Optimization of Comminution Circuit Throughput and Product Size Distribution by Simulation and Control

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Abstract

The goal of this project was to improve energy efficiency of industrial crushing and grinding operations (comminution). Mathematical models of the comminution process were used to study methods for optimizing the product size distribution, so that the amount of excessively fine material produced could be minimized. The goal was to save energy by reducing the amount of material that was ground below the target size, while simultaneously reducing the quantity of materials wasted as “slimes” that were too fine to be useful. Extensive plant sampling and mathematical modeling of the grinding circuits was carried out to determine how to correct this problem. The approaches taken included (1) Modeling of the circuit to determine process bottlenecks that restrict flowrates in one area while forcing other parts of the circuit to overgrind the material; (2) Modeling of hydrocyclones to determine the mechanisms responsible for retaining fine, high-density particles in the circuit until they are overground, and improving existing models to accurately account for this behavior; and (3) Evaluation of the potential of advanced technologies to improve comminution efficiency and produce sharper product size distributions with less overgrinding. The mathematical models were used to simulate novel circuits for minimizing overgrinding and increasing throughput, and it is estimated that a single plant grinding 15 million tons of ore per year saves up to 82.5 million kW-hr/year, or 8.6×10^{11} BTU/year. **Implementation of this technology in the midwestern iron ore industry, which grinds an estimated 150 million tons of ore annually to produce over 50 million tons of iron ore concentrate, would save an estimated 1×10^{13} BTU/year.**

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Introduction

Comminution is any process where particles are crushed, ground, or otherwise broken to reduce their particle size. Crushing and grinding of feedstocks is a critical operation in mining and in a range of other industries. It is necessary to liberate valuable minerals from waste constituents so that they can be concentrated, and for producing products with the correct particle sizes for use. However, comminution is both energy-intensive and expensive, with tremendous room for improvement. Comminution operations in 1981 used 32.7 billion kilowatt-hours of electric power, which approached 2% of total U.S. electric power consumption that year, and so improvements in the efficiency of these processes would be of great economic significance. However, optimization of full-scale comminution processes by direct experiment is difficult and expensive because of the cost of modifying and operating the circuits to conduct these experiments. Mathematical simulation of the process is therefore necessary in order to identify the most promising routes for optimizing comminution performance.

There are two significant sources of grinding inefficiency that are caused by non-optimum circuit design: (1) Underutilization of equipment, due to bottlenecks in the system that prevent all parts of the circuit from operating at full capacity; and (2) Overgrinding, due to material being retained in the circuit after it has reached its target size, and therefore being ground to finer than the target size.

Underutilization of equipment is an important factor leading to energy inefficiency in tumbling-media grinding mills because a large component of the power draw of these mills is the power needed simply to rotate and agitate the grinding media. If the mill does not contain enough ore to be ground to utilize all of this energy, then the excess is wasted as extra noise and heat that does not actually result in additional particle breakage. It is therefore important to make certain that the tumbling-media mills contain their full capacity of material to be ground at all times to minimize this waste of energy.

Overgrinding is a particular concern, because it not only results in energy being wasted in unnecessary size reduction, but also can produce fines that are much smaller than can be processed efficiently (slimes). When slimes are produced, they are largely lost to the tailings, which not only wastes materials, but also results in the loss of all of the energy used in producing the slime particles.

The objective of this project was therefore to: (1) Examine grinding circuits to determine where in the circuit energy was being lost due to undercapacity operation and/or overgrinding; (2) Simulate the comminution circuits to investigate potential approaches to reduce these problems, and therefore increase energy efficiency; and (3) Consider the potential of inclusion of advanced comminution technologies, particularly high-pressure grinding rolls, to improve the overall circuit performance and energy efficiency.

Executive Summary

The goal of a comminution circuit is to grind particles to their liberation size, so that the valuable minerals are completely broken free from the gangue minerals. For the best grinding efficiency, all particles should ideally be ground only to the liberation size, and no finer. In order to accomplish this, the circuit should have the following characteristics:

1. All grinding mills should operate at their design capacity, to ensure that sufficient material is present in the mills to absorb all of the breakage energy available. If the mills are operated at less than design capacity, then a larger fraction of energy is wasted on frictional losses and on fracture of grinding media.
2. Material should be removed from the circuit as soon as it is sufficiently fine, otherwise particles retained in the circuit have the opportunity to break multiple times after they have already reached the target grind size.
3. Breakage at the coarser sizes should be maximized, as a coarse particle breaking does not produce as many extremely fine particles as a finer particle breaking will produce.
4. All particles of the target size should have equal probability of being removed by the classification system, regardless of factors such as particle density.

The bulk of the study was carried out in an iron ore processing plant operated by one of the industrial co-sponsors of the project. This plant was particularly suitable because there was extensive operating data available collected over a period of years, and also because the grinding circuit was a high-volume autogenous/pebble mill circuit of a type that is responsible for much of the comminution energy used by the U.S. mining industry. This circuit also included an advanced high-pressure grinding roll (HPGR), and therefore provided an opportunity to examine the effects of incorporating this new, highly energy-efficient grinding technology into existing grinding circuits.

Throughput and energy consumption studies of the plant determined that the circuit was only rarely operating in a balanced state where both the primary and secondary grinding mills were operating at full design capacity. It was found that changes in ore grindability caused shifts of the relative amount of grinding energy used by each stage of grinding. In general, when the ore was more difficult to grind the primary mill needed to provide a greater share of the grinding, resulting in the secondary mills running undercapacity. When the ore was easier to grind, the primary mill was running at less than capacity while the circuit throughput was limited by the secondary mills. To address this problem, a plan was devised where a screen between the primary and secondary mills could send a varying amount of material back to the primary mill for additional grinding as needed. This would make it possible to make use of excess primary mill capacity when the ore was easier to grind, while allowing load to be removed from the primary mill when the ore was harder to grind.

It was also determined that the incorporation of the HPGR unit into the grinding circuit was of great benefit in improving the throughput of the primary mill, particularly when the ore was harder to grind. The HPGR made the primary mill feed finer in size, thereby reducing the amount of grinding that the unit needed to carry out. In addition, the HPGR was specifically used to grind the “critical size” particles that the primary mill would normally have a great deal of difficulty in grinding, allowing the primary mill energy to be concentrated on grinding the particle sizes where it was most effective.

While some energy inefficiency in the plant was due to comminution units often operating at less than design capacity, a much more serious effect was the overgrinding of ore caused by incorrect particle sizing by the hydrocyclone classifiers. Extensive plant and laboratory hydrocyclone studies determined that the overgrinding was due to the fact that hydrocyclones separate particles based, not on their diameter alone, but on their settling rates in water. This settling rate is a function of both the particle diameter and particle density, and as a result a hydrocyclone processing a mixture of particles with two different densities will always separate the denser particles at a finer size than the less-dense particles. This has classically resulted in a perceived trade-off in grinding circuit operations, where the operators were forced to choose between grinding the denser particles to an excessively fine size (resulting in a waste of energy), or not grinding the less-dense particles enough (resulting in more locked particles, higher losses of valuable material to the tailings, and reduced product quality).

Using the laboratory and plant hydrocyclone results, it was possible to model the hydrocyclone performance and to determine methods for preventing the dense particles from being overground without being forced to accept excessive amounts of locked particles. A novel comminution circuit was developed with the following features:

1. It uses a pair of hydrocyclones to separate the particles which are just slightly coarser than the target size into a separate stream, which consists of very nearly monosized particles.
2. This stream of intermediate size particles, instead of recirculating to a closed-circuit mill, is diverted to a mill operating in open circuit. The particles going through the open circuit mill are subjected to only a very limited number of breakage events, and so the probability of a particle being broken more than once, and overground as a result, is minimal. Since the particles are nearly monosized, the product from the open circuit mill remains very narrow.

As a result of the novel features of this flowsheet, the intermediate-sized particles that would normally make up a large fraction of the circulating load (and would therefore both tend to be overground, and would tend to decrease the mill capacity by increasing its circulating load) is prevented from interfering with the grinding of the coarser fraction.

Simulations of this circuit indicated that this new configuration would have roughly 50% greater throughput than a conventional circuit using the same number of mills, without changing the level of material in the mill.

The combination of these three approaches (improved balancing of comminution loads throughout the circuit, incorporation of HPGR to assist primary grinding, and improved hydrocyclone/secondary mill layout to drastically reduce circulating loads and minimize overgrinding) promises to produce major benefits in improving the energy efficiencies of a wide variety of grinding circuits.

Energy Savings

Plant studies determined that the retention of high-density iron oxide particles in the grinding circuit by the hydrocyclones resulted in a waste of 1.57 kW-hr for every ton of ore ground. Since the plant grinds approximately 15 million tons of ore per year, the **direct overgrinding losses from this source are 23 million kW-hr/year, or 2.4×10^{11} BTU/year for this single plant.**

It was determined using circuit simulations that the fine iron oxide particles were not only consuming energy while being overground, but were also making up the bulk of the 253% recirculating load in the secondary grinding circuit. These fine particles were only slowly broken by the mills, and were reducing the mill volume available for breakage of coarser particles. When the circuit was redesigned to minimize overgrinding, the fine iron oxide particles were prevented from being trapped in the circuit and the circulating load dropped to only 42.5%. This allowed the throughput of the mills to increase by 50% without increasing the level of material in the mills. Since the energy consumption of a tumbling-media grinding mill is almost entirely a function of the level of material in the mill, this would correspond to a reduction of the specific energy consumption (kilowatt-hours per ton) of material ground in the mill by as much as 50%. The plant currently uses an estimated 16.4 kW-hr/ton in the secondary grinding circuit, which would be reduced to an estimated 10.9 kW-hr/ton in the modified circuit, for a savings of 5.5 kW-hr/ton. **For the 15 million tons of ore ground in this single plant annually, the energy savings is 82.5 million kW-hr/year, or 8.6×10^{11} BTU/year.** This plant produces less than one-tenth of the iron ore concentrate generated in the U.S. annually, and so **applying this approach throughout the entire midwestern iron ore industry will result in a total savings of nearly 1×10^{13} BTU/year.**

Background

The project examined two causes of comminution energy inefficiency: 1) Unbalanced loads between the various stages of comminution, and 2) inefficient or incorrect classification causing retention of fines in the grinding circuit.

Unbalanced Loads

Grinding circuits are normally designed so that, when the circuit is operating at its design capacity, all of the individual stages of comminution are also at the optimum capacity for each comminution machine. This initial circuit design is based on the grindability of the ore used in the design and pilot studies for the plant. If the grindability of an entire ore body was perfectly consistent for the life of the processing plant, this would be perfectly adequate. However, since ores are natural products that vary from point to point in the ore body, sometimes radically, it is normal that much of the time the ore being processed will have different comminution behavior than the ore used in the design of the plant.

When the grindability of an ore changes, the amount of comminution energy required in primary comminution can change at a different rate than the energy required in secondary comminution. This results in one stage becoming overloaded, causing a bottleneck in the circuit and forcing the other stages to be underutilized. Underutilization of tumbling-media grinding mills, such as ball, rod, autogenous, or pebble mills, is a significant waste of energy because the energy input to the mills depends almost entirely on the mass and volume of the mill charge, which is dominated by the quantity of grinding media in the mill (Kawatra, 2006; Taggart, 1945). If there is not sufficient ore present to be ground and absorb the energy imparted to the grinding media by the rotation of the mill, this leads to incomplete utilization of the energy and process inefficiency.

In order to be able to keep the mills in a grinding circuit all operating at their optimum capacity in the face of changing ore characteristics, two things are necessary. First, it is necessary to determine how the relative capacities of the different comminution stages changes when the ore characteristics (hardness, grindability, density, etc.) vary from point to point in the mine. Second, a means is needed to be able to shift grinding load between stages, so that when one stage has excess capacity, it can take on more of the grinding load to relieve the bottleneck at the overloaded stage.

Inefficient or incorrect classification

In grinding circuits, energy is wasted when particles are ground to a size finer than is necessary. Such overgrinding can be caused when particles are retained in the circuit too long, and continue to be ground even after they have reached the target product size of the circuit. Overgrinding can be caused by inefficient or incorrect classification returning a

large fraction of the finest particles to the circuit, leading to excessive mill retention times causing particles to be broken multiple times before they are discharged.

A significant cause of overgrinding is when an ore contains minerals of different densities, and a hydrocyclone or other classifier is used to control product size from the circuit grinding the ore. The most important case of this is iron ore grinding, where the iron oxides have densities of approximately 5 g/cm³ while the gangue minerals are less than 3 g/cm³, and the iron oxides make up a large fraction of the mass of the raw ore. Since classifiers separate based on both size and density, the higher-density iron oxides continue to report to the classifier coarse product even when they are already finer than the lower-density gangue particles that report to the classifier fine product. This results in the iron oxides being ground to a much finer size than necessary.

The performance of hydrocyclone classifiers is determined using efficiency curves, which show the probability of a particle reporting to the hydrocyclone underflow as a function of its size. This is expressed using selectivity functions, $S(d)$, which represents the fraction of the feed material in size fraction “d” that reports to the hydrocyclone underflow. This is normally divided into (a) a bypass fraction that is taken to be equal to the value of the water split, R_f , and represents the material that does not undergo classification; and (b) a classification function, $C(d)$, which has a smooth, “S” shape, starting at 100% at the coarse end and decreasing to zero at the fine end as shown in Figure 1. The classification function can be expressed closely by equations such as (Plitt, 1976)

$$C(d) = 1 - e^{-0.693 \left(\frac{d}{d_{50c}} \right)^m} \quad (1)$$

Or (Lynch and Rao, 1968)

$$C(d) = \frac{e^{\alpha \left(\frac{d}{d_{50c}} \right)} - 1}{e^{\alpha \left(\frac{d}{d_{50c}} \right)} + e^{\alpha} - 2} \quad (2)$$

Where $C(d)$ = classification function for particles of size d after correcting for particles that bypass classification; d = particle size in micrometers; d_{50c} = particle size that has equal probability of reporting to the overflow or the underflow, after correcting for the particles that bypass classification; and m, α = measures of the sharpness of separation.

Figure 1. Shape of an ideal corrected hydrocyclone efficiency curve.

If a hydrocyclone is expected to produce a standard “S” shaped efficiency curve, then it is possible to predict the corrected d50 size using a relationship such as the one shown in Equation 3 (Plitt, 1976):

$$d50_c = \frac{50.5 D_c^{0.46} D_i^{0.6} D_o^{1.21} \exp(0.063\phi)}{D_u^{0.71} h^{0.38} Q^{0.45} (\rho_s - \rho)^{0.5}} \quad (3)$$

where $d50_c$ = corrected d50 (μm); ϕ = volumetric fraction of solids in feed; D_c = cyclone diameter (cm); h = free vortex height (cm); D_i = inlet diameter (cm); Q = volumetric flowrate of feed (L/min); D_o = overflow diameter (cm); ρ_s = solid density (g/cm^3); D_u = underflow diameter (cm); ρ = liquid density (g/cm^3)

However, there are a number of cases where the efficiency curve does not follow this uniform, easily modeled shape. The curve can show inflections where the slope changes abruptly, and in extreme cases the curve can reverse direction as shown in Figure 2. This is commonly referred to as a “fish-hook” in the literature. The term comes from the fact that, when the smallest size measured corresponds to the portion where the curve is rising, it resembles a hook.

A number of possible causes of efficiency curve inflections have been proposed in the literature. Laplante and Finch (1984) explained the coarse inflection as being a result of a combination of density and size distribution effects. The causes of the fine inflection are less clear, and a variety of agglomeration, heavy media, entrainment, and fines recirculation mechanisms have been proposed (Heiskanen, 1993). To date, no investigators have reported observing both the coarse inflection and the fine inflection in a single efficiency curve, which has tended to contribute to confusion as to which type of inflection is being addressed in any given paper.

Figure 2. A hydrocyclone efficiency curve showing two inflections, one at the coarse end and one at the fine end.

When inflections occur at either coarse or fine sizes, they can make it difficult to unambiguously determine the d50c size for the hydrocyclone, which greatly complicates efforts to accurately model comminution circuits that incorporate hydrocyclones.

Coarse Inflections

From equation 3, it is evident that the cut size is a function of the particle density. The higher the density of a material, the smaller the d50 (Lynch and Rao, 1968). With a heterogeneous feed containing particles that vary in density, each feed component follows the appropriate efficiency curve for particles of its density. The effect of this on a synthetic mixture of low-density and high-density particles is exemplified in Figure 3, which includes the efficiency curves for pyrite (specific gravity = 5.0) and non-sulfides

(specific gravity 2.7), as well as the combined bulk efficiency curve for the mixture. Both of the pure components clearly follow portions of classic S-shaped efficiency curves, but the combined curve deviates markedly from an S-shape.

Figure 3. Example of an inflection at the coarse end of a hydrocyclone efficiency curve due to the presence of particles of very different densities (after Laplante and Finch, 1984)

In order for the differences in particle density to produce a non-S-shaped efficiency curve, the particles must not only differ in density, but also must differ in size distribution, with the heavier mineral being concentrated in the finer size fractions (Laplante and Finch, 1984). This results in the bulk efficiency curve following the shape of the low-density component at the coarser sizes, and then switching to following the high-density component at the finer sizes. If the relative percentages of high-density particles and low-density particles in each size fraction are known, then it is possible to calculate the bulk efficiency curve based on the weighted averages according to the respective weight percentages of dense and light components contained in the feed stream (Heiskanen, 1993). This is calculated as shown in Equation 4 (Napier-Munn et al., 1996);

$$S(d) = f_L(d)S_L(d) + [1-f_L(d)]S_H(d) \quad (4)$$

where $S(d)$ = bulk efficiency curve value for size fraction d ; $S_L(d)$ = efficiency curve value of light feed component for size fraction d ; $S_H(d)$ = efficiency curve value of heavy

(dense) feed component for size fraction d ; $f_L(d)$ = weight fraction that is the light feed component for size fraction d ; d = particle size (μm)

The degree to which this effect occurs depends on the difference in density between types of particles. The most extreme case is reported by Banishi et al. (1991), where metallic gold particles in a silicate ore produce a pronounced inflection at an unusually fine size, due to the extremely high density of gold (19.3 g/cm^3)

Fine Inflections

While efficiency curve inflections at the coarser sizes are satisfactorily explained by differences in particle density and size distribution, the inflections at the finer sizes are a completely different phenomenon that has resisted reliable measurement or modeling. In the case of the fine inflections, there are numerous possible proposed mechanisms, which have not been conclusively demonstrated. It has also not been demonstrated whether the fine inflection is a significant consideration, since it occurs at the very fine particle sizes where there is often only a very small fraction of the total material present. It has even been proposed that the phenomenon of fine inflections does not even exist, and that its appearance is due mainly to the difficulty of accurately measuring particle sizes and weights at very fine sizes. The effect is sometimes elusive, and some authors have not observed the fine inflection even after extensive experiments (Coelho and Medronho, 2001).

The fine inflection is well known in air classifiers, and can be expressed mathematically as the result of internal recycling and reclassification of material in the classifier (Luckie and Austin, 1975). However, air classifiers have a significantly different design than hydrocyclones, with areas that can clearly be identified as producing two stages of classification. It is the interaction of these two stages that leads to the inflection in air classifier efficiency curves. This analysis therefore does not apply directly to hydrocyclones, which lack the internal structures needed to produce multiple stages of classification within a single unit.

A fine inflection can be modeled by assuming that it is due to fines entrainment in the hydrocyclone underflow (Finch, 1983; Del Villar and Finch, 1992; Kelly, 1991). The model based on this assumption becomes:

$$S(d) = C(d) + a(d) \quad (5)$$

Where $C(d)$ is the fraction of the feed of size d that is recovered to the underflow as a result of classification forces only, and $a(d)$ is the fraction of the feed of size d that is carried to the underflow independently of the classification. The bypass function $a(d)$ is normally calculated from the water bypass fraction, R_f , as follows:

$$a(d) = R_f(1-C(d)) \quad (6)$$

If it is assumed that, instead, the value of $a(d)$ instead increases linearly with decreasing values of d , and becomes equal to the water bypass fraction (R_f) when $d=0$, then the Plitt equation for the uncorrected efficiency curve becomes:

$$S(d) = 1 - e^{-0.693 \left(\frac{d}{d_{50c}} \right)^m} + R_f \left(\frac{d_0 - d}{d_0} \right) \quad (7)$$

Where $S(d)$ = selectivity function for particles of size d , uncorrected for the bypass fraction; R_f = fraction of the water entering with the feed that reports to the hydrocyclone underflow; and d_0 = the maximum particle size that is entrained by the water flow into the underflow while bypassing classification.

This model does predict a small inflection in the efficiency curve as long as the value of d_0 is quite fine, as shown by the example curve given in Figure 4. It is consistent with the observation that, in many cases, the fine inflection is most pronounced when the value of R_f is quite high (Pasquier and Cilliers, 2000). However, the model does not predict two features that are sometimes reported for the fine inflection: (1) it does not allow the value of $S(d)$ for the small particles to exceed the value of R_f , and (2) it does not account for cases where $S(d)$ for the fine particles first rises, and then falls again.

A commonly proposed mechanism for the fine inflection is fines agglomerating to the coarse solids and being preferentially carried to the underflow, as shown in Figure 5. (Heiskanen, 1993; Finch, 1983). While this mechanism is quite plausible, it is difficult to conclusively prove and even more difficult to reliably model. In any case, the fine inflection has been observed even in cases where measures have been taken to ensure thorough dispersion (Rouse et al., 1987). Therefore, while fines agglomeration is a likely contributing factor in many cases, it is unlikely to be the cause of all observed cases of the fine inflection. In this situation, it is expected that the fine inflection would be most pronounced when there are many coarse particles and few fine particles, as then there would be significant surface area available to carry the fines. It would also be expected that the magnitude of the fine inflection would vary depending on the properties of the coarse particles that could lead to agglomeration.

Figure 4: Selectivity curve for a 25 cm hydrocyclone, sized by micro-sieving to 25 microns, and modeled using Equation 6 with the following parameters: $d(50)c = 70 \text{ } \mu\text{m}$; $n = 3.2$; $d_0 = 100 \text{ } \mu\text{m}$, $R_f = 0.47$. In addition to the selectivity function $S(d)$, the entrainment function $a(d)$ and the classification function $C(d)$ are also shown. (after Finch, 1983)

Figure 5: Schematic of the fines agglomeration effect, with a coarse particle carrying fines that are attached to its surface.

Another possible mechanism is based on fine particles being entrained in the wake behind coarser particles, as shown in Figure 6 (Neesse et al., 2004; Kraipech et al., 2002). This is supported by the observation that when particles of different sizes are settling together, the settling rates of the finer particles is increased compared to the settling rate that they have when no coarse particles are present. This effect can be quantified, and models based on it can show a quite pronounced fine inflection. It is reported that this effect becomes experimentally noticeable at particle sizes less than $3.5\text{ }\mu\text{m}$ for lime particles, glass beads, and dusts suspended in water (Kraipech et al., 2002). In this situation, the quantity of fines that could be carried to the underflow would depend on the volume of coarse particle wakes, and would therefore increase as the number of coarse particles increased. It would also be expected that there would be relatively little dependence on the coarse particle properties, and the effect would depend only on the size and settling velocity of the coarse particles.

Figure 6: Fine particles trapped in the wake of coarse particle, and being carried along by it to the hydrocyclone underflow.

It is normally expected that there is a tradeoff between overgrinding and the presence of locked particles. If the circuit product size is made coarser to reduce the overgrinding, this typically leads to an increase in the top size of the product as well, with the top size particles being poorly liberated. This reduces the grade that can be produced in the processing that follows comminution. In order to reduce this trade-off, it is necessary to determine how to change the grinding circuit to produce a narrower size distribution, which will reduce overgrinding while simultaneously controlling the quantity of coarse locked particles leaving the circuit.

A plant using the flowsheet shown in Figure 7, which was operated by one of the industrial co-sponsors of this project, was chosen for the initial plant surveys. This plant includes a number of features common to the industry. It has also recently incorporated a high-pressure grinding roll unit which provided an opportunity to determine how such inclusion of advanced technologies can affect the overall grinding circuit performance.

Samples were collected from numerous points in the plant by MTU and industrial personnel, and selected process streams were analyzed to determine size distribution by sieving. Each sieve size was chemically analyzed to determine the iron content, which could be used to calculate the relative quantities of magnetite and silica. The specific gravities of the particles in each size fraction were also measured. These samples, along with process flowrates compiled from the plant's data management system and mass balances calculated from the flowrates and analyses, were used to develop and validate circuit models for the plant.

Figure 8. Configuration of the hydrocyclone/pebble mill circuit sampled for this study. The total flowrates of the various streams for the particular circuit sampled were: New Circuit Feed – 120.6 long tons of dry solids per hour (LTPH) or 122.5 metric tons/hour (mt/hr); Cyclone underflow – 299.3 LTPH (304 mt/hr); Pebble mill discharge – 306.9 LTPH (312 mt/hr); Circuit product – 128.1 LTPH (130.1 mt/hr). The hydrocyclone feed was 13.2% solids

Plant Hydrocyclones

The hydrocyclones in the plant were operated in banks of 16, with 14 cyclones operating at any one time and two spares. The hydrocyclone/pebble mill configuration was as shown in Figure 8.

Samples were collected from the cyclone feed, cyclone underflow, and circuit product as part of an overall survey of the plant performance. Size analyses of the samples were carried out by three methods: 1. Wet-sieving at 25 μm using a woven-wire test sieve, followed by dry sieving of the +25 μm particles using woven-wire test sieves in a Ro-Tap sieve shaker; 2. Micro-sieving of dry powders using electroformed nickel-foil sieves in a Sonic Sifter apparatus, which allowed sieving down to 10 μm particle size; and 3. Microtrac® laser diffraction particle size analysis to measure particle sizes down to 1 μm , primarily as a check on the accuracy of the sieve analyses. For the sieved samples, each individual size fraction was assayed using a dichromate titration method to determine the iron assay of each sieved size fraction for each stream. The size distribution and assay data was then mass balanced, and the magnetite concentration in each size fraction was calculated from the iron assays.

Laboratory Hydrocyclone Experiments

In the course of the plant sampling and analysis of the plant data, a number of unusual features were noted in the hydrocyclone performance that would strongly affect their classification behavior, but did not appear to be fully accounted for by mathematical models of the hydrocyclone. In particular, the curves were seen to show inflections which were believed to be due to the presence of both high-density and low-density particles in the hydrocyclone feeds.

In order to more closely examine these features, and to determine how changes in feed properties and hydrocyclone operating parameters would affect them, a series of laboratory experiments were carried out using controlled size distributions and feed compositions for the hydrocyclone feed. Also, while the plant sampling results appeared to exhibit a coarse inflection, analysis of plant samples was unable to determine whether a fine inflection was also occurring, and so the laboratory experiments were designed to be able to detect both types of inflection.

Equipment

A 10.2 cm diameter Krebs hydrocyclone was used, mounted on a test rig with a variable-speed pump, pressure gauge, ultrasonic flowmeter, and sampling mechanism for collecting simultaneous overflow and underflow samples. The hydrocyclone dimensions were :

Feed inlet: 3.5 cm

Vortex finder diameter: 3.18 cm

Spigot diameter: 1.59 cm

The hydrocyclone test rig consisted of a sump with a variable-speed centrifugal pump circulating material continuously through the hydrocyclone. An oil-filled pressure gauge was used to monitor hydrocyclone inlet pressure. Flowrates in the hydrocyclone test rig were monitored using an ultrasonic Doppler flowmeter. Samples were collected using a specially designed sample cutter that simultaneously collected samples from both the cyclone overflow and underflow, so that the relative flowrates of the two streams could be accurately measured. Particle size distributions were determined using a Leeds and Northrup Microtrac laser diffraction particle size analyzer.

Materials and Procedures

Materials used were finely-ground quartz sand, and magnetite concentrate. Experiments were first conducted using magnetite alone and quartz alone, followed by experiments with mixtures of magnetite and quartz. All samples were collected in triplicate at each hydrocyclone operating condition, and each of the triplicate samples were analyzed separately to determine the random variations from test to test. In all experiments, the random variation in the fraction of each size reporting to the underflow was less than ± 0.01 units. This is smaller than the magnitude of the inflections that were observed, demonstrating that the inflections were actually present and not simply due to experimental error.

Experiments with Single Minerals. For experiments using magnetite alone, and using quartz alone, the size distributions of the hydrocyclone feeds were as shown in Figure 9. For these experiments the size distributions of magnetite and quartz were reasonably similar, and in particular they had similar quantities of particles finer than 10 μm . By this it was determined whether there were differences in the fine inflections for the two minerals that could be attributed to differences in the mineral properties.

Experiments were run at three different percent solids for each mineral (2.5, 5.5, and 16.5 % solids for the magnetite; 2.3, 8.4, and 16.9% solids for the quartz). At each percent solids, the cyclone inlet pressure was varied over the range of 5-20 psi to alter the hydrocyclone flowrate.

Experiments with Magnetite/Quartz Mixtures. Additional experiments were conducted with 50:50 mixtures by weight of magnetite and quartz. Two sets of these experiments were conducted, with the size distributions of the quartz and magnetite being very different. This was done in order to produce the maximum coarse inflection. In the first set of experiments, the quartz was the underflow (coarse) product from the cyclone experiments using quartz alone, while the magnetite was the overflow (fine) product from the cyclone experiments using magnetite alone. This resulted in a moderate amount of overlap between the magnetite and quartz size distributions, as can be seen in Figure 10.

Figure 9: Magnetite and quartz size distributions used in laboratory hydrocyclone experiments using magnetite alone and quartz alone.

The second set of 50:50 magnetite:quartz experiments used quartz that had been further processed by sedimentation to remove the finest particles, and magnetite that had been processed by sedimentation to remove the coarsest particles, with the coarse magnetite particles then being ground using an attrition mill and added back to produce a finer magnetite size distribution. This resulted in much less overlap between the magnetite and quartz size distributions, and the magnetite being significantly finer than in the first set of experiments, as shown in Figure 10.

For these experiments, the percent solids was varied from as high as 30% solids to as low as 2.5% solids, and the cyclone operating pressure was varied from 7 to 20 psi to alter the flowrates.

Figure 10: Magnetite and quartz size distributions used for experiments with a 50:50 mixture by weight of magnetite and quartz. Two sets of experiments were run, with the second set having less material in the “overlap” region than the first set did.

Mathematical Modeling of Comminution Circuits and Circuit Components

A tremendous body of literature exists on the topic of comminution modeling, and has lead to the development of powerful steady-state simulation packages. One of the most advanced, USIM-PAC v. 3.0.7.0 (BRGM, 2004), was used in this project. This package allowed simulation of all of the major components of the comminution circuits under study, and included the capability of independently tracking the behavior of the high-density magnetite phase and the lower-density silicate phase, in addition to predicting the overall behavior of the bulk ore. This was important for examining the effects of preferential retention of high-density particles in the circuit by the action of the hydrocyclones.

Results and Discussion

Plant Sampling Results

Representative analytical results from plant sampling are given in Tables 1-4, for the streams that were considered most critical for studying the comminution performance: Pebble mill discharge, Cobber concentrate, Cyclone underflow, and Cyclone overflow.

Table 1: Pebble Mill discharge sample analysis, including size distribution, iron content and specific gravity for each particle size.

Size		Individual		Cumulative		Iron Content		Spec. Grav.	
Mesh	Microns	Weight (g)	Ind %	% Pass	% Retained	Avg	Stdev	Avg	Stdev
+6#	3327	2	0.21	0.21	99.79	19.80	0.06	2.76	0.23
-6#..+8#	2362	3	0.32	0.53	99.47	29.72	0.10	2.84	0.30
-8#..+10#	1651	2.3	0.24	0.77	99.23	30.42	0.12	2.95	0.54
-10#..+14#	1168	1.9	0.20	0.97	99.03	32.01	0.08	2.91	0.13
-14#..+20#	833	2.9	0.31	1.28	98.72	35.09	4.14	3.04	0.02
-20#..+28#	589	3.1	0.33	1.60	98.40	32.71	0.38	3.36	0.27
-28#..+35#	417	1.7	0.18	1.78	98.22	NA	NA	3.04	0.14
-35#..+48#	295	6	0.63	2.41	97.59	32.80	0.41	2.99	0.17
-48#..+65#	208	6.3	0.66	3.08	96.92	33.03	0.11	3.00	0.19
-65#..+100#	147	11.6	1.22	4.30	95.70	33.48	0.14	3.15	0.09
-100#..+150#	104	17.4	1.83	6.14	93.86	33.41	0.27	3.32	0.12
-150#..+200#	74	38.6	4.07	10.21	89.79	33.96	0.27	2.97	0.12
-200#..+270#	52	53.7	5.66	15.87	84.13	33.66	0.54	3.01	0.39
-270#..+325#	44	53.5	5.64	21.51	78.49	37.14	0.20	2.68	1.13
-325#..+500#	25	225.6	23.79	45.30	54.70	45.15	0.25	3.87	0.25
-500#		518.7	54.70	100.00	0.00	46.31	11.11	3.60	0.26

Table 2: Cobber Concentrate sample analysis, including size distribution, iron content and specific gravity for each particle size.

Size		Individual		Cumulative		Iron Content		Spec. Grav.	
Mesh	Microns	Weight (g)	Ind %	% Pass	% Retained	Avg	Stdev	Avg	Stdev
+6#	3327	0.00	0.00	0.00	100.00	NA	NA	NA	NA
-6#..+8#	2362	0.00	0.00	0.00	100.00	NA	NA	NA	NA
-8#..+10#	1651	1.50	0.08	0.08	99.92	37.31	2.10	2.82	1.78
-10#..+14#	1168	32.40	1.77	1.85	98.15	33.72	0.61	3.31	0.64
-14#..+20#	833	117.60	6.42	8.27	91.73	34.37	0.21	3.26	0.49
-20#..+28#	589	147.00	8.02	16.29	83.71	34.77	0.30	3.18	0.35
-28#..+35#	417	147.10	8.03	24.32	75.68	34.29	1.07	3.08	0.30
-35#..+48#	295	156.10	8.52	32.83	67.17	35.69	0.26	3.30	0.35
-48#..+65#	208	158.10	8.63	41.46	58.54	36.09	0.22	3.27	0.11
-65#..+100#	147	118.10	6.44	47.91	52.09	36.65	0.19	3.12	0.30
-100#..+150#	104	127.50	6.96	54.86	45.14	37.24	0.19	3.23	0.16
-150#..+200#	74	110.40	6.02	60.89	39.11	38.79	0.48	3.20	0.14
-200#..+270#	52	46.10	2.52	63.41	36.59	39.40	0.61	3.65	0.50
-270#..+325#	44	36.80	2.01	65.41	34.59	42.16	1.24	3.63	0.05
-325#..+500#	25	136.70	7.46	72.87	27.13	49.92	1.59	3.81	0.17
-500#		497.10	27.13	100.00	0.00	63.45	0.27	4.47	0.13

Table 3: Cyclone Underflow sample analysis, including size distribution, iron content and specific gravity for each particle size.

Size		Individual		Cumulative		Iron Content		Spec. Grav.	
Mesh	Microns	Weight (g)	Ind %	% Pass	% Retained	Avg	Stdev	Avg	Stdev
+6#	3327	3.90	0.16	0.16	99.84	25.59	5.83	2.88	0.46
-6#..+8#	2362	7.10	0.29	0.44	99.56	29.38	1.02	2.94	0.27
-8#..+10#	1651	7.00	0.28	0.72	99.28	29.94	0.60	3.11	0.24
-10#..+14#	1168	22.60	0.91	1.63	98.37	17.10	17.61	3.26	0.24
-14#..+20#	833	74.00	2.97	4.60	95.40	33.86	0.53	2.69	0.83
-20#..+28#	589	91.60	3.68	8.28	91.72	34.36	0.52	3.16	0.15
-28#..+35#	417	99.90	4.01	12.29	87.71	34.62	0.50	3.20	0.16
-35#..+48#	295	100.10	4.02	16.31	83.69	35.22	0.16	2.97	0.25
-48#..+65#	208	106.50	4.28	20.59	79.41	35.18	0.32	2.88	0.38
-65#..+100#	147	88.70	3.56	24.15	75.85	35.79	0.41	3.54	0.90
-100#..+150#	104	122.20	4.91	29.05	70.95	35.02	0.56	3.52	0.07
-150#..+200#	74	168.00	6.75	35.80	64.20	35.06	0.72	3.56	0.19
-200#..+270#	52	120.90	4.85	40.65	59.35	34.18	1.24	3.04	0.86
-270#..+325#	44	106.00	4.26	44.91	55.09	37.16	0.92	3.45	0.18
-325#..+500#	25	604.10	24.26	69.16	30.84	51.63	1.02	3.83	0.39
-500#		768.00	30.84	100.00	0.00	67.50	0.70	4.26	0.06

Table 4: Cyclone Overflow sample analysis, including size distribution, iron content and specific gravity for each particle size.

Size		Individual		Cumulative		Iron Content		Spec. Grav.	
Mesh	Microns	Weight (g)	Ind %	% Pass	% Retained	Avg	Stdev	Avg	Stdev
+6#	3327	0.00	0.00	0.00	100.00	NA	NA	NA	NA
-6#..+8#	2362	0.10	0.01	0.01	99.99	NA	NA	NA	NA
-8#..+10#	1651	0.20	0.02	0.03	99.97	NA	NA	NA	NA
-10#..+14#	1168	0.20	0.02	0.05	99.95	NA	NA	NA	NA
-14#..+20#	833	0.20	0.02	0.07	99.93	NA	NA	NA	NA
-20#..+28#	589	0.20	0.02	0.10	99.90	NA	NA	NA	NA
-28#..+35#	417	0.20	0.02	0.12	99.88	NA	NA	NA	NA
-35#..+48#	295	0.20	0.02	0.14	99.86	NA	NA	NA	NA
-48#..+65#	208	0.30	0.03	0.17	99.83	NA	NA	NA	NA
-65#..+100#	147	0.20	0.02	0.19	99.81	NA	NA	NA	NA
-100#..+150#	104	0.80	0.09	0.28	99.72	NA	NA	NA	NA
-150#..+200#	74	4.00	0.43	0.70	99.30	25.53	0.03	3.26	0.24
-200#..+270#	52	5.40	0.58	1.28	98.72	27.09	0.14	3.14	0.07
-270#..+325#	44	8.90	0.95	2.23	97.77	27.47	0.15	3.19	0.05
-325#..+500#	25	97.10	10.34	12.57	87.43	33.33	0.42	3.28	0.05
-500#		821.00	87.43	100.00	0.00	43.16	0.63	3.58	0.23

In the grinding circuit shown in Figure 7, ore was initially ground in fully-autogenous grinding mills, and then screened into three fractions: (1) A coarse 2 inch x ½ inch (50.8 mm x 12.7 mm) fraction, (2) ½ inch x 0.04 inch (12.7 mm x 1 mm) particles, and (3) a fine product passing 0.04 inch (1 mm). The -1 mm material was passed through a magnetic separator (cobber) to reject liberated nonmagnetic silicates, and the magnetic fraction was then classified using a hydrocyclone and sent to pebble mills for final grinding. Pebbles for the pebble mills were removed from the coarse screen product, and the remainder of the +1 mm material was recirculated to the primary mill.

A cone crusher and high pressure grinding roll (HPGR) were included in the circuit primarily to deal with the “critical size” material, which are particles that are too coarse to be efficiently ground by the primary mill, and too fine to act as efficient grinding media in this unit. For this ore, the critical size was in the 2 inch x ½ inch (50.8 mm x 12.7 mm) size fraction. By passing this size fraction through the cone crusher, or through both the cone crusher and the HPGR, it could be reduced to a particle size that would be ground more effectively. In the process, this reduced the amount of grinding energy that needed to be provided by the primary mill. It was therefore expected that, as the amount of material passed through the cone crusher/HPGR increased, the capacity of the mill would also increase.

Projection of Circuit Throughput

The first concern was to determine an equation that could be used to calculate projected circuit throughput, based on ore characteristics and circuit operation. Operating data for the circuit had been collected over a period of three years, and was available in the form of daily values for a number of parameters. Those judged most relevant to this study were: 1) total circuit throughput; 2) cone crusher throughput; 3) cone crusher operating hours; 4) HPGR throughput; 5) HPGR operating hours; 6) estimated feed work index; 7) primary mill power draw; 8) pebble mill power draw; 9) feed silica content; and 10) feed magnetic iron content.

Initially, it was anticipated that the primary bottleneck in the circuit would be the primary mill. In this case, circuit capacity would be dependent on the primary mill capacity, which in turn was expected to be a function of (1) the work index, (2) the quantity of material crushed by the cone crusher, and (3) the quantity of material crushed by the HPGR. Upon examining the data, it was found that the circuit throughput did not correlate with the measured values of these three variables, because much of the time the primary autogenous mills were not operating at full capacity. This showed that the assumption that the primary mill was the bottleneck was not always valid. When the primary mill was not at full capacity, the pebble crusher/roll press operating time and ore characteristics are not controlling throughput, and therefore an equation intended only to project the primary mill capacity cannot predict the overall circuit throughput.

A substantial fraction of the time, the throughput was being limited by the capacity of the pebble mills. In order to determine when the circuit was operating in each of these two

regimes (pebble mill limited versus primary mill limited), the pebble mill power draw was plotted against the primary mill power draw for a 4-month subset of the operating data to produce the graph shown in Figure 11. In this graph, it is seen that there were distinct periods where the throughput was being limited by the pebble mill, while at other times the throughput was clearly controlled by the primary mill.

Figure 11: Operating regimes for the grinding circuit. By plotting the pebble mill horsepower versus the primary mill horsepower, it is seen that there are distinct regions where the pebble mill power draw is nearly constant while the primary mill power draw varies substantially, indicating that the pebble mill is the throughput bottleneck in this region. Similarly, there is a region where the primary mill is clearly the limiting factor.

The immediate question was then, what was causing the circuit to change from being primary-mill limited to being pebble-mill limited? It was believed that the cause was a change in the ore work index. Unfortunately, the work index measurements as they were performed at the plant are difficult to correlate with the feed that actually enters the mill on a given day, since the work index values are measured on samples collected from the mine before the ore is transported to the plant. As a result, the error bars for the work index measurements are quite large. However, the average work-index results shown in Figure 12 suggest that the grinding circuit tends to be primary-mill limited when the work index is high and the ore is harder to grind, and that it becomes pebble-mill limited as the work index decreases and the ore becomes softer.

Based on Figure 11, it was clear that two equations were needed for projecting the circuit throughput: one equation for when the throughput was controlled by the primary mill, and a second equation for when the throughput was controlled by the secondary mill. Two subsets of plant data were therefore extracted from the overall data set, one corresponding to only those days when the circuit throughput was definitely being limited by the primary

mill capacity, and the second corresponding to days when it was definitely being limited by the secondary mill capacity.

Figure 12: Average work index for each of the operating regimes of the grinding circuit. These are averages of daily measurements of the ore work index, performed on samples collected directly from the mine. The work index values are averages for all of the points in each operating regime, as marked in Figure 11.

The following variables were selected as being most likely to correlate with either the primary mill or secondary mill throughput:

PCFR = mean daily pebble crusher feedrate, long tons/hour.

RPFR = mean daily roll press feed rate, long tons/hour

WI = Feed work index, Kw-hr/long ton

MAG = Percent magnetic iron in feed

SI = Percent SiO₂ in feed

These variables were selected for the following reasons:

1. The PCFR and RPFR reduce the effective particle size of the feed to the primary mill, and therefore the amount of grinding that the primary mill must carry out is reduced. Therefore, increasing these feedrates will tend to increase overall circuit throughput.

2. The WI is a measure of the hardness of the ore, and as a result an increase in work index was expected to decrease the throughput of both the primary mill and the secondary mill.

3. The percent magnetic iron (MAG) in the feed determines the quantity of material that the cobber will recover and send to the secondary mill. Increasing MAG values were therefore expected to increase the load on the secondary mill for any given circuit throughput, and so the MAG values were expected to be inversely proportional to throughput.

4. The percent SiO₂ in the feed was expected to both be inversely proportional to the MAG value, and directly related to the WI value.

Linear equations were used to relate these variables to circuit throughput. The equation coefficients were determined using Systat 10.2 (SSI, 2003). This package calculates the significance ranges for each coefficient and determines the predictive power of each variable. This information was used to choose the variables which were of greatest value in calculating the projected throughput.

Equation for Primary-Mill Limited Condition

The significance of each of the variable coefficients described above for predicting circuit throughput under the primary-mill limited condition are given in Table 5.

Table 5: Significance of variables for predicting overall circuit feedrate when the circuit is in the primary-mill limited condition. This is for a linear equation of the form $FR = A*PCFR + B*RPFR + C*MAG + D*WI + E*SI + \text{Constant}$. A value of “P” smaller than 0.150 is considered to show that a given independent variable is useful for predicting the dependent variable.

Variable	Coefficient	Standard Error of coefficient	% Standard Error	“P”
Constant	413.97	80.82	18.7	0.000
PCFR	0.0026	0.0007	26.9	0.001
RPFR	0.0028	0.0007	25.0	0.000
MAG	-7.489	2.697	36.0	0.010
WI	-3.735	3.123	83.6	0.243
SI	1.577	7.07	446.5	0.825

From this analysis, the variables with the greatest predictive power are PCFR and RPFR, while the work index (WI) and silica content (SI) have little predictive power and very high standard errors. The magnetite content (MAG) was marginal. It is desirable to have the model be as simple as possible, and so the equation was reduced to as few variables as possible while still maintaining a good correlation coefficient. Since it was found that even without the inclusion of the MAG value the correlation coefficient was still 0.9229,

the equation was reduced to just two variables, with the coefficients recalculated to produce the equation below:

$$FR = 0.00267PCFR + 0.00242RPFR + 205.6 \quad (8)$$

It was originally expected that the work index would be an important factor in producing an accurate projected circuit feedrate, but it was found to be of limited value, due to the difficulty in matching the work index measured from samples collected from the ore body with the actual work index of the incoming ore on a specific day. The correlation of actual grinding circuit throughput to the predicted throughput using this equation was very good, as can be seen in Figure 13. It is important to note that this equation is only valid when the primary mill is controlling the throughput. When the pebble mill is controlling circuit throughput, a different equation will be needed that will take into account the fraction of the ore that is rejected by the magnetic cobbler, as a high magnetic iron content will result in a greater quantity of feed being sent to the pebble mills at the same overall circuit throughput.

Figure 13: Correlation of predicted feedrate with measured feedrate in the regime where the primary mill capacity controls throughput, using Equation 8. In this regime, the operation of the cone crusher and the HPGR are the primary factors controlling capacity.

Equation for Secondary-Mill-Limited Condition

Using the data set where the circuit feedrate was limited by the pebble mill, the following results were obtained from the curve-fit and the statistical analysis, as shown in Table 6.

Table 6: Significance of variables for predicting overall circuit feedrate when the circuit is in the secondary-mill limited condition.				
Variable	Coefficient	Standard Error of coefficient	% Standard Error	"P"
Constant	233.1	64.434	27.6	0.001
PCFR	0.0032	0.0006	18.8	0.000
RPFR	0.0018	0.0004	22.2	0.000
MAG	-1.197	1.926	160.9	0.540
WI	-3.837	2.255	58.8	0.102
SI	11.548	6.048	52.4	0.068

Examining the values for the standard error and the "P" values, the variables with the most predictive power were, again, the pebble crusher and roll press feedrates. However, the correlation was not as good as was achieved with the primary mill model (Equation 8), and so the silica content, which also showed considerable predictive power, was included in the final equation. Once these variables were chosen, the final fitted equation became:

$$FR = 0.00372PCFR + 0.00176RPFR + 9.977SI + 154.0 \quad (9)$$

The correlation coefficient (R) for this equation was 0.867, and the correlation is shown in Figure 14.

Figure 14: Correlation of predicted feedrate with measured feedrate in the regime where the secondary mill capacity controls throughput, using Equation 9.

It is expected that the best method for predicting plant throughput using these equations is to calculate the throughput with both models. The lower of the two throughputs can then be taken as the overall plant throughput. This will also indicate which operating regime the circuit is expected to be in.

Load Balancing in the Grinding Circuit

In order to fully utilize all of the available capacity in the grinding circuit, it is necessary to be able to shift the grinding load to the point in the circuit where extra capacity is available. The circuit described can currently do this to some extent, by using the cone crusher and HPGR to provide extra grinding capacity in the primary mill. However, another method is needed to allow load to be shifted from the pebble mills when the load imbalance becomes more severe.

Figure 15: Proposed location of supplemental screening stage for shifting grinding load from the pebble mill back to the primary mill (primary screen, cone crusher and HPGR omitted for clarity).

To some extent, it is possible to shift grinding load back to the primary mill by recycling uncrushed pebbles. It has been observed that when all of the recycled pebbles are crushed using the cone crusher and HPGR, the primary mill product is approximately 30% passing 25 μm , but when only the cone crusher is used, the fineness increases to 35% passing 25 μm . When no pebbles are crushed, the primary mill product is still finer, at 40% passing 25 μm . This increased product fineness will reduce the amount of grinding that will need to be done by the secondary mill, at the expense of increasing the power draw for the primary mill. However, the amount of load shifting that can be accomplished by this means is limited. Also, it is believed that the energy required to grind uncrushed pebbles is greater than the energy needed to comminute them using the

cone crusher/HPGR combination, and that therefore this is an inefficient use of extra grinding capacity in the primary mill.

The most promising approach for load balancing in this circuit would be the introduction of a screening stage immediately after the cobber, as shown in Figure 15. Analysis of the size distribution of the cobber concentrate, shown in Figure 16, determined that if the cobber concentrate were screened at 48 mesh, this would allow 31% of it to be recirculated back to the primary mill. The advantage of screening at this point in the circuit is that the cobber has already rejected liberated silica from the total feedstream at this point, and so the screen needs much less capacity than would be necessary if it were installed as part of the main screen deck immediately following the primary mill.

Figure 16: Size distribution of the cobber concentrate feed stream, which is the feed to the pebble mills. 31% of the material is coarser than 48 mesh, which is a screen size that is practical for use in plant operations.

The amount of power draw that can be shifted from the pebble mill to the primary mill by this means can be easily calculated using the Bond relationship (Weiss, 1985);

$$W = 10W_i \left(\frac{1}{\sqrt{P}} - \frac{1}{\sqrt{F}} \right) \quad (10)$$

where W_i is the work index, P is the 80% passing size of the product in micrometers, and F is the 80% passing size of the feed in micrometers.

The cobber concentrate flowrate was 120 +/- 7 LTPH for each pebble mill, at 48.8 +/- 3.7 % solids. The +48 mesh material is therefore $120 \times 0.31 = 37.2 \pm 2.2$ LTPH, and it is 80% passing 1046 μm . The average Work Index for the ore is 11.2 kw-hr/lt, and the screen undersize product is 80% passing 137 μm

From the Bond equation, the power required to grind the +48 mesh material to a size that will pass the screen is therefore

$$W = 10 \times 11.2 \times \left(\frac{1}{\sqrt{137}} - \frac{1}{\sqrt{1046}} \right)$$

$$= 6.1 \text{ kw-hr/lt.} \quad (11)$$

When this was recirculated to the primary mill, the added primary mill load was then $37.2 \times 6.1 = 227 \text{ Kw} = 304 \text{ Hp}$ per pebble mill feed stream recirculated. Since there were two pebble mills per primary mill in the circuit, this comes to 608 Hp for both pebble mills recirculating +48 mesh material.

The design capacity of the primary mills was 8650 HP, and so sufficient capacity was available for full recirculation whenever mill power draw was less than 8000 HP. Recirculation from one pebble mill could be used when the power draw was between 8000 and 8300 HP

Figure 17: Histogram of average primary mill horsepower for all days that the plant was operating over a period of three years.

Analysis of the historical data for the plant showed that the power draw for the primary mill varied considerably, as can be seen in the bar graph shown in Figure 17. The power draw was <8000 HP 45% of the time, and was between 8000 and 8300 HP 13% of the

time, and so recirculation from at least one pebble mill feed stream could have been used 58% of the time without exceeding the primary mill design capacity.

In order to determine the practicality of screening the cobber concentrate at 48 mesh with the necessary capacity, pilot-scale screen testing was carried out by Derrick Inc. using both their standard low-profile screens, and their Stack-Sizer multi-deck screen. From the results shown in Table 7, it can be seen that the cobber concentrate can easily be screened at this size with high efficiency using either of these units. The size distributions of the screen products as they compare with the original cobber concentrate size distribution are shown in Figure 18.

Table 7: Pilot-scale screening results for two different types of Derrick screens processing a sample of the cobber concentrate

Test No.	Feed		Oversize	Undersize	Efficiency at 48 mesh				
	Dry Solids	Solids (%)	Plus 48 mesh	Plus 48 mesh	Plus 48 mesh	Oversize (%)	Undersize (%)	Overall (%)	
	(LTPH)		(%)	(%)	(%)				(%)
Low Profile Screening Machine Results									
1	44.6	50.3	47.7	92.7	5.2	94.4	93.3	93.8	
2	41.0	50.3	47.7	91.5	5.3	94.4	92.0	93.2	
3	48.2	50.3	47.7	88.7	5.2	94.7	89.0	91.7	
4	44.1	55.5	47.2	89.4	5.0	94.8	89.9	92.2	
5	48.3	55.5	47.2	89.9	4.6	95.2	90.4	92.7	
6	52.5	55.5	47.2	87.3	4.5	95.4	87.6	91.3	
Stack Sizer® Screening Machine Results									
7	220	55.5	47.2	90.8	2.6	97.2	91.1	94.0	
8	241	55.5	47.2	90.8	3.1	96.8	91.3	93.9	
9	262	55.5	47.2	91.3	3.2	96.6	91.7	94.0	
10	215	51.8	53.5	92.7	4.3	96.4	91.3	94.0	
11	234	51.8	53.5	92.6	4.5	96.3	91.1	93.9	
12	253	51.8	53.5	93.8	5.1	95.7	92.8	94.3	

Hydrocyclones

Plant Cyclone Results

The results from samples collected around the hydrocyclones were used to calculate efficiency curves, such as the curves shown in Figure 19. It was particularly interesting to note the difference in behavior between the magnetite and the quartz in the hydrocyclone. While the hydrocyclone begins to remove quartz from the circuit at a fairly coarse size ($d_{50} = 39\mu\text{m}$), it does not remove the magnetite until a significantly finer size ($d_{50} = 20\mu\text{m}$). The hydrocyclone underflow between approximately $20\mu\text{m}$ and $39\mu\text{m}$ therefore predominantly magnetite. This material is then returned to the grinding circuit and re-ground until it finally becomes fine enough to be removed. This results in a substantial accumulation of magnetite in the hydrocyclone underflow in this size range, as shown in Figure 20.

Figure 18: Feed, oversize, and undersize particle size distributions for the pilot-scale screen tests with the cobber concentrate. All of the tests produced essentially identical size distributions.

The effect of this retention of the higher-density fraction in the circuit is seen in the overall efficiency curve of Figure 19. While the magnetite and quartz efficiency curves are very close to the ideal “S” shape, the overall curve shows an inflection. This was due to the cyclone feed containing approximately equal quantities of magnetite and quartz at sizes coarser than 39 μm , but at finer than this size it was primarily composed of magnetite, as can be seen in Figure 20. As a result, the overall efficiency curve was initially the average of the magnetite and quartz efficiency curves, but then rapidly switched to following the overall efficiency curve once it reached sizes where the quartz had been removed from the circuit. The result was an inflection in the overall efficiency curve.

The practical significance is that such an inflection only occurs when the cyclone feed has become enriched in the denser mineral in the fine sizes. If the high-density and low-density minerals have the same size distribution, then the overall bulk efficiency curve will simply be the average of the curves for the high- and low-density minerals. The inflection can only be seen when the overall curve switches from following the low-density curve to following the high-density curve, which can only happen if there is a difference in size distributions between the two minerals.

Figure 19. Hydrocyclone efficiency curves determined from overflow and underflow samples collected from an operating hydrocyclone in the magnetite concentrator studied. The d50 sizes were 20 μ m for the homogeneous magnetite efficiency curve and 39 μ m for the homogeneous quartz curve. The coarse inflection in the overall curve is circled. This phenomenon was observed for numerous samples collected from this plant.

Laboratory Cyclone Results

Single Mineral Results. The results for the individual minerals did not show a coarse inflection, due to the feed being composed of minerals of a single density. However, both the quartz and magnetite did show a fine inflection.

For quartz, the fine inflection was relatively small, as shown by the example curves in Figure 21, but was still much larger than the random variations between triplicate samples, demonstrating that it was a real effect and not due to random error. The fine inflection was only slightly affected by percent solids, and was completely unaffected by flowrate, although the d50 size for the remainder of the curve varied as predicted by Equation 3.

Figure 20: Quantities of magnetite and quartz by size fraction in the hydrocyclone underflow collected from an operating hydrocyclone in a magnetite concentrator plant. The particles larger than 20 μ m and smaller than 39 μ m were primarily magnetite.

For pure magnetite, however, the fine inflection shown in Figure 22 was much more pronounced than the fine inflection for silica shown in Figure 21. The magnetite feed had a size distribution similar to the quartz feed, and so the magnitude of the fine inflection would be expected to be similar in both cases if the same mechanism was responsible for the fine inflection. This very large difference between the quartz and the magnetite, and the fact that the magnetite is much more sensitive to flowrate and percent solids, clearly indicates that different mechanisms are responsible in each case.

Mineral Mixture Results. Experiments with mixtures of minerals were able to show both the coarse inflection and the fine inflection. In the first set of experiments, with a relatively large overlap of the magnetite and quartz particle sizes, the coarse inflection in the efficiency curves was not observed, although the fine inflection was present, shown in Figure 23. The second set of experiments, with very little overlap between the magnetite

and quartz size distributions, showed a coarse inflection that varied with percent solids, being very visible at 5.8% solids but disappearing at greater than 21% solids, as can be seen in Figure 24. This coarse inflection was clearly visible in all three of the triplicate samples in each of the experiments where it appeared, and was significantly larger than the random variations between triplicates, indicating that it is a real effect and not due to random errors. In these experiments, it can be seen that as the percent solids increased, the d50 also increased as would be expected from Equation 3. This caused the efficiency curve to sweep through the size where the feed changes from being entirely quartz to being entirely magnetite. As a result, we see the inflection occurring near the top of the curve at the lowest percent solids, and the inflection moves down the curve as the percent solids increases and the d50 size coarsens. It is interesting to note that the coarse inflection is most easily visible when it is near the top of the curve. However, it is difficult to distinguish at all when it is near the middle or bottom of the curve, as it simply appears to be a decrease in the sharpness of separation.

Figure 21: Efficiency curves for pure silica as a function of percent solids at a fixed flowrate (110 l/min). At each percent solids, samples were collected in triplicate and analyzed independently to ensure reproducible results.

Figure 22: Efficiency curves for pure magnetite as a function of flowrate. Flowrate was varied from 109 l/m to 202 l/m, at a constant percent solids of 5.5%. The fine inflection for pure magnetite was significantly affected by flowrate. At each flowrate, samples were collected in triplicate and analyzed independently to ensure reproducible results.

The fine inflection can also be seen in the results shown in both Figure 23 and Figure 24. It is very interesting to note that, even though all of the material producing the fine inflection in these experiments is magnetite, the behavior is much more similar to that of the pure quartz curve (Figure 21) than the pure magnetite curve (Figure 22). This shows that the magnitude of the fine inflection is controlled by the nature of the coarse particles, not by the nature of the fine particles. This is not consistent with the entrainment model for the fine inflection (Finch, 1983), as in that case the fine inflection would be controlled entirely by the characteristics of the fine particles.

Figure 23: Representative efficiency curves for the first set of mixed-mineral experiments. Inlet pressure was 12 psi, providing a flowrate of approximately 200 l/m, and the percent solids was varied as indicated. For each experiment, samples were collected and analyzed in triplicate to ensure reproducible results. The coarse inflection is difficult to see because of the relatively large overlap in the quartz and magnetite size distributions, but the fine inflection is prominent.

The behavior of the fine inflection is also not entirely consistent with the hydrodynamic model (Neesse et al., 2004), as this model would not predict such a large alteration in behavior simply by changing the density of the coarse particles. The result is much more consistent with agglomeration of fine particles onto the coarse surfaces. In this particular system, there is potential for magnetic agglomeration of fine magnetite particles onto the coarse magnetite, which would account for the much higher value of the fine inflection for pure magnetite than for pure quartz. When there is no coarse magnetite present to carry the fine magnetite particles, but only non-magnetic quartz particles, then the behavior of the fine magnetite becomes much more similar to that of the fine quartz.

Figure 24: Variations in efficiency curves for a quartz/magnetite mixture with very little overlap between the magnetite and quartz size distributions, as the percent solids in the cyclone feed changes. For each experiment, samples were collected and analyzed in triplicate to ensure reproducible results. The coarse inflection is only easily seen at the two lowest percent solids. The increased quantity of magnetite in the fine fraction also resulted in a smaller fine inflection than was seen in the experimental results shown in Figure 23.

Overgrinding Analysis and Simulations

An additional phenomenon that was noted with the mixed-mineral experiments is that, when the magnetite was made finer and the quantity of material in the particle size range from 1-5 micrometers increased, the magnitude of the fine inflection was reduced. This can be seen by comparing Figure 23 (which had a smaller amount of material in the fine fraction and exhibited a large fine inflection) with Figure 24 (which had a larger amount of material in the fine fraction and exhibited a small fine inflection). This indicates that only a portion of the fine material can follow the fine inflection, with the remainder behaving as would be predicted by conventional classification theory. If the fine inflection is being caused by the fine particles agglomerating onto the coarser particles, then there is a limited amount of fine material that can be recovered before all of the available coarse particle surface area is used up. Similarly, if the fine inflection is caused

by fine particles being carried in the wakes of the coarse particles, there is a limited amount of wake volume available to carry the particles. As a result, the fine inflection is only significant when there is relatively little material present in the finest size range. As the material in the finest sizes increases, the behavior of the fines that are not being transported by coarse particles swamps the effects of the fines being carried by the coarse particles, and the fine inflection becomes negligible. It is interesting to note that, in the results reported here, even when there is a very large fine inflection, the size distributions shown in Figures 9 and 10 show that less than 10% of the total weight of the feed was affected, and so the fine inflection has a minimal effect on the actual composition of the bulk cyclone products.

The results for the pebble mill circuit were as shown in Table 8. It should be noted that the -25 μm fraction of the cyclone underflow/mill feed is 30.8% of the material being returned to the pebble mill for grinding, even though it is already finer than the target grind size of 25 μm . Further, this fraction of the pebble mill feed is unusually enriched in magnetite, and consists of 92.9% magnetic material. From this information, it is clear that a great deal of the pebble mill feed consists of fine magnetite that is being retained in the circuit by the hydrocyclones, and as a result is being overground. This is a very large contribution to the circulating load, which is 250% of the circuit new feed.

Table 8: Results of plant sampling. “Overall” percentages are the percent of each size fraction in the stream, with the sum of + 25 μm and -25 μm values equaling 100%. “Magnetic” and “Nonmagnetic” percentages are the assays for each size fraction.

Stream	Size fraction	Overall	Magnetic	Nonmagnetic
Circuit New Feed (CNF)– 120.6 lt/hr	+ 25 μm	87.9 lt/hr 72.9%	45.4 lt/hr 51.7%	42.5 lt/hr 48.3%
	- 25 μm	32.7 lt/hr 27.1%	28.6 lt/hr 87.4%	4.1 lt/hr 12.6%
Cyclone U'Flow/Mill Feed (CUF) – 299.3 lt/hr	+ 25 μm	207.1 lt/hr 69.2%	115.8 lt/hr 55.9%	91.3 lt/hr 44.1%
	- 25 μm	92.2 lt/hr 30.8%	85.6 lt/hr 92.9%	6.6 lt/hr 7.1%
Pebble Mill Discharge (PMD) – 306.9 lt/hr	+ 25 μm	139.0 lt/hr 45.3%	76.6 lt/hr 55.1%	62.4 lt/hr 44.9%
	- 25 μm	167.9 lt/hr 54.7%	107.1 lt/hr 63.8%	60.8 lt/hr 36.2%
Cyclone O'flow/ Circuit Product (COF) – 128.1 lt/hr	+ 25 μm	16.1 lt/hr 12.6%	7.0 lt/hr 43.5%	9.1 lt/hr 56.5%
	- 25 μm	112.0 lt/hr 87.4%	66.5 lt/hr 59.4%	45.5 lt/hr 40.6%

The Bond equation can be used to estimate the energy wasted in overgrinding the fine magnetite. The target size was 80% passing 25 μm , but size analysis of the magnetite product from the circuit showed that it was 80% passing 20 μm . Given that the average work index of the ore was 11.6 kW-hr/ton, the energy wasted can be calculated as

$$W = 11.6 \left(\frac{10}{\sqrt{20}} - \frac{10}{\sqrt{25}} \right) = 2.74 \text{ kW-hr/ton} \quad (1)$$

Since the overall circuit product was 57.4% magnetite, the energy wasted per ton of total circuit feed was $(0.57)(2.74) = 1.57 \text{ kW-hr/ton}$. The plant grinds approximately 15 million tons of ore per year, and so the overgrinding losses from this source are 23 million kW-hr/year, or $2.4 \times 10^{11} \text{ BTU/year}$ for a single plant.

These results show that there is a significant benefit to be gained if the plant can be redesigned to prevent overgrinding. This is an ideal situation for simulation studies, as it is necessary to consider radical rearrangements of the circuit which would not be feasible to conduct on a speculative basis in actual plant studies.

Base Circuit Simulations

In order to simulate the pebble mill portion of the grinding circuit, a pebble mill model and hydrocyclone model were needed. Both of these models were implemented using USIM-PAC 3.0.

The pebble mill model used was based on the following mill characteristics:

• Number of mills in parallel	1
• Mill diameter inside shell (m)	4
• Length/diameter ratio	2.1
• Fraction of critical speed	0.8785
• Mill discharge	Overflow
• Filling of the mill (%)	43
• Reference size for the wear function (mm)	15.875
• Wear coefficient (0=surface, 1=volume)	0
• Wear rate of pebbles (1/h)	3.45
• Reference size class for the selection function	10

The forms of the selection and breakage functions used in the model were based on work by Austin and Herbst (BRGM, 2004; Austin et al., 1987; Kinneberg and Herbst, 1984);

Breakage function:

$$B_{ij} = \Phi(x_{i-1}/x_j)^\gamma + (1 - \Phi)(x_{i-1}/x_j)^\beta \quad (2)$$

Where B_{ij} is the fraction of the mass of broken particles from size fraction i that reports to size fraction j ; and x_i is the top size limit of size fraction i ;

The breakage function parameters that were found to give the best fit to plant data were:

- Φ 0.096
- β 3.93
- γ 0.608

Selection function:

$$S_i = S_1^E \exp[a_1 \ln(d_i/d_{i(ref)}) + a_2 (\ln(d_i/d_{i(ref)}))^2] \quad (3)$$

Where S_i is the fraction of particles in size fraction i that are broken; d_i is the geometric mean particle diameter of size fraction i ; and $d_{i(ref)}$ is the reference particle size class

The selection function parameters that were found to give the best fit to plant data were:

- S_1^E 0.75
- a_1 -1.5
- a_2 -0.5

The simulations did not incorporate a liberation model. The proportions of magnetite and quartz by size were determined by chemical analysis of sieve fractions collected from the plant, and it was assumed that, since the target grind size was being held constant, the degree of liberation at each size would also be essentially constant and would therefore not have a major effect on the predictions of the model.

Using these parameters to simulate the pebble mill, the results shown in Figure 25 were obtained, which shows an excellent match between the predicted and measured mill discharge. Several different sets of data from the same plant at different flowrates were used to validate the model parameters, with similar results.

Figure 25: Simulation results for the pebble mill model as compared with actual grinding mill performance.

The hydrocyclone model used was that developed by Plitt (1976), which performed adequately in predicting the cyclone performance in this circuit.

$$C(d) = 1 - e^{-0.693 \left(\frac{d}{d_{50c}} \right)^m} \quad (4)$$

Where $C(d)$ = classification function for particles of size d after correcting for particles that bypass classification; d = particle size in micrometers; d_{50c} = particle size that has equal probability of reporting to the overflow or the underflow, after correcting for the particles that bypass classification; and m = measure of the sharpness of separation.

The corrected d_{50} size was calculated using the formula:

$$d_{50c} = \frac{50.5 D_c^{0.46} D_i^{0.6} D_o^{1.21} \exp(0.063\phi)}{D_u^{0.71} h^{0.38} Q^{0.45} (\rho_s - \rho)^{0.5}} \quad (5)$$

where d_{50c} = corrected d_{50} (μm); ϕ = volumetric fraction of solids in feed; D_c = cyclone diameter (cm); h = free vortex height (cm); D_i = inlet diameter (cm); Q = volumetric flowrate of feed (L/min); D_o = overflow diameter (cm); ρ_s = solid density (g/cm^3); D_u = underflow diameter (cm); ρ = liquid density (g/cm^3)

Simulations of the overall pebble mill/hydrocyclone circuit reproduced the tendency of the circuit to retain fine magnetite in the pebble mill feed, as can be seen in Figure 26. Here, it can be seen that the non-magnetic material was much coarser than the magnetics, and the overall size distribution of the mill feed was dominated by the presence of fine magnetite that was being returned to the mill and overground.

Figure 26: Size distributions of the magnetic and non-magnetic material in the hydrocyclone underflow/mill feed, as calculated by simulations. These results were consistent with the observed performance of the grinding circuit.

It is particularly noteworthy that the magnetics are concentrated in the size range of 25 to 50 μm . If this size fraction could be segregated into a separate process stream to be managed separately, it would allow the particles coarser than 50 μm to be ground without excessive production of overground material.

Modified Circuit Simulations

The results for the unmodified circuit suggested a novel flowsheet to deal with the problem of overgrinding the magnetite fines, shown in Figure 27. If the particles that are near the target size are separated from the rest of the pebble mill feed by using two-stage cycloning, then they will be prevented from being trapped in the circulating load, allowing the first pebble mill to concentrate on grinding the particles that are definitely coarse enough to require vigorous grinding.

Two-stage classification has been used in the past to improve the performance of classifiers (Dahlstrom and Kam, 1987; Hukki et al., 1977). The most effective

applications of two-stage classification have been in cases where the bypass fraction of a single-stage classifier is large, allowing a large quantity of the fine particles to bypass classification and be returned to the grinding circuit. With a two-stage cyclone circuit of proper configuration and water additions, the fine-particle bypass can be decreased, resulting in lower circulating loads and the possibility of higher feed capacity (Peterson and Herbst, 1984). However, this approach is of little effect if the hydrocyclones are already being operated with a very low bypass fraction, as is often the case in iron ore concentrators (Weldum, 2003).

When fine, high-density particles are in fact undergoing classification, and not bypassing classification, they are then classified into the coarse product due to their density. Since the settling rate of fine, dense particles is the same as that of coarser, less dense particles, the fine magnetite tends to concentrate in the hydrocyclone underflow as a direct result of correct operation of the hydrocyclone. In this situation, the reduction of the bypass fraction will not cause these particles to be removed from the coarse product, and an attempt to use two-stage classification will simply result in a stream of trapped, narrowly-sized high-density particles continuously circulating between the two hydrocyclone classifiers. This stream of near-size particles then continuously increases in volume until the hydrocyclones become overloaded (Weldum, 2003).

If the near-size particles can be separated as a sufficiently narrow size distribution, the possibility arises of grinding this stream from two-stage cycloning in an open-circuit mill. This would eliminate the possibility of near-size particles being trapped in the circulating load and overground. Normally, open-circuit grinding will produce a broad size distribution due to the lack of particle size control (Kinneberg and Herbst, 1984). However, it is also normal for the feed to an open-circuit mill to itself have a broad size distribution, which would broaden still further during open-circuit grinding.

In previous work in this plant (Weldum, 2003), it had been determined that the use of two-stage cycloning could produce one stream with a very narrow size distribution. If such a closely-sized feed is ground in open circuit, the size distribution would be expected to remain moderately narrow. In addition, it is normal for the coarsest particles to have a higher probability of breaking than the finer particles in a tumbling-media mill (Teke et al., 2002), and so a pass through an open-circuit mill would be expected to preferentially break the coarsest particles.

Based on these considerations, a new circuit was developed and simulated as shown in Figure 27. The primary considerations in this circuit were:

- Use 2-stage cycloning to produce three product streams: (1) Coarse particles for grinding in a closed-circuit mill; (2) Fine particles that are definitely fine enough to be removed from the grinding circuit as product; and (3) Near-size particles that are only slightly coarser than the target product size, and which have an extremely narrow size distribution.

- Use open-circuit grinding to reduce the top size of the near-size particle stream. Since material only makes a single pass through this mill, there is no opportunity to continuously recycle and regrind high-density particles. The narrow size distribution of the near-size particle stream will allow the open-circuit mill to produce a moderately closely-sized product.
- As far as possible, use only equipment that is already present in the existing circuit. By reconfiguring the mills and existing cyclones, it would be possible to implement the circuit shown in Figure 27 directly from the circuit shown in Figure 7, with the only major equipment addition being the dewatering cobber added to remove excess water from the open-circuit mill feed.

Figure 27. Modified circuit that was produced from the circuit shown in Figure 1. This is largely a rearrangement of existing equipment, with the only major added unit being the dewatering cobber for the open circuit pebble mill.

Since the product size of an open circuit mill is quite sensitive to the feedrate (Teke et al., 2002), it was first necessary to ensure that an appropriate amount of material was sent to this mill by the two hydrocyclones. It was determined that, in order to produce the target feed size distribution, the open circuit mill needed to receive 81 tons/hr of feed. In order to produce this quantity of material, the hydrocyclones were operated with the efficiency curves shown in Figure 28.

Figure 28: Efficiency curves for the two-stage cyclones used in the combined closed circuit/open circuit flowsheet, including a comparison with the efficiency curve for the single stage of cycloning used in the original, unmodified circuit. The close spacing between the efficiency curves ensured a very sharp size distribution in the feed to the open-circuit mill, while still providing the 81 tons/hr of solids needed to produce the desired size distribution.

The two mill feed products produced by the two-stage cyclone processing are shown in Figure 29. The feed to the closed-circuit mill is much coarser than it had been for the unmodified circuit. In addition, the difference between the magnetite and quartz size distributions is greatly reduced, so that there is much less tendency to overgrind the magnetite while reducing the size of the nonmagnetic quartz. For the feed to the open-circuit mill, the hydrocyclones are successfully producing a very narrowly-sized stream where the size distributions for the magnetite and quartz are very similar. Again, this similarity of the size distribution will help to prevent overgrinding of the magnetite.

Two products are produced by the two-stage circuit: a primary cyclone overflow stream, and an open circuit mill discharge stream. The size distributions predicted by the simulation for these streams are shown in Figure 30, along with the sum of the two product streams, and the size distribution of the final product of the original single-stage circuit for comparison. The open circuit mill discharge stream can be seen to be maintaining a narrow size distribution, due to the fact that the mill feed consisted of closely-sized particles. It should be noted that the size distribution of the open circuit mill feed was considerably finer than the plant feeds that were originally available for used in determining and validating the model parameters. The model is therefore not fully

validated for the open circuit mill conditions, and it is possible that the predicted size distribution from the simulation will not be identical to the actual size distribution that would be produced in the plant. However, since there is no circulating load from the open-circuit mill, any errors from this portion of the circuit simulation will not propagate back to the rest of the circuit.

Figure 29. Size distributions of the feed streams produced by the two-stage cyclone arrangement for the Closed Circuit (CC) mill and the Open Circuit (OC) mill shown in Figure 27.

When the two products from the two-stage mill are added together, the size distribution predicted is nearly identical to that produced by the original single-stage grinding circuit. This indicates that the two-stage grinding circuit is fully capable of reaching the target grind size. The main benefit is seen when the relative flowrates of material through the circuit are compared, as shown in Table 9. The use of the two-stage circuit allows the circulating load to be drastically reduced, from a 253% circulating load to only 42.5%. This makes available a great deal of extra hydrocyclone and grinding mill capacity, allowing the capacity of the circuit to be increased by 50%. This reduction in circulating load is a direct result of the large reduction in the recirculation and overgrinding of the fine, high-density magnetite particles. It should be noted that the “Circulating Load” and “Primary Cyclone Feed” streams for the two-stage-circuit contained, respectively, only 66.0% and 64.5% magnetite, compared to 89.2% and 82.0% magnetite for the corresponding streams in the single-stage closed grinding circuit. This shows that the

two-stage open/closed grinding circuit was removing the near-size magnetite from the circulating load and diverting it to the open circuit mill feed for removal from the circuit.

Figure 30. Size distributions for the two products produced by the two-stage open/closed circuit grinding simulation, compared with the product from the original single-stage closed-circuit grinding process.

Table 9: Solids flowrates and percent magnetite of process streams in the simulated two-stage open/closed grinding circuit, as compared to the two single-stage closed grinding circuits that it would replace.

Process Stream	Pair of Single-Stage Closed Grinding Circuits	Two-Stage Open/Closed Grinding Circuit
Circuit Feed	240 t/h solids 63.9% magnetite	360 t/h solids 63.9% magnetite
Circulating Load	606 t/h solids (253%) 89.2% magnetite	153 t/h solids (42.5%) 66.0% magnetite
Primary Cyclone Feed	846 t/h solids 82.0% magnetite	513 t/h solids 64.5% magnetite
Open Circuit Mill Feed	--	81 t/h solids 86.5% magnetite
Primary Cyclone Overflow	240 t/h solids 63.8% magnetite	273 t/h solids 58.4% magnetite

High Pressure Grinding Roll Simulations

High pressure grinding rolls (HPGR) are a recent development in comminution that have the potential to significantly improve comminution energy efficiency. These units were first applied to grinding cement clinker in the cement industry, where they were found to increase overall grinding circuit energy efficiency by 20-30% when they were used in conjunction with ball mills. It is estimated that an HPGR operating alone is 40-50% more energy-efficient than a ball mill grinding to the same particle size (McIvor, 1997).

Figure 31. Implementation of a high pressure grinding roll (HPGR) in the primary grinding circuit of an iron ore processing plant.

HPGRs are well suited for inclusion into comminution circuits to process material that is troublesome to process using other types of mills. For example, an HPGR can be included in a primary autogenous grinding circuit to assist in reducing the critical size material, as shown in Figure 31. The critical size consists of ore lumps that are too large to be easily broken down in the primary mill, and too small to be effective grinding media for the feed sizes being processed in the portion of the circuit. The traditional solution to critical size material consists of screening out the critical size, and using a cone crusher to reduce it to a fine enough size that the primary autogenous mill can finish grinding it. If the second stage of grinding is a pebble mill, then a portion of the critical size particles can be used as grinding media for the fine-size particles entering the pebble mill.

Figure 32: Implementation of an HPGR in an autogenous grinding circuit for iron ore.

The crushing of the critical size material provides a suitable location for introducing an HPGR into the grinding circuit. In the plant under study, an HPGR was installed to grind the critical size particles from three primary autogenous mills, as shown in Figure 32. This was judged to be the configuration that would give the plant the greatest benefit from a single HPGR unit.

Operating data for the HPGR in the plant was provided for this project by plant personnel, and was used to create and validate a mathematical model of the circuit in USIM-PAC.

The model simulations of the primary mill circuit incorporating the HPGR produced very good agreement with actual plant performance, as can be seen in Figure 33.

Figure 33: Simulated and observed primary mill feed and primary mill discharge in a circuit incorporating a high pressure grinding roll.

Figure 34: Variations in primary mill feed and primary mill discharge size distributions observed in the plant as the fraction of critical size passed through the HPGR was increased.

Figure 34 shows how the performance of the grinding circuit changed as the plant altered the fraction of the critical-size material that was ground using the HPGR. The plant was operating scheme would increase the amount of material to the HPGR as the ore became

harder to grind, which is why the throughput was lower at 100% HPGR use than at 0% HPGR use.

The immediate effect of sending more of the critical size material to the HPGR was that the primary mill feed became finer, since the HPGR was pre-grinding the product from the cone crusher before it was recycled to the primary mill. This resulted in the primary mill product also becoming finer. With more fines being produced by the primary mill at approximately the same flowrate, this corresponds to increased energy efficiency by the mill.

Conclusions

The distribution of load in a grinding circuit will shift as the ore characteristics change. In the circuit studied, it appears that increasing ore work index tends to increase load on the primary mills more than it increases load on the secondary mills, so the circuit has three distinct operating regimes: primary-mill limited for hard ores, pebble-mill limited for softer ores, and a mixed condition for intermediate ores. It is therefore important to know the characteristics of the incoming ore so that the circuit can be operated accordingly.

In order to maintain the maximum throughput through the circuit, it is important to use all of the available capacity in each comminution stage. To do this, it is necessary to have a method for shifting load between the various grinding unit operations, so that load can be taken off of whichever operation is currently the bottleneck. This can be effectively done by at least two methods: 1) Selectively removing particle sizes that are difficult for a given stage to process, and using a different comminution technique to process it. An example of this is the use of high-pressure grinding rolls to deal with critical-size material from an autogenous mill. 2) Adjustment of screening to recirculate a portion of the feed at the fine end of the circuit back to the coarse end. This is particularly advantageous if there is an intermediate separation step between stages of grinding, so that the load on the screen can be reduced. Analysis of historical data for the plant examined showed that the use of intermediate screening to shift grinding load could have been used at least 58% of the time to increase the overall throughput of the circuit.

There are two distinct phenomena that lead to inflections in hydrocyclone efficiency curves. The coarse inflection is caused by the presence of fine, high-density particles combined with coarser, low-density particles in the cyclone feed. This results in the efficiency curve being dominated by low-density materials at the coarser sizes and high-density materials at the finer sizes, with the inflection occurring when the curve switches from one to the other. In a comminution circuit, such an inflection is an indicator that high-density particles are being preferentially retained in the grinding circuit, and are being overground. Since overgrinding is a significant waste of energy and can cause losses of valuable minerals, the appearance of a coarse inflection in a hydrocyclone efficiency curve in a plant is a sign that there is a problem that needs to be corrected.

The fine inflection, on the other hand, has been explained by a number of theories, many of which do not fully account for the behavior seen in the results reported here. The observed behavior is most consistent with an agglomeration mechanism, where fine particles agglomerate onto coarse particles and are carried into the cyclone underflow. This mechanism can only carry a limited quantity of fines, and so when there are very large quantities of fine particles present, the majority of them follow the theoretical curve because there is insufficient coarse particle surface to carry them all. As a result, the fine inflection does not appear likely to be of much industrial significance, as it is only clearly seen in cases where there is very little of the cyclone feed at the affected sizes.

Overgrinding of valuable minerals is often caused by the tendency of hydrocyclones to retain dense minerals in a closed grinding circuit until they are ground to an excessively fine size. This is a particular problem for iron ore concentrators, where a large fraction of the mass of the ore consists of the higher-density iron oxide minerals. The classical solution to this problem was to use screens for product size control rather than classifiers, but this is not practical when the target size is very fine due to limited screen capacity and high maintenance costs.

Simulations of a magnetite ore grinding circuit indicated that a re-configuration of the circuit can greatly reduce the overgrinding problem. The use of two-stage hydrocycloning can concentrate the near-size high-density magnetite particles into a closely-sized single stream that requires only a very small amount of grinding to reach the target size. Open-circuit grinding of this stream was predicted to preferentially grind the coarsest particles, leaving a product that has the necessary size distribution to be a finished product. This introduction of open-circuit grinding greatly reduced the circulating load of the grinding circuit (from 253% to only 42.5%), since the fine magnetite was no longer being continuously returned to the mill for repeated regrinding. The simulations indicated that this change increased the circuit capacity by as much as 50% while still making the target grind specification. Since more material is being passed through the mill per unit time, but the mill filling (and therefore mill power draw) is unchanged, this corresponds to a 50% improvement in energy efficiency. Applying this approach throughout the entire midwestern iron ore industry will result in a total savings of nearly 1×10^{13} BTU/year.

The introduction of high pressure grinding rolls (HPGR) to assist the primary mill also has significant energy benefits. The use of the HPGR allowed the primary mill throughput to be maintained even as the ore became more difficult to grind, and resulted in a finer primary mill product. This corresponded to an increase in energy efficiency, since the primary mill was carrying out more grinding with essentially the same energy input.

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