauge actions (Lüscher-Weisz, Iwasaki, DBWZ)

In numerical simulations it has been seen that these improved gauge actions (especially DBW2) reduce the chiral violations
Chiral violations

How small can $N_s$ be while still avoiding large values of the residual mass?

To investigate this problem, we have computed various matrix elements for several choices of $N_s$ and of the domain-wall height $M$.

A thorough exploration of large regions in the two-dimensional space spanned by $N_s$ and $M$ would be quite expensive for Monte Carlo simulations.

Perturbation theory remains then often the more practical and cheaper way for gathering hints of what is happening when the parameters move in this space.

We want to study the dependence of $m_{res}$ (and other quantities that can act as indicators of chiral violations) on $M$ and $N_s$.

We have then carried out some selected one-loop calculations using the Feynman rules which exactly correspond to the theory at finite $N_s$.

This is then not the situation of past calculations where, in place of the exact quark propagators, their asymptotic expressions for large $N_s$ were used.

The purpose of this work is to calculate with the exact Feynman rules the deviations from the $N_s = \infty$ results in the case where $N_s$ is limited to small values, of $O(10)$.
Tadpoles

The tadpole contributions present wide variations with $N_s$ and $M$, so that sometimes they turn out to be small while sometimes they are large.

This suggests that some care should be used when talking about tadpole dominance in relation to domain-wall fermions.

Both tadpoles (of $\Sigma_0$ and $\Sigma_1$) even decrease toward zero for $M \to 0$ or $M \to 2$.

A central point also is that there are two kinds of tadpoles in the game here:

- the tadpole of order zero in $p$, which tends to zero for $N_s \to \infty$, which contributes to $\Sigma_0$ and the residual mass.
- the tadpole of order $ap$, which tends to its Wilson value for $N_s \to \infty$, which contributes to $\Sigma_1$ and the renormalization factors.

They really behave differently, in the large $N_s$ case as well as in the exact case for any finite $N_s$.

Essentially, the $i\gamma$ in the first order flips the chirality of the damping factors, and they then combine in a different way.
Residual mass

Residual mass in lattice units at $\beta = 5.2$ in Landau gauge.

<table>
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<th>$M$</th>
<th>$N_s = 8$</th>
<th>$N_s = 12$</th>
<th>$N_s = 16$</th>
<th>$N_s = 20$</th>
<th>$N_s = 24$</th>
<th>$N_s = 28$</th>
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Conclusions

- Our perturbative calculations show that the numerical deviations from the case of infinite $N_s$ depend, apart from $N_s$ and to a smaller extent from the bare coupling $g_0$, very strongly on the choice of $M$.
- These deviations can become rather pronounced when $M$ is close to the borders of the region of allowed values.
- The values of $am_{res}^{(1)}$ are positive only for $M \geq 1.2$.
- Looking at the numbers for $N_s = 8$, our results would indicate that the minimal amount of chiral violations is attained for $M \sim 1.2$.
- The pattern of the deviations from the case of exact chirality turns out to be approximately the same for all quantities studied.
- For $M = 1.8$, a standard choice in Monte Carlo simulations, chiral violations are still not small for $N_s = 16$.
- For example, $m_{res}$ for a lattice spacing of $2 \text{GeV}$ is equal to about $100 \text{MeV}$ in the quenched case and about $120 \text{MeV}$ in full QCD.
- For the difference between the vector and axial-vector renormalization constants as well as for the power-divergent mixing, the chiral violations are instead of about $2 - 3 \text{MeV}$, suggesting that they are of higher order in $m_{res}$. 

Y. Kikukawa
Institute of Physics, University of Tokyo

**Electroweak theory on the lattice with exact gauge invariance and its applications**

In this talk, I discuss what one can do with lattice gauge theory to study the electroweak theory.

I first show that by using Domain wall fermion, or, the overlap Dirac operator satisfying the Ginsparg-Wilson relation, it is possible to formulate the electroweak theory on the lattice, keeping exact gauge invariance. Our construction is for both infinite and finite four-dimensional lattice. It covers all SU(2) topological sectors with vanishing U(1) magnetic fluxes. Our result provides the first gauge-invariant regularization of the electroweak theory.

Then I discuss about two possible applications of this formulation, which seem feasible numerically. The first one is a computation of the effect of quarks and leptons to the sphaleron rate at finite temperature. The second one is a lattice construction of a model of dynamical electroweak symmetry breaking.
Construction of SU(2) x U(1) Electroweak theory (II)

finite volume case

\[ \Gamma_4 = \{ x = (x_0, \cdots, x_3) \in \mathbb{Z}^4 \mid 0 \leq x_\mu < L \} = \mathbb{L}^4 \]

\[ \| 1 - U_\square^{(2)} \| \leq \epsilon \quad \| 1 - \{U_\square^{(1)}\}^{6Y} \| \leq \epsilon \quad \epsilon < \frac{1}{30} \]

\[ U_\mu(x) = e^{iA^T_\mu(x)}g(x)g(x + \hat{\mu})^{-1}U_{[w]}(x, \mu)V_{[m]}(x, \mu) \]

\[ F_{\mu\nu}(x) = \partial_\mu A^T_\nu(x) - \partial_\nu A^T_\mu(x) + \frac{2\pi m_{\mu\nu}}{L^2} \]

a pair of doublets (a,b) measure defined globally!

\[ v_j^{(a)}(x) = v_j(x) \]

\[ v_j^{(b)}(x) = (\gamma_5 C^{-1} \otimes i\sigma_2) [v_j(x)]^* \]

cf. Nuberger (98) Bar-Campos (00)
Construction of SU(2) x U(1) Electroweak theory (II)

\[ \eta_\mu(x) = \eta^{(2)}_\mu(x) \oplus \eta^{(1)}_\mu(x) \quad U^{(1)}_t(x, \mu) = e^{it\hat{A}_\mu(x)} U_{[w]}(x, \mu) \quad t \in [0,1] \quad (m_{\mu\nu} = 0) \]

\[ \mathcal{L}_\eta = i \sum_{i} (\nu_i, \delta_\eta \nu_i) = \sum_x \eta_\mu(x) j_\mu(x) \]

\[ = i \int_0^1 dt \text{Tr} \left\{ \hat{P}_- \left[ \partial_t \hat{P}_-, \delta_\eta \hat{P}_- \right] \right\} + \delta_\eta \int_0^1 dt \sum_{x \in \Gamma_4} \left\{ \hat{A}_\mu(x) k_\mu(x) \right\} + \mathcal{L}_\eta \big|_{U(1)=U_{[w]}, U^{(2)}} \]

**local counter term!**  **Wilson line contr.**  **Kadoh-YK in prep.**  **cf. Luscher(98)**

**gauge anomaly cancellation**

- cohomological analysis in \( \Gamma_4 \quad x \in \Gamma_4 \)
- \( q(x) = \text{tr} \left\{ \gamma_5 (1 - aD)(x, x) \right\} \big|_{U(1), U^{(2)}} \)

\[ \sum_\alpha Y_\alpha q(x) \big|_{U(1) \to \{U(1)\} Y_\alpha} \]

\[ = \sum_\alpha Y_\alpha q(x) \big|_{U(x)} + \sum_\alpha Y_\alpha^3 \frac{1}{32\pi^2} \epsilon_{\mu\nu\lambda\rho} F_{\mu\nu}(x) F_{\lambda\rho}(x + \hat{\mu} + \hat{\nu}) + \delta_\mu^* k_\mu(x) \]

\[ = \partial_\mu^* k_\mu(x) \]

**Suzuki et al. (01) Kadoh-Nakayama-YK(04)**
Construction of SU(2) x U(1) Electroweak theory (II) 
finite volume case

- covers all SU(2) topological sectors with vanishing U(1) magnetic fluxes
- global integrability can be proved rigorously
  even number of SU(2) doublets, U(1) Wilson line parts
- explicit with two simplifications  cf. U(1), Luscher (98)
  ★ direct proof of gauge anomaly cancellation in \( \mathbb{L}^4 \)
  ★ separate treatment of the Wilson line
- some non-perturbative applications?

based on:
D.-Kadoh and Y.K., in preparation
possible applications of lattice EW theory (II)

a construction of a model of
dynamical EW symmetry breaking

\[ \text{SU}(2)_L \times U(1)_Y \rightarrow U(1)_{EM} \]

\[ \text{SU}(2)_{TC} \quad \text{"minimal walking technicolor model"} \]

Dietrich, Sannino, Tuominen (05)

4 x SU(2)_{TC} adjoint \quad 4 x SU(2)_{TC} singlet

\[
\begin{align*}
\lambda_1^a & \quad 3 \times \text{SU}(2)_L \text{ doublet} \\
\lambda_2^a & \quad 1 \times \text{SU}(2)_L \text{ doublet} \\
\lambda_3^a & \\
\lambda_4^a & \\
\psi_1 & \\
\psi_2 & \\
\psi_3 & \\
\psi_4 &
\end{align*}
\]

with DW / overlap fermions,
lattice construction is possible

if \( Y(3,2) = 0 \), just like EW theory

cf. \( N=1 \) SYM \quad \text{Nishimura}(97)

\quad \text{Neuberger}(98) \text{ Kaplan}(99) \quad \cdot

chiral sym. & breaking \( \langle \lambda \lambda \rangle \)

\[
\begin{align*}
\text{SU}(4) & \quad \rightarrow \quad \mathbf{O}(4) \\
\text{SU}(2)_L \times \text{SU}(2)_R & \quad \rightarrow \quad \text{SU}(2)_V
\end{align*}
\]

if \( g_Y = 0 \), numerical
simulation is possible !

*global issue; Neuberger(98), Bar(02)
Properties claimed

- almost conformal; “walking coupling”
- chiral symmetry breaking
  \[ \text{SU(4) ----> O(4)} \]
  matching to CRMT \[ \sum F_\pi \]
  cf. Toublan-Vervaarshot (99)
  the order of finite temp. rest.
- consistent with the (usually severe)
  constraints from EW precision measurements
  \[ O(p^4) \] low energy coupling \[ L_{10} \]
- Light composite Higgs scalar
  \[ M_H \sim 150 \text{ GeV} \]

Check needed by a non-perturbative method
These are the problems familiar in lattice QCD,
although tough
Should we change the topology at all?

Tetsuya Onogi (YITP, Kyoto Univ.) for JLQCD collaboration

RBRC Workshop: "Domain Wall Fermions at Ten Years"
March 16 2007 at BNL

1. Topology in unquenched simulation
2. QCD vacuum
3. $\theta$ and Q dependence of the observables
4. Topological susceptibility
5. Summary
The aim of this talk

- Topology change in unquenched QCD is a serious problem. One should carefully think which strategy should be taken: Enforce the topology change? or fix the topology?

- Review the theoretical understanding of QCD in \( \theta \) vacuum and QCD at fixed topology.

- Claim that the fixed Q effect is a finite size effect, which can be removed in large volume or correctly estimated.

- Give a proposal to measure topological susceptibility at fixed topology.

Talk by T-W. Chiu
3. \( \theta \) and Q dependence of the observables

+discussions with S.Aoki, H. Fukaya and S. Hashimoto

- \( Z(\theta) = \exp[-TE_0(\theta)] \) : partition function in \( \theta \) vacuum
  \[ Z_Q = \frac{1}{2\pi} \int d\theta Z(\theta) \exp(i\theta Q) \] : partition function at fixed Q

- \( G(\theta) = \langle O_1 \cdots O_n \rangle_{\theta} \) : observable in \( \theta \) vacuum
  \[ G_Q = \frac{1}{Z_Q} \frac{1}{2\pi} \int d\theta Z(\theta) G(\theta) \exp(i\theta Q) \] : observable at fixed Q

The partition function and observable at fixed Q can be obtained from those in \( \theta \) vacuum using saddle point approximation for large V (volume)
Parameterizing the vacuum energy as

\[ E(\theta) = L^3 \left[ \frac{1}{2} \chi_t \theta^2 + O(\theta^4) \right], \quad \chi_t : \text{topological susceptibility} \]

one obtains

\[
\begin{align*}
Z_Q &= \exp \left( -\frac{Q^2}{2V\chi_t} + O(V^{-2}) \right) \\
G_Q &= G(0) + G^{(2)}(0) \frac{1}{2V\chi_t} \left( 1 - \frac{Q^2}{V\chi_t} \right) + \text{higher order (for } G=\text{CP even)} \\
G_Q &= G^{(1)}(0) \frac{iQ}{V\chi_t} + \text{higher order (for } G=\text{CP odd)}
\end{align*}
\]

"(n)" means n-th derivative in \( \theta \)

- Difference of observables with fixed \( Q \) and in \( \theta \) vacuum can be estimated as \( 1/V \) correction and higher order.
- Topological susceptibility as well as higher moments are the key quantities.
- One can also obtain the \( \theta \) dependence of CP-odd observable.

EDM can be obtained.
More sytematic proof

Aoki, Fukaya, Hashimoto, Onogi in progress

Consider the topological charge density correlator.

Using formula

\[
\langle \omega(x)\omega(0) \rangle_Q = \langle \omega(x)\omega(0) \rangle_{\theta=0} + \langle \omega(x)\omega(0) \rangle^{(2)}_{\theta=0} \frac{1}{2V\chi t} \left(1 - \frac{Q^2}{V\chi t}\right)
\]

where

\[
\langle \omega(x)\omega(0) \rangle^{(2)}_{\theta=0} = -\langle \omega(x)\omega(0)Q^2 \rangle_{\theta=0} + \langle \omega(x)\omega(0) \rangle_{\theta=0} \langle Q^2 \rangle_{\theta=0}
\]

Using the clustering property

\[
\lim_{|x| \to \infty} \langle \omega(x)\omega(0) \rangle_{\theta=0} = 0
\]

\[
\lim_{|x| \to \infty} \langle \omega(x)\omega(0) \rangle^{(2)}_{\theta=0} \quad = \quad \lim_{|x| \to \infty} 2 \int_{V_x} dy \langle \omega(y)\omega(x) \rangle_{\theta=0} \\
\times \int_{V_0} dz \langle \omega(z)\omega(0) \rangle_{\theta=0} \\
= \quad 2\chi_t^2
\]

\[
\lim_{|x| \to \infty} \langle \omega(x)\omega(0) \rangle_Q = \frac{1}{V} \left(\frac{Q^2}{V} - \chi t\right) + O(V^{-2})
\]
5. Summary

- The effect of fixing the topology is a finite size effect, which can be removed in large volume or correctly estimated by the topological susceptibility and suitable effective theory.

- Fortunately, the pion mass receives the largest correction but other quantities receives only subleading corrections through pion mass.

- The theta dependence of CP-odd observable can also be extracted from fixed topology simulation.

- Topological susceptibility can be measured by the asymptotic values of single pseudoscalar 2pt function at fixed topology.

  Talk by T-W. Chiu

- Systematic study of next-leading order (partially quenched) ChPT is needed.
In this talk I discuss the chiral behavior of non-singlet pseudo-Goldstone boson (PGB) masses and decay constants in mixed action calculations and present preliminary results from actual simulations. Here, mixed action stands for overlap valence quarks on \( N_f = 2 + 1 \) seas of tree-level \( O(a) \)-improved Wilson quarks. One of the main goals of these mixed-action simulations is to determine phenomenologically important weak matrix elements to a few percent accuracy, with controlled extrapolations to the physical limit of QCD. In addition, these studies should yield many interesting “side-products”, such as quark masses, chiral Lagrangian low-energy constants, etc. Moreover, the \( N_f = 2 + 1 \) gauge configurations generated can be used to study many other quantities, associated with the hadron spectrum or charm and beauty physics, for example.

The talk is divided into two main parts. After motivating the use of mixed actions, I briefly review finite-volume, partially-quenched, mixed-action chiral perturbation theory and discuss next-to-leading (NLO) results for the PGB masses and decay constants. It is assumed that quark masses and lattice volumes are chosen such that PGB masses and momenta are comparable in size and are small compared to the chiral cutoff (i.e. that the finite-volume, chiral \( p \)-regime counting of Gasser and Leutwyler is applicable). It is also assumed that \( O(\alpha_s \alpha) \) discretization errors are counted like meson mass or momentum squared, but are sufficiently small for the system to be in the usual chiral phase. I then present preliminary results from ongoing mixed-action simulations and very preliminary fits of NLO chiral expressions to these results.

At this early stage in our investigation, it is clear that more detailed analyses and more data at low masses and at other lattice spacings are needed to determine the extent to which we will be able to reliably make contact with mixed-action chiral perturbation theory to extrapolate our results to the physical limit of QCD in a model-independent way. Nevertheless, the good performance of the algorithms used and our preliminary results make our mixed action approach, with overlap valence quarks on improved Wilson seas, look very promising.
Motivation

Determine weak matrix elements to few percent accuracy with controlled extrapolations to the physical limit of QCD:

\[ M_\pi \rightarrow 135 \text{ MeV}, \quad a \rightarrow 0, \quad L \rightarrow \infty \]

- Recent algorithmic (multiple time-scale integration, Hasenbusch acceleration, RHMC, DDHMC . . .) (Sexton & Weingarten '92, Hasenbusch '01, Clark et al '06, Lüscher '05, Urbach et al '06, . . .) and hardware advances
  \( \Rightarrow N_f = 2 + 1 \) QCD with e.g. \( M_\pi \sim 250 \text{ MeV}, \ a \sim 0.065 \text{ fm} \) and \( L \sim 3.1 \text{ fm} \)
  becoming accessible to Wilson fermions
  \( \Rightarrow \) near-continuum chiral \( p \)-regime w/out conceptual pbs of staggered fermions
- Overlap inversions are numerically feasible on these backgrounds
  \( \Rightarrow \) full \( \chi \text{S} \) (in valence sector) w/out cost of dynamical overlap fermions
  \( \Rightarrow \) simplified renormalization
  \( \Rightarrow \) full \( O(a) \) improvement w/ only NP \( O(a) \)-improved Wilson sea action
- To extrapolate to physical limit \( \rightarrow \) finite-volume mixed action \( \text{PQ}\chi\text{PT} \) (Sharpe '90 '92, Bernard & Golterman '92 '94, Sharpe & Shoresh '00 '01, Sharpe & Singleton '98, Aoki '03, Bär et al '03 '04, Sharpe '06, . . .)
GW valence on Wilson seas

\[ (M_{12}^{Q})^{\text{NLO}}_\Omega = M_{12}^2 \left\{ 1 + \frac{1}{(4\pi F)^2} \left[ \text{PQ } \chi\text{-logs}(\mu, M_{11}, M_{22}, \bar{M}_{\ell\ell}, \bar{M}_{ss}) + \text{FV} \right. \right. \]
\[ \left. \left. + 8 \left( (2\alpha_6 - \alpha_4)(\mu)(2\bar{M}_{\ell\ell}^2 + \bar{M}_{ss}^2) + (2\alpha_8 - \alpha_5)(\mu)M_{12}^2 \right) \right] + A_1^2 M_a^2 \right\} \]

\[ (F_{12}^{Q})^{\text{NLO}}_\Omega = F \left\{ 1 + \frac{1}{2(4\pi F)^2} \left[ \text{PQ } \chi\text{-logs}(\mu, M_{11}, M_{22}, \bar{M}_{\ell\ell}, \bar{M}_{ss}) + \text{FV} \right. \right. \]
\[ \left. \left. + 8 \left( \alpha_4(\mu)(2\bar{M}_{\ell\ell}^2 + \bar{M}_{ss}^2) + \alpha_5(\mu)M_{12}^2 \right) + A_2^2 M_a^2 \right\} \]

- \( M_{qq'} \) and \( \bar{M}_{qq'} \) are the non-singlet pseudo-Golstone boson masses at LO
- Terms in \( M_a^2 = 2aW \), with \( W \sim \Lambda_{QCD}^3 \), due to breaking of \( \chi S \) by Wilson fermions
- With Wilson seas: \( A_1^2, A_2^2 = O(1) \); with tree-level \( O(a) \)-improved seas:
  \( A_1^2, A_2^2 = O(\alpha_s) \); and with non-perturbatively \( O(a) \)-improved seas: \( A_1^2, A_2^2 \equiv 0 \)
Simulation ingredients

- Gauge action: tree-level Symanzik improved
- Sea quarks: smeared-link, tree-level $O(a)$-improved Wilson fermions
- Valence quarks: smeared-link overlap fermions
- Algorithm: Rational HMC with even-odd preconditioning, multiple time-scale Omelyan integration and Hasenbusch acceleration (Clark et al '06, Sexton & Weingarten '92, Omelyan et al '03, Hasenbusch '01, Urbach et al '06)
- Renormalization: will be non-perturbative à la Rome-Southampton
- Parameters:
  - $a \sim 0.09$ fm
  - $M_\pi \sim 300, 400, 490, 560$ MeV with $M_\pi L \gtrsim 4$
  - $O(20\%)$ overestimated $m_s$
  - Overlap roughly matched with Wilson
  - $O(20)$ configs at 300 MeV and $O(100)$ at other points
  - Other $a$ and $M_\pi$ are being investigated

Calculations performed on BG/L's at FZ Jülich and clusters at the University of Wuppertal, Eötvös University and CPT Marseille
No metastabilities and stable algorithm

Worst case scenario according to $\sigma \approx a/\sqrt{\Omega}$ criterion (Del Debbio et al '05):
$a \sim 0.15 \text{ fm, } \Omega/a^4 = 16^3 \times 32$

$(M_\pi \simeq 300 \text{ MeV})$
Very preliminary fit to the PBG decay constants

- $aF_{12}$ obtained using AWI $\rightarrow$ no renormalization needed
- 6 points with $M_{\ell\ell} \lesssim M_K$ fitted to NLO expression for $F_{12}$

- $aF_\pi$ from self-consistent extrapolation to physical point gives $a \sim 0.09\text{fm}$
- Fit results in right ball park
- Need more detailed analyses and more data at low masses and other lattice spacings to determine extent to which we can match onto mixed-action $\chi$PT
Tunneling HMC algorithm

Yigal Shamir (tel-Aviv)

with Maarten Golterman (SFSU)

We propose a variant of the HMC algorithm, dubbed Tunneling HMC (THMC) algorithm, which allows for real eigenvalues of a fermion matrix to change sign during the molecular dynamics evolution. We describe two implementations: for overlap and for domain-wall fermions. The partition function is first augmented by the determinant-squared of the corresponding super-critical Wilson operator, which is beneficial. However, with ordinary HMC, the price is that the global topology cannot change. When the new algorithm is applied to the "auxiliary" Wilson determinant, the tunneling between different topological sectors is made possible.

* not tested yet!
Why?

Near zero Wilson eigenvalues – a problem for DWF and overlap
DWF: bigger $m_{res}$
overlap: higher cost

solution (Vranas, JLQCD):
include $\det(D^\dagger D)$ in the path integral
where $D =$ supercritical Wilson operator
$\Rightarrow$ suppresses near zero Wilson eigenvalues "surgically"!

new problem:
global topology is frozen
(changing it requires a Wilson eigenvalue to go through zero).

THMC: recover global topology change
The basic idea

Partition function:

\[ Z = \int \mathcal{D}U \exp(-S_g)\det(D^\dagger D) \]
\[ = \int \mathcal{D}U \mathcal{D}\phi^* \mathcal{D}\phi \exp(-S_g - S_{pf}) \det(A) \]

\[ S_{pf} = \phi^\dagger \mathcal{M}^{-1} \phi \]
\[ \mathcal{M} = D^\dagger D + \alpha \sum_{i=1}^{n} |\chi_i\rangle \langle \chi_i|, \]

\[ (A^{-1})_{ij} = \alpha^{-1} \delta_{ij} + \langle \chi_i| (D^\dagger D)^{-1} |\chi_j\rangle \]

choose \[ |\chi_i\rangle \approx |\psi_i\rangle \] hence MD is blind to the near zero eigenvalues of \( \psi_i \)

\[ H_{\text{Molecular Dynamics}} = S_\pi + S_g + S_{pf} \]

\[ H_{\text{accept/reject}} = H_{\text{Molecular Dynamics}} + S_{\text{zero modes}} \]
Overlap implementation  (set $\chi_i = \psi_i$)

$$Z = \int \mathcal{DU} \exp(-S_g) \det(D^\dagger D)$$

$$= \int \mathcal{DU} D\phi^* D\phi \exp(-S_g - S_{pf}) \det(A)$$

$$S_{pf} = \phi^\dagger M^{-1} \phi \quad \quad M = D^\dagger D + \alpha \sum_{i=1}^{n} |\psi_i\rangle \langle \psi_i|,$$

$$D^\dagger D \psi_i = \lambda_i \psi_i \quad \quad (\hat{A}^{-1})_{ij} = (\alpha^{-1} + \lambda_i^{-1}) \delta_{ij}$$

$$H_{\text{Molecular Dynamics}} = S_{\pi} + S_g + S_{pf}$$

$$H_{\text{accept/reject}} = H_{\text{Molecular Dynamics}} - \log \det(A)$$

Requires $\psi_i$ and $\delta\psi_i/\delta U$ at each MD step; but they are computed anyway in an overlap simulation!
DWF implementation

Want to keep "zero modes' lifter" fixed during each MD evolution.

New stochastic degrees of freedom: \( \chi_i \) (much like pseudo-fermions)

\[
Z = \int \mathcal{D}\mathcal{U} \exp(-S_g) \det(D\dagger D)
\]

\[
= \int \mathcal{D}\mathcal{U}\mathcal{D}\phi^* \mathcal{D}\phi \mathcal{D}\chi^* \mathcal{D}\chi \exp(-S_g - S_{pf} - S_{ker}) \det(A)
\]

\( S_{pf} = \phi^\dagger \mathcal{M}^{-1} \phi \quad \mathcal{M} = D\dagger D + \alpha \sum_{i=1}^{n} |\chi_i\rangle \langle \chi_i| , \)

\[
(A^{-1})_{ij} = \alpha^{-1} \delta_{ij} + \langle \chi_i | (D\dagger D)^{-1} |\chi_j\rangle
\]

Kernel action: \( S_{ker} = \gamma \sum_{i=1}^{n} \langle \chi_i - \psi_i | \chi_i - \psi_i \rangle \)

\[
H_{\text{Molecular Dynamics}} = S_{\pi} + S_g + S_{pf}
\]

\[
H_{\text{accept/reject}} = H_{\text{Molecular Dynamics}} + S_{ker} - \log \det(A)
\]
(Reduction in) acceptance — a crude model

Consider overlap implementation (Ignore $S_{ker}$)

Assume exact energy conservation $H_{MD}(U', \pi') = H_{MD}(U, \pi)$

In principle: $P_{accept} = \int_0^1 dx dy \ P_1(x) P_2(y|x) \ \min\{1, y/x\}$

where $x = \det(A(U))$, $y = \det(A(U'))$

In THMC, molecular dynamics is blind to near zero modes; not unreasonable to assume the same fixed distribution $\hat{P}$ for both $x$ and $y$.

Get easily: $P_{accept} = \int_0^1 dx dy \ \hat{P}(x) \hat{P}(y) \ \min\{1, y/x\} \geq 1/2$
Simulation of Lattice Gauge Action from the Overlap Operator

Keh-Fei Liu
University of Kentucky

- Gauge Field Tensor and Gauge Action from the Overlap Dirac Operator
- Monte Carlo Simulation of Lattice QCD
Lattice QCD Action

- Gauge action:
  \[ \text{tr}_{cs} (D_\nu^0 (x, x) - D_\nu^0 (x, x)) \xrightarrow{a \to 0} a^4 F_{\mu \nu}^a F_{\mu \nu}^a (x) + O(a^6); \]

\[ S_g = \frac{1}{2cg^2} \text{Tr} (D_\nu - D_\nu^0) \]

\[ \gamma_5 \text{ hermicity and G-W relation} \]

\[ S_g = \frac{1}{2cg^2} \text{Tr} D_\nu = \frac{1}{4cg^2} \text{Tr} (D_\nu + D_\nu^+) = \frac{1}{4cg^2} \text{Tr} (D_\nu^+ D_\nu) \]

- Partition function

\[ Z = \int DUD \overline{\psi} D\psi \ e^{- \frac{1}{4cg^2} \text{Tr} D_\nu^+ D_\nu + \sum_{f=1}^{N_f} \overline{\psi}_f D_\nu (m_f) \psi_f} \]
MC Simulation

\[ Z = \int DU \det e^{\frac{-1}{4c_g^2 \Omega} D_{ov}^+ D_{ov}} \prod_{f=1}^{N_f} \det D_{ov}(m_f) \]

- Auxiliary fermion:

\[ Z = \int DUD\psi_f D\bar{\psi}_f D\bar{\psi}_g D\psi_g e^{\frac{-1}{4c^2 \Omega} \bar{\psi}_g (e^{4c^2 \Omega} \psi_g + \sum_{f=1}^{N_f} \bar{\psi}_f D_{ov}(m_f) \psi_f} \]

- Pseudofermions:

\[ Z = \int D Ud\phi_f^* d\phi_f e^{-\sum_{f=1}^{N_f} \phi_f^* e^{\frac{1}{4cN_f g^2 D_{ov}^+ D_{ov}}} \phi_f} \]
MC Simulation

- Two examples with HMC algorithm:

  \[ [D_{ov}, D_{ov}^+] = 0 \]

  \[
  \phi_f^* e^{\frac{D_{ov}^+ D_{ov}}{4cN_f g^2}} D_{ov}^{-1}(m_f) \phi_f = \phi_f^* e^{\frac{D_{ov}^+ D_{ov}}{8cN_f g^2}} D_{ov}^{-1}(m_f) e^{\frac{D_{ov}^+ D_{ov}}{8cN_f g^2}} \phi_f
  \]

  Chebyshev polynomials

  \[
  e^{\frac{D_{ov}^+ D_{ov}}{8cN_f g^2}} \approx \sum_{i=1}^{M} c_i (D_{ov}^+ D_{ov})^i; \quad e^x = (e^{x/N})^N
  \]
MC Simulation

- Rational HMC:
  - two flavors
    \[ \phi^* e^{6cg^2D_{ov}D_{ov}} (D_{ov}^+D_{ov}(m))^{-1} \phi \approx \phi^* \sum_{i=1}^{N} \frac{a_i}{D_{ov}^+D_{ov} + b_i} \phi, \]
  - one flavor
    \[ \phi^* e^{12cg^2D_{ov}D_{ov}} (D_{ov}^+D_{ov}(m))^{-1/2} \phi \approx \phi^* \sum_{i=1}^{N} \frac{c_i}{D_{ov}^+D_{ov} + d_i} \phi, \]
Remarks

- Gauge action and fermion action based on the same overlap operator with the same pseudofermion approximation.
- Gauge action serves as an UV-filter for the fermion action.
- Gauge action is not ultra-local (chirally smeared).
- Gauge action is not reflection positive.
- Negativity of local topological charge correlator (due to reflection positivity)
  \[ \langle q(x)q(0) \rangle_{|x| \neq 0} \leq 0 \]
Spectral sum rules and duality violations
Maarten Golterman (SFSU)
work with Oscar Catà and Santi Peris
(BNL workshop Domain-wall fermions at 10 years)

We study the issue of duality violations in the $W-AA$ vacuum polarization
Function in the chiral limit, with the help of a model with an expansion
in inverse powers of the number of colors, $N_c$, allowing us to consider
resonances with a finite width. Due to these duality violations, the Operator
Product Expansion (OPE) and the moments of the spectral function do not
match at finite momentum, and we analyze this difference. We perform
a comparative study of different methods proposed in the literature for
the extraction of the OPE parameters and find that, when applied to our model,
they all fare quite similarly. The model strongly suggests that a significant
improvement in precision can only be expected after duality violations are
included.
Physics from (the OPE of) $\Pi_{LR}$:

$$i \int d^4 x \, e^{i q x} \langle J^{V,A}_\mu(x) J^\dagger_\nu V,A(0) \rangle = (q_\mu q_\nu - g_{\mu\nu} q^2) \Pi_{V,A}(q^2)$$

$$\Pi_{LR}(q^2) = \frac{1}{2} (\Pi_V(q^2) - \Pi_A(q^2))$$

1) In the chiral and large-$N_c$ limits

$$\Pi_{LR}(q^2 = -Q^2) = -8\pi\alpha_s \langle \bar{\psi}\psi \rangle^2 / Q^6 + O(1/Q^8)$$

and $\langle (\pi\pi)_{I=2} | Q_8 | K^0 \rangle$ is proportional to the $1/Q^6$ coefficient,

while $\langle (\pi\pi)_{I=2} | Q_7 | K^0 \rangle$ is an integral over $\Pi_{LR}(Q^2)$.

2) The OPE part of $\Pi_V(Q^2) + \Pi_A(Q^2)$ "contaminate" the determination of $\alpha_s$ from $\tau$ decays.

(Braaten, Narison and Pich)
Getting OPE coefficients from data:

The OPE for $\Pi(Q^2) = \Pi_{LR}(q^2 = -Q^2)$ is an asymptotic expansion for large $Q^2$

$\pi \rho(t) = \Im \Pi(t)$ known from data up to a scale $s_0 = m_r^2$

Cauchy's theorem: ($P$ any polynomial)

$$\int_0^{s_0} dt \ P(t) \frac{1}{\pi} \Im \Pi(t) = -\frac{1}{2\pi i} \oint_{|q^2|=s_0} dq^2 \ P(q^2) \ \Pi(q^2)$$

Idea: substitute $\Pi(Q^2) \rightarrow \Pi_{OPE}(Q^2)$ on the right-hand side ("duality")

Assumption: $s_0$ already in the asymptotic regime

Problem: not valid even for large $s_0$ near positive real axis!
Our model at finite $N_c$  

(Blok, Shifman and Zhang)

$$
\Pi(q^2) = \frac{F^2_\rho}{-q^2 + M^2_\rho} + \frac{F^2_0}{q^2} + \sum_{n=0}^{\infty} \frac{F^2}{-q^2 + M^2_V(n)} - \sum_{n=0}^{\infty} \frac{F^2}{-q^2 + M^2_A(n)}
$$

Replace $-q^2 - i\varepsilon$ by $z = (-q^2 - i\varepsilon)\zeta$, $\zeta = 1 - a(\pi N_c)$ and $\Pi(q^2)$ by

$$
\Pi(q^2) = \frac{1}{\zeta} \left\{ \frac{F^2_0}{z} + \frac{F^2_\rho}{z + M^2_\rho} + F^2 \left[ \psi(z + m^2_A) - \psi(z + m^2_V) \right] \right\}
$$

Expand in $1/N_c \Rightarrow$ width $\Gamma(n) = aM(n)/N_c$  

(Breit-Wigners near poles)

$\Pi(q^2)$ analytic for all $q^2$ except cut along the positive real axis

(note: no multi-particle continuum)
Tests:

1) Finite-energy sum rules (Peris et al., Bijnens et al.) determine duality point $s_0^*$ from $M_{0,1}(s_0) \approx 0$, and predict

$$A_6(s_0^*) \equiv a_6 + b_6 \log s_0^* + \frac{b_8}{s_0^*} + ...$$

$$A_8(s_0^*) \equiv a_8 + b_8 \log s_0^* - b_6 s_0^* + \frac{b_{10}}{s_0^*} + ...$$

$s_0^* = 1.472 \text{ GeV}^2$: $A_6 = -4.9 \times 10^{-3} \text{ GeV}^6$, $A_8 = 9.3 \times 10^{-3} \text{ GeV}^8$

$s_0^* = 2.363 \text{ GeV}^2$: $A_6 = -2.0 \times 10^{-3} \text{ GeV}^6$, $A_8 = -1.6 \times 10^{-3} \text{ GeV}^8$

exact: $A_6 = -2.8 \times 10^{-3} \text{ GeV}^6$, $A_8 = 3.4 \times 10^{-3} \text{ GeV}^8$

Note: 2nd duality point only sets $M_0(s_0) = 0$, not $M_1(s_0)$

$b_6 s_0^* = -1.4 \times 10^{-3} \text{ GeV}^8$ at 2nd duality point! (Smaller in QCD?)
Can we do better?

try model the duality violations: \[
\frac{1}{\pi} \text{Im} \Delta(t) = \kappa e^{-\gamma t} \sin(\alpha + \beta t)
\]

fit to (range $1.5 < s_0 < 3.5 \text{ GeV}^2$) \[
\int_0^{s_0} dt \, \rho(t) , \quad \int_0^{s_0} dt \, t \, \rho(t)
\]

find $\kappa = 0.026$, $\gamma = 0.591 \text{ GeV}^{-2}$, $\alpha = 3.323$, $\beta = 3.112 \text{ GeV}^{-2}$

with this, predict duality points for higher moments,

find $s_0^* = 2.350 \text{ GeV}^2$ for $n = 2$, $s_0^* = 2.307 \text{ GeV}^2$ for $n = 3$, etc.

and $A_6 = -2.5 \times 10^{-3} \text{ GeV}^6$, $A_8 = 3.3 \times 10^{-3} \text{ GeV}^8$

(exact: $A_6 = -2.8 \times 10^{-3} \text{ GeV}^6$, $A_8 = 3.4 \times 10^{-3} \text{ GeV}^8$)

order 10% errors up to $A_{16}$ \[\Rightarrow\] worth trying in QCD?
Schrödinger Functional Boundary Conditions for Domain Wall Quarks

Stefan Sint, Trinity College Dublin

* The Schrödinger functional, a short reminder
* Chirally twisting the Schrödinger functional
* Orbifold technique
* Symmetries
* Conclusions and outlook
SF boundary conditions and chiral rotations

Consider isospin doublets $\chi'$ and $\bar{\chi}'$ satisfying homogeneous SF boundary conditions ($P_{\pm} = \frac{1}{2}(1 \pm \gamma_0)$),

$$P_+ \chi'(x)|_{x_0=0} = 0, \quad P_- \chi'(x)|_{x_0=T} = 0,$$
$$\bar{\chi}'(x) P_-|_{x_0=0} = 0, \quad \bar{\chi}'(x) P_+|_{x_0=T} = 0.$$

perform a chiral field rotation,

$$\chi' = \exp(i\alpha\gamma_5 \tau^3/2)\chi, \quad \bar{\chi}' = \bar{\chi} \exp(i\alpha\gamma_5 \tau^3/2),$$

the rotated fields satisfy chirally rotated boundary conditions

$$P_+(\alpha)\chi(x)|_{x_0=0} = 0, \quad P_- (\alpha)\chi(x)|_{x_0=T} = 0,$$
$$\bar{\chi}(x)\gamma_0 P_-(\alpha)|_{x_0=0} = 0, \quad \bar{\chi}(x)\gamma_0 P_+(\alpha)|_{x_0=T} = 0,$$

with the projectors

$$P_{\pm}(\alpha) = \frac{1}{2} \left[ 1 \pm \gamma_0 \exp(i\alpha\gamma_5 \tau^3) \right].$$
Special cases of $\alpha = 0, \pi/2$:

$$P_\pm(0) = P_\pm, \quad P_\pm(\pi/2) \equiv Q_\pm = \frac{1}{2}(1 \pm i\gamma_0\gamma_5\tau^3),$$

The chiral rotation introduces a mapping between renormalised correlation functions

$$\langle O[\chi, \bar{\chi}] \rangle_{P_\pm} = \langle \tilde{O}[\chi, \bar{\chi}] \rangle_{P_\pm(\alpha)}$$

with

$$\tilde{O}[\chi, \bar{\chi}] = O[\exp(i\alpha\gamma_5\tau^3/2)\chi, \bar{\chi}\exp(i\alpha\gamma_5\tau^3/2)],$$

Boundary quark fields are included by replacing

$$\bar{\zeta}(x) \leftrightarrow \bar{\chi}(0, x)P_+ \quad \zeta(x) \leftrightarrow P_-\chi(0, x)$$

Note: The chirally rotated framework is only chosen for technical convenience. Using the above dictionary any standard SF correlator can be easily translated to this rotated framework (for an even number of fermions)
achievement: the construction is completely analogous to the Wilson-Dirac case (S.'05); a block structure (in time) of the Wilson-Dirac operator is obtained and inherited by the overlap operator

⇒ the Neuberger operator in the interval is simply obtained by using the corresponding orbifolded Wilson-Dirac kernel:

\[
D_N = 1 - A(A^\dagger A)^{-1/2}, \quad A = 1 - aD_W
\]

with the kernel \(D_W\),

\[
aD_W \chi(x) = -U(x,0)P_-\chi(x + a\hat{\Theta}) + (K\psi)(x) - U(x - a\hat{\Theta})^\dagger P_+\chi(x - a\hat{\Theta}),
\]

where we have set \(\chi(x) = 0\) for \(x_0 < 0\) and \(x_0 > T\), and

\[
K = 1 + \frac{1}{2} \sum_{k=1}^{3} \left\{ a(\nabla_k^* + \nabla_k^*)^\gamma_5 - a^2 \nabla_k^* \nabla_k^* \right\} + \delta_{x_0,0} i \gamma_5 \tau^3 P_- + \delta_{x_0,T} i \gamma_5 \tau^3 P_+
\]
Symmetries

In a massless theory in finite volume the identification of flavour and chiral symmetries is a mere convention!

- take the standard Schrödinger functional with projectors $P_{\pm}$ as SU(2) flavour symmetric reference basis

- in the rotated SF, the SU(2) flavour symmetry is realised à la Ginsparg-Wilson:

\[
\gamma_5 \tau^{1,2} \mathcal{D}_N + \mathcal{D}_N \gamma_5 \tau^{1,2} = \mathcal{D}_N \gamma_5 \tau^{1,2} \mathcal{D}_N \\
\tau^3 \mathcal{D}_N - \mathcal{D}_N \tau^3 = 0
\]

Note that the flavour algebra closes $[\hat{\gamma}_5 = \gamma_5 (1 - a \mathcal{D}_N)]$:

\[
\hat{T}^1 = \hat{\gamma}_5 \tau^2 / 2, \quad \hat{T}^2 = -\hat{\gamma}_5 \tau^1 / 2, \quad \hat{T}^3 = \tau^3 / 2, \quad [\hat{T}^a, \hat{T}^b] = i \epsilon^{abc} \hat{T}^c
\]
• Chiral symmetry is broken by the SF boundary conditions: expect: the standard GW relation is violated by terms which decrease exponentially with the distance from the boundaries.

• Form of non-singlet chiral symmetries:

\[ [\tau^{1,2}, D_N] \neq 0, \quad \{\gamma_5 \tau^3, D_N\} \neq aD_N\gamma_5\tau^3D_N. \]

expect: both flavour components of \( D_N \) become equal and the GW relation holds up to corrections which decrease exponentially with the distance from the boundaries (checked at tree level).

• GW versions of parity and time reversal, e.g.:

\[ P : \chi(x) \rightarrow i\gamma_0\gamma_5\tau^3\chi(\bar{x}), \quad \bar{x} = (x_0, -x), \quad D_N P + P D_N = D_N P D_N \]

• In contrast to the case of Wilson quarks, parity and flavour are realised exactly, expect no extra counterterms!
Non-perturbative renormalization of 4 fermi operator with Schrödinger functional scheme

Y. Taniguchi for PACS-CS collaboration
University of Tsukuba

We evaluate renormalization factor $\tilde{Z}_{B_K}$ which convert bare $B_K^{(0)}$ on lattice with DWF to renormalization group invariant $\tilde{B}_K$. By using quenched data by CP-PACS we get $\tilde{B}_K = 0.783(11)$ for a preliminary value.
Purpose of this talk

- $\tilde{Z}_{BK}$: Bare $B_K^{(0)}$ on lattice with DWF
  $\Rightarrow$ Renormalization group invariant (RGI) $\tilde{B}_K$

- Three steps for RGI $\tilde{B}_K$
  1. From lattice to renormalized: $B_K^{(0)} \Rightarrow B_K^{(SF)}(a\mu)$
     - lattice artifact: $O(a\mu) \Rightarrow a\mu \ll 1$
  2. Non-pert. RG evolution: $B_K^{(SF)}(a\mu_{\text{min}}) \Rightarrow B_K^{(SF)}(a\mu_{\text{max}})$
  3. Pert. transformation to RGI $B_K^{(SF)}(a\mu_{\text{max}}) \Rightarrow \tilde{B}_K$

- 2. 3. steps are done by Alpha-collab. for $O_{VA+AV}$
- We need to evaluate 1. for DWF with $O_{VA+AV}$
Renormalization of $O_{VA+AV}$ (Alpha-collab.)

- Four point function

$$F(x_0) = \frac{1}{L^3} \langle \mathcal{O}_1 \mathcal{O}_2 O_{VA+AV}(x) \mathcal{O}_3' \rangle$$

$$\mathcal{O}[\Gamma] = a^6 \sum_{\bar{x}, \bar{y}} \bar{\zeta}(\bar{x}) \Gamma \zeta(\bar{y})$$

- Boundary operator is renormalized with wave function renormalization factor, which is cancelled dividing by two point function

$$f_1 = -\frac{1}{2L^6} \langle \mathcal{O}'[\gamma_5] \mathcal{O}[\gamma_5] \rangle$$

- Definition of renormalization factor

$$h(x_0) = \frac{F(x_0)}{f_1^{3/2}}, \quad Z_{VA+AV}(g_0, a\mu) h^{(0)}(\frac{T}{2}) = h^{(\text{tree})}(\frac{T}{2})$$
Numerical simulation

- Quenched simulation data for $B_K$ with Iwasaki action and DWF ($N_s = 16$)
  at $\beta = 2.6$, $\beta = 2.9$, $\beta = 3.2$
  corresponding to $a^{-1} = 2$, $a^{-1} = 3$, $a^{-1} = 4$ GeV
- We need renormalization factor $Z_{VA+AV}(a\mu)$
  at $\mu_{\text{min}} = 1/2L_{\text{max}} = 1/(1.436r_0) \sim 275$ MeV
  $a\mu_{\text{min}} = 0.17 \pm 0.05$
- Need to fine tune $\beta$ such that $2aN_T = 1.436r_0$

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<td>$\beta$</td>
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<td>2.63</td>
<td>2.79</td>
<td>2.92</td>
<td>3.03</td>
<td>3.13</td>
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- $\mu_{\text{max}} = 2\mu_{\text{min}} \sim 35.2$ GeV
- Values at $\beta = 2.6$, $\beta = 2.9$, $\beta = 3.2$ by interpolation
Renormalized $B_K$

- RG$\bar{\Lambda}$ $\bar{B}_K = 0.783(11)$ (preliminary)

- Renormalized $B_K$ in $\overline{\text{MS}}$ NDR scheme at $\mu = 2$ GeV
  
  $B_K^{\overline{\text{MS}}}(2 \text{ GeV}) = 0.5633(81)$ (preliminary)
Conclusion

- $\tilde{Z}_{B_K}$: Bare $B_K^{(0)}$ on lattice with DWF
  $\Rightarrow$ Renormalization group invariant (RGI) $\tilde{B}_K$

- Schrödinger functional scheme for DWF
  with orbifolding construction

- RGI $\tilde{B}_K = 0.783(11)$ (preliminary).

- $\overline{\text{MS}}$ $B_{K}^{\overline{\text{MS}}}(2 \text{ GeV}) = 0.5633(81)$ (preliminary)

- $O(a)$ behaviour depends on domain-wall height $M$
  - $M$ should be tuned such that $M - \delta M_{\text{tadpole}} \sim 1$
Future of Chiral Extrapolations with Domain Wall Fermions

What are the constraints on $a$, $m$, $L$, and particularly $m_{\text{reS}}$, such that one can successfully calculate phenomenologically interesting quantities?

Stephen R. Sharpe

University of Washington
Implications of $m_{\text{res}} \neq 0$ in PGB sector

The leading $m_{\text{res}}/a$ effect has been absorbed into quark mass, but what about contributions from Pauli term, suppressed by $(a\Lambda)^2$?

- Symanzik effective Lagrangian for DWF ($q$ are boundary fields):

$$L_{\text{Sym.}} \sim \bar{q}(\mathcal{D} + m)q + \frac{m'_{\text{res}}}{a} \bar{q}q + ac\bar{q}(\sigma \cdot F)q + \ldots$$

  $\triangleright$ Same form as for Wilson fermions, but here $c \sim m'_{\text{res}} \ll 1$

- Match onto chiral effective theory [SS & Singleton]

$$L_{\chi} = \frac{f^2}{4} \text{tr}(\partial_{\mu} \Sigma \partial_{\mu} \Sigma^\dagger) - \frac{f^2 B}{2} \text{tr}(M_q \Sigma + \Sigma^\dagger M_q) + \ldots$$

$$M_q = m + \frac{m'_{\text{res}}}{a} + ac\Lambda^2$$

  $\triangleright$ Since $m_\pi^2 \propto M_q$, it must be that $M_q = m_q = m + m_{\text{res}}/a$ and so

$$m_{\text{res}} = m'_{\text{res}} + ca^2 \Lambda^2$$

- Conclusion: No $m_{\text{res}} a\Lambda^2$ term in leading order $L_{\chi}$ if use standard $m_{\text{res}}$
Implications for PGBs (continued)

This "trick" does not work for higher order terms:

- PGB matrix elements of $\bar{q}\sigma \cdot Fq$ and $\bar{q}q$ not proportional at higher order
- Extra terms obtained by replacement $m_q \rightarrow m_{\text{res}}a\Lambda^2$

$$\frac{m_{\pi}^2}{m_q} \sim f_{\pi} \sim \text{const.} \left[1 + O(m_q/\Lambda) + O(m_{\text{res}}a\Lambda) + O(a^2\Lambda^2) + \ldots\right]$$

- Here $m_{\text{res}}$ indicates order of magnitude—dependence on $L_5$ may differ
- **Numerically tiny and subdominant to $a^2\Lambda^2$ term:**

$$m_{\text{res}}a\Lambda \approx 0.003 \left(\frac{0.1 - 0.5 \text{ GeV}}{1.6 \text{ GeV}}\right) \lesssim 10^{-3} \ll (a\Lambda)^2 \approx 0.004 - 0.1$$

- Similar $O(a)$ term present in all hadronic quantities
Enhanced $m_{\text{res}}$ effects: condensate

- For the $\chi$SB induced by $m_{\text{res}}$ to be problematic, must be enhanced
  - This can be due either to power divergences or mixing with operators with less suppressed chiral behavior.

- Most extreme case is quark condensate:
  - Symanzik expansion of scalar
    \[
    (\bar{q}q)^{\text{DWF}} \sim (\bar{q}q)^{\text{cont.}} + \frac{m + x m_{\text{res}} / a}{a^2} + \ldots
    \]
  - $x = \mathcal{O}(1)$ but $x \neq 1$ because term arises from UV momenta where cannot use Symanzik action
  - Thus do not remove divergence by $m_q = m + m_{\text{res}} / a \to 0$ [Blum et al.]
    \[
    \lim_{m_q \to 0} \lim_{L \to \infty} \langle \bar{q}q \rangle_{\text{DWF}} = \langle \bar{q}q \rangle_{\text{cont.}} + (x - 1) \frac{m_{\text{res}}}{a^3} + \ldots
    \]

- Relative correction is large
  \[
  \frac{\delta \langle \bar{q}q \rangle}{\langle \bar{q}q \rangle} \sim \frac{m_{\text{res}}}{a^3 \Lambda^5} \sim \frac{3 \times 10^{-3}}{(0.1)^3} \sim \mathcal{O}(1)
  \]
  - Cannot calculate physical condensate directly
Impact on $\epsilon'/\epsilon$ (continued)

- With DWF get additional lower-dimension operators to subtract

$$m_{\text{res}} \bar{s}d, \quad \frac{m_{\text{res}}}{a} \bar{s}\sigma \cdot Fd, \quad \frac{m_{\text{res}} (m_s^2 + m_d^2)}{a} \bar{s}d, \quad \frac{m_{\text{res}} (m_s^2 - m_d^2)}{a} \bar{s}\gamma_5 d$$

- How do they impact the calculation?

- $m_{\text{res}}\bar{s}d/a^3$ removed by RBC “slope” method, while $m_{\text{res}}\bar{s}\sigma \cdot Fd/a$ leads to relative correction no larger than omitted NNLO terms:

$$\frac{\delta ME}{ME} \sim \frac{m_{\text{res}}}{a\Lambda} \sim \frac{3 \times 10^{-3}}{0.06 - 0.3} \sim 0.01 - 0.05$$

- Contributions of other operators suppressed by further $m_s/\Lambda \approx 1/4$

- Conclusion: Present parameters probably adequate, though worth investigating methods for subtracting additional operator

Though need to then do some extra work to implement Laiho-Soni.
Implications of $m_{\text{res}}$ for $B_K$

- Want to calculate $B_K$ to at least 5% precision [Talk by Soni]
- No power divergent mixing, so effects of $m_{\text{res}}$ suppressed by $a\Lambda$ as for spectral quantities
- However, L-L operator can mix with chirally unsuppressed L-R operator: does this enhance the usual $m_{\text{res}}a\Lambda$ corrections?
- No! Mixing comes at cost of $m_q$ and gain of $1/m_q$. Net result is

$$\frac{\delta B_K}{B_K} \sim m_{\text{res}} \times (a\Lambda) \lesssim 10^{-3}.$$ 

- Conclusion: present parameters are adequate for precision calculation of $B_K$
# RBRC Workshop: Domain Wall Fermions at Ten Years

**March 15-17, 2007**

**Physics Department Large Seminar Room**

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Domain Wall Fermions at 10 Years

March 14 – 17, 2007

Physics Department
Large Seminar Room

Wednesday, March 14

Welcome Reception – 6:00 to 7:30 – Berkner Hall Lobby

Thursday, March 15

08:00 Registration / Continental Breakfast / Collection of Proceedings Pkgs (Large Seminar Room Lounge)
08:30 Welcome (Nicholas Samios, Director, RBRC; Thomas Blum; Amarjit Soni)
08:40 TBA (David Kaplan)
09:20 2+1 Flavor DWF QCD: Zero & Finite Temperature Ensembles and Delta $S = 1$ Physics (Robert Mawhinney)
10:00 Dynamical QCD with Exact Chiral Symmetry --- from the p-Regime to the ε-Regime (Shoji Hashimoto)
10:40 Coffee Break
11:10 QCD Simulations at Realistic Quark Masses: Probing the Chiral Limit (Gerrit Schierholz)
11:50 Chiral Phase Transition in Large N QCD with Overlap Fermions (Rajamani Narayanan)
12:30 Lunch
02:00 Domain Wall Fermions and Other Five-Dimensional Algorithms (Anthony Kennedy)
Spreading up Domain Wall Fermion Algorithms using QCDLAB (Artan Borici)
Moebius Domain Wall Fermion (Richard Brower)
3:00 Gap Domain Wall Fermions (Pavlos Vranas)
Fat Links for Dynamical Chiral Fermions (Stefan Schaefer)
Approaching the Chiral Limit with Dynamical Overlap Fermions (Takashi Kaneko)
04:00 Coffee Break
04:30 Topological Susceptibility in the Trivial Sector with Dynamical Overlap (Ting-Wai Chiu)
Baryons from 2+1 Flavor DWF QCD (Christopher Maynard)
Spectrum Study of $N_f=2$ QCD with Overlap Fermions (Junichi Noaki)
05:30 Isospin Breaking Study with $N_f=2$ Domain-Wall QCD + Quenched QED Simulation (Takumi Doi)
Chiral Extrapolations for Domain Wall Fermions (Meifeng Lin)
Quark Condensate from Finite-Size Study (Silvia Necco)
Friday, March 16

08:00  Continental Breakfast

08:30  Domain Wall Filters (Herbert Neuberger)
09:10  Low-Energy Couplings from Lattice Simulations in the $\epsilon$ Regime (Pilar Hernandez)
09:50  Spectrum and Decay Constants of Light Mesons with 2+1 Flavors of DWF (Brian Pendleton)

10:30  Coffee Break

11:00  Hadron Structure with Domain Wall Fermions on a Staggered Sea (John Negele)
11:40  Status of the 2+1 Flavor Wilson-Clover Simulation of QCD by PACS-CS Collaboration (Yoshinobu Kuramashi)

12:20  Lunch

02:00  Hadron Scattering from Lattice QCD (Kostas Orginos)
Semi-leptonic Hyperon Decay in Full Lattice QCD (Huey-Wen Lin)
Nucleon Structure from 2+1 Flavor DWF Simulations (Takeshi Yamazaki)

03:00  Domain Wall Fermions, Approximate Chiral Symmetry and Weak Matrix Elements (Norman Christ)
$B_K$ with two Flavors of Dynamical Overlap Fermions (Norikazu Yamada)
$B_K$ with 2+1-Flavour Domain Wall Fermions (Saul Cohen)

04:00  Coffee Break

04:30  Update on $B_K$ with a Mixed Action (Jack Laiho)
K13 Form Factor with 2+1 Flavours of Domain Wall Fermions (James Zanotti)
Chiral Violations in Perturbative Domain Wall QCD (Stefano Capitani)

05:30  Electroweak Theory on the Lattice (Yoshio Kikukawa)
Should We Change the Topology at All? (Tetsuya Onogi)
Chiral Behavior in Mixed Action Calculations with 2+1 Sea Quark Flavors (Laurent Lellouch)

Workshop Dinner – 7:00 to 9:00 – Brookhaven Center South Room

Saturday, March 17

08:00  Continental Breakfast

08:30  Tunneling HMC Algorithm (Yigal Shamir)
09:10  Simulation of Gauge Action from the Overlap Operator (Keh-Fei Liu)
09:50  Spectral Sum Rules, Duality Violations and Low-Energy Constants (Maarten Golterman)

10:30  Coffee Break

11:00  Schrodinger Functional Boundary Conditions for Domain Wall Quarks (Stefan Sint)
11:20  Non-Perturbative Renormalization of 4 Fermi Operator with Schroedinger Functional Scheme (Yusuke Taniguchi)
11:40  Future of Chiral Extrapolations with DWF (Stephen Sharpe)
12:20  Meeting Adjourns
Additional RIKEN BNL Research Center Proceedings:

Volume 83 – QCD in Extreme Conditions, July 31-August 2, 2006 – BNL-76933-2006
Volume 81 – Parton Orbital Angular Momentum (Joint RBRC/University of New Mexico Workshop) February 24-26, 2006 – BNL-75937-2006
Volume 80 – Can We Discover the QCD Critical Point at RHIC?, March 9-10, 2006 – BNL-75692-2006
Volume 79 – Strangeness in Collisions, February 16-17, 2006 – BNL-
Volume 78 – Heavy Flavor Productions and Hot/Dense Quark Matter, December 12-14, 2005 – BNL-76915-2006
Volume 77 – RBRC Scientific Review Committee Meeting – BNL-52649-2005
Volume 76 – Odderon Searches at RHIC, September 27-29, 2005 – BNL-75092-2005
Volume 75 – Single Spin Asymmetries, June 1-3, 2005 – BNL-74717-2005
Volume 71 – Classical and Quantum Aspects of the Color Glass Condensate – BNL-73793-2005
Volume 69 – Review Committee – BNL-73546-2004
Volume 68 – Workshop on the Physics Programme of the RBRC and UKQCD QCDOC Machines – BNL-73604-2004
Volume 67 – High Performance Computing with BlueGene/L and QCDOC Architectures – BNL-
Volume 65 – RHIC Spin Collaboration Meetings XXVII (July 22, 2004), XXVIII (September 2, 2004), XXX (December 6, 2004) - BNL-73506-2004
Volume 64 – Theory Summer Program on RHIC Physics – BNL-73263-2004
Volume 63 – RHIC Spin Collaboration Meetings XXIV (May 21, 2004), XXV (May 27, 2004), XXVI (June 1, 2004) – BNL-72397-2004
Volume 60 – Lattice QCD at Finite Temperature and Density – BNL-72083-2004
Volume 59 – RHIC Spin Collaboration Meeting XXI (January 22, 2004), XXII (February 27, 2004), XXIII (March 19, 2004)– BNL-72382-2004
Volume 58 – RHIC Spin Collaboration Meeting XX – BNL-71900-2004
Volume 57 – High pt Physics at RHIC, December 2-6, 2003 – BNL-72069-2004
Volume 56 – RBRC Scientific Review Committee Meeting – BNL-71899-2003
Additional RIKEN BNL Research Center Proceedings:

Volume 53 - Theory Studies for Polarized \( pp \) Scattering - BNL-71747-2003
Volume 52 - RIKEN School on QCD "Topics on the Proton" - BNL-71694-2003
Volume 51 - RHIC Spin Collaboration Meetings XV, XVI - BNL-71539-2003
Volume 50 - High Performance Computing with QCDOC and BlueGene - BNL-71147-2003
Volume 49 - RBRC Scientific Review Committee Meeting - BNL-52679
Volume 48 - RHIC Spin Collaboration Meeting XIV - BNL-71300-2003
Volume 47 - RHIC Spin Collaboration Meetings XII, XIII - BNL-71118-2003
Volume 46 - Large-Scale Computations in Nuclear Physics using the QCDOC - BNL-52678
Volume 45 - Summer Program: Current and Future Directions at RHIC - BNL-71035
Volume 44 - RHIC Spin Collaboration Meetings VIII, IX, XI - BNL-71117-2003
Volume 43 - RIKEN Winter School - Quark-Gluon Structure of the Nucleon and QCD - BNL-52672
Volume 42 - Baryon Dynamics at RHIC - BNL-52669
Volume 41 - Hadron Structure from Lattice QCD - BNL-52674
Volume 40 - Theory Studies for RHIC-Spin - BNL-52662
Volume 39 - RHIC Spin Collaboration Meeting VII - BNL-52659
Volume 38 - RBRC Scientific Review Committee Meeting - BNL-52649
Volume 37 - RHIC Spin Collaboration Meeting VI (Part 2) - BNL-52660
Volume 36 - RHIC Spin Collaboration Meeting VI - BNL-52642
Volume 35 - RIKEN Winter School - Quarks, Hadrons and Nuclei - QCD Hard Processes and the Nucleon Spin - BNL-52643
Volume 34 - High Energy QCD: Beyond the Pomeron - BNL-52641
Volume 33 - Spin Physics at RHIC in Year-1 and Beyond - BNL-52635
Volume 32 - RHIC Spin Physics V - BNL-52628
Volume 31 - RHIC Spin Physics III & IV Polarized Partons at High \( Q^2 \) Region - BNL-52617
Volume 30 - RBRC Scientific Review Committee Meeting - BNL-52603
Volume 29 - Future Transversity Measurements - BNL-52612
Volume 28 - Equilibrium & Non-Equilibrium Aspects of Hot, Dense QCD - BNL-52613
Volume 27 - Predictions and Uncertainties for RHIC Spin Physics & Event Generator for RHIC Spin Physics III - Towards Precision Spin Physics at RHIC - BNL-52596
Volume 26 - Circum-Pan-Pacific RIKEN Symposium on High Energy Spin Physics - BNL-52588
Volume 25 - RHIC Spin - BNL-52581
Volume 24 - Physics Society of Japan Biannual Meeting Symposium on QCD Physics at RIKEN BNL Research Center - BNL-52578
Volume 23 - Coulomb and Pion-Asymmetry Polarimetry and Hadronic Spin Dependence at RHIC Energies - BNL-52589
Volume 22 - OSCAR II: Predictions for RHIC - BNL-52591
Volume 21 - RBRC Scientific Review Committee Meeting - BNL-52568
Volume 20 - Gauge-Invariant Variables in Gauge Theories - BNL-52590
Volume 19 - Numerical Algorithms at Non-Zero Chemical Potential - BNL-52573
Additional RIKEN BNL Research Center Proceedings:

Volume 18 — Event Generator for RHIC Spin Physics — BNL-52571
Volume 17 — Hard Parton Physics in High-Energy Nuclear Collisions — BNL-52574
Volume 16 — RIKEN Winter School - Structure of Hadrons - Introduction to QCD Hard Processes — BNL-52569
Volume 15 — QCD Phase Transitions — BNL-52561
Volume 14 — Quantum Fields In and Out of Equilibrium — BNL-52560
Volume 13 — Physics of the 1 Teraflop RIKEN-BNL-Columbia QCD Project First Anniversary Celebration — BNL-66299
Volume 12 — Quarkonium Production in Relativistic Nuclear Collisions — BNL-52559
Volume 11 — Event Generator for RHIC Spin Physics — BNL-66116
Volume 10 — Physics of Polarimetry at RHIC — BNL-65926
Volume 9 — High Density Matter in AGS, SPS and RHIC Collisions — BNL-65762
Volume 8 — Fermion Frontiers in Vector Lattice Gauge Theories — BNL-65634
Volume 7 — RHIC Spin Physics — BNL-65615
Volume 6 — Quarks and Gluons in the Nucleon — BNL-65234
Volume 5 — Color Superconductivity, Instantons and Parity (Non?)-Conservation at High Baryon Density — BNL-65105
Volume 4 — Inauguration Ceremony, September 22 and Non-Equilibrium Many Body Dynamics — BNL-64912
Volume 3 — Hadron Spin-Flip at RHIC Energies — BNL-64724
Volume 2 — Perturbative QCD as a Probe of Hadron Structure — BNL-64723
Volume 1 — Open Standards for Cascade Models for RHIC — BNL-64722
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