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Abstract

The dynamics of photoevaporated molecular clouds is determined by the ablative pressure acting on the ionization front. An important step in the understanding of the ensuing motion is to develop the linear stability theory for the initially flat front. Despite the simplifications introduced by the linearization, the problem remains quite complex and still draws a lot of attention. The complexity is related to the large number of effects that have to be included in the analysis: acceleration of the front, possible temporal variation of the intensity of the ionizing radiation, the tilt of the radiation flux with respect to the normal to the surface, and partial absorption of the incident radiation in the ablated material. In this paper, we describe a model where all these effects can be taken into account simultaneously, and a relatively simple and universal dispersion relation can be obtained. The proposed phenomenological model may prove to be a helpful tool in assessing the feasibility of the laboratory experiments directed towards scaled modeling of astrophysical phenomena.

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1. Introduction

The shape of photoevaporated molecular clouds (e.g., [1,2]) is most probably caused by a variety of hydrodynamical processes occurring under the action of the ablation force. Some of the models relate the observed structures to the existence of large initial density perturbations (see, e.g., [3-5]). The others (see below) attribute the shape to a development of instabilities of the initially slightly perturbed fronts. In the present paper, we consider several aspects of this second approach, concentrating on the linear stage of instability. We present a simple phenomenological model that allows one to describe, in a unified way, all the stabilizing and destabilizing factors which have been studied thus far in a piecemeal fashion.

The linear analyses of the ablation front instability can be traced back to the papers by Spitzer [6] and Frieman [7] where the instability was associated with the Rayleigh-Taylor (RT) instability of an accelerating interface. Kahn [8] has argued that the partial absorption of the ionizing radiation in the ablated material should lead to a stabilization of the RT instability. Vandervoort [9] developed a detailed theory of the ionization front instability, with the radiation tilt included, at a zero acceleration (i.e., this was an instability different from the RT instability). In the limit of the zero density ratio, $\eta \equiv \rho_{abl}/\rho \rightarrow 0$ (where ρ_{abl} and ρ is the density in the ablation flow and in the molecular cloud, respectively), the instability is present only for the non-zero tilt; it can be called the "tilted radiation" (TR) instability. Axford [10] and Sysoev [11] included effect of absorption into the stability analysis of a non-accelerating front and normal incidence and have found a generally stabilizing effect. Williams [12] included both the radiation tilt and absorption (but no acceleration) and made conclusion that the radiation tilt makes the system more unstable at all wavelengths. Ryutov et al [13] considered the TR instability in the presence of acceleration (but without absorption in the ablation flow). In numerical simulations by Kane et al [14] and Mizuta et al [15, 16], which contained both linear and nonlinear stages, there were acceleration and absorption present, but no radiation tilt. It was found that, in such a situation, the absorption has a strong stabilizing effect on the linear RT instability but non-linear perturbations would grow [16].

In the present paper, which is limited entirely to the linear theory, we include in the analysis all three factors: acceleration, radiation tilt, and absorption in the ablation flow. We discuss also the "impulsive acceleration" instability. By the latter we mean the situation where the intensity of the photo-ionizing radiation comes as a short pulse, with the time-scale shorter than the dynamical time of the system. This scenario is of some interest because the light curves of the young OB stars may indeed have a substantial spike early in time [17, 18].

2. Basic assumptions

We assume that the radiation comes from a direction that forms an angle θ with the normal to the unperturbed planar surface (Fig. 1). The absorption coefficient along the ray is assumed to be just a constant number κ, so that the intensity along the ray varies according to equation $dI^*/ds = -\kappa I^*$, where s is a coordinate along the ray. We use an asterisk to designate the energy flux at the plane normal to the direction of the rays. We denote this intensity at the unperturbed surface of the cloud as I_0^*. When radiation reaches the molecular cloud, the absorption is assumed to occur in a zero-thickness layer. In this last respect, our model is identical to that used in Refs. [6, 13].

The ablation pressure is determined by the energy flux I through the surface of the cloud. Following the model used in Ref. [8], we assume that the ablation pressure is some growing function of I,

$$p_a = p_a(I). \tag{1}$$

In the unperturbed state this energy flux is $I_0 = I_0^* \cos\theta$.

In this brief communication we discuss only the simplest model of the cloud, within which the cloud material is considered as an incompressible fluid. As was shown in Ref. [11], the model of an incompressible fluid yields the results that are very close to a more sophisticated model of the cloud as a compressible ideal gas.

The ablation pressure accelerates the cloud in the negative direction of the axis z (Fig. 1). The absolute value g of the acceleration is equal to

$$g = \frac{p_a}{\rho h}, \tag{2}$$

where h is the cloud thickness. The effective gravity force in the frame attached to the unperturbed ablation front is directed towards $z>0$.

We assume that the density in the ablation flow is much smaller than ρ, and present results corresponding to the limit $\eta \equiv \rho_{abl}/\rho \to 0$. We work in the frame moving together with the unperturbed ablation front. In this frame, the cloud material flows through the surface of the ablation front with the velocity

$$\mathrm{v} = \eta\sqrt{p_{abl}/\rho_{abl}} \tag{3}$$

In the limit of $\eta \to 0$ that we consider in the most of this paper, this velocity is negligible and we ignore it.

3. Equations for perturbations

Perturbation of the interface between the cloud and the ablation flow leads to the perturbation of the energy flux *I* through the perturbed surface. There are two sources for this perturbation. First, if the surface gets tilted with respect to its original orientation, the angle between the rays and the surface changes, thereby leading to change of *I*. If the plane of incidence of the incoming radiation is the *xz* plane, as shown in Fig. 1, then the corresponding change of *I* is $\delta I = I_0^* \sin\theta \partial\xi / \partial x$, where $\xi(x,y)$ is the displacement of the surface in the *z* direction. Second, if certain element of the surface is displaced, the intensity changes because of the change of the absorption along the ray. This contribution is, obviously, $\delta I = \kappa\xi$, so that he total perturbation of intensity is $\delta I = I_0^*\left(\sin\theta \partial\xi / \partial x + \kappa\xi\right)$. This leads to the perturbation of the ablation pressure,

$$\delta p_a = Cp_a\left(\sin\theta \frac{\partial\xi}{\partial x} + \kappa\xi\right), \tag{4}$$

where $C>0$ is a coefficient of order of unity: $C = (I_0^* / p_a)[\partial p_a(I_0) / \partial I_0]$.

At this point, it is convenient to perform a Fourier transform in the *xy* plane, and separate the spatial and temporal variables. In other words, perturbation will have the following dependence on *x,y,* and *t*: $\exp(-i\omega t + ik_x x + ik_y y)$. An instability would correspond to $\Gamma \equiv \mathrm{Im}\,\omega > 0$. We use also notation α for the angle between the two-dimensional wave vector $\boldsymbol{k}$ and the axis *x* (Fig. 1), so that $k_x = k\cos\alpha$. For such perturbations, according to Eq. (4),

$$\delta p_a = Cp_a\left(ik\sin\alpha \sin\theta + \kappa\right)\xi. \tag{5}$$

By considering the dynamics of perturbations inside the slab, one can relate the pressure perturbation at the inner side of the perturbed interface to the displacement of the interface. This can be done in a standard way (in particular, see the corresponding derivation in Ref. [13]). As the pressure perturbation at the inner side of the perturbed surface has to be equal to δp_a, we obtain that (Cf. Eq. (10) in Ref. [13])

$$\xi = \frac{k\delta p_a}{\rho}\left[\frac{1}{\left(1-e^{2kh}\right)\left(\omega^2 - kg\right)} - \frac{1}{\left(1-e^{-2kh}\right)\left(\omega^2 + kg\right)}\right]. \tag{6}$$

Then, from Eqs. (5) and (6), one obtains the following dispersion relation, that contains effects of radiation tilt, radiation absorption, and acceleration:

$$\omega^4 - \omega^2 kghC\left(ikh\sin\theta\cos\alpha + \kappa h\right)\coth kh - k^2 g^2\left[1 - C\left(ikh\sin\theta\cos\alpha + \kappa h\right)\right] = 0 \tag{7}$$

4. The analysis of the dispersion relation

It is instructive to see what this dispersion relation predicts in the limiting cases that have been analyzed in the past. To consider a situation of a semi-infinite cloud with no acceleration (as it was done in Refs. [9-12]), one has to replace *g* in Eq. (7) by its expression (2) and take the limit of large *h*. One then obtains that

$$\omega^2 - k(p_a/\rho)C\left(ik\sin\theta\cos\alpha + \kappa\right) = 0 \qquad (8)$$

In the limit of no absorption (κ=0), we essentially recover the results by Vandervoort (for a low-density ablation flow, $\eta \rightarrow 0$): no instability at the normal incidence (θ=0), and instability in the presence of the radiation tilt, with the growth rate proportional to the wave number,

$$\mathrm{Im}\omega = \pm k\sqrt{\frac{Cp_a\sin\theta\,|\cos\alpha|}{2\rho}}. \qquad (9)$$

If we include absorption, then, at a normal incidence, one obtains non-damped oscillations, whereas in the presence of the tilt, the instability is present at arbitrary large absorption coefficient. The latter result corresponds to that obtained in the linear analysis [12]. In the limit of large absorption, $\kappa >> k$, the growth rate is equal to:

$$\mathrm{Im}\omega = \pm k\sin\theta\cos\alpha\sqrt{\frac{Cp_a k}{\rho\kappa}} \qquad (10)$$

Development of perturbations in the presence of acceleration and absorption, was studied numerically in Ref. [16] for normal incidence. In this case, our Eq. (7) yields:

$$\omega^4 - \omega^2 k\kappa g h^2 C\coth kh - k^2 g^2\left(1 - C\kappa h\right) = 0 \qquad (11)$$

For large-enough absorption coefficients such that $C\kappa h$>1, the system becomes stable. This agrees with the results of Ref. [16]. Dependence of the growth rate on the absorption coefficient in the unstable domain ($C\kappa h$<1) is illustrated by Fig. 2. The real part of frequency of the unstable modes is equal to zero, i.e., in this regard, they behave as standard RT perurbations.

Finally, if we include all the ingredients, i.e., absorption, tilt, and gravity (i.e., return to the general equation (7)), we find an instability that exists at any absorption coefficient (at a non-zero tilt). This is illustrated by Fig. 3, where the normalized growth rate is presented for the case of κh=2, where the system would be stable at a normal incidence (θ=0). Unlike the "standard" RT instability, perurbations here have a final real frequency (i.e., a final phase velocity along the surface) – a feature that can be exploited to experimentally identify the mode in possible laboratory experiment [13].

5. Impulsive irradiation

It was noted by Pound [19] that the dynamical time of evolution of the Eagle Nebula is much shorter than the evolutionary time of the typical O-type star, the ones that produce the ionizing radiation. This circumstance points at a possibility that the stars are still in a transient stage of their formation, and their luminosity may have varied significantly during the past years. Such variations, including non-monotonous variations, with the luminosity passing through a maximum, is a common phenomenon in the evolution of very young stars (e.g., [17, 18]). To get some insights into thepossible implications of this effect, we consider the following simple model: that the ablative pressure "turns on" at t=0, reaches the maximum and "turns off" at some $t=t_0$, which is much shorter that the growth time of perturbations. This model corresponds to the model of "impulsive acceleration," which is sometimes used to imitate the Richtmyer-Meshkov instability. In order for our model of absorption to work, the time t_0 should, on the other hand, exceed the transit time of the ablated gas over the distance of the order of $1/k$. We will assume that this condition is satisfied.

To make further simplification, we consider only perturbations with kh>2-3. So that we can neglect the feed-through to the back surface of the cloud and concentrate on what is going on at the front surface. For the time-dependent ablative pressure, one cannot any more consider the $\exp(-i\omega t+ik_x x+ik_y y)$ dependence of perturbations on time and has to seek perturbations of the form $f(t)\exp(ik_x x+ik_y y)$. Quite analogously to Eq. (8) but with the acceleration effects included, one then obtains for the function $\xi(t)$:

$$\frac{\partial^2 \xi}{\partial t^2} = k\frac{p_a(t)}{\rho}\left(h^{-1} - ikC\sin\theta\cos\alpha - \kappa C\right)\xi \qquad (12)$$

In the absence of radiation tilt and absorption, one recovers the standard RT equation for the acceleration depending on time (see Eq. (2)). Assuming that the initial condition $\xi(t=0)=\xi_0$, $\dot{\xi}(t=0)=0$, one readily finds that for the very short pulse

$$\xi(t) = \xi_0 k\Delta v t\left(1 - ikhC\sin\theta\cos\alpha - \kappa hC\right) + \xi_0 \qquad (13)$$

where $\Delta v = (1/\rho h)\int_0^{t_0} p_a dt$. With no tilt and no absorption, one finds a standard result for the impulsive acceleration. For normal incidence, the large-enough absorption (κhC>1) causes the front inversion. If a substantial (θ~1) tilt is present, the second term in the bracket becomes dominant. It causes a 90^0 phase shift in the x direction.

The impulsive acceleration just after the "lighting up" of the OB-type stars may be an additional mechanism for launching a subsequent evolution of molecular clouds.

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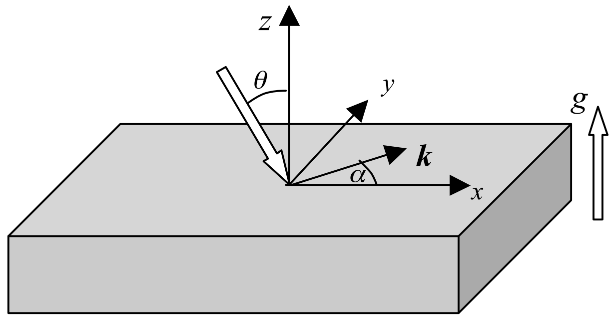

Fig. 1. The geometry of the problem

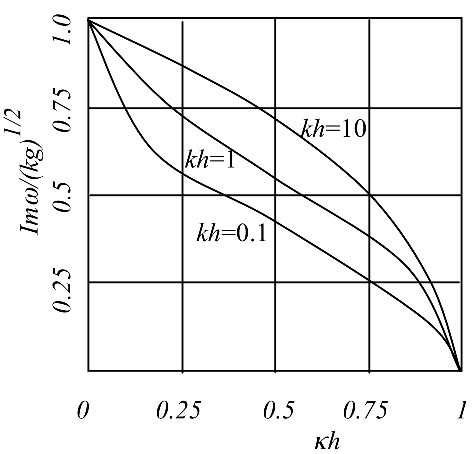

Fig. 2 The normalized growth rate at a zero tilt vs the normalized absorption coefficient. At κh>1 the RT instability ceases to exist.

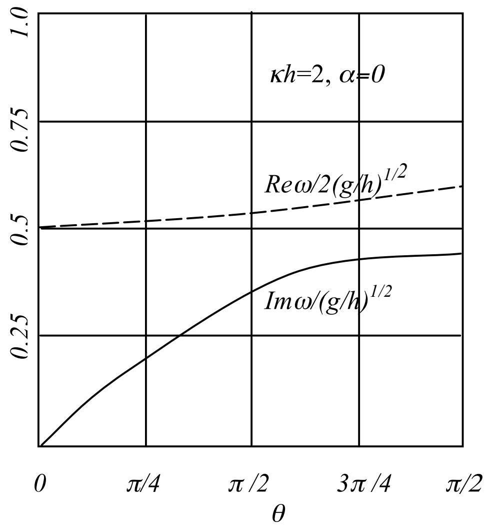

Fig. 3. The growth rate (solid line) and real frequency (dashed line) for κh=2, C=1, and kh=1 vs the tilt angle θ. Note the different normalization of the real and imaginary parts.