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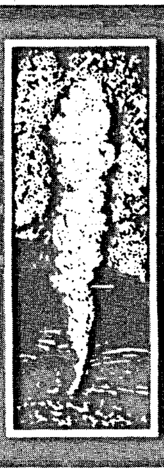

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Application of the Spread-Spectrum Technique in Well Logging

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Abstract—This paper presents the novel concept of employing the noise insensitive spread-spectrum technique in well logging. The proposed design of a spread-spectrum device improves the performance of well logging tools, particularly within highly noisy environments. The heart of the device is a shift register which generates a pseudorandom binary code sequence. A coder is connected to the transmitter and codes the probing signal by utilizing the pseudorandom sequence. A decoder is connected to the receiver and correlates the return signal to the same sequence, which is used as a sliding reference. Shifts as small as a fraction of a bit are unambiguously resolvable, and distance resolution of the order of micrometers is achievable. Spread-spectrum well logging tools can operate even with coded signal-to-noise ratio below zero-dB. The spread-spectrum device can be interfaced with any available wave transmitting logging tool. However, tools employing acoustic waves are favorable because the acoustic wave propagation velocity is low and allows the use of inexpensive electronics. The problems associated with high temperatures which are commonly encountered in geothermal reservoirs are bypassed, since the spread-spectrum device can be located either inside the well logging tool or together with the supporting electronics on the surface.

Introduction

Many well logging techniques utilize electromagnetic, acoustic or optical waves. Acoustic waves are advantageous because their propagation velocity is five to six orders of magnitude lower than that of electromagnetic or optical waves. The time intervals involved in acoustic waves require slow or moderately fast processing electronics. Therefore, the cost is lower, the reliability is higher, and complicated real-time data processing is possible. The methods described in this treatment apply to all wave techniques, but the examples as well as the references are drawn from the acoustic well logging field.

Acoustic logging is a continuous record of the amplitude and time required for an elastic wave to travel a given distance of subsurface formation immediately adjacent to the borehole versus depth. Since rock formation is not ideally elastic, part of the energy of an elastic wave is absorbed and dispersed in the rocks. Absorption occurs due to the mutual friction of rock particles and dispersion due to the heterogeneity of the rock.

Waves are characterized by two basic parameters, amplitude and frequency. The first parameter is related to attenuation and wave shape, while the second is related to phase and timing. Since the velocity and attenuation of sound waves in a reservoir depend on the formation properties, acoustic data may be used to calculate, detect or estimate among others the porosity, lithology, synthetic seismograms, fractures, borehole ellipticity, as well as casing defects and perforations.

Acoustic logging is usually carried out with the transmission and reflection procedures. In the transmission method, a transmitter emits acoustic energy, and part of this energy is guided by the formation or casing and is sensed by one or several receivers. In the reflection method a transducer emits acoustic energy that is beamed horizontally toward the borehole wall. Part of this energy is reflected back and sensed by the same transducer.

This paper introduces the conceptual design of a well logging devise utilizing the noise insensitive spread spectrum technique, and summarizes some acoustic well logging tools which can utilize such technique. The theoretical background of spread spectrum systems is presented in a non-mathematical fashion, so that complex and sophisticated derivations can be avoided. The advantages of the spread-spectrum technique and expected performance of the improved well logging tools are also discussed.

Common Acoustic Logging Tools

Tools available for acoustic logging include among others the single-transmitter sonde, the borehole compensated system, the long-spacing sonic, the circumferential sonde, and the borehole televiewer. The continuous velocity sonde contains one large transmitter and one or two receivers (Summers and Broding, 1952). Consequently, these tools are known as single or dual receiver systems. The transmitter generates an acoustic wave which travels through the borehole fluid to the formation. At the borehole wall the transmitted wave gives rise to compressional, shear and fluid waves which return to the receiver following different pathways. A *single receiver system* measures the time required for the wave to travel from the transmitter to the receiver. Obviously, the total time measured by the system exceeds the desired value by the amount equal to the travel time in the borehole fluid. The *dual receiver system* measures the time required for the wave to travel the distance between receivers. This method eliminates the effect of the travel time in the borehole fluid, provided that the time intervals from the two receivers to the formation are equal. The technique can give incorrect results if the tool is tilted in the hole, or if the receivers are staggered across a cave boundary.

To overcome the problems of the single transmitter sondes, the *borehole compensated acoustic tool* was developed (Kokesh et al., 1965). This tool consists of two transmitter-receiver arrays, one inverted relative to the other. Two values of acoustic wave travel time are measured and averaged. The result of this averaging technique is the elimination of incorrect measurements. Typically, a compensated acoustic tool has a transmitter-receiver spacing of 3-ft and receiver-receiver spacing of 2-ft.

The *long-spacing sonic tool* consists of two transmitters spaced 2-ft apart at the bottom of the tool, and two receivers spaced 2-ft apart at the top with 8-ft separation between the nearest transmitter and receiver. The long transmitter-receiver spacing is required in order to obtain wave travel time measurements from the formation further from the borehole, to allow for adequate time separation between the various arrivals, as well as to achieve a good signal-to-noise ratio and minimum signal distortion. This tool records the entire received wave-form, not only the first arrival, from which compressional, shear and fluid arrivals can be distinguished and separated. To assure correct well logging the tool's internal bias level must be set such as to detect the difference between noise and first wave arrival time. The ability to separate compressional and shear wave velocities is very useful in lithological formation investigations (Pickett, 1963).

The *Circumferential sonde* designed by Vogel and Worrell (1977) is a device employing ultrasonic transmitters and receivers mounted on four wall-contacting arms placed 90° apart along the circumference of the borehole wall. The tool is approximately 8.5-ft long and 4-in in diameter when the arms are closed. The transmitters and receivers are cylindrical in shape, perpendicularly fixed to the tool axis. It is reported that the tool can be used for vertical fracture detection because the transmitted shear and guided fluid waves are impaired by open fracture of sufficient width (Vogel and Herolz, 1981).

The *Borehole Televiewer* is a logging tool that records acoustic waves reflected from the borehole wall (Zemanek et al., 1969). The tool contains a piezoelectric transducer that acts as both the transmitter and receiver. The transducer is rotated at 3 revolutions per second by a motor, and it probes the borewall with pulses 500 times each revolution. A magnetic compass is used to generate a trigger pulse that sets the direction reference. The amplitude of the reflected acoustic signal is displayed as a gray-scale image of the borehole wall. A smooth surface reflects better than a rough one, and a hard surface produces larger reflections than a soft. Therefore, fractures and other rough parts of the wall are displayed as dark lines. If the televiewer is off center or tilted in a circular borehole, a low amplitude is recorded and images exhibit two vertical stripes. If the borehole is elliptical, acoustic waves are reflected away from the tool and only reflections from the major and minor axes are recorded. These stripes may be interpreted as a fractured zone, or can make the precise fracture identification difficult (Georgi, 1985).

Recent advances have increased the interest of the borehole televiewer utilization in well logging applications. An acoustic borehole televiewer for operation in geothermal environments with temperatures up to 280°C has also been developed (Heard and Bauman, 1983). Pasternack and Goodwill (1983) have shown that digital borehole televiewer data provide more informative images than analog televiewer images. Taylor (1983) has developed a method for recording not only a picture of amplitude versus azimuth and depth, but also a picture showing the arrival time of the reflected wave. Rambow (1984) reports that combining amplitude and travel time improves the data interpretation. Although the borehole televiewer has been used extensively in several investigations of hydraulic fracturing stress and other applications (Zoback et al., 1985; Stock at al., 1985), not all of the information provided by the tool is utilized to full advantage (Paillet et al., 1985).

Instrumentation capable of operating sufficiently in the hostile geothermal environments had not been available until quite recently. Some of the advances in geothermal instrumentation are presented by Adorni et al. (1985). The existing acoustic well logging tools emit proper signals, record the associated distorted reflections and apply a series of processing techniques to clean the received signals and extract the desired information. The complexity of the return signal and the low signal-to-noise ratio make the processing difficult and the final results partially reliable. Recently, digital techniques improved the signal processing but there are still inaccuracies inherent to the process of signal sensing.

Spread Spectrum Signal Coding

A fundamentally different approach of well logging tool operation is to transmit a series of pulses instead of one pulse at a time. A set of such pulses forms a digital codeword with internal structure. Consequently, a coded signal is transmitted and the received word-echoes are compared to the transmitted word. The observed differences between the sent and received signals are indicative of the properties of the medium in which the signal propagates. The codeword structure makes it possible for the tool to operate at low signal-to-noise ratio, and still have good performance. Therefore, for a given signal-to-noise ratio the tool response is by far superior when coded signals are used.

There are many digital coding techniques (Blahut, 1983; Wozenkraft and Jacobs, 1965), but the most appropriate for well logging applications is the *spread-spectrum.* This coding technique maps a single binary pulse onto a long string of pulses which constitute the code sequence. The two basic properties of code sequences are the syntax of ones and zeroes and the shape of their correlation (auto-correlation and cross-correlation) functions (Dixon, 1984). A shift register with ℓ-stages generates sequences of variable length depending on the feedback connections. In sequences with maximum length, $(2^\ell - 1)$ bits, the amounts of ones and zeroes differ only by one. The positions where these two values appear in the sequences are described by a pseudorandom probability density function, and their correlation has a unique shape. The decoding process in the receiver is based on the correlation between the reference (transmitted) signal and the received signal. The output of the correlator is a signal relatively free of noise, even if the environment of the propagated coded word is very noisy. In fact, spread spectrum techniques can give valuable information even at coded signal-to-noise ratios far below zero-dB. When the code sequence and its identical image are perfectly synchronized, their correlation reaches an absolute maximum. In maximum length sequences a shift more than one bit from the complete overlap reduces the correlation function by a factor $G_p = 2^\ell$, where G_p is the processing gain. Figure 1, shows such a correlation function.

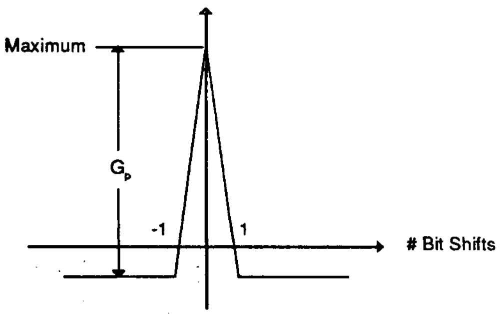

Figure 1. Correlation function of a maximum length code sequence generated by an ℓ-stage shift register.

(a) Noise Insensitivity

The disturbance, $d(t)$, that actually reaches the receiver consists of two parts, namely the signal, $s(t)$, and the random noise, $n(t)$:

$$d(t) = s(t) + n(t). \qquad (1)$$

The receiver performs the detection by correlating the incoming disturbance to the transmitted signal reference. When the relative shift of the signals is k bits, then the correlation R_k in terms of discrete representation is:

$$R_k = \sum_{j=1}^{M} s_j(s_{j-k} + n_{j-k})$$

$$= \sum_{j=1}^{M} s_j s_{j-k} \qquad (2)$$

where M is the number of bits in the sequence. The signal and the random noise are uncorrelated and the $(s_j n_{j-k})$ products are zero. When perfect alignment is achieved the correlation reaches its maximum value of $\sum_{j=1}^{M} s_j^2 = M$. In any detection system a vital parameter is the signal-to-noise ratio. At the receiver input the signal-to-noise power ratio is

$$\left(\frac{S}{N}\right)_{input} = \alpha \quad [dB] \qquad (3)$$

while the signal-to-noise ratio at the correlator output, in terms of the processing gain, is G_p times higher than that of the received coded signal

$$\left(\frac{S}{N}\right)_{output} = \alpha + 10\ell og_{10} G_p \quad [dB]. \qquad (4)$$

The basic trade off is between processing gain and required bandwidth. A processing gain of G_p implies a bandwidth increase of G_p.

Accurate time measurements are accomplished by sliding slowly the reference code sequence until its correlation with the received coded signal is maximum. From the amount of sliding the round trip delay is calculated. The temporal information is combined with the known propagation velocity to calculate the travel distance. Accuracy as high as a small fraction (1/8 to 1/10) of a bit length is easily accomplished (Dixon, 1984). Figure 2, describes the measuring process and shows the shift register. This particular sift register generates a sequence of $2^5 - 1 = 31$ maximum length, and yields a processing gain of 32 or approximately 15dB.

The length of the code sequence used for distance measurements must be such that when the receiver starts to receive back the codeword's leading bits, the transmitter still emits the same sequence. This is accomplished whenever round-trip delay is shorter than the sequence duration. In this way the time reference for the round trip delay is precisely known.

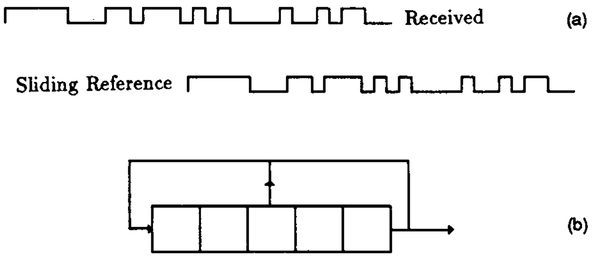

Figure 2. (a) Schematic representation of the time measuring process, (b) Five stage shift register and feedback connections.

The distance resolution measurement is given by the following equation:

$$\Delta s = up\frac{T_b}{2} \qquad (5)$$

where Δs is the distance resolution, u is the wave propagation velocity, p ($p \leq 1$) is the resolvable fraction of a bit length, and T_b is the duration of a single pulse.

It has been shown that systems using spread spectrum techniques achieve high spatial resolution, and can operate with very poor signal-to-noise ratio. This noise immunity results from the decision making process of the electronic detector which is quite independent of the received noise power. In other words, factors such as reflectivity amplitude variation do not degrade the accuracy of the measurement. Spread spectrum decouples amplitude degradations from temporal data, which is very advantageous for spatial measurements.

(b) Multiple Signal Interference Suppression

In typical well logging applications, the transmitted signal gives rise to a multiplicity of returned signals. Compressional, shear, and delayed echoes are mixed with reflections at different interfaces and the compound signal that reaches the receiver becomes highly distorted and degraded by noise. Furthermore, the different amplitudes associated with each one of these signals make their identification difficult. In contrast, the digital spread-spectrum coded signals are insensitive to signal-to-noise ratio fluctuations, and multiple coded signals are easily separable as long as the spacing between consecutive interfaces is longer than the distance resolution limit, Δs.

Signal identification can be achieved either in real time by using more than one time shifted correlators, or by postprocessing of the recorded data. Discrimination of signals can also be obtained by compound sequences, such as the Jet Propulsion Laboratory (JPL)–sequences. These sequences are long, and consist of shorter ones. They have multiple correlation peaks, one for every component–sequence, which are advantageous in signal discrimination and synchronization.

The Conceptual Design

The proposed spread spectrum device consists of two units, the coder and the decoder. The coder is placed before the transmitter and the decoder after the receiver. The coder consists of a local oscillator that provides the carrier, a shift register that generates the code sequence and a modulator that imposes the code on the carrier wave. The output of the coder drives the transmitter that emits the signal wave–form. The decoder consists of a mixer that multiplies the locally generated coded carrier with the incoming signal, a correlator and a bandpass filter that rejects the out of band unwanted signals. The equipment required to build the device are commercially available (Rappaport and Grieco, 1984; Shreve, 1985). The device can be interfaced to the present well logging tools without any major modifications. It is light–weight with characteristic dimensions of a few inches, and possibly can be located either inside the tool or together with the supporting electronics on the surface. Figure 3, shows a block diagram of the spread spectrum device.

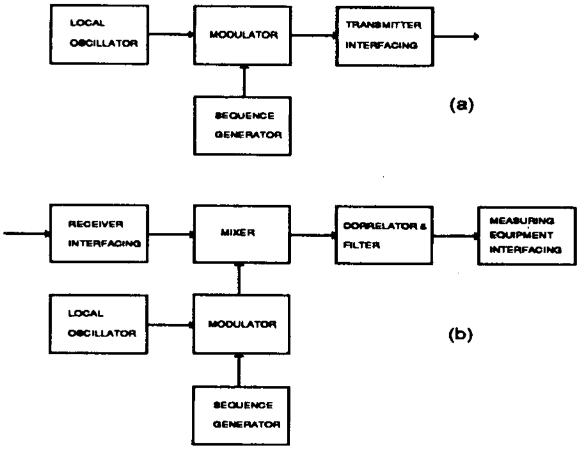

Figure 3. The spread–spectrum device, (a) Transmitter and (b) Receiver.

Discussion

One distinct advantage of well logging tools employing the spread–spectrum device is the ability to operate in noisy environments with excellent performance. Because of this noise immunity, signals can propagate longer distances, experience high attenuations, and still the weak return signal can reveal the carried information. Amplitude and timing information are virtually decoupled and variations of the amplitude do not degrade the tool performance.

Parameters such as formation porosity, fracture locations, and borehole geometry are commonly obtained by the present well logging tools through wave travel distance measurements. The dramatic increase of the distance or time measurement resolution achieved by well logging tools combined with the spread spectrum device leads to much more accurate determination of formation characteristics. A wide variety of devices can be built depending on the requirements. However, for high operating speeds fast electronics and transducers are required, but formation microdetails can be observed.

The ability of the spread spectrum device to discriminate closely spaced or overlapping signals, independently of relative amplitudes, indicates that signals propagated with different velocities or reflected from different surfaces can be resolved with minimum ambiguity. Standard well logging tools may yield inaccurate data measurements when more than one signals are received simultaneously.

Besides accurate temporal information, penatration increase is another advantage of the spread spectrum coding. Since the technique offers a gain of G_p, for a given level of transmitted power, the acoustic wave can travel further and experience higher roundtrip attenuation compared to uncoded waves. Figure 4 shows the penetration advantage when the 5–stage shift register coder discussed previously is used for several typical rocks. Attenuation and velocity data for the calculations of Figure 4 were obtained from Knopoff [1965], and Tittman [1986]. The wave penetration depth increases as the carrier frequency decreases or as the number of shift register stages increases.

If more information is desired from well logging, the device can transmit several code sequences concur-

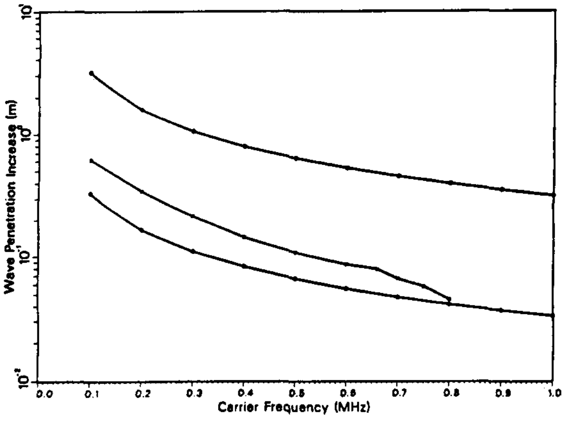

Figure 4. Wave penetration increase versus carrier frequency for (a) Limestone, (b) Westerly Granite, and (c) Sandstone.

rently using only one transmitter–receiver pair and process all of them simultaneously. The cross–correlation function shape of different, suitably selected sequences guarantees the unambiguous recovery of the information carried by each one of them.

Since the information is retrieved from the correlation function of the digital signal any amplitude variation is masked. If amplitude information is necessary, the received signal can be monitored before it enters the correlator by using analog processing techniques. Therefore, amplitude information can be gained from each and every one of the returned pulses.

Conclusions

Well logging can be improved tremendously with the new device utilizing spread spectrum techniques. Overlapping signals are discriminated and signals propagated with different velocities or reflected from different surfaces are resolved accurately. Distance resolution of the order of micrometers is achievable. Parameters related to the device such as shift register length, code sequence length, resolution, dynamic range, and noise immunity can be tailored according to the specific requirements. Constrains imposed by the existing transducers can also be met by proper selection of these parameters. The device can be interfaced with any presently available well logging tool or can be incorporated into any newly designed tool. Operational problems due to high temperatures and pressures can be avoided by locating the spread–spectrum device at the surface together with the other electronics. The cost of the device is only a small fraction of the total cost of a well logging tool, because the necessary components are commercially available and relatively inexpensive. Some of the fields where upgraded well logging tool performance is applicable are: geothermal, oil and gas, lithology, and seismology.

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USE OF DERIVATE FOR DETECTING LINEAR IMPERMEABLE BARRIERS BY TRANSIENT PRESSURE TEST

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ABSTRACT

The goal of this paper is to show a numerical method for evaluate distance at an impermeable barrier, using buildup and drawdown pressure test analysis.

This technical needs of derivates evaluation in transition zone defined between two classics semilog straight lines with slopes "m" and "2m" that are present in these systems.

Complementary to shis study the authors propose type curves for application in alternatives analysis.

An illustrative example is showed using the proposed method.

INTRODUCTION

The identification of heterogeneities in geothermal reservoir takes an important place in evaluation of these. Accurate predictions from simulation works require of true conceptual models and propper caracterizations.

The transient well testing are very useful tools to evaluate the hydraulic parameters of reservoir and identify of heterogeneities of this, for example the impermeable barriers that are the main objective of this paper.

Horner (1951) was the first which pointed out that from buildup well testing is possible to detect impermeable boundaries, he reported that the graphic of Pws vs. log ($t/(t_p+ t)$) shows two straight lines with slopes "m" and "2m".

Davis and Hawkins (1963), Witherspoon et. al. (1976) and Erlougher (1977) gave methods to evaluate the distance to impermeable barriers using the profile of the straight lines. Nevertheless these methods are restricted if the lines are not developed completely.

Gray (1965) established that the difference between the extrapolated pressure data of the first semilog straight line gives a useful method to obtain the distance to barrier and Martinez and Cinco (1983) extended further this method.

The objective of this study to carry out an interpretation algorithm, oriented in the transition zone between one and other semilog straight using the numerical derivates of pressure. The method proposed does not require definition of the second straight line.

BASIC EQUATIONS

The dimensionless pressure drop caused by one well at constant rate producing in an infinite medium, homogeneous and isotropic, can be given by the line source solution of the diffusivity equation and is expressed in terms of exponential integral as:

$$P_D = -0.5Ei(-r_D^2 / (4t_D)) \quad \text{..............(1)}$$

Where the dimensionless variables are defined with conventional form by:

$$P_D = (\rho kh\Delta p)/(\beta W\mu)$$

$$t_D = (\alpha kt)/(\phi \mu c_t r_w^2)$$

$$r_D = r/r_w \quad \text{.............(2)}$$

The effect of one impermeable barrier can be simulated from the images theory, locating one production well at the other side of the barrier and simetric to the real well, this is showed in figure 1. The solution for this system is given by:

$$P_D = -0.5(Ei(-r_{D1}^2/(4t_D)) + Ei(-r_{D2}^2/(4t_D)))\text{...(3)}$$

The r_{D1} and r_{D2} are dimensionless distance between the observation point (x,y) and the real well and image well respectively, and are defined as:

$$r_{D1} = r_1/r_W$$

$$r_{D2} = r_2/r_W \quad \text{................(4)}$$

For the system of figure 1, the following relations are valid:

$$r_{D1}^2 = x_D^2 + y_D^2$$

$$r_{D2}^2 = (2d_D - x_D)^2 + y_D^2 \quad \text{...............(5)}$$