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CONTENTS

	Page
1. PURPOSE.....	3
2. METHOD	3
3. ASSUMPTIONS.....	4
4. USE OF COMPUTER SOFTWARE	5
5. CALCULATION	6
5.1 MATERIAL PROPERTIES	6
5.1.1 Calculations for Fracture Toughness	7
5.1.2 Drip Shield Top Plate and WP Shell Dimensions	8
5.1.3 Calculations for True Measures of Ductility.....	8
5.2 DESIGN METHODOLOGY	9
5.2.1 Comparison of Two Design Approaches	10
5.2.2 Fracture Mechanics of Dynamic Impacts	12
6. RESULTS	13
7. CONCLUSIONS.....	14
8. REFERENCES	14
9. ATTACHMENTS.....	16

1. PURPOSE

The objective of this activity is to show that the use of the traditional strength of materials approach to the drip shield and the waste package (WP) designs is bounding and appropriate when compared to the fracture mechanics approach. The scope of this activity is limited to determining the failure assessment diagrams for the two materials at issue: Ti-7 and Alloy 22. This calculation is intended for use in support of the license application design of the drip shield and the WP. This activity is associated with the drip shield and the WP designs. The activity evaluation for work package number P3212234F2, included in *Technical Work Plan for: Waste Package Design Description for LA* (Ref. 1, p. A-6), has determined that the development of this document is subject to *Quality Assurance Requirements and Description* requirements. The control of the electronic management of data is accomplished in accordance with the methods specified in Reference 1, Section 10. AP-3.12Q, *Design Calculations and Analysis* (Ref. 2), is used to develop and document the calculation.

2. METHOD

A closed-form solution for determination of the failure characteristics of brittle and ductile materials is used to assess the failure mechanisms of Ti-7 and Alloy 22. A quasi-static solution for failure assessment is obtained for both materials. This approach considers three different closed-form solutions: compact specimen, single edge notched bend (SENB) specimen, and single edge notched tension (SENT) specimen. One of the reasons for selecting these three specimen geometries is to use the most realistic geometries with closed form solution that are similar to the design basis event of a rock fall over a drip shield top plate or a WP shell. Similar results are expected for all three specimens, which will also increase the level of confidence in the results of the calculation.

The results of this calculation are reported in terms of the failure assessment diagrams. These diagrams are used to assess the material behavior at failure to ascertain whether the material experiences a brittle fracture, ductile collapse, or a combination of the two. If the failure mode of these two materials is dominated by the ductile collapse, then the use of the traditional strength of materials approach for the failure determination of the drip shield and the WP designs is bounding and appropriate when compared to the fracture mechanics approach.

Originator: Initials: Z.C. Date: 4/30/02
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3. ASSUMPTIONS

In the course of developing this document, assumptions were made regarding the drip shield and the WP structural calculations. These are identified below.

- 3.1 V-notch impact energy and elongation as a function of temperature are not available for either Ti-7 or Alloy 22. Therefore, the room temperature material properties are assumed for these materials. The impact of using such material properties at room temperature is anticipated to be small. The rationale for this assumption is that the V-notch impact energy and, therefore, the fracture toughness tend to increase with increasing temperature. Therefore, the use of the room temperature yields slightly conservative results. An increase in temperature also means reduction in strength of these materials. So, from the strength of materials point of view, the use of the room temperature yields in slightly nonconservative results. From the elongation point of view, the use of room temperature for elongation results in slightly conservative results due to less ductile behavior. Considering the overall effect of these properties, it is reasonable to assert that the impact of using such material properties at room temperature is anticipated to be small. The reason for this assessment is that the change in the material properties in question does not have a dominant impact on the calculation results at the temperature of interest in this calculation. This assumption does not require any further confirmation. This assumption is used in Section 5.1.
- 3.2 Some of the rate-dependent material properties are not available for Ti-7 and Alloy 22. The material properties obtained under the static loading conditions are assumed for all materials. The impact of using material properties obtained under static loading conditions is anticipated to be small. The rationale for this assumption is that the mechanical properties of subject materials do not significantly change at the peak strain rates that occur during the low velocity impacts caused by a rock fall event. Furthermore, ductile fracture of metals is strain controlled; therefore, they often experience an increase in toughness at high strain rates (Ref. 10, p. 211). The use of static properties is, therefore, slightly conservative. This assumption does not require any further confirmation. This assumption is used in Section 5.1.
- 3.3 The chemical compositions of Ti-7 and commercially pure Ti-50A are essentially the same (see Ref. 6, Section II-B, SB-265, Table 2, and Ref. 7, p. 6); therefore, the same V-notch impact energy is used for both material specifications. Furthermore, the lower-end of the typical range of V-notch impact energy of Ti-7 is used as a bounding value [30 ft-lb (at 20 °C) (Ref. 7)] for the purpose of obtaining a bounding result in this calculation. The rationale for this assumption is that the lower values of the V-notch impact energy corresponds to lower values of the material fracture toughness, which results in an increased brittle fracture

Originator: Initials: Z.C. Date: 4/30/02
Checker: Initials: VB Date: 4/30/02

response of the material. Therefore, a bounding result is obtained by this assumption, which does not require any further confirmation. This assumption is used in Section 5.1.1.

- 3.4 V-notch impact energy of Alloy 22 [260 ft-lb (at 20 °C) (Ref. 9, p. 14)] is used in this calculation. The rationale for this assumption is that this nickel-alloy is a material with an extremely high fracture toughness and the fact that the specimen did not break after the test implies that this impact energy value can be used as a lower bound for Alloy 22. Therefore, this V-notch impact energy value is adequate for the purpose of evaluating its failure characteristics. This assumption does not require any further confirmation. This assumption is used in Section 5.1.1.
- 3.5 The uniform strain of Ti-7 is not available in literature. Therefore, it is assumed that the uniform strain is equal to the elongation. The rationale for this assumption is that larger tensile strength results in larger flow stress, and subsequently, slightly conservative results because of a decrease in ductile behavior. This assumption does not require any further confirmation. This assumption is used in Section 5.1.3.
- 3.6 The uniform strain of Alloy 22 is not available in literature. Therefore, it is conservatively assumed that the uniform strain is 90% of the elongation. The rationale for this assumption is the character of stress-strain curve for Alloy 22 (Ref. 5, Attachment 2). This assumption does not require any further confirmation. This assumption is used in Section 5.1.3.
- 3.7 Poisson's ratio of Alloy 22 is not available in literature. Poisson's ratio of Alloy 625 (SB-443 N06625) is assumed for Alloy 22. The impact of this assumption is anticipated to be negligible. The rationale for this assumption is that the chemical compositions of Alloy 22 and Alloy 625 are similar (see Ref. 6 [Sect. II-B, SB-575, Table 1] and Ref. 15, p. 143, respectively). This assumption does not require any further confirmation. This assumption is used in Section 5.1.

4. USE OF COMPUTER SOFTWARE

Section 9 contains computations using the standard functions of a commercial-off-the-shelf software program, Mathcad 2000 Professional. The results of the computations can be reproduced and checked by hand; therefore, the software use is considered exempt from the requirements of AP-SI.1Q (Ref. 3). These computations are performed using Mathcad 2000 Professional on a personal computer. The filenames for each computation are documented in the header for the attachments. Formulas, algorithms, listings of inputs and outputs, and numerical solution techniques are described in comments in the computation files or in the calculation itself, as applicable. The form of the

Originator: Initials: Z.C. Date: 4/30/02
Checker: Initials: VB Date: 4/30/02

computation files is such that the calculations lend themselves to easy verification by visual inspection.

5. CALCULATION

5.1 MATERIAL PROPERTIES

Material properties used in this calculation are listed in this section. Some of the temperature-dependent and rate-dependent material properties are not available for Alloy 22 and Ti-7. Therefore, room temperature material properties obtained under the static loading conditions are used for the subject materials (Assumptions 3.1 and 3.2).

Ti-7 (Titanium Grade 7) (SB-265 R52400, drip shield plate material, Ref. 4, Section 1.2.1.18):

- Yield strength = 276 MPa (40 ksi at 20 °C) (Ref. 6, Section II-D, Table Y-1)
- Tensile strength = 345 MPa (50 ksi at 20 °C) (Ref. 6, Section II-D, Table U)
- Modulus of elasticity = 107 GPa (15.5×10^6 psi at 20 °C) (Ref. 6, Section II-D, Table TM-5)
- Density = 4512 kg/m³ (0.163 lb/in³) (Ref. 6, Section II-D, Table NF-2)
- Elongation = 0.2 (20 °C) (Ref. 6, Section II-B, SB-265, Table 1)
- Poisson's ratio = 0.34 (20 °C) (Ref. 14, p. 621)

Alloy 22 (SB-575 N06022, waste package outer shell material, Ref. 8, Section 1.2.1.4):

- Yield strength = 310 MPa (45 ksi at 20 °C) (Ref. 6, Section II-D, Table Y-1)
- Tensile strength = 690 MPa (100 ksi at 20 °C) (Ref. 6, Section II-D, Table U)
- Modulus of elasticity = 206 GPa (20 °C) (Ref. 9, p. 14)
- Density = 8690 kg/m³ (0.314 lb/in³) (Ref. 6, Section II-B, SB-575, Section 7.1)
- Elongation = 0.45 (20 °C) (Ref. 6, Section II-B, SB-575, Table 4)
- Poisson's ratio = 0.278 (20 °C) (Ref. 15, p. 143) (see Assumption 3.7)

Originator: Initials: Z.C. Date: 4/30/02

Checker: Initials: JB Date: 4/30/02

5.1.1 Calculations for Fracture Toughness

The fracture toughness of Ti-7 and Alloy 22 can be determined using the relation given below:

$$\left(\frac{K_{Ic}}{S_y} \right)^2 = 5 \left(\frac{CVN}{S_y} - 0.05 \right) \quad (\text{Ref. 6, Section VIII, Division 3, Appendix D - Nonmandatory Fracture Mechanics Calculations, Article D-600})$$

where:

K_{Ic} = fracture toughness [ksi (in)^{1/2}]

S_y = yield strength (ksi)

CVN = V-notch impact energy (ft-lb)

This equation is re-arranged to give the following:

$$K_{Ic} = S_y \left[5 \left(\frac{CVN}{S_y} - 0.05 \right) \right]^{1/2}$$

Therefore, for Ti-7:

$$K_{Ic} = 40 \left[5 \left(\frac{30}{40} - 0.05 \right) \right]^{1/2} \quad (\text{see Section 5.1 and Assumption 3.3})$$

$$K_{Ic} = 74.8 \text{ ksi (in)}^{1/2}$$

$$K_{Ic} = 82 \text{ MPa (m)}^{1/2}$$

Originator: Initials: Z.C. Date: 4/30/02

Checker: Initials: JB Date: 4/30/02

Similarly, for Alloy 22:

$$K_{Ic} = 45 \left[5 \left(\frac{260}{45} - 0.05 \right) \right]^{1/2} \quad (\text{see Section 5.1 and Assumption 3.4})$$

$$K_{Ic} = 240.8 \text{ ksi (in)}^{1/2}$$

$$K_{Ic} = 265 \text{ MPa (m)}^{1/2}$$

5.1.2 Drip Shield Top Plate and WP Outer Shell Dimensions

Drip shield top plate thickness = 15 mm (Ref. 4, Section 1.2.1.18)

WP outer shell thickness = 20 mm (Ref. 8, Section 1.2.1.4)

5.1.3 Calculations for True Measures of Ductility

The material properties in Section 5.1 refer to engineering stress and strain definitions: $s = P/A_0$ and $e = L/L_0 - 1$ (see Ref. 12), where P stands for the force applied during a static tensile test, L is the length of the deformed specimen, and L_0 and A_0 are the original length and cross-sectional area of the specimen, respectively. The engineering stress-strain curve does not give a true indication of the deformation characteristics of a material during plastic deformation since it is based entirely on the original dimensions of the specimen. In addition, ductile metal that is pulled in tension becomes unstable and necks down during the test. Hence, this calculation requires input in terms of true stress and strain definitions: $\sigma = P/A$ and $\varepsilon = \ln(L/L_0)$ (see Ref. 12).

The relationships between the true stress and strain definitions and the engineering stress and strain definitions, $\sigma = s \cdot (1 + e)$ and $\varepsilon = \ln(1 + e)$, can be readily derived based on constancy of volume ($A_0 \cdot L_0 = A \cdot L$) and strain homogeneity during plastic deformation (see Ref. 12). These expressions are applicable only in the hardening region of the engineering stress-strain curve that is limited by the onset of necking.

The following parameters are used in subsequent calculations:

$s_y \approx \sigma_y$ = yield strength

s_u = engineering tensile strength

Originator: Initials: ZC

Date: 4/30/02

Checker: Initials: VB

Date: 4/30/02

σ_u = true tensile strength

$e_y \approx \varepsilon_y$ = strain corresponding to yield strength

e_u = engineering strain corresponding to tensile strength (engineering uniform strain)

ε_u = true strain corresponding to tensile strength (true uniform strain)

For Ti-7, in absence of data on the uniform strain in available literature, the uniform strain is assumed to be the same as the material elongation (strain corresponding to rupture of the tensile specimen) (Assumption 3.5).

In the case of Ti-7, the true measures of ductility are

$$\varepsilon_u = \ln(1 + e_u) = \ln(1 + 0.2) = 0.18$$

$$\sigma_u = s_u \cdot (1 + e_u) = 345 \cdot (1 + 0.2) = 414 \text{ MPa (see Section 5.1)}$$

Therefore, the true strength of Ti-7 at 20 °C is 414 MPa.

The literature indicates that the stress-strain curve for Alloy 22 does not manifest three-stage deformation character (Assumption 3.6). Therefore, elongation reduced by 10% (to take into account the specimen-failure part of the stress-strain curve) is assumed for uniform strain.

For Alloy 22 ($e_u = 0.9 \cdot \text{elongation} = 0.405$), the true measures of ductility are

$$\varepsilon_u = \ln(1 + e_u) = \ln(1 + 0.405) = 0.34$$

$$\sigma_u = s_u \cdot (1 + e_u) = 690 \cdot (1 + 0.405) = 970 \text{ MPa (see Section 5.1)}$$

Hence, the true tensile strength of Alloy 22 at 20 °C is 970 MPa.

5.2 DESIGN METHODOLOGY

An impact load over the drip shield or the WP may occur due to rock fall. Such dynamic loads might induce rapid crack propagation in brittle materials, or might result in ductile collapse in ductile materials, or might result in any combination of these two extreme cases. This section first discusses the two basic design approaches used in solid mechanics. Then, certain aspects of dynamic fracture mechanics are described that are not present in quasi-static fracture mechanics.

Originator: Initials: Z.C Date: 4/30/02

Checker: Initials: VB Date: 4/30/02

5.2.1 Comparison of Two Design Approaches

There are two basic design approaches in solid mechanics: the traditional strength of materials approach and the fracture mechanics approach. The former involves a comparison of the applied stress with a design criterion, which is usually based on the material tensile strength. The latter contains a three-way comparison between the applied stress, flaw size, and the stress intensity. Any two of these parameters can be fixed to determine the third one, which is then compared to an allowable value for the design.

In order to design a safe structure, the determination of the mode of failure is imperative to all designs. The failure of structures is governed by the behavior of the specific material used in the design, under prescribed loading conditions. An overloading of a brittle material results in fractures, whereas that of a highly ductile material causes a plastic collapse. For the materials that fall between the two characteristics, the point of failure may be reached by the combination of both fracture and collapse.

The failure mode of a specific material can be identified by performing an evaluation of a specific diagram that is referred to as the failure assessment diagram (FAD). The FAD for Ti-7 and Alloy 22 are given in Figures I-2 and II-2, respectively. The FAD is plotted using:

$$K_r = S_r \left[\frac{8}{\pi^2} \ln \sec \left(\frac{\pi}{2} S_r \right) \right]^{-1/2} \quad (\text{Ref. 10, p. 475})$$

where:

K_r = stress intensity ratio = K_I / K_{Ic}

S_r = stress ratio = σ / σ_c

K_I = stress intensity

K_{Ic} = fracture toughness

σ = applied stress

Originator: Initials: Z.C. Date: 4/30/02

Checker: Initials: VB Date: 4/30/02

The work hardening of materials can be taken into account by using an average of yield and tensile strength:

$$\sigma_c = \text{collapse stress} = \sigma_{\text{flow}} = \text{flow stress} = (\text{yield strength} + \text{true ultimate strength}) / 2$$

A point on the FAD is defined by two parameters: the stress intensity ratio and the stress ratio. Any point that falls inside the curve defined by the above equation indicates that the design is safe. However, any point outside the subject region indicates a failure.

A quasi-static solution for failure assessment is performed for both Ti-7 and Alloy 22. This approach considers three different closed-form solutions: compact specimen, SENB specimen, and SENT specimen. The reason for selecting these three specimen geometries is to use the most realistic geometries with closed-form solution that are similar to the design basis event of a rock fall over a drip shield top plate or a WP shell [see Ref. 10, p. 603 and 604, cases (a), (c), and (f)]. Since the failure curve in the FAD is independent of the geometry of the problem (Ref. 10, p. 475), the use of the plate formulations is deemed acceptable.

The point of possible failure is determined by plotting a line extending from origin to the failure curve, with an angle measured from horizontal. The slope of the line is defined by the ratio of the stress intensity ratio to the stress ratio. A critical value of this angle is observed on the failure curve at the point where the curve starts to take an asymptotic approach to the axis of the stress ratio. This angle is calculated as 31° [$\arctan(0.6/1) = 31^\circ$, see Figures I-1 through VI-1]. Hence, any angle less than 31° is deemed to indicate a ductile collapse. Figures I-1 through I-3 show the FADs for Ti-7; Figures IV-1 through VI-1 show the FADs for Alloy 22. Each FAD includes a failure curve and two separate lines. The dash line and the dotted line indicate the solution for the plane stress and the plane strain cases, respectively. These two cases provide lower and upper bounds for the real three-dimensional problem.

The goal of this evaluation is to determine the failure mode, i.e., whether the material behavior is brittle or ductile. If the lines of the plane stress and plane strain have a slope that is close to the vertical, then the material shows a brittle behavior and a fracture occurs. On the other hand, if the lines extend into the region of failure with a slope closer to the horizontal (less than 31°), then the material is said to be ductile and the failure occurs by plastic (ductile) collapse. The specific structural behavior of both Ti-7 and Alloy 22 under increased loading is determined by the use of FADs in Attachments I through VI; the results are discussed in Section 6.

Originator: Initials: Z.C. Date: 4/30/02
Checker: Initials: VB Date: 4/30/02

5.2.2 Fracture Mechanics of Impacts

Dynamic fracture mechanics contains three complicating aspects in comparison to the quasi-static fracture mechanics:

1) Inertia effects:

Inertia effects are important if there is a potential for a rapid crack growth. However, a rapid crack growth does not occur ordinarily (environment-assisted deterioration of material toughness and ductility, such as neutron irradiation embrittlement, is outside the scope of this study) in highly ductile materials such as Alloy 22, or in moderately ductile materials such as Ti-7 at temperatures and strain rates that are of interest in this study. Such behavior of ductile materials is substantiated in Reference 10, page 211. It is known that dynamically loaded materials, in general, show two different phases in their time-history response diagrams. The first phase is the short-term transitional period where inertia effects, such as the kinetic energy, dominate prior to the transition time. However, the deformation energy dominates at times significantly larger than the transition time, which is also called the long-term response. The same reference also indicates that for a three point bend specimen, a quasi-static approach can be used as long as the fracture occurs at times larger than the transition time. This requirement is relatively easy to meet in impact tests on ductile materials (Ref. 10, page 211). Thus, inertia effects are eliminated for the materials considered in this document.

2) Reflected stress waves:

Quasi-static formulations of fracture mechanics do not include the effects of discrete stress waves; therefore, this formulation is only valid after stress waves have traversed the width of the specimen several times. Reference 10, page 211 indicates that the transition time is approximately 300 μ s and fracture occurs after 600 μ s. This is relatively easy to meet in impact tests on ductile materials. Shell thicknesses of 15 mm and 20 mm for Ti-7 and Alloy 22, respectively, were specified in Section 5.1.2. In solids, the longitudinal velocity of the elastic stress wave V with modulus of elasticity E , mass density ρ , and Poisson's ratio ν is calculated using:

$$V = ((1-\nu) * E / ((1+\nu) (1-2\nu) * \rho))^{1/2} \text{ (see Ref. 11, p. 37). Therefore:}$$

$$V \text{ (Ti-7)} = ((1 - 0.34) * 107 * 10^9 / ((1 + 0.34) (1 - (2 * 0.34)) * 4512))^{1/2} = 6042 \text{ m/s (see Section 5.1)}$$

$$V \text{ (Alloy 22)} = ((1 - 0.278) * 206 * 10^9 / ((1 + 0.278) (1 - (2 * 0.278)) * 8690))^{1/2} = 5492 \text{ m/s (see Section 5.1)}$$

Originator: Initials: Z.C. Date: 4/30/02

Checker: Initials: VB Date: 4/30/02

The ratio of the specimen width to the speed of sound gives the time (t) required for the stress waves to transverse the specimen width.

$$t(\text{Ti-7}) = 0.015 / 6042 = 2.5 \mu\text{s}$$

$$t(\text{Alloy 22}) = 0.02 / 5492 = 3.6 \mu\text{s}$$

These calculations indicate that the elastic stress waves will travel 120 times ($300 \mu\text{s} / 2.5 \mu\text{s} = 120$) and 83 times ($300 \mu\text{s} / 3.6 \mu\text{s} = 83$) through the thickness of Ti-7 and Alloy 22 shells, respectively, before the transition time is reached from inertial effect dominance to strain energy dominance. Hence, the effect of reflected waves can also be eliminated for the purpose this analysis; a quasi-static approach is deemed acceptable.

3) Strain rate effect:

The fracture behavior of ductile metals is primarily strain controlled (see Ref. 10, p. 211). Therefore, the fracture toughness of these metals, such as Ti-7 and Alloy 22, shows tendency to increase at high loading rates because more energy is required to reach a given strain value. Furthermore, most metals are not sensitive to moderate variations in strain rate near ambient temperature (see Ref. 10, p. 205). Hence, the effect of strain rate is considered to be small for the materials of concern in this document.

Regardless of the three complicating aspects stated above, this calculation considers the effect of strain hardening. When strain hardening is taken into account, the shape of the FAD changes such that the safe region for the design increases along the axis of the stress ratio (see Ref. 10, p. 487). A linear formulation approach to the problem is conservative in this case because a shell made from a strain hardening material can withstand greater stresses. Therefore, the FAD used in this analysis is slightly conservative and appropriate for the design.

6. RESULTS

The results of the calculation are provided in Figures I-1 through VI-1. These figures depict the FADs for both Ti-7 and Alloy 22, which were used in determining the drip shield and the WP material failure characteristics. In these diagrams, the horizontal axis represents the stress ratio; the vertical axis represents the stress intensity ratio. The solid curve indicates the threshold for failure, which encloses the region of safety. The point of possible failure is determined by plotting a line extending from origin to the failure curve, with an angle measured from horizontal. A critical value of the angle between this line and the horizontal is 31° (see Section 5.2.1). Any angle less than 31° is deemed to indicate a ductile collapse. Figures I-1 through III-1 (Ti-7) and Figures IV-1 through

Originator: Initials: Z.C. Date: 4/30/02
Checker: Initials: vB Date: 4/30/02

VI-1 (Alloy 22) indicate the range of possible failure between the plane stress and plane strain cases, which are represented by straight lines. These two cases provide lower and upper bounds for the problem; the real solution for a three-dimensional case lies between them.

7. CONCLUSIONS

The FADs of both Ti-7 and Alloy 22 (see Figures I-1 through VI-1) indicate that the angles of all straight lines for both the plane stress and plane strain cases are less than 31° . Therefore, any failure due to excessive loading is dominated by ductile collapse. This conclusion was reached by consideration of the fact that the failure lines cross the failure curve within a range of the limit angle (0° to 31°), which represents a ductile material behavior under external loading (FAD curve asymptotically approaches the plastic collapse limit $S_r = 1$). Since the failure mode is dominated by the ductile collapse, it is also concluded that the use of the traditional strength of materials approach for the failure determination (such as the Tresca criterion) of the drip shield and the WP designs is bounding and appropriate when compared to the fracture mechanics approach.

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Originator: Initials: Z.C. Date: 4/30/02

Checker: Initials: JB Date: 4/30/02

9. ATTACHMENTS

Attachment I (4 pages):	Drip Shield Top Plate Failure Assessment Calculations (compact specimen)
Attachment II (4 pages):	Drip Shield Top Plate Failure Assessment Calculations (single edge notched bend specimen)
Attachment III (4 pages):	Drip Shield Top Plate Failure Assessment Calculations (single edge notched tension specimen)
Attachment IV (4 pages):	Waste Package Outer Shell Failure Assessment Calculations (compact specimen)
Attachment V (4 pages):	Waste Package Outer Shell Failure Assessment Calculations (single edge notched bend specimen)
Attachment VI (4 pages):	Waste Package Outer Shell Failure Assessment Calculations (single edge notched tension specimen)

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Date: 4/30/02

Title: Comparison of the Traditional Strength of Materials Approach to Design with the Fracture Mechanics Approach

**Drip Shield Top Plate Failure Assessment Calculations
(Compact Specimen)**

Ti-7 material properties:

$\epsilon_u := 0.18$	true strain, see Section 5.1.3
$\sigma_u := 414 \cdot 10^6 \cdot \text{Pa}$	true tensile strength, see Section 5.1.3
$\sigma_y := 276 \cdot 10^6 \cdot \text{Pa}$	(at 20 degrees C, low temperature results in more brittle behavior, see Section 5.1)
$\sigma_{\text{flow}} := \frac{(\sigma_y + \sigma_u)}{2}$	flow stress, see Reference 10, p. 609
$K_{Ic} := 82 \cdot 10^6 \cdot \text{Pa} \cdot \sqrt{\text{m}}$	fracture toughness value for Ti-7 (see Section 5.1.1)

Compact specimen (Ref.10, p. 603, case (a)):

Ti-7 plate applicable dimensions for the compact specimen:

ratio := 0.5	standard value for "a/W" ratio for test specimens (Ref. 13, p. 14)
W := 0.015·m	top plate thickness (see Section 5.1.2)
a := ratio·W	calculated crack depth
b := W - a	distance from crack tip to material surface
B := 0.1·m	compact specimen length; this parameter has no effect on results since it cancels out subsequently
P := 1·N	applied force, this parameter has no effect on results since it cancels out subsequently

Calculate stress intensity, KI (see Ref. 10, p. 605, case (a)):

$$KI := \left(\frac{P}{B \cdot \sqrt{W}} \right) \cdot \frac{\left(2 + \frac{a}{W} \right) \cdot \left[0.886 + 4.64 \cdot \left(\frac{a}{W} \right) - 13.32 \cdot \left(\frac{a}{W} \right)^2 + 14.72 \cdot \left(\frac{a}{W} \right)^3 - 5.6 \cdot \left(\frac{a}{W} \right)^4 \right]}{\left(1 - \frac{a}{W} \right)^{\frac{3}{2}}}$$

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Date: **4/30/02**
Date: **4/30/02**

Title: Comparison of the Traditional Strength of Materials Approach to Design with the Fracture Mechanics Approach

Calculate limit load for plane stress (Ref. 10, p. 609, case (a)):

$$\eta := \sqrt{\left(2 \cdot \frac{a}{b}\right)^2 + \left(4 \cdot \frac{a}{b}\right) + 2} - \left[\left(2 \cdot \frac{a}{b}\right) + 1\right]$$

$$\sigma := \frac{P}{1.072 \cdot \eta \cdot B \cdot b}$$

Calculate FAD parameters:

$$S_r := \frac{\sigma}{\sigma_{\text{flow}}} \quad K_r := \frac{K_I}{K_{Ic}}$$

$$\frac{K_r}{S_r} = 0.433$$

Calculate angle of the loading path from horizontal, α :

$$\alpha := \text{atan}\left(\frac{K_r}{S_r}\right) \quad \alpha = 0.409 \text{ radians}$$

$$\text{deg}\alpha := \left(\frac{180}{\pi}\right) \cdot \alpha \quad \text{deg}\alpha = 23.409 \text{ degrees}$$

Plot failure assessment diagram:

Plot a line "deg α " degrees from horizontal:

$$i := 0..101 \quad S_{r_i} := 0.01 \cdot i \quad K_{r_i} := S_{r_i} \cdot \tan(\alpha)$$

Define failure curve:

$$j := 0..99 \quad S_{r_j} := 0.01 + 0.01 \cdot j$$

$$K_{r_p} := \left[S_{r_p} \cdot \left[\left(\frac{8}{\pi^2} \right) \cdot \ln \left[\sec \left[\left(\frac{\pi}{2} \right) \cdot S_{r_p} \right] \right] \right]^{\frac{-1}{2}} \right]$$

Reference 10, p.475

$$S_{r_{100}} := 1 \quad K_{r_{100}} := 0 \quad \text{the last data point specified manually, due to the round-off limits in preceding calculation}$$

Originator: Initials: Z.C.
Checker: Initials: VB

Date: 4/30/02
Date: 4/30/02

Title: Comparison of the Traditional Strength of Materials Approach to Design with the Fracture Mechanics Approach

Calculate limit load for plane strain (Ref. 10, p. 609, case (a)):

$$\sigma := \frac{P}{1.455 \cdot \eta \cdot B \cdot b}$$

Calculate FAD parameters:

$$Sr1 := \frac{\sigma}{\sigma_{flow}} \quad Kr1 := \frac{K1}{K1c}$$

$$\frac{Kr1}{Sr1} = 0.588$$

Calculate angle of the loading path from horizontal, α :

$$\alpha1 := \text{atan}\left(\frac{Kr1}{Sr1}\right) \quad \alpha1 = 0.531 \text{ radians}$$

$$\text{deg}\alpha1 := \left(\frac{180}{\pi}\right) \cdot \alpha1 \quad \text{deg}\alpha1 = 30.438 \text{ degrees}$$

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Date: 4/30/02
Date: 4/30/02

Title: Comparison of the Traditional Strength of Materials Approach to Design with the Fracture Mechanics Approach

Plot failure assessment diagram:

Plot a line "deg α 1" degrees from horizontal:

$$i := 0..101 \quad S_{r1_i} := 0.01 \cdot i \quad K_{r1_i} := S_{r1_i} \cdot \tan(\alpha 1)$$

Figure I-1. Failure assessment diagram for Ti-7 (compact specimen)

The figure given above shows the failure curve and the two loading paths defined by the angles calculated above. The dash line and the dotted line indicate the solution for the plane stress and the plane strain cases, respectively. These two cases provide lower and upper bounds for the problem; the real solution lies between them. The figure shows that the bounding range for the solution falls within a region of failure that is dominated by ductile collapse, not brittle fracture (see Ref. 10, p. 476).

Originator: Initials: Z.C.
Checker: Initials: VB

Date: 4/30/02
Date: 4/30/02

Title: Comparison of the Traditional Strength of Materials Approach to Design with the Fracture Mechanics Approach

**Drip Shield Top Plate Failure Assessment Calculations
(Single Edge Notched Bend [SENB] Specimen)**

Ti-7 material properties:

$\epsilon_u := 0.18$	true strain, see Section 5.1.3
$\sigma_u := 414 \cdot 10^6 \cdot \text{Pa}$	true tensile strength, see Section 5.1.3
$\sigma_y := 276 \cdot 10^6 \cdot \text{Pa}$	(at 20 degrees C, low temperature results in more brittle behavior, see Section 5.1)
$\sigma_{\text{flow}} := \frac{(\sigma_y + \sigma_u)}{2}$	flow stress, see Reference 10, p. 609
$K_{Ic} := 82 \cdot 10^6 \cdot \text{Pa} \cdot \sqrt{\text{m}}$	fracture toughness value for Ti-7 (see Section 5.1.1)

SENB specimen (Ref.10, p. 603, case (c)):

Ti-7 plate applicable dimensions for the SENB specimen:

ratio := 0.5	standard value for "a/W" ratio for test specimens (Ref. 13, p. 14)
W := 0.015 · m	top plate thickness (see Section 5.1.2)
a := ratio · W	calculated crack depth
b := W - a	distance from crack tip to material surface
S := 1 · m	distance between two support points; this parameter has no effect on results since it cancels out subsequently
B := 0.1 · m	compact specimen length; this parameter has no effect on results since it cancels out subsequently
P := 1 · N	applied force, this parameter has no effect on results since it cancels out subsequently

Calculate stress intensity, KI (see Ref. 10, p. 605, case (c)):

$$KI := \left(\frac{P}{B \cdot \sqrt{W}} \right) \cdot \frac{3 \cdot \left(\frac{S}{W} \right) \cdot \sqrt{\frac{a}{W}} \cdot \left[1.99 - \left(\frac{a}{W} \right) \cdot \left(1 - \frac{a}{W} \right) \cdot \left[2.15 - 3.93 \cdot \left(\frac{a}{W} \right) + 2.7 \cdot \left(\frac{a}{W} \right)^2 \right] \right]}{2 \cdot \left(1 + \frac{2 \cdot a}{W} \right) \cdot \left(1 - \frac{a}{W} \right)^{\frac{3}{2}}}$$

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Checker: Initials: v B

Date: 4/30/02
Date: 4/30/02

Title: Comparison of the Traditional Strength of Materials Approach to Design with the Fracture Mechanics Approach

Calculate limit load for plane stress (Ref. 10, p. 609, case (b)):

$$\sigma := \frac{P \cdot S}{1.072 \cdot B \cdot b^2}$$

Calculate FAD parameters:

$$S_r := \frac{\sigma}{\sigma_{\text{flow}}} \quad K_r := \frac{K_I}{K_{Ic}}$$

$$\frac{K_r}{S_r} = 0.368$$

Calculate angle of the loading path from horizontal, α :

$$\alpha := \text{atan}\left(\frac{K_r}{S_r}\right) \quad \alpha = 0.352 \text{ radians}$$

$$\text{deg}\alpha := \left(\frac{180}{\pi}\right) \cdot \alpha \quad \text{deg}\alpha = 20.188 \text{ degrees}$$

Plot failure assessment diagram:

Plot a line "deg α " degrees from horizontal:

$$i := 0..101 \quad S_{r_i} := 0.01 \cdot i \quad K_{r_i} := S_{r_i} \cdot \tan(\alpha)$$

Define failure curve:

$$j := 0..99 \quad S_{r_j} := 0.01 + 0.01 \cdot j$$

$$K_{r_p} := \left[S_{r_p} \cdot \left[\left(\frac{8}{\pi^2} \right) \cdot \ln \left[\sec \left[\left(\frac{\pi}{2} \right) \cdot S_{r_p} \right] \right] \right]^{\frac{-1}{2}} \right]$$

Reference 10, p.475

$$S_{r_{100}} := 1 \quad K_{r_{100}} := 0$$

the last data point specified manually, due to the round-off limits in preceding calculation

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Date: **4/30/02**
 Date: **4/20/02**

Title: Comparison of the Traditional Strength of Materials Approach to Design with the Fracture Mechanics Approach

Calculate limit load for plane strain (Ref. 10, p. 609, case (b)):

$$\sigma := \frac{P \cdot S}{1.455 \cdot B \cdot b^2}$$

Calculate FAD parameters:

$$Sr1 := \frac{\sigma}{\sigma_{flow}} \quad Kr1 := \frac{Kl}{Klc}$$

$$\frac{Kr1}{Sr1} = 0.499$$

Calculate angle of the loading path from horizontal, α :

$$\alpha1 := \operatorname{atan}\left(\frac{Kr1}{Sr1}\right) \quad \alpha1 = 0.463 \text{ radians}$$

$$\deg \alpha1 := \left(\frac{180}{\pi}\right) \cdot \alpha1 \quad \deg \alpha1 = 26.521 \text{ degrees}$$

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Checker: Initials: VB

Date: 4/30/02
Date: 4/30/02

Title: Comparison of the Traditional Strength of Materials Approach to Design with the Fracture Mechanics Approach

Plot failure assessment diagram:

Plot a line "deg α 1" degrees from horizontal:

$$i := 0..101 \quad S_{rI_i} := 0.01 \cdot i \quad K_{rI_i} := S_{rI_i} \cdot \tan(\alpha 1)$$

Figure II-1. Failure assessment diagram for Ti-7 (SENB specimen)

The figure given above shows the failure curve and the two loading paths defined by the angles calculated above. The dash line and the dotted line indicate the solution for the plane stress and the plane strain cases, respectively. These two cases provide lower and upper bounds for the problem; the real solution lies between them. The figure shows that the bounding range for the solution falls within a region of failure that is dominated by ductile collapse, not brittle fracture (see Ref. 10, p. 476).

Originator: Initials: Z.C.
Checker: Initials: VB

Date: 4/30/02
Date: 4/30/02

Title: Comparison of the Traditional Strength of Materials Approach to Design with the Fracture Mechanics Approach

Drip Shield Top Plate Failure Assessment Calculations (Single Edge Notched Tension [SENT] Specimen)

Ti-7 material properties:

$\epsilon_u := 0.18$	true strain, see Section 5.1.3
$\sigma_u := 414 \cdot 10^6 \cdot \text{Pa}$	true tensile strength, see Section 5.1.3
$\sigma_y := 276 \cdot 10^6 \cdot \text{Pa}$	(at 20 degrees C, low temperature results in more brittle behavior, see Section 5.1)
$\sigma_{\text{flow}} := \frac{(\sigma_y + \sigma_u)}{2}$	flow stress, see Reference 10, p. 609
$K_{Ic} := 82 \cdot 10^6 \cdot \text{Pa} \cdot \sqrt{\text{m}}$	fracture toughness value for Ti-7 (see Section 5.1.1)

SENT specimen (Ref.10, p. 604, case (f)):

Ti-7 plate applicable dimensions for the SENT specimen:

ratio := 0.5	standard value for "a/W" ratio for test specimens (Ref. 13, p. 14)
W := 0.015·m	top plate thickness (see Section 5.1.2)
a := ratio·W	calculated crack depth
b := W - a	distance from crack tip to material surface
B := 0.1·m	compact specimen length; this parameter has no effect on results since it cancels out subsequently
P := 1·N	applied force, this parameter has no effect on results since it cancels out subsequently

Calculate stress intensity, KI (see Ref. 10, p. 606, case (f)):

$$K_I := \left(\frac{P}{B \cdot \sqrt{W}} \right) \cdot \frac{\sqrt{2 \cdot \tan\left(\frac{\pi \cdot a}{2 \cdot W}\right)} \cdot \left[0.752 + 2.02 \cdot \left(\frac{a}{W}\right) + 0.37 \cdot \left(1 - \sin\left(\frac{\pi \cdot a}{2 \cdot W}\right)\right)^3 \right]}{\cos\left(\frac{\pi \cdot a}{2 \cdot W}\right)}$$

Originator: Initials: Z.C.
Checker: Initials: VB

Date: 4/30/02
Date: 4/30/02

Title: Comparison of the Traditional Strength of Materials Approach to Design with the Fracture Mechanics Approach

Calculate limit load for plane stress (Ref. 10, p. 609, case (d)):

$$\eta := \sqrt{1 + \left(\frac{a}{b}\right)^2} - \frac{a}{b}$$

$$\sigma := \frac{P}{1.072 \cdot \eta \cdot B \cdot b}$$

Calculate FAD parameters:

$$S_r := \frac{\sigma}{\sigma_{flow}} \quad K_r := \frac{K_I}{K_{Ic}}$$

$$\frac{K_r}{S_r} = 0.405$$

Calculate angle of the loading path from horizontal, α :

$$\alpha := \text{atan}\left(\frac{K_r}{S_r}\right) \quad \alpha = 0.385 \text{ radians}$$

$$\text{deg}\alpha := \left(\frac{180}{\pi}\right) \cdot \alpha \quad \text{deg}\alpha = 22.062 \text{ degrees}$$

Plot failure assessment diagram:

Plot a line "deg α " degrees from horizontal:

$$i := 0..101 \quad S_{r_i} := 0.01 \cdot i \quad K_{r_i} := S_{r_i} \cdot \tan(\alpha)$$

Define failure curve:

$$j := 0..99 \quad S_{r_j} := 0.01 + 0.01 \cdot j$$

$$K_{rp} := \left[S_{rp} \cdot \left[\left(\frac{8}{\pi^2} \right) \cdot \ln \left[\sec \left[\left(\frac{\pi}{2} \right) \cdot S_{rp} \right] \right] \right]^{\frac{-1}{2}} \right]$$

Reference 10, p.475

$$S_{rp100} := 1 \quad K_{rp100} := 0 \quad \text{the last data point specified manually, due to the round-off limits in preceding calculation}$$

Originator: Initials: Z.C.
Checker: Initials: VB

Date: 4/30/02
Date: 4/30/02

Title: Comparison of the Traditional Strength of Materials Approach to Design with the Fracture Mechanics Approach

Calculate limit load for plane strain (Ref. 10, p. 609, case (d)):

$$\sigma := \frac{P}{1.455 \cdot \eta \cdot B \cdot b}$$

Calculate FAD parameters:

$$SrI := \frac{\sigma}{\sigma_{flow}} \quad KrI := \frac{KI}{KIc}$$

$$\frac{KrI}{SrI} = 0.550$$

Calculate angle of the loading path from horizontal, α :

$$\alpha I := \operatorname{atan}\left(\frac{KrI}{SrI}\right) \quad \alpha I = 0.503 \text{ radians}$$

$$\deg \alpha I := \left(\frac{180}{\pi}\right) \cdot \alpha I \quad \deg \alpha I = 28.815 \text{ degrees}$$

Originator: Initials: *z c*
Checker: Initials: *v B*

Date: *4/30/02*
Date: *4/30/02*

Title: Comparison of the Traditional Strength of Materials Approach to Design with the Fracture Mechanics Approach

Plot failure assessment diagram:

Plot a line "deg α 1" degrees from horizontal:

$$i := 0..101 \quad S_{r1_i} := 0.01 \cdot i \quad K_{r1_i} := S_{r1_i} \cdot \tan(\alpha 1)$$

Figure III-1. Failure assessment diagram for Ti-7 (SENT specimen)

The figure given above shows the failure curve and the two loading paths defined by the angles calculated above. The dash line and the dotted line indicate the solution for the plane stress and the plane strain cases, respectively. These two cases provide lower and upper bounds for the problem; the real solution lies between them. The figure shows that the bounding range for the solution falls within a region of failure that is dominated by ductile collapse, not brittle fracture (see Ref. 10, p. 476).

Originator: Initials: Z.C.
Checker: Initials: VB

Date: 4/30/02
Date: 4/30/02

Title: Comparison of the Traditional Strength of Materials Approach to Design with the Fracture Mechanics Approach

Waste Package Outer Shell Failure Assessment Calculations (Compact Specimen)

Alloy 22 material properties:

$\epsilon_u := 0.34$	true strain, see Section 5.1.3
$\sigma_u := 970 \cdot 10^6 \cdot \text{Pa}$	true tensile strength, see Section 5.1.3
$\sigma_y := 310 \cdot 10^6 \cdot \text{Pa}$	(at 20 degrees C, low temperature results in more brittle behavior, see Section 5.1)
$\sigma_{\text{flow}} := \frac{(\sigma_y + \sigma_u)}{2}$	flow stress, see Reference 10, p. 609
$K_{Ic} := 265 \cdot 10^6 \cdot \text{Pa} \cdot \sqrt{\text{m}}$	fracture toughness value for Alloy 22 (see Section 5.1.1)

Compact specimen (Ref.10, p. 603, case (a)):

Alloy 22 plate applicable dimensions for the compact specimen:

ratio := 0.5	standard value for "a/W" ratio for test specimens (Ref. 13, p. 14)
W := 0.02·m	outer shell thickness (see Section 5.1.2)
a := ratio·W	calculated crack depth
b := W - a	distance from crack tip to material surface
B := 0.1·m	compact specimen length; this parameter has no effect on results since it cancels out subsequently
P := 1·N	applied force, this parameter has no effect on results since it cancels out subsequently

Calculate stress intensity, KI (see Ref. 10, p. 605, case (a)):

$$KI := \left(\frac{P}{B \cdot \sqrt{W}} \right) \cdot \frac{\left(2 + \frac{a}{W} \right) \cdot \left[0.886 + 4.64 \cdot \left(\frac{a}{W} \right) - 13.32 \cdot \left(\frac{a}{W} \right)^2 + 14.72 \cdot \left(\frac{a}{W} \right)^3 - 5.6 \cdot \left(\frac{a}{W} \right)^4 \right]}{\left(1 - \frac{a}{W} \right)^{\frac{3}{2}}}$$

Originator: Initials: Z.C.
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Date: 4/30/02
Date: 4/30/02

Title: Comparison of the Traditional Strength of Materials Approach to Design with the Fracture Mechanics Approach

Calculate limit load for plane stress (Ref. 10, p. 609, case (a)):

$$\eta := \sqrt{\left(2 \cdot \frac{a}{b}\right)^2 + \left(4 \cdot \frac{a}{b}\right) + 2} - \left[\left(2 \cdot \frac{a}{b}\right) + 1\right]$$

$$\sigma := \frac{P}{1.072 \cdot \eta \cdot B \cdot b}$$

Calculate FAD parameters:

$$Sr := \frac{\sigma}{\sigma_{flow}} \quad Kr := \frac{Kl}{Klc}$$

$$\frac{Kr}{Sr} = 0.287$$

Calculate angle of the loading path from horizontal, α :

$$\alpha := \text{atan}\left(\frac{Kr}{Sr}\right) \quad \alpha = 0.279 \text{ radians}$$

$$\text{deg}\alpha := \left(\frac{180}{\pi}\right) \cdot \alpha \quad \text{deg}\alpha = 16.011 \text{ degrees}$$

Plot failure assessment diagram:

Plot a line "deg α " degrees from horizontal:

$$i := 0..101 \quad Sr_i := 0.01 \cdot i \quad Kr_i := Sr_i \cdot \tan(\alpha)$$

Define failure curve:

$$j := 0..99 \quad Sr_{pj} := 0.01 + 0.01 \cdot j$$

$$Krp := \left[Sr_p \cdot \left[\left(\frac{8}{\pi^2} \right) \cdot \ln \left[\sec \left[\left(\frac{\pi}{2} \right) \cdot Sr_p \right] \right] \right]^{\frac{-1}{2}} \right]$$

Reference 10, p.475

$$Sr_{p100} := 1 \quad Kr_{p100} := 0 \quad \text{the last data point specified manually, due to the round-off limits in preceding calculation}$$

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Date: 4/30/02
Date: 4/30/02

Title: Comparison of the Traditional Strength of Materials Approach to Design with the Fracture Mechanics Approach

Calculate limit load for plane strain (Ref. 10, p. 609, case (a)):

$$\sigma := \frac{P}{1.455 \cdot \eta \cdot B \cdot b}$$

Calculate FAD parameters:

$$Sr1 := \frac{\sigma}{\sigma_{flow}} \quad Kr1 := \frac{KI}{KIc}$$

$$\frac{Kr1}{Sr1} = 0.389$$

Calculate angle of the loading path from horizontal, α :

$$\alpha1 := \tan^{-1}\left(\frac{Kr1}{Sr1}\right) \quad \alpha1 = 0.371 \text{ radians}$$

$$\deg \alpha1 := \left(\frac{180}{\pi}\right) \cdot \alpha1 \quad \deg \alpha1 = 21.28 \text{ degrees}$$

Originator: Initials: Z.C.
Checker: Initials: VB

Date: 4/30/02
Date: 4/30/02

Title: Comparison of the Traditional Strength of Materials Approach to Design with the Fracture Mechanics Approach

Plot failure assessment diagram:

Plot a line "deg α 1" degrees from horizontal:

$$i := 0..101 \quad Sr1_i := 0.01 \cdot i \quad Kr1_i := Sr1_i \cdot \tan(\alpha 1)$$

Figure IV-1. Failure assessment diagram for Alloy 22 (compact specimen)

The figure given above shows the failure curve and the two loading paths defined by the angles calculated above. The dash line and the dotted line indicate the solution for the plane stress and the plane strain cases, respectively. These two cases provide lower and upper bounds for the problem; the real solution lies between them. The figure shows that the bounding range for the solution falls within a region of failure that is dominated by ductile collapse, not brittle fracture (see Ref. 10, p. 476).

Originator: Initials: **ZC**
 Checker: Initials: **VB**

Date: **4/30/02**
 Date: **4/30/02**

Title: Comparison of the Traditional Strength of Materials Approach to Design with the Fracture Mechanics Approach

Waste Package Outer Shell Failure Assessment Calculations (Single Edge Notched Bend [SENB] Specimen)

Alloy 22 material properties:

$\epsilon_u := 0.34$	true strain, see Section 5.1.3
$\sigma_u := 970 \cdot 10^6 \cdot \text{Pa}$	true tensile strength, see Section 5.1.3
$\sigma_y := 310 \cdot 10^6 \cdot \text{Pa}$	(at 20 degrees C, low temperature results in more brittle behavior, see Section 5.1)
$\sigma_{\text{flow}} := \frac{(\sigma_y + \sigma_u)}{2}$	flow stress, see Reference 10, p. 609
$K_{Ic} := 265 \cdot 10^6 \cdot \text{Pa} \cdot \sqrt{\text{m}}$	fracture toughness value for Alloy 22 (see Section 5.1.1)

SENB specimen (Ref.10, p. 603, case (c)):

Alloy 22 plate applicable dimensions for the SENB specimen:

ratio := 0.5	standard value for "a/W" ratio for test specimens (Ref. 13, p. 14)
W := 0.02·m	outer shell thickness (see Section 5.1.2)
a := ratio·W	calculated crack depth
b := W - a	distance from crack tip to material surface
S := 1·m	distance between two support points; this parameter has no effect on results since it cancels out subsequently
B := 0.1·m	compact specimen length; this parameter has no effect on results since it cancels out subsequently
P := 1·N	applied force, this parameter has no effect on results since it cancels out subsequently

Calculate stress intensity, KI (see Ref. 10, p. 605, case (c)):

$$KI := \left(\frac{P}{B \cdot \sqrt{W}} \right) \cdot \frac{3 \cdot \left(\frac{S}{W} \right) \cdot \sqrt{\frac{a}{W}} \cdot \left[1.99 - \left(\frac{a}{W} \right) \cdot \left(1 - \frac{a}{W} \right) \cdot \left[2.15 - 3.93 \cdot \left(\frac{a}{W} \right) + 2.7 \cdot \left(\frac{a}{W} \right)^2 \right] \right]}{2 \cdot \left(1 + \frac{2 \cdot a}{W} \right) \cdot \left(1 - \frac{a}{W} \right)^{\frac{3}{2}}}$$

Originator: Initials: Z.C.
Checker: Initials: vB

Date: 4/30/02
Date: 4/30/02

Title: Comparison of the Traditional Strength of Materials Approach to Design with the Fracture Mechanics Approach

Calculate limit load for plane stress (Ref. 10, p. 609, case (b)):

$$\sigma := \frac{P \cdot S}{1.072 \cdot B \cdot b^2}$$

Calculate FAD parameters:

$$Sr := \frac{\sigma}{\sigma_{flow}} \quad Kr := \frac{KI}{K_{Ic}}$$

$$\frac{Kr}{Sr} = 0.244$$

Calculate angle of the loading path from horizontal, α :

$$\alpha := \text{atan}\left(\frac{Kr}{Sr}\right) \quad \alpha = 0.239 \text{ radians}$$

$$\text{deg}\alpha := \left(\frac{180}{\pi}\right) \cdot \alpha \quad \text{deg}\alpha = 13.697 \text{ degrees}$$

Plot failure assessment diagram:

Plot a line "deg α " degrees from horizontal:

$$i := 0..101 \quad Sr_i := 0.01 \cdot i \quad Kr_i := Sr_i \cdot \tan(\alpha)$$

Define failure curve:

$$j := 0..99 \quad Srp_j := 0.01 + 0.01 \cdot j$$

$$Krp := \left[Srp \cdot \left[\left(\frac{8}{\pi^2} \right) \cdot \ln \left[\sec \left[\left(\frac{\pi}{2} \right) \cdot Srp \right] \right] \right]^{\frac{-1}{2}} \right]$$

Reference 10, p.475

$$Srp_{100} := 1 \quad Krp_{100} := 0$$

the last data point specified manually, due to the round-off limits in preceding calculation

Originator: Initials: Z.C.
Checker: Initials: VB

Date: 4/30/02
Date: 4/30/02

Title: Comparison of the Traditional Strength of Materials Approach to Design with the Fracture Mechanics Approach

Calculate limit load for plane strain (Ref. 10, p. 609, case (b)):

$$\sigma := \frac{P \cdot S}{1.455 \cdot B \cdot b^2}$$

Calculate FAD parameters:

$$Sr1 := \frac{\sigma}{\sigma_{flow}} \quad Kr1 := \frac{KI}{K_{Ic}}$$

$$\frac{Kr1}{Sr1} = 0.331$$

Calculate angle of the loading path from horizontal, α :

$$\alpha1 := \text{atan}\left(\frac{Kr1}{Sr1}\right) \quad \alpha1 = 0.319 \text{ radians}$$

$$\text{deg}\alpha1 := \left(\frac{180}{\pi}\right) \cdot \alpha1 \quad \text{deg}\alpha1 = 18.303 \text{ degrees}$$

Originator: Initials: Z.C.
Checker: Initials: VB

Date: 4/30/02
Date: 4/30/02

Title: Comparison of the Traditional Strength of Materials Approach to Design with the Fracture Mechanics Approach

Plot failure assessment diagram:

Plot a line "deg α 1" degrees from horizontal:

$$i := 0..101 \quad S_{r1_i} := 0.01 \cdot i \quad K_{r1_i} := S_{r1_i} \cdot \tan(\alpha 1)$$

Figure V-1. Failure assessment diagram for Alloy 22 (SENB specimen)

The figure given above shows the failure curve and the two loading paths defined by the angles calculated above. The dash line and the dotted line indicate the solution for the plane stress and the plane strain cases, respectively. These two cases provide lower and upper bounds for the problem; the real solution lies between them. The figure shows that the bounding range for the solution falls within a region of failure that is dominated by ductile collapse, not brittle fracture (see Ref. 10, p. 476).

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**Waste Package Outer Shell Failure Assessment Calculations
(Single Edge Notched Tension [SENT] Specimen)**

Alloy 22 material properties:

$\epsilon_u := 0.34$	true strain, see Section 5.1.3
$\sigma_u := 970 \cdot 10^6 \cdot \text{Pa}$	true tensile strength, see Section 5.1.3
$\sigma_y := 310 \cdot 10^6 \cdot \text{Pa}$	(at 20 degrees C, low temperature results in more brittle behavior, see Section 5.1)
$\sigma_{\text{flow}} := \frac{(\sigma_y + \sigma_u)}{2}$	flow stress, see Reference 10, p. 609
$K_{Ic} := 265 \cdot 10^6 \cdot \text{Pa} \cdot \sqrt{\text{m}}$	fracture toughness value for Alloy 22 (see Section 5.1.1)

SENT specimen (Ref.10, p. 604, case (f)):

Alloy 22 plate applicable dimensions for the SENT specimen:

ratio := 0.5	standard value for "a/W" ratio for test specimens (Ref. 13, p. 14)
W := 0.02·m	outer shell thickness (see Section 5.1.2)
a := ratio·W	calculated crack depth
b := W - a	distance from crack tip to material surface
B := 0.1·m	compact specimen length; this parameter has no effect on results since it cancels out subsequently
P := 1·N	applied force, this parameter has no effect on results since it cancels out subsequently

Calculate stress intensity, KI (see Ref. 10, p. 606, case (f)):

$$K_I := \left(\frac{P}{B \cdot \sqrt{W}} \right) \cdot \frac{\sqrt{2 \cdot \tan\left(\frac{\pi \cdot a}{2 \cdot W}\right)} \cdot \left[0.752 + 2.02 \cdot \left(\frac{a}{W}\right) + 0.37 \cdot \left(1 - \sin\left(\frac{\pi \cdot a}{2 \cdot W}\right)\right)^3 \right]}{\cos\left(\frac{\pi \cdot a}{2 \cdot W}\right)}$$

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Date: 4/30/02
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Calculate limit load for plane stress (Ref. 10, p. 609, case (d)):

$$\eta := \sqrt{1 + \left(\frac{a}{b}\right)^2} - \frac{a}{b}$$

$$\sigma := \frac{P}{1.072 \cdot \eta \cdot B \cdot b}$$

Calculate FAD parameters:

$$S_r := \frac{\sigma}{\sigma_{\text{flow}}} \quad K_r := \frac{KI}{KI_c}$$

$$\frac{K_r}{S_r} = 0.269$$

Calculate angle of the loading path from horizontal, α :

$$\alpha := \text{atan}\left(\frac{K_r}{S_r}\right) \quad \alpha = 0.262 \text{ radians}$$

$$\text{deg}\alpha := \left(\frac{180}{\pi}\right) \cdot \alpha \quad \text{deg}\alpha = 15.037 \text{ degrees}$$

Plot failure assessment diagram:

Plot a line "deg α " degrees from horizontal:

$$i := 0..101 \quad S_{r_i} := 0.01 \cdot i \quad K_{r_i} := S_{r_i} \cdot \tan(\alpha)$$

Define failure curve:

$$j := 0..99 \quad S_{r_j} := 0.01 + 0.01 \cdot j$$

$$K_{r_j} := \left[S_{r_j} \cdot \left[\left(\frac{8}{\pi^2} \right) \cdot \ln \left[\sec \left[\left(\frac{\pi}{2} \right) \cdot S_{r_j} \right] \right] \right]^{\frac{-1}{2}} \right]$$

Reference 10, p.475

$S_{r_{100}} := 1 \quad K_{r_{100}} := 0$ the last data point specified manually, due to the round-off limits in preceding calculation

Originator: Initials: Z.C.
Checker: Initials: VB

Date: 4/30/02
Date: 4/30/02

Title: Comparison of the Traditional Strength of Materials Approach to Design with the Fracture Mechanics Approach

Calculate limit load for plane strain (Ref. 10, p. 609, case (d)):

$$\sigma := \frac{P}{1.455 \cdot \eta \cdot B \cdot b}$$

Calculate FAD parameters:

$$SrI := \frac{\sigma}{\sigma_{flow}} \quad KrI := \frac{KI}{KIc}$$

$$\frac{KrI}{SrI} = 0.365$$

Calculate angle of the loading path from horizontal, α :

$$\alpha I := \operatorname{atan}\left(\frac{KrI}{SrI}\right) \quad \alpha I = 0.35 \text{ radians}$$

$$\deg \alpha I := \left(\frac{180}{\pi}\right) \cdot \alpha I \quad \deg \alpha I = 20.032 \text{ degrees}$$

Originator: Initials: Z.C.
Checker: Initials: VB

Date: 4/30/02
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Title: Comparison of the Traditional Strength of Materials Approach to Design with the Fracture Mechanics Approach

Plot failure assessment diagram:

Plot a line "deg α 1" degrees from horizontal:

$$i := 0..101 \quad Srl_i := 0.01 \cdot i \quad Krl_i := Srl_i \cdot \tan(\alpha 1)$$

Figure VI-1. Failure assessment diagram for Alloy 22 (SENT specimen)

The figure given above shows the failure curve and the two loading paths defined by the angles calculated above. The dash line and the dotted line indicate the solution for the plane stress and the plane strain cases, respectively. These two cases provide lower and upper bounds for the problem; the real solution lies between them. The figure shows that the bounding range for the solution falls within a region of failure that is dominated by ductile collapse, not brittle fracture (see Ref. 10, p. 476).

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