Super-Resolution Algorithms

for Nondestructive Evaluation Imaging

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Go Boilers!!!

Purdue’s “All-American” Marching Band
Agenda

• Problem Definition:
  - Ultrasonic NDE measurements
  - The spatial resolution problem

• Impulse Response Estimation for Enhancing Spatial Resolution
  - Mitigate “ringing” due to the transducer and propagation paths

• Bandlimited Spectrum Extrapolation for Super-Resolution

• Examples of Processing Results
Ultrasonic Pulse-Echo Signals (A-Scans) Are Distorted By the Transducer and the Propagation Paths ("Ringing")

The Ideal Reflection Is An Impulse Sequence

Time $\propto$ Distance
Ultrasonic Pulses Are *Bandlimited* by the Transducer

==>

The Pulses “*Ring*”, Reducing Spatial Resolution

\[ y(t) = \text{Reflected Pulse} \]

Front Reflection:  

Flaw Reflection:  

Back Reflection:

\[ |Y(f)|^2 = \text{DFT of the Reflected Pulse} \]
We Define Ultrasonic $A$, $B$, and $C$-Scans Used in Nondestructive Evaluation (NDE) Studies:

**A-Scan** $x(t)$
*(A Single Waveform)*

**B-Scan**
*(Family of A-Scans)*

**C-Scan**
*(Horizontal Slice At Depth $z$: Use A Time Gate)*

**3D Volume**
*(Family of B-Scans)*

$Time \propto Distance$
The Reference Scatterer is Chosen to Provide the Transducer / Path Response in the Absence of a Flaw

Desired properties of the reference scatterer:
• Reflects back most of the energy
• Resembles some feature associated with the flaw environment

Reference Signal
\[ x(t) \]
\[ y(t) \]

Front or Back Surface Reference

Corner Reflector Reference
We Use a Reference Scatterer to Help Remove Distortion: Conceptually, This is a “System Identification” Problem

**Experiment to Measure the Scattered Signal** \( Y(f) \)

\[
G(f) \xrightarrow{\text{Sending Transducer}} TF(f) \xrightarrow{\text{Forward Propagation Path (Beam Spreading)}} PF(f) \xrightarrow{\text{Scattering From Flaw}} H(f) \xrightarrow{\text{Return Propagation Path (Spherical Spreading)}} PR(f) \xrightarrow{\text{Receiving Transducer}} Y(f)
\]

**Experiment to Measure the Reference Signal** \( X(f) \)

\[
G(f) \xrightarrow{\text{Sending Transducer}} TF(f) \xrightarrow{\text{Forward Propagation Path (Beam Spreading)}} PF(f) \xrightarrow{\text{Reference Scatterer}} HR(f) \approx 1 \xrightarrow{\text{Return Propagation Path (Spherical Spreading)}} PR(f) \xrightarrow{\text{Receiving Transducer}} X(f)
\]

**Conceptually:**

\[
\frac{Y(f)}{X(f)} = \frac{TF(f)PF(f)H(f)PR(f)TR(f)}{TF(f)PF(f)(1)PR(f)TR(f)} \approx H(f) \xrightarrow{F^{-1}} h(t)
\]
System Identification: Estimate the Impulse Response $\hat{h}(t)$

Given: $x(t)$ and $u(t)$  Estimate: $\hat{h}(t)$

$$e(t) = u(t) - \hat{u}(t)$$
The Inverse Problem Is Very Difficult

We Must Regularize the Problem

- Ill-Posed
  (Infinite Number of possible solutions)
- Bandlimited Transducer Spectral Response
- Ill-Conditioned - Numerical Errors Due to Spectral Zeros
The **System Model** and **Processing Algorithms** Are Summarized in Block Diagrams

**System Model**

\[ x(t) \rightarrow \text{System } h(t) \rightarrow y(t) \rightarrow u(t) \]

\[ n(t) \]

\[ h(t) \rightarrow \text{Front} \rightarrow \text{Flaw} \rightarrow \text{Back} \]

The Ideal Impulse Response is a Series of Delta Functions

**Processing Algorithms**

\[ x_0(t) \rightarrow \text{Pre-Processing} \rightarrow x(t) \rightarrow \text{System Identification (Wiener)} \rightarrow \hat{h}(t) \rightarrow \text{Band-Limited Spectrum Extrapolation} \rightarrow \hat{h}_e(t) \]

\[ u_0(t) \rightarrow u(t) \]

Estimated Impulse Response

Spectrum Extrapolated Est. of Impulse Response

Grace Clark, Ph.D.
Our Objective is to Improve Temporal Resolution by **Extrapolating Spectra**

- The transducer bandlimits our signals
  - System identification solutions are not unique
  - System identification solutions are valid only in a finite frequency interval \([f_1, f_2]\).
    They give us the optimal least squares solution, given the bandwidth of the transducer.
  - We can never obtain narrow impulses in the time domain

- We wish to extrapolate spectra beyond \([f_1, f_2]\).
  - This can allow us to obtain better approximations to impulses in the time domain.

- We propose to extrapolate the spectra of:
  \[ u(t) \quad \text{The measured pulse-echo signal} \]
  \[ \hat{h}(t) \quad \text{The estimated impulse response} \]
We Use *Bandlimited Spectrum Extrapolation* To Improve *Spatial Resolution*

<table>
<thead>
<tr>
<th>Ideal</th>
<th>Measured or Estimated</th>
</tr>
</thead>
<tbody>
<tr>
<td>$h(t)$ <em>Ideal Impulse Response</em></td>
<td>$\hat{h}(t)$ <em>Estimated Impulse Response</em></td>
</tr>
<tr>
<td>$</td>
<td>H(f)</td>
</tr>
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We Trust Measured or Estimated Spectrum on the Region $f_1$ to $f_2$.
Complex Variable Theory Gives Us a Solid Theoretical Basis for Spectrum Extrapolation

- Our temporal signals have \textit{bounded support}:
  - They are transient (finite length) signals in the time domain

- The Fourier Transform of a signal with bounded support is \textit{ANALYTIC} (continuous, all derivatives exist).

- If any analytic function in the complex plane is known exactly in an arbitrarily small (but finite) region of that plane, then the \textit{entire function} can be found \textit{(uniquely)} by \textit{ANALYTIC CONTINUATION}.

\[ \hat{H}(f) \]

Region We Trust

\[ f_1 \quad f_2 \quad f \]
Analytic Continuation Algorithms are Hypersensitive to Noise - *Must Regularize*

- Prior knowledge can be used as constraints to regularize the problem

- Iterative algorithms (*method of successive approximations*) are slow, not unique, but can incorporate constraints.

- Non-iterative algorithms are faster, but can’t usually incorporate constraints.

- Often, it is not necessary to determine the inverse of the distortion operator
  - Good for nonlinear or time-varying operators
We Use an Iterative Algorithm for Regularized Analytic Continuation

• Estimate the impulse response at the next iteration as a function $F$ of the impulse response at the last iteration:

$$h_{k+1}(t) = Fh_k(t), \quad \text{for } k = 0, 1, 2, \ldots$$

• Iterate between the time and frequency domains

  \textit{(Method of Alternating Orthogonal Projections)}

• Convergence is proved using contraction mapping theorems from functional analysis

• Use an \textit{“adaptive algorithm”} that assumes the impulse response to be a sequence of impulses - \textit{constrain the time domain signal to be an impulse train}:

$$
\begin{align*}
  h(t) &= \sum c_i \delta(t - t_i) \\
  u(t) &= \sum c_i x(t - t_i) + n(t)
\end{align*}
$$

\[ \text{Ideal Impulse Response} \]
We Constrain the Temporal and Spectral Support Using Projection Operators

**Temporal Projection Operator**

\[ d(t) \]

\[ -T \quad 0 \quad T \]

**Spectral Projection Operators**

\[ P_T(f) = \text{Envelope}\left\{ \frac{|X(f)|}{\max|X(f)|} \right\} \]

\[ P_T(f) = \text{Envelope}(X(f)) \]

\[ P_R(f) \]

\[ -f_2 \quad -f_1 \quad 0 \quad f_1 \quad f_2 \]
ith Iteration of the Spectrum Extrapolation Algorithm: 
**Alternating Orthogonal Projections, w/Adaptive Algorithm**

\[ n = \text{Time Index} = -(N/2-1), \cdots, -2, -1, 0, 1, 2, \cdots, N/2-1 \]

\[ k = \text{Frequency Index} = -(N/2-1), \cdots, -2, -1, 0, 1, 2, \cdots, N/2-1 \]

\[ P(k) = (-k_1, -k_2) \cup (k_1, k_2) \]

\[ d(n) = [-T, T] \]
We Constructed a “Phantom” Part - *Aluminum Block* Containing *Flat-Bottom Holes*
We Can Combine CAD Models With 3-D Data To Clarify Ultrasonic Evaluation Results

3-D Ultrasonic Data Det

3-D data and CAD Model-Solid

3-D data and CAD Model-Lines
Processing Results Show Great \textit{Reduction of Ringing}, and \textit{Enhancement of Range Resolution}

The Measured Pulse-Echo Signal Contains Transducer \textit{Ringing}, Which Limits Resolution

The \textit{Estimated Impulse Response} Shows the Optimal Ringing Reduction Possible, Using the Band-Limited Transducer Spectrum

The \textit{Spectrum-Extrapolated Impulse Response Estimate} Allows \textit{Super-Resolution} Because We Now Have a Broader Effective Signal Spectrum
System Identification and Spectrum Extrapolation Results Are Summarized for the Flat-Bottom Hole Phantom Signals

Original Wiener BSE

Amplitude

Time

Time

Time

Time

Time

Time

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Time

Time

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Time

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Time

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Time

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The Processed 3D Volume Shows Greatly-Enhanced Spatial Resolution (System ID Only)

Raw 3D Volume

Processed 3D Volume (System ID Only)
Ultrasonic Pulse-Echo Signals Are Distorted by the Transducer and the Propagation Paths
Ultrasonic Pulse-Echo Signals Are Distorted by the Transducer and the Propagation Paths

Welds Are Scanned for Penetration Thickness

Example: W79 Weld

W79 Weld Signals

Raw Signal

Estimated Impulse Response

Bandwidth-Extrapolated Impulse Response

Time (Proportional to Distance)