The Influence of Wafer Elasticity on Acoustic Waves During LIGA Development

Aili Ting

Prepared by
Sandia National Laboratories
Albuquerque, New Mexico 87185 and Livermore, California 94550

Sandia is a multiprogram laboratory operated by Sandia Corporation, a Lockheed Martin Company, for the United States Department of Energy's National Nuclear Security Administration under Contract DE-AC04-94AL85000.

Approved for public release; further dissemination unlimited.
The Influence of Wafer Elasticity on Acoustic Waves During LIGA Development

Aili Ting
Fluid and Thermal Science Department
Sandia National Laboratories
P.O. Box 969
Livermore, California 94551-0969

ABSTRACT

During acoustically stimulated LIGA development, a wafer receives sound waves from both sides at a wide variety of incidence angles that vary in time depending on the orientation of the wafer relative to the multiple transducers that are typically actuated in a periodic sequence. It is important to understand the influence of these variables on the transmission of energy through the wafer as well as the induced motion of the wafer itself because these processes impact the induced acoustic streaming of the fluid within features, the mechanism presently thought responsible for enhanced development of LIGA features. In the present work, the impact of wafer elasticity on LIGA development is investigated. Transmission waves, wafer bending waves, and the related concepts such as critical bending frequency, mechanical impedance, coincidence, and resonance, are discussed.

Supercritical-frequency incident waves induce supersonic bending waves in the wafer. Incident wave energy is channeled into three components, transmitted, reflected and energy deposited to the wafer, depending on the wafer material, thickness and wave incidence angle. Results show at normal incidence for a 1-mm PMMA wafer, about 47% of the wave energy is deposited in the wafer. The wafer gains almost half of the incident energy, a result that agrees well with the Bankert et al measurements.

In LIGA development, transmitted waves may sometimes produce strong acoustic motion of the developer on the wafer backside, especially for the so-called coincidence case in which almost all incident wave energy transfers to the backside. Wafer bending waves cause wafer oscillation at high frequency, promoting the development process, but features shaking may weaken their attachments to the substrate. Resonance is not likely for the entire wafer, but may occur in short and wide wafer feature columns, which are least likely to break away from the substrate, perhaps resulting in good agitation of the fluid in adjacent feature cavities.
CONTENTS

1. INTRODUCTION ................................................................. 6
2. WAVES IN SOLIDS EXCITED BY ACOUSTIC WAVE ................. 7
3. WAVE SPEEDS ........................................................................ 8
4. CRITICAL BENDING WAVE FREQUENCY ................................ 9
5. WAVE IMPEDANCE OF FLUID AND MECHANICAL IMPEDANCE OF STRUCTURE ........................................ 12
6. COINCIDENCE ANGLE .......................................................... 16
7. TRANSMISSION LOSS AND ENERGY DEPOSITION IN A WAFER .................. 17
8. FORCED AND FREE BENDING WAVES IN FINE PLATE .......... 26
9. EXAMPLES OF NATURAL FREQUENCIES AND NATURAL MODES ........... 27
10. DISCUSSION ......................................................................... 30
ACKNOWLEDGMENTS .............................................................. 32
REFERENCES ............................................................................. 33

TABLES

Table 1. Response of Rigid Wafer and Elastic Wafer to Acoustic Waves .................. 6
Table 2. Maximum PMMA Thickness without a Coincidence Angle ...................... 17
Table 3. PMMA Feature Column Base Natural Frequencies ................................. 28
Table 4. PMMA Wafer’s First Three Natural Frequencies .................................. 29

FIGURES

Figure 1. Layout of Development Tank ............................................... 7
Figure 2. Plate Deformation Patterns by Waves ....................................... 8
Figure 3a. Dispersion Curves for Bending Waves in PMMA Wafer .................. 10
Figure 3b. Dispersion Curves for Bending Waves in Silicon Wafer ................. 10
Figure 4. Wave Numbers for Bending Waves in PMMA Wafer .................... 10
Figure 5a. PMMA Wafer Bending Wave Speed and Critical Frequency .......... 11
Figure 5b. Silicon Wafer Bending Wave Speed and Critical Frequency .......... 11
Figure 6. Acoustic Wave Interacts with an Infinite Plate ............................... 12
Figure 7a. PMMA Wafer Mechanical Impedance to 1 MHz Acoustic Waves .... 15
Figure 7b. Silicon Wafer Mechanical Impedance to 1 MHz Acoustic Waves .... 15
Figure 8. Regions of Wafer Mechanical Impedance for Wave Incidence Angle At a Single Critical Frequency ................................................................. 16
Figure 9. PMMA and Silicon Wafer Thickness Changes with Coincidence Angle 17
Figure 10. Transmission and Reflection of Acoustic Wave .............................. 18
Figure 11. Energy Distribution in Transmission, Reflection and Energy Deposition on Wafer ........................................................ 20
Figure 12. PMMA Wafer Transmission, Reflection, and Wafer Energy Coefficients for 1 MHz Waves ................................................................. 21
Figure 13. Silicon Wafer Transmission, Reflection, and Wafer Energy Coefficients for 1 MHz Waves ................................................................. 23
Figure 14a. PMMA Wafer Transmission Coefficient for 1 MHz Acoustic Waves 24
Figure 14b. Silicon Wafer Transmission Coefficient for 1 MHz Acoustic Waves 24
Figure 15a. PMMA Wafer Sound Reduction Index for 30° Wave incidence Angle 25
Figure 15b. Silicon Wafer Sound Reduction Index for 30° Wave incidence Angle 25
Figure 16a. Sound Reduction Index for 1-mm PMMA Wafer in Liquid Developer 26
Figure 16b. Sound Reduction Index for 1-mm Silicon Wafer in Liquid Developer 26
Figure 17. Natural Flexural Modes of a Beam with one Clamped and one Free End 29
Figure 18. The First Five Natural Flexural Modes for a Square Plate with One Clamped Side 30
**INTRODUCTION**

In previous work, we modeled the acoustic agitation in a LIGA development tank for a rigid wafer and obtained the induced acoustic pressure wave pattern in the tank which agreed well with the measured power intensity distribution.\[^{1,2}\] However, a rigid wafer does not have the same response as an elastic wafer. Actually, the response of an elastic wafer is far more complicated, resulting not only in reflected waves as for a rigid wafer, but also in transmitted waves and forced wafer bending waves that depend on the wafer stiffness and inertia. In addition, the finite wafer size produces natural modes and natural frequencies that may interact with the incident waves resulting in resonance. The related concepts such as critical frequency, mechanical impedance, and coincidence, play important roles. Table 1 lists a comparison between the response of rigid and elastic wafers for various aspects of wave-wafer interaction.

**Table 1. Response of Rigid Wafer and Elastic Wafer to Acoustic Waves**

<table>
<thead>
<tr>
<th>Compared Items</th>
<th>Rigid Wafer</th>
<th>Elastic Wafer</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Response Waves in Fluid</strong></td>
<td>Full reflection wave</td>
<td>Reflection and transmission waves</td>
</tr>
<tr>
<td><strong>Energy</strong></td>
<td>Incident energy = Reflection energy</td>
<td>Incident energy = Reflection energy + Transmission energy + Energy deposition to wafer</td>
</tr>
<tr>
<td><strong>Wave in Wafer</strong></td>
<td>None</td>
<td>Bending waves may be induced</td>
</tr>
<tr>
<td><strong>Wafer Movement</strong></td>
<td>None</td>
<td>Wafer particle vibration</td>
</tr>
<tr>
<td><strong>Wafer Critical Frequency</strong></td>
<td>None</td>
<td>Supercritical and subcritical cases</td>
</tr>
<tr>
<td><strong>Wave Impedance At Interface</strong></td>
<td>Fluid impedance at rigid body becomes (\infty)</td>
<td>Fluid impedance and Mechanical impedance</td>
</tr>
<tr>
<td><strong>Wafer Stiffness and Inertia</strong></td>
<td>N/A</td>
<td>Stiffness or mass controlled response</td>
</tr>
<tr>
<td><strong>Coincidence</strong></td>
<td>N/A</td>
<td>Stiffness offsets Inertia</td>
</tr>
<tr>
<td><strong>Resonance</strong></td>
<td>N/A</td>
<td>Natural frequencies are excited</td>
</tr>
<tr>
<td><strong>Transmission Loss or Sound Reduction Index</strong></td>
<td>No transmission</td>
<td>Provides Reflection energy and Energy deposition to wafer</td>
</tr>
<tr>
<td><strong>Wafer Back Agitation</strong></td>
<td>Some areas may not be reached by reflected waves from tank and free surface</td>
<td>By transmitted waves, wafer bending waves, and reflected waves from tank and free surface</td>
</tr>
</tbody>
</table>

In the present work, the impact of wafer elasticity on LIGA development is investigated through analysis and discussion of the issues outlined in Table 1.

Acoustically stimulated development tanks typically have several rectangular transducer bars mounted in the floor or the sidewall of the tank. To avoid overheating of these acoustic drivers they are usually operated in sequence with each one powered for only a
few seconds at a time. The wafer is usually held either vertically or horizontally at a fixed location, as illustrated schematically in Figure 1. Thus, over the course of a transducer cycle a vertical wafer receives waves from both sides at a wide variety of incidence angles. As a result, it is important to understand the transmission of energy through the wafer to fluid on the other side as well as the induced motion of the wafer itself. Both of these processes impact the induced acoustic streaming of the fluid within features, the mechanism presently thought responsible for enhanced development of LIGA features. In addition to this primary enhancement mechanism it is also possible that other processes such as acoustic bubble growth and collapse or high frequency substrate or feature motion may play a role that is not yet understood.

The remainder of this paper is organized as follows. The next two sections provide a brief introduction to wave motions and wave speeds in solid materials. This is followed by three sections (4. Critical Bending Frequency, 5. Wave Impedance, and 6. Coincidence) which lay the background needed to fully understand section 7. (Transmission Losses and Energy Deposition in a wafer). All of these results are based upon wave transmission in plates of infinite extent. Sections 8 and 9 address the issue of the finite wafer size and also apply the same methodology to the scale of a single feature element such as a free standing PMMA column. Section 10 summarizes all of the results. Some readers may wish to read lightly over Sections 4 through 6, and focus their attention on the applications in Sections 7 through 10.

WAVES IN SOLIDS INDUCED BY ACOUSTIC WAVES

During the LIGA development process, acoustic waves incident on a rigid wafer would result in full reflection of acoustic energy from the wafer. However, this is not true for an elastic wafer. In fact, elastic waves are induced in the wafer; this energy exchange between acoustic waves and an elastic wafer may accelerate the development process, but may also have negative effects. To study this, we consider an elastic wafer and examine the elastic waves that can be excited by an incident acoustic wave.

Two basic waves, longitudinal waves and shear waves, and many of their hybrid combinations are possible in solids. Bending waves (flexural waves or transverse waves) can only be exited in an elastic wafer by incident acoustic waves and exchanges of energy with the fluid.31

Figure 1 shows the wave patterns and particle movements of the two basic waves and the bending wave. Longitudinal waves and acoustic waves are compressive waves enabled by the compressibility of the media, while shear wave can only exist in solids since fluids cannot withstand shear stresses or store shear energy.
No net transport of the medium will result in wave propagation, because particles of the medium only vibrate around their equilibrium positions. The particle displacements in a longitudinal wave are along the direction of wave propagation except for the Poisson contraction or expansion in a quasi-longitudinal wave, as shown in figure 2. Even though the shapes of the shear wave and bending wave look alike, the particle displacements in a bending wave are along the direction perpendicular to the wave propagation and include the effects of rotary inertia and shear deformation. Note that the deformation in the figure is exaggerated to be clearly seen.

Figure 2. Plate Deformation Patterns by Waves

Among the waves propagating in bars, beams, and plates, bending waves play a significant role in structure-fluid interactions. When the plate is acoustically excited, acoustic waves transfer energy to the plate by inducing a bending wave propagating in the plate. When a plate is mechanically excited, bending waves radiate acoustic energy into the fluid. Since the particle displacements and velocities are along a direction perpendicular to wave propagation, they exchange energy effectively with the adjacent fluid particles.

Although the wafer placed in the development bath typically consists of a patterned PMMA layer bonded to a silicon substrate, the important principle will be illustrated for a wafer having uniform material properties.

**Waves Speeds**

The speeds of wave propagation are \( C_L = (E/\rho)^{1/2} \) for longitudinal waves in a beam, \( C_L = (E/\rho (1 - v^2))^{1/2} \) for quasi-longitudinal waves in a plate, and \( C_s = (G/\rho)^{1/2} \) for shear waves (where \( E \) is Young’s modulus, \( G \) is the shear modulus, \( \rho \) is the density, and \( v \) is Poisson’s ratio). These wave speeds depend only on the material and thermal properties of the media and are independent of frequency. Thus, they are non-dispersive waves, so the wave shapes will always remain the same for any wave frequency. The speed of non-dispersive waves may also be expressed as, \( \omega/k \), the ratio of circular frequency to wave number where \( k = 1/\lambda \) and \( \lambda \) is the wavelength.
Unlike non-dispersive waves, the bending wave speed, \( C_B = \left[ \pi C_L f h / (3)^{1/2} \right]^{1/2} \) (frequency, \( h \) thickness) and hence \( C_B \sim (fh)^{1/2} \), depends not only on medium properties but also on wave frequency (or wave number) and the plate (or beam) thickness. Therefore, the wave speed is greater for a thick wafer than a thin wafer due to the stiffness of the wafer, and is greater for a high frequency wave than a low frequency wave due to the more efficient energy exchange. Such waves are said to be dispersive because different frequency components travel at different speeds.

The speed of wave energy propagation is the group speed, \( C_g = d\omega / dk \). It is the same as the wave speed for a non-dispersive wave \( (d\omega / dk = \omega / k) \) but is different from the wave speed for a dispersive wave. The group speed of bending waves can be obtained from the expression for \( C_B \) as \( C_g = 2 C_B \), twice as fast as the bending wave speed, so energy transport of bending waves is very efficient, especially for high-frequency waves and for thick wafers. The dispersive nature of bending waves is especially important in fluid-structure coupling, as in the following example.

The LIGA group at Sandia Laboratories used Good Fellow Perspex brand CQ grade PMMA sheets having the following properties: \( E = 2.4-3.3 \times 10^9 \text{ Pa} \), \( \rho = 1190 \text{ kg/m}^3 \), \( \nu = 0.4 \), so \( C_t = 1600 \text{ m/s} \), \( C_L = 1800 \text{ m/s} \), and \( C_s = 950 \text{ m/s} \). For PMMA at \( f = 1 \text{ MHz} \), \( C_B = 1800 \text{ m/s} \), \( C_g = 3600 \text{ m/s} \) for \( h = 1 \text{ mm} \), and \( C_B = 402.5 \text{ m/s} \), \( C_g = 805 \text{ m/s} \) for \( h = 50 \text{ \mu m} \). The ratio of bending wave speed to quasi-longitudinal wave speed is \( C_B / C_L = (\pi f h / C_L (3)^{1/2})^{1/2} \), which becomes \( (10^3 f h)^{1/2} \) for PMMA. Therefore, at \( f = 1 \text{ MHz} \), a 1-mm wafer is more efficient in transferring wave energy than a 50-\( \mu \text{m} \) thick feature column since \( C_B = C_L \) for the wafer and \( C_B = C_L / (20)^{1/2} \) for the feature column, indicating two very different wave energy propagation speeds. Similarly, at \( h = 1 \text{ mm} \), a 1 MHz wave is more efficient in transferring wave energy than a 0.5 MHz wave. The corresponding properties of the silicon substrate are \( E = 1.3-1.9 \times 10^{11} \text{ Pa} \), \( \rho = 2330 \text{ kg/m}^3 \), \( \nu = 0.2 \), so \( C_t = 8542 \text{ m/s} \), \( C_L = 8718 \text{ m/s} \), and \( C_s = 5241 \text{ m/s} \). The qualitative observations made for PMMA also hold for a silicon wafer.

**CRITICAL BENDING WAVE FREQUENCY**

In this section we show that 1 MHz sound waves can produce bending waves only in PMMA wafers having a thickness exceeding 689 \( \mu \text{m} \) and in silicon wafers having a thickness exceeding 142 \( \mu \text{m} \). An incident acoustic wave may induce a bending wave having the same wave speed and frequency in an infinitively large plate when the incident wave frequency is greater than the plate’s critical frequency. A bending wave critical frequency of a solid plate, \( f_C \), is defined as the frequency at which the bending wave speed and frequency are equal to that of an acoustic wave in the adjacent fluid, so \( f_C = c^2 (3)^{1/2} / (\pi C_L h) \) since \( C_B = c, f_B = f \). Therefore, \( f_C \) is inversely proportional to the thickness \( h \) of the plate (or beam), and the product \( h f_C \) depends only on the properties of material and medium. When incident acoustic waves have multiple frequencies, then the lowest critical frequency is important. No bending wave with a frequency lower than this critical frequency will be induced. This will be explained after the following paragraph.
Figure 3 shows bending wave-speed curves, changing with wave frequency for a variety of PMMA and silicon wafer thickness compared with acoustic, longitudinal, and shear wave speeds. Critical frequency is the frequency at which the bending wave curve for each PMMA thickness intersects the acoustic wave line. Acoustic wave speed is taken as 1500 m/s (the value from water), a value between a higher longitudinal wave speed and a lower shear wave speed (the Young’s modulus $E$ is generally much greater than the shear modulus $G$). Unlike the constant wave speeds (non-dispersive) for acoustic, longitudinal, and shear waves, the bending wave speed spreads widely and is proportional to $(hf)^{1/2}$. Note that longitudinal wave speed is too high to plot in Figure 3b.

![Figure 3a. Dispersion Curves for Bending Waves in PMMA Wafer](image)

![Figure 3b. Dispersion Curves for Bending Waves in Silicon Wafer](image)

Similarly, figure 4 presents the wave number curves. The large wave number results in the small wave speed, so the bending wave curves sequenced from top to bottom in Figure 3a correspond to the curves from bottom to top in Figure 4. In the figures, the critical wave frequency is 575 kHz for a 1.2-mm PMMA plate and 1723 kHz for a 0.4-mm plate, so the thick wafer has the low critical frequency due to the wafer stiffness $(h)^{-1}$. The critical frequency of a 50 or 100-μm PMMA feature column is far beyond 1 MHz, indicating that no induced bending waves in the small feature column by an incident 1–MHz acoustic waves. However, small feature columns may differ from wide plates because their dimensions are much smaller than the acoustic wavelength. On the other hand, the critical frequencies of a 400 or 1200-μm silicon wafer is far below 1 MHz, as indicated in Figure 3b.

![Figure 4. Wave Numbers for Bending Waves](image)
Figure 5a gives critical frequency and bending wave speed changing with PMMA and silicon wafer thickness, respectively. At the upper left corner of the Figure 5a, quasi-longitudinal wave speeds of Good Fellow CQ PMMA sheet are presented. The critical frequency decreases as the thickness of PMMA increases due to the \( h^{-\frac{1}{2}} \) law. For any PMMA sheet thickness, draw a vertical line upward to intersect the critical frequency curve \( f_c \) and the appropriate bending wave curve \( C_b \). Whether or not bending waves will be induced in the PMMA plate depends on the point on \( f_c \) that is lower or higher than the point on \( C_b \). In Figure 5b, critical frequencies for silicon are much lower due to its high longitudinal wave speed. Thus, bending waves can only be induced by 1 MHz waves in a PMMA wafer having thickness greater than 689 \( \mu \)m and in a silicon wafer having thickness greater than 142 \( \mu \)m.

![Figure 5a. Bending Wave Speed and Critical Frequency of PMMA Wafer \((C_b=1800 \text{ m/s})\)](image)

![Figure 5b. Bending Wave Speed and Critical Frequency of Silicon Wafer \((C_b=8718 \text{ m/s})\)](image)

To explain this, the wavelength ratio of bending and acoustic waves, \( \lambda_b / \lambda = (f / f_c)^{\frac{1}{2}} \), is a very important parameter in the fluid-solid wave interaction for any incident wave frequency \( f \) and its wavelength \( \lambda \). A bending wave wavelength is larger than the acoustic wavelength as acoustic frequency is higher than the critical frequency: \( \lambda_b > \lambda \), for \( f > f_c \), and smaller as acoustic frequency is lower than the critical frequency: \( \lambda_b < \lambda \), for \( f < f_c \). In the latter case, bending waves are theoretically impossible.

Consider an incident acoustic wave encountering an infinite plate (x-axis) with an incidence angle \( \theta \) (see Figure 6). If an induced bending wave of the same frequency is propagating in the plate, then at the fluid-plate interface, the acoustic wave number vector \( k \) is decomposed into a bending wave number \( k_b \), and a wave number \( k_z \), denoting a wave perpendicular to the wafer. Thus \( k^2 = k_b^2 + k_z^2 \) and \( k_b = k \sin \theta \). This requires \( k_b < k \), implying \( C_b > c \), \( \lambda_b > \lambda \) and hence \( f > f_c \). Therefore, only an acoustic wave having frequency higher than the critical frequency of the plate is able to induce a forced bending wave. This induced bending wave is always supersonic, and its wavelength is always greater than the acoustic wavelength, as shown in figure 6. In other words, no
bending wave will be induced in the plate when the acoustic wavelength is greater than the bending wavelength since a real bending wave number $k_B$ does not exist. The special case in which $k_B = k_\lambda, \theta = \pi/2$, is excluded and will be explained later.

To explain the above statement rigorously, assuming that at the interface the acoustic pressure can be expressed approximately to a plane wave pressure,

$$p(x, y, t) = p_{\text{max}} \exp(-i k_B x) \exp(i k_{xy}) \exp(i \omega t).$$

By the linear $Y$-momentum equation of the fluid and the fluid-plate interface matching condition, it follows that

$$u_y = (u_y)_{\text{fluid}} = \frac{-\partial p / \partial y}{i \omega \rho_0} = \frac{k \rho_0 (u_y)_{\text{max}}}{(k^2 - k_B^2)^{1/2}}.$$ 

Note that here $k_{xy} = -(k^2 - k_B^2)^{1/2}$ is toward the plate. When $f > f_C, k_B < k, \lambda_B > \lambda$, $p_{\text{max}}$ is real, and the acoustic wave at the interface does induce a forced bending wave in the plate. When $f < f_C, k_B > k, \lambda_B < \lambda$, $p_{\text{max}}$ is imaginary and the $\exp(i k_{xy})$ term in $p(x, y, t)$ becomes $\exp[-(k_B^2 - k^2)^{1/2} y]$ that does not represent a wave, but a fast decaying factor toward the plate. In this case, only a surface wave is formed on top of the plate, i.e., no bending wave is induced in the plate or no energy exchanges with the plate. When $f = f_C$ at critical frequency, $k_B = k, \lambda_B = \lambda$, $\sin \theta = 1, \theta = \pi/2$, wave propagates parallel to the plate, so $p_{\text{max}} \to \infty$. In reality, no bending wave will be possible in this case because the extremely large wave pressure indicates the extremely large resistance for energy transfer from the fluid to the plate.

For a plate (or beam) with a finite size, the situation becomes more complicated. Excited by acoustic pressure, the induced forced bending wave vibrates at the incident acoustic frequency that is independent of natural frequencies. The natural frequencies are a function of the plate properties that depend on its mass or inertia, stiffness or elasticity, and boundary conditions. Resonance will be encountered when the acoustic frequency coincides with one of the natural frequencies. Therefore, although a finite plate having length scale very much greater than the incident acoustic wavelength can be approximately treated as an infinite plate, a wafer of 3-in radius comparing to wavelengths of 1.5 or 2 mm (for 1 or 0.75 MHz waves, respectively) the feature columns on the wafer may still need to consider its boundary conditions and its natural frequencies.

**Wave Impedance of Fluid and Mechanical Impedance of Structure**

In this section we review the concepts of wave impedance and coincidence angle and how they are related to energy transmission at a fluid–solid interface. In wave-solid
interaction, both fluid acoustic impedance and mechanical impedance of a solid body must be included to investigate the response of either the solid body or the fluid when a wave in one of them excites the other. Wave impedance quantifies the response of a fluid to acoustic waves, while mechanical impedance quantifies the response of a solid to incident acoustic waves by its inertia and stiffness. Both inertia and stiffness are closely related to incident frequency by the interface conditions. In general, high impedance waves do not transmit acoustic energy efficiently but may still be efficient at transmitting vibrational energy because of high particle velocities. Low impedance waves, in contrast, allow a matching between solid body waves and acoustic waves in the adjacent fluid. If the mechanical impedance of a structure matches the acoustic impedance of the fluid, then the energy can be effectively exchanged between the two.

The fluid acoustic impedance is defined as the ratio of the complex wave pressure and velocity at any interface for a given frequency, $Z_f = (p / V)_{\text{interface}}$. The inverse of the impedance is the mobility. By using the exponential representation of harmonic time dependence, the real part of the impedance indicates the system resistance to the wave, while the imaginary part indicates the system reactance to the wave, or the phase difference between the pressure and the velocity fields. Therefore, $Z_f$ describes how well the velocity field will cooperate with the pressure field. The unit of impedance is the same as the unit of a fluid flux [kg/m²s].

As in the example of previous section, fluid acoustic impedance of an infinitely large flat fluid-solid interface can be derived from the same linear momentum equation of the fluid, projecting in the direction perpendicular to the interface,

$$Z_t = \omega \rho_0 / (k \cos \theta) = \rho_0 c / \cos \theta = \rho_0 \omega / (k^2 - k_B^2)^{1/2} \quad \text{as } k > k_B,$$

$$Z_t \to \rho_0 c \quad \theta \to 0, \quad \text{as } k_B \to 0, k \to k,$$

$$Z_t \to \infty \quad \theta \to \pi / 2, \quad \text{as } k_B \to k, k \to 0,$$

$$Z_t = i \rho_0 \omega / (k_B^2 - k^2)^{1/2} \quad \text{as } k < k_B, \text{ no } \theta \text{ available, no wave}.$$

From above expression, the fluid acoustic impedance is the fluid mass flux the waves are subjected to along the path of their propagation. The minimum fluid acoustic impedance for the LIGA developer is about $1.5 \times 10^6$ [kg/m²s] in normal incident case $\theta = 0$. This impedance value is large due to the high acoustic wave speed, so the order of magnitude of fluid impedance-fluid flux ratio is $Z_f / \rho_0 v = \rho_0 c / \rho_0 v = c / v \approx 1$.

Similar to the discussion for critical frequency, when $k > k_B$, $C_B$ is supersonic, $k$ and $Z_t$ are real, so pressure and velocity are in-phase and thus $Z_t$ is a fluid resistance to the wave. High wave speed or high frequency, as well as high fluid density leads to high resistance to the waves. Two special cases are the normal incident waves and the parallel incident waves. In the normal incident case, the wave travels as a plane progressive wave, and the fluid wave impedance becomes $\rho_0 c$, the smallest, as $\theta$ approaches zero. In the parallel incident case, the fluid wave impedance will become infinitively large as $\theta$ approaches $\pi/2$. In this case, fluid impedance will become extremely large, so no bending wave will be induced.
In the case, as \( k < k_b \), similar to the discussion about critical frequency, \( \cos \theta \) and \( Z_r \) becomes imaginary, so, again, no bending wave will be induced in the solid if the plate is acoustically excited. The following discussion about mechanical impedance shows that if a plate is mechanically excited, as \( k < k_b \), the wave impedance in the fluid behaves only like an additional mass controlled mechanical impedance from a solid layer with thickness of \((k_b^2 - k^2)^{1/2}\). This can be understood with an effective density as \( \rho_b / (k_b^2 - k^2)^{1/2} \). Therefore, the fluid layer vibrates with the solid and produces no wave in the fluid.\(^4\)

The mechanical impedance of a solid body is defined as \( Z_m = F / V \), in which applied force per unit area of interface replaces the pressure in the fluid wave impedance, indicating how well the solid particle velocity will cooperate to the applied force. The real part of \( Z_m \) is the mechanical damping to the vibration, while the imaginary part represents the mass and the stiffness acting on the vibration to increase or reduce the phase angle difference between solid particle velocity and the fluid wave pressure if no other applied force is included. For a bending wave in a plate (or beam) excited by an acoustic wave from the adjacent fluid, the applied force on the plate surface is fluid pressure, so the mechanical impedance of the plate is derived from the bending vibration equation\(^3\),

\[
D V^4 \eta + m c^2 \eta / \partial t^2 = -p \quad \eta \text{ transverse displacement of solid particle, } \partial \eta / \partial x = v \text{ velocity, and } D = Eh^3/(1-\nu^2)/12 \text{ bending stiffness, } m = \rho h \text{ plate mass per unit area, } \rho, \text{ plate density). From this equation,}
\]

\[
Z_m = -i(D k_b^4 - m \omega^2) / \omega = -i(m/\omega)(\omega_b^2 - \omega^2), \quad \omega_b = (D/m)^{1/2} k_b^2,
\]

\[Z_m = 0, \quad \text{as } \omega = \omega_b \quad \text{coincidence}\]

\[Z_m = -id k_b^4 / \omega, \quad \text{as } \omega < \omega_b, \quad \text{stiffness control}\]

\[Z_m = im \omega, \quad \text{as } \omega > \omega_b, \quad \text{mass control}\]

Without including other damping, \( Z_m \) is purely imaginary, a reactance, so pressure and velocity are always out-of-phase. Resistance can only come from damping (not included here). The mass, or inertia, \( m \omega \), increases the phase difference between pressure and velocity and retards the pressure-velocity cooperation, while the stiffness, \(-D k_b^4 / \omega\), reduces the phase difference and helps the cooperation. Here \( \omega_b \) is the coincidence frequency, depending on wafer stiffness, mass, thickness and the wafer bending wave number. The impedance will become stiffness controlled as \( \omega < \omega_b \) and inertia or mass controlled as \( \omega > \omega_b \). As the wafer stiffness balances its inertia, \( \omega = \omega_b \).

Three parameters in mechanical impedance are the plate thickness \( h \), the wave frequency \( \omega \), and the wave incidence angle \( \theta \). High wave frequency results in mass controlled response (high inertia and low stiffness), while large incidence angle results in stiffness controlled response (large stiffness). The plate thickness \( h \) has great influence on mechanical impedance. For a given wave frequency, the large stiffness (\( \sim h^2 \)) of a thick plate is able to overcome the inertia (\( \sim h \)) resulting in stiffness control for a range of incidence angles that is impossible for thin plates. In contrast, a thin (\(< 0.7 \text{ mm})\) PMMA plate or beam excited by 1 MHz acoustic waves has positive imaginary \( Z_m \) indicating that
inertia or mass is controlling since the thin-plate stiffness is too weak to overcome the inertia. Therefore, no coincidence is possible for when small feature columns with thickness less than 0.7 mm are excited by 1 MHz waves.

Figure 7a and b presents the variation of PMMA and silicon mechanical impedance with wave incidence angle for 1 MHz acoustic waves, respectively. $Z_m$ is divided by $\sqrt{-1}$ to make it real. The upper part is the mass controlled region where $\text{imag} (Z_m) > 0$, while the lower part is the stiffness controlled region, $\text{imag} (Z_m) < 0$. The point where each curve crosses $Z_m = 0$ corresponds to the coincidence angle $\phi_c$, discussed in next section. The curves for thicker PMMA wafers (0.8, 1.0, and 1.2 mm) have coincidence angles, and the incidence angle smaller or greater than its coincidence angle determines the impedance is mass or stiffness controlled. The curves with thinner PMMA wafers (0.6, 0.4, and 0.2 mm) have no coincidence angles and are always mass controlled. All curves for silicon wafers have smaller coincidence angles (around 20° for 1.2-mm to around 60° for 0.2 mm), but they are hard to distinguish due to silicon’s very strong bending stiffness, even though silicon inertia is about two times larger than that of PMMA.

Fluid acoustic impedance $Z_f$ is plotted in the small figure inside Figure 7a. $Z_f$ is always resistance, and the greater the incidence angle, the greater the impedance. In the case of normal incidence, surprisingly, $Z_f$ is most favorable, while $Z_m$ is the worst to retard the wave pressure-particle velocity cooperation. Therefore, coincidence is more favorable than normal incidence for transferring wave energy to the fluid on the other side, but does not effectively transfer wave energy to the plate itself. At incidence angles close to 90°, fluid wave impedance approaches infinitively large while mechanical impedance becomes very favorable to the wave pressure and wafer particle velocity cooperation. In this case, wave energy does not transfer to the fluid due to the mismatch of the fluid wave impedance and the plate mechanical impedance.
The concept of Impedance originated in electrical engineering, where electrical impedance or resistance is equal to electrical potential divided by electrical current. In the acoustic analog, the wave pressure corresponds to the electric potential, and the induced velocity corresponds to electrical current, so acoustic impedance corresponds to the electrical resistance. Thus, when many media are involved, the simple electric-circuit analog can often be used to get the interface wave pressure and particle velocity.

**COINCIDENCE ANGLE**

Transmission of wave energy through a wafer is the greatest when the angle of incidence is equal to the coincidence angle. The coincidence angle is the wave incidence angle at which the plate inertia offsets the stiffness, resulting in in-phase wave pressure and velocity. This is only possible when the acoustic frequency is supercritical. In fact, in the competition between wave frequency $\omega$ and plate coincidence frequency $\omega_c$, a small wave incidence angle favors mass-controlled impedance whereas a large wave incidence angle favors stiffness-controlled impedance, as mentioned before. In between, there is a coincidence angle or critical angle at which the stiffness just balances the inertia. In this case, the fluid will transfer energy efficiently through the plate. Only damping, if there is any, will dominate the response. Although the critical frequency, $2\pi \omega_c = f_c$, is a special coincidence frequency ($k_B = k$ and $\theta = \pi/2$), this case is excluded as discussed before.

The sine of the coincidence angle, $\phi_c$, describes the portion of acoustic wave number contributing to the bending wave number at the interface such that the stiffness of the plate cancels the inertia. Therefore,

\[
k_B = (\omega^2 m / D)^{1/4},
\]

\[
sin^2 \phi_c = sin^2 \theta = k_B^2 / k^2 = [m \omega^2 / (D k^4)]^{1/2} = (m / D)^{1/2} c^2 / \omega.
\]

or

\[
sin^2 \phi_c = k_B^2 / k^2 = c^2 / c_B^2 = (2(3)^{1/2} / \omega C_f) (c^2 / h).
\]

Because of $sin \phi_c \sim (\omega h)^{-1/2}$, the thicker the wafer or the higher the wave frequency, the smaller the coincidence angle will be, in agreement with previous discussion.

Figure 8 shows the regions of mass control, stiffness control, and damping control by wave incidence angle $\theta$ at a supercritical frequency for a given plate thickness. Again, we see the response of the plate is in the mass or inertia controlled region at small wave incidence angle, in stiffness control region at large wave incidence angle, and in damping control region $\omega > \omega_c$. 

![Figure 8. Regions of Wafer Mechanical Impedance for Wave Incidence Angle at a Single Critical Supercritical frequency](image-url)
immediately around coincidence angle. Energy transfer through the plate is the greatest in the damping controlled region but weak for normally incidence waves because mass controlled impedance retards the pressure-velocity cooperation.

Figure 9 gives the coincidence angle, $\phi_{\text{co}}$, variation with PMMA and silicon thickness for a variety of acoustic frequencies. At 1 MHz wave, the coincidence angles for a PMMA plate with a thickness of 1.2, 1.0, and 0.8, are about 50°, 56°, 69°, respectively, corresponding to the ratio of their frequencies to the critical frequency of 1.728, 1.440, and 1.152, respectively. The coincidence angle for the 0.7 mm plate becomes 90° that should be excluded as before. These coincidence angles depend mainly on their frequency ratios as indicated in the small plot inside Figure 9, where the influence of thickness is implicitly included in $\omega/\omega_c$. This single curve is valid for any material, and it is tangential to the line $\omega/\omega_c=1$.

![Figure 9. PMMA and Silicon Wafer Thickness Changes with Coincidence Angle](image)

<table>
<thead>
<tr>
<th>Frequency</th>
<th>PMMA Microns</th>
<th>Silicon Microns</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.5 MHz</td>
<td>459</td>
<td>95</td>
</tr>
<tr>
<td>1.2 MHz</td>
<td>574</td>
<td>119</td>
</tr>
<tr>
<td>1.0 MHz</td>
<td>689</td>
<td>142</td>
</tr>
<tr>
<td>0.8 MHz</td>
<td>861</td>
<td>178</td>
</tr>
<tr>
<td>0.6 MHz</td>
<td>1149</td>
<td>237</td>
</tr>
</tbody>
</table>

Table 2. Max PMMA and Silicon Thickness without a Coincidence Angle

No coincidence angle is available for thin PMMA and silicon plates at sub-critical frequency, as shown in Figure 9. Each curves has a minimum plate thickness that corresponds to the critical frequency. As some examples listed in the Table 2, no coincidence angle is possible when the PMMA thickness is less than about 689 $\mu$m for a 1 MHz wave, and less than 1.16 mm for a 600 kHz wave because these frequencies are just the critical frequencies of a plate with the corresponding thickness. The minimum plate thickness for silicon are about four times smaller because the much higher flexural wave speeds. Thus, for megasonic waves (0.7–1 MHz), coincidence angle generally exists only for wafer, not for PMMA feature column with thickness less than those values.

**Transmission Loss and Energy Deposition in the Wafer**

Four waves are involved in transmission of acoustic waves through a plate: the incident wave and reflected wave at one side of the plate, the transmitted wave at the other side,
and the induced bending wave in the plate, as shown in Figure 10. Here subscript ‘I’, ‘R’, ‘T’ and ‘p’ are used to identify incident, reflected, transmitted waves and plate, respectively.

Figure 10. Transmission and Reflection of Acoustic Wave

The matching conditions at fluid-solid interface are continuity of the pressure and normal (to the plate) particle velocity. They are

\[ p_T = p_I - p_R \]
\[ p_p = p_I + p_R - p_T = 2p_R \]
\[ u_I \cos \theta - u_R \cos \theta = u_T \cos \theta = v_p \]

where \( \theta \) is the incident wave angle, \( v_p \) and \( p_p \) are the normal particle velocity and the pressure of plate, respectively. For the special case of rigid plate, no transmitted waves and no plate movement will result. As applied in previous work for a rigid wafer, they become

\( p_T = 0, \ p_I = p_R, \ p_p = 2p_I, \ v_p = u_T = 0, \ u_I = -u_R \)

On the other hand, it follows from linear momentum equation after canceling \( \cos \theta \) from both sides of each equation,

\[ u_I = p_I / (\rho_0 c), \quad u_R = p_R / (\rho_0 c), \quad u_T = p_T / (\rho_0 c), \]

The mechanical impedance per unit area of the plate thus becomes

\[ Z_p = p_p / v_p = 2(p_I - p_T) / u_T \cos \theta = (p_I / p_T - 1)(2\rho_0 c) / \cos \theta, \]

Therefore, the pressure ratio of transmitted wave to incident wave is

\[ p_T / p_I = 1 / [1 + Z_p \cos \theta / (2\rho_0 c)]. \]

Expressing this pressure ratio in terms of fluid acoustic impedance, we obtain the important expression:

\[ p_T / p_I = 1 / [1 + Z_p / (2 Z_f)] = 2 Z_f /[2 Z_f + Z_p]. \]
This expression indicates that the pressure ratio of the transmitted wave to the incident wave is the ratio of total impedance without the plate to total impedance including plate. The pressure ratios for reflected waves and for the plate bending wave thus are

\[ p_R / p_t = 1 - p_T / p_t = Z_p / [2 Z_f + Z_p]. \]
\[ p_p / p_t = 2 (1 - p_T / p_t) = 2 (p_R / p_t) = 2 Z_p / [2 Z_f + Z_p]. \]

Therefore, the reflection pressure ratio is the ratio of the plate mechanical impedance to the total impedance, and the plate pressure is twice of the reflection pressure, instead of twice of the incident wave pressure as in rigid wafer case.

The above equations illustrate the benefit of using impedances to calculate the response at the interface without knowing other details of the fluid field and the plate movement. Wave energy is redistributed when waves impinge on the plate. The transmission coefficient \( \tau \), and reflection coefficient \( r \) are defined as the energy intensity ratio of transmitted wave to incident wave and reflected wave to incident wave, respectively. The plate energy coefficient \( E_p \) is the ratio of deposited plate energy to incident wave energy. The average acoustic power intensity, the time average power intensity, becomes proportional to \( p^2 \) or \( v^2 \) because

\[ I = (1/T) \int_0^T pv \, dt = (1/2) \text{Re}(pv^*) = 0.5 \text{Re}(Z_f v v^*) \]
\[ = 0.5v^2 \text{Re}(Z_f) = 0.5 p^2 \text{Re}(1/Z_f^*) = 0.5 p^2 \text{Re}(1/Z_f). \]

where the superscript * means conjugate, \( I \) the power intensity, \( T \) the period, \( t \) the time. Therefore,

\[ \tau = \left| \frac{p_T}{p_t} \right|^2 = 1 / \left| 1 + Z_p \cos \theta / (2 \rho_o c) \right|^2 = \left| 2 Z_p / (2 Z_f + Z_p) \right|^2. \]
\[ r = \left| \frac{p_R}{p_t} \right|^2 = \left| 1 - \frac{p_T}{p_t} \right|^2 = \left| Z_p / (2 Z_f + Z_p) \right|^2. \]
\[ E_p = \left| \frac{p_p}{v_p} \right| / \left| (p_t^2 \cos \theta / \rho c) \right| = 2 (1 - \left| \frac{p_T}{p_t} \right|) \left| \frac{p_T}{p_t} \right| = 4 \frac{Z_p Z_f}{(2 Z_f + Z_p)^2} = 2 \sqrt{\tau r}. \]

Note that the plate energy coefficient is not simply the square of the plate pressure ratio because the plate pressure does not obey the fluid momentum equation. This relation indicates the wafer will obtain wave energy only if transmission and reflection coexist. At the coincidence angle, the plate mechanical impedance becomes zero as if the plate does not exist. In this case, \( \tau = 1, r = E_p = 0 \) indicating that 100% of the wave energy is being transferred to the fluid on the wafer’s back side. For a rigid wafer, no transmission is possible, \( \tau = E_p = 0, r = 1 \). The incident wave energy is transferred directly to the reflected wave energy.

Addition of the three wave coefficients results in unity, showing the wave energy is conserved. Therefore, the incident wave energy is split into three different parts: transmission, reflection, and deposition in the plate. The percentage of each depends completely on the fluid and plate impedance and the incidence angle.
$\tau + r + E_p = (|p_T/p_I| + |1 - p_T/p_I|)^2 = (\sqrt{\tau} + \sqrt{r})^2 = 1$

Together with $p_T/p_I + p_R/p_I = \sqrt{\tau} + \sqrt{r} = 1$, Figure 11 displays all the possible variations of $\tau$, $r$, $E_p$, and interface pressure ratio values, $p_T/p_I$, $p_R/p_I$. Here, $p_T/p_I$ is independent variable, $\tau = \sqrt{p_T/p_I}$, $r = \sqrt{1 - p_T/p_I}^2$, $p_R/p_I = 1 - p_T/p_I$. Note that $p_T/p_I$ is not included since it can be larger than 1, and its value is just the twice of the $p_R/p_I$. The maximum transmission $\tau = 1$ is realized only by zero $r$ and $E_p$. However, the maximum $E_p = 1/2$ is realized only by nonzero $\tau$ and $r$, in which $\tau = 1/4$, $r = 1/4$. In this case, $p_T/p_I = p_R/p_I = 1/2$.

In LIGA development, a large transmission coefficient provides great agitation to the fluid on the backside of the wafer, i.e., good for back agitation. The large plate energy coefficient means the wafer obtains large portion of incident acoustic energy to maintain the forced bending waves. The feature cavities of the wafer will get better agitation by wafer particle movement. However, the shaking of the slender features may weaken the base.

Figure 12 presents the transmission coefficient of a PMMA wafer for 1MHz waves at various incidence angles; (a) through (h) are for wafer thickness of 1.2, 1.0, 0.8, 0.6, 0.4, 0.2, 0.1, and 0.05 mm, respectively.
Figure 12. PMMA Wafer Transmission, Reflection, and Wafer Energy Coefficients ($\tau$, $r$, $E_p$) For 1 MHz Waves
In (a) through (c), coincidence angles can be identified as $56^\circ$, $52^\circ$, and $69^\circ$, respectively, at which $\tau$ approaches unity, $r$ and $E_p$ approach zeros, so all of the incident wave energy is transmitted though the wafer. There is no coincidence angle in (d) through (h), because the thickness of wafer is so thin that 1 MHz is less than the critical frequency. In (a) and (b), as the incidence angle increases, the transmission coefficient increases before reaching the coincidence angle and decreases after the coincidence angle. This is because the plate mechanical impedance first decreases toward zero, and then increases, while the reflection coefficient and plate bending energy coefficient have just the same trend as the plate mechanical impedance, an opposite trend to the transmission coefficient. However, at $\theta = \pi / 2$, $\tau \rightarrow 1$ is not realized because of infinitively large fluid impedance. In (c) through (h) $\tau$ increases monotonically while $r$ and $E_p$ decrease monotonically since no coincidence angle is involved. Among these figures, (g) and (h) show that $\tau = 1$ and $r = 0$, $E_p = 0$ for feature column thickness less than 100 $\mu$m, indicating an almost completed transmission without reflection and plate energy deposition because the feature is so thin that the plate mechanical impedance is much smaller than fluid wave impedance.

As for the pressure ratio, at normal incidence of case (a), where $h = 1$ mm, the wave pressure ratios $p_T / p_i$, $p_R / p_i$, and $p_p / p_i$ are 0.37, 0.63, and 1.26, respectively. Here, the incident wave pressure is divided between transmitted wave pressure and reflected wave pressure, while plate pressure is double the reflected wave pressure, as a result of all three waves—incident, transmitted, and reflected waves interact with the plate. The energy ratios become $\tau = 0.14$, $r = 0.39$, and $E_p = 0.47$. In this case, the plate gains almost a half of incident energy, and the reflection energy is about 40% leaving the transmission energy of only about 14%, as shown in Figure 12 (a) as $\theta = 0$. Comparing all (a) and (h) at normal incidence, the thinner the wafer, the larger the transmission coefficient and the smaller the reflection coefficient. The plate energy coefficient is around 50% that is the maximum when the plate thickness is 0.6-0.8 mm. This result agrees well with the Bankert et al measurements: energy probe readings drop 54% near the tank top due to presence of the wafer. For a 0.05-mm wafer in (h), greater than 98.5% energy is transmitted, no reflection, and less than 1.5% plate energy deposition.

Figure 13 presents the similar three coefficients of a silicon wafer for 1 MHz waves at various incidence angles. Coincidence angles can be identified in (a) through (f).
Figure 13. Silicon Wafer Transmission, Reflection, and Wafer Energy Coefficients ($\tau, r, E_p$) For 1 MHz Waves
In Figure 13 (a)-(d), the distinct wave energy transfer characteristics of silicon from PMMA show that for thick plates, wave energy does not go through the plate very much but reflects strongly from the plate except at the coincidence angles. This is due to the mismatch of the fluid wave impedance and the plate’s mechanical impedance. For very thin plates that are thinner than the minimum thickness required for a coincidence angle at this wave frequency, most of the wave energy goes through the plates, and surprisingly, (g) and (h) for 0.1 and 0.05 mm are very similar to (f) and (g) in Figure 12 for 0.2 and 0.1 mm PMMA plates. Silicon plate gain about 30-50% wave energy at small incidence angles for (a) through (h) cases, and at almost all incidence angles for (c) case.

Suppose a PMMA resist having a 1-mm silicon substrate. The total mechanical impedance of this multilayer will be the sum of the impedances of PMMA and silicon. Since the 1-mm silicon mechanical impedance is stronger in stiffness and inertia, it dominates the multilayer impedance for all the PMMA thickness considered here. The effective impedance curves almost overlap as one curve, and the corresponding energy coefficient curves are very similar to those of silicon. In contrast, if the silicon substrate is thin, then it will not affect very much since the mechanical impedance $Z_m \sim h$.

Figure 14a and b presents the variation of transmission coefficient with incidence angle for a frequency of 1 MHz and for a few of PMMA and silicon wafer thickness, respectively, ranging from thick to thin wafers. The coincidence angle shifts to the right when increasing incidence angle for thick wafer and disappears for the thin wafer. Again, case $\theta = \pi/2, \tau = 1$ is unrealistic. Again, silicon has smaller coincidence angle for the same thickness compared with PMMA.

![Figure 14a. Transmission Coefficient of PMMA Wafer (f = 1MHz)](image)

![Figure 14b. Transmission Coefficient of Silicon Wafer (f = 1MHz)](image)

The transmission loss ($TL$) or refraction index ($R$) is defined as $TL = 10 \log_{10} (1 / \tau)$. The unit is decibel (dB), representing the wave intensity loss by transmission in decimal log-scale (3 dB of TL means transmitted power about a half, 6 dB to a quarter, etc.) For example, $\tau = 0.14$ in above normal incident example corresponds to $TL = 8.54$ dB. From expression of $\tau$,
\[ TL = 10 \log_{10} \left[ 1 + Z_p \cos \theta / (2\rho_c) \right]^2 \]
\[ = 10 \log_{10} \left[ 1 + \left( D k_B^4 - m\omega^2 \right) \cos \theta / (2\rho_c \omega) \right]^2 \]
\[ = 10 \log_{10} \left[ 1 + m (\omega^2 - \omega_B^2) \cos \theta / (2\rho_c \omega) \right]^2, \]

where the plate bending stiffness \( D = Eh^3/(1-\nu^2)/12 \), Poisson ratio \( \nu \), and \( \omega_B = (D/m)^{1/2}k_B^2 \).

Regarding the transmission loss varied with frequency, Figure 15a and b presents the variation of transmission loss with wave frequency up to 2 MHz for a 30° incidence angle and for a few PMMA and silicon wafer thickness, respectively. A typical damping that is about 2% of the stiffness (usually including interface and material damping) was included to make the results more realistic. At 1 MHz, waves transmitted through a 1-mm PMMA wafer at 30° incidence have a transmission loss of about 6.58 dB (\( \tau = 0.22 \)), which is greater than for all other thinner wafers, but smaller than in the normal incidence for the same wafer where \( TL = 8.54 \) dB (\( \tau = 0.14 \)), as in Figure 12a. For wave frequencies lower than 1.5 MHz, a thicker PMMA wafer results in the greater loss, as in previous two figures. At higher frequency, the curves for thick wafers go down from their maximum values toward the coincidence cases at which 30° will become the coincidence angle at corresponding frequencies and the loss are the minimum. At 1 MHz, all silicon wafers at 30° incidence have large transmission losses except at the frequencies close to its coincidence frequency due to silicon’s large mechanical impedance.

Figure 15a. Sound Reduction Index of PMMA Wafer for 30° Wave Incidence Angle

Figure 15b. Sound Reduction Index of Silicon Wafer for 30° Wave Incidence Angle

Figure 16a and b presents the variation of transmission loss of 1-mm PMMA and silicon wafer with frequency ratio (to critical frequency) for several incident wave angles, respectively, for frequency up to 2 MHz. Silicon has much smaller \( \omega \), and much larger transmission loss. Again, a typical damping that is equal to 2% of the stiffness was included. Transmission losses for normally incident waves are greater than for waves...
with small incidence angles as $\omega/\omega_c < 2.2$ (\textless \text{coincidence angle}) but smaller for waves with large incidence angles (\textgreater \text{coincidence angle}), as frequency ratio greater than 2.2.

For incidence angle greater than 40° for PMMA and 20° for silicon, the transmission loss achieves a minimum (coincidence case) for a frequency ratio of about 2.5 (PMMA) and 8.5 (silicon) and then increases at greater frequencies. For a 1 MHz wave that has a frequency ratio of 1.44 for PMMA and 7.6 for silicon transmitting through a 1-mm wafer, the transmission loss is close to 9 dB and 14.6 dB, respectively, for a normally incident case, which agrees with the previous PMMA transmission loss value. Again, the case with 90° should be excluded.

**FORCED AND FREE BENDING WAVES IN A FINITE PLATE**

All of the previous discussion is based on unbounded fluid and an infinite plate. However, in the actual process of LIGA development, a finite wafer is immersed in a finite tank of liquid developer. The Sandia LIGA group uses a megasonic cleaning tank made by PCT Systems. The molded quartz tank is stress free, and has very low natural frequencies (< 2 kHz), so no vibration of the tank will result from response to megasonic acoustic frequencies (> 700 kHz). Therefore, the tank can be approximated as a rigid tank, but the natural frequencies of the finite wafer may need to be considered.

The response of a finite plate to an incident acoustic wave is much more complex than that for an infinite plate. The natural frequencies and modes of the plate may be excited in a resonance or non-resonance response. In fact, the incident wave frequency, the plate critical frequency, and the lowest (fundamental, or base) natural frequencies of the plate all play critical rolls in wave-plate interaction. The situation is similar to some extent in a single-degree-of-freedom system (mass-spring-damper). Whether a finite system response to a disturbance is mass dominant, stiffness dominant, or damping dominant,
depends on whether the incident frequency is higher, or lower, or very close to (resonance case) the base natural frequency. In the present case, the critical plate frequency is also a significant parameter. At supercritical frequencies for PMMA wafers and subcritical frequencies for wafer’s features, megasonic frequencies are greater than the base natural frequencies of both a PMMA wafer and its feature columns.

Because of $\lambda_w / \lambda = (f / f_c)^{1/2}$, when the incident wave frequency is greater than the plate critical frequency, $f > f_c$, and hence $\lambda_w > \lambda$, $k > k_w$, a supersonic forced bending wave is induced in the plate with the incident wave frequency, the same as for an infinite plate. If the wave frequency overlaps one of the natural frequencies, resonance will occur. For example, for a rectangular plate, resonance occurs when $f = f_{mn}, k_w = k_{mn}$ where subscript $mn$ corresponds to natural frequency. Free bending wave, or non-resonance bending waves may also be induced if the incident wave and the plate bending waves match spatially (wavelength), instead of frequency-wise.\(^3\) In this case, mass dominates the bending wave, and damping dominates the resonance.

When the incident wave frequency is smaller than the plate critical frequency, $f < f_c$, and hence $\lambda_w < \lambda$, $k < k_w$, no bending wave is induced in an infinite plate. For a finite plate, however, this is no longer true due to the boundary conditions. If the wave frequency is greater than the base natural frequency, $f_0 < f$, a subsonic forced bending wave can still be induced. An acoustic wave transmission will pass through the plate, and the response will be mass controlled. Looking at the wave number for a rectangular plate, even if $k < k_{mn} = \sqrt{(k_1 + k_2)}$, one of $k_1$ and $k_2$ is still possibly smaller than $k$, making the subsonic bending wave propagating possible. However, this wave energy cannot be transmitted efficiently, or it is relatively weak and slow.

Transmission losses can be generally separated into four classes: stiffness, resonance, mass, and coincidence controlled. In our cases, the wave frequency is higher than wafer bending critical frequency, but is lower than wafer feature bending frequency. Therefore, the wafer transmission loss increases with incidence angle due to mass control when the wave incidence angle is less than coincidence angle, or decreases due to stiffness control when the wave incidence angle is larger than coincidence angle, or drops sharply due to coincidence (damping) control. However, the likelihood of resonance for the wafer is zero (see next section examples). Unlike the infinite case, subsonic bending waves can be induced in wafer features, and the transmission loss is usually mass controlled. If resonance happens, features will get most of the energy.

### EXAMPLES OF NATURAL FREQUENCIES AND NATURAL MODES

From the flexural vibration equation without a load, a beam with a clamped end at $x = 0$ has the solution:\(^6\)

$$\eta = C_1 (\cos kx - \cosh kx) + C_2 (\sin kx - \sinh kx)$$
where \( \eta \) is the amplitude of the motion and \( k^4 = \omega^2(D/m) \). With a free end at \( x = l \), it is required that

\[
\cos kl \cosh kl = -1,
\]

so \( k, l = 1.875, k, l = 4.694, k, l = 7.855, k, l = 10.996, k, l = 14.137, k, l = 17.279 \ldots \) The base frequency becomes

\[
f_1 = \omega/2\pi = k^2(D/m)^{1/2}/2\pi = (1.875/l)^2 \{Eh^3/[12(1-v^2)\rho h]\}^{1/2}/2\pi \text{ so } f \sim h/l^2.
\]

Here \( l \) and \( h \) denote length and thickness, respectively. The thicker the plate or beam, the higher the natural frequency will be due to the bending stiffness; and the longer the length, the lower the natural frequency will be due to the smaller wave number.

Table 3 presents the first two frequencies for varied PMMA wafer feature column thickness and length. The base natural frequencies for PMMA feature columns having a thickness up to 200 \( \mu m \) and a length from 500–1000 \( \mu m \) are less than 250 kHz and are far less than megasonic frequencies of interest. However, the second natural frequency for lengths of 500–600 \( \mu m \) can reach over 700 kHz when the thickness is as large as 100–200 \( \mu m \), comparable to megasonic frequencies. For higher modes, the natural frequencies are even higher. However, interface damping and material damping greatly reduce the amplitude of the higher natural modes.

<table>
<thead>
<tr>
<th>( h/ \mu m )</th>
<th>500 ( \mu m )</th>
<th>600 ( \mu m )</th>
<th>800 ( \mu m )</th>
<th>1000 ( \mu m )</th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
<td>55.953</td>
<td>38.856</td>
<td>21.857</td>
<td>13.988</td>
</tr>
<tr>
<td></td>
<td>350.68</td>
<td>243.52</td>
<td>136.98</td>
<td>87.669</td>
</tr>
<tr>
<td>100</td>
<td>111.91</td>
<td>77.712</td>
<td>43.713</td>
<td>27.976</td>
</tr>
<tr>
<td></td>
<td>701.35</td>
<td>487.05</td>
<td>273.97</td>
<td>175.34</td>
</tr>
<tr>
<td>150</td>
<td>167.86</td>
<td>116.57</td>
<td>65.57</td>
<td>41.465</td>
</tr>
<tr>
<td></td>
<td>1052</td>
<td>730.57</td>
<td>410.95</td>
<td>263.01</td>
</tr>
<tr>
<td>200</td>
<td>223.81</td>
<td>155.42</td>
<td>87.426</td>
<td>55.953</td>
</tr>
<tr>
<td></td>
<td>1402.7</td>
<td>974.10</td>
<td>547.93</td>
<td>350.68</td>
</tr>
</tbody>
</table>

To illustrate the natural flexural modes of a beam with one clamped end and one free end, Figure 17 includes sketches of the first five vibration modes.\(^3\) Notes that the bending shapes of the beam are over exaggerated in transverse direction to be clearly seen. These forced or free natural modes may cause the feature column to break off the wafer base even though the frequencies are not high.
Using a square PMMA and silicon wafer model that has a larger thickness \((h = 500–1200 \mu m)\) and very much larger length \((l = 0.1–0.15 m)\) than a feature column, together with the one clamped edge and three free edges, the natural frequencies of the wafer are at least three orders of magnitudes smaller than that for feature columns because of \(f \sim h/l^2\). Thus, the first several frequencies can be ignored comparing with the megasonic acoustic wave frequencies and the higher modes are strongly suppressed by damping. Table 4 presents the first three natural frequencies in Hz instead of kHz for PMMA wafer thickness from 800-1000 \(\mu m\) and wafer length from 10 to 20 cm. Although a silicon wafer has about five times higher natural frequencies due to \(f \sim D^{1/2} \sim C_l\), these frequencies are still very low. Figure 18 gives the corresponding mode shapes for first five natural frequencies. The dash lines indicate the traces of the flexion points. Therefore, for the high frequency modes, the mode shapes become very complicated, and the flexion points spread over the wafer.

For a more realistic circular wafer with a free edge except for a single clamped point, the order of magnitudes of the natural frequencies and the natural modes should be very similar to that for a square plate. Moreover, the mathematical problem becomes much more difficult to solve without requiring a numerical approach such as ABAQUS. For these reasons and the expectation of very low natural frequencies, there is little reason for further investigation.

<table>
<thead>
<tr>
<th>(h) [(\mu m)]</th>
<th>(l) [cm]</th>
<th>10 cm</th>
<th>15 cm</th>
<th>20 cm</th>
</tr>
</thead>
<tbody>
<tr>
<td>1200</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>816.2</td>
<td></td>
<td>33.37</td>
<td>14.83</td>
<td>8.34</td>
</tr>
<tr>
<td>204.74</td>
<td></td>
<td></td>
<td>36.25</td>
<td>20.40</td>
</tr>
<tr>
<td>1000</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>68.02</td>
<td></td>
<td>27.80</td>
<td>12.36</td>
<td>6.95</td>
</tr>
<tr>
<td>170.61</td>
<td></td>
<td></td>
<td>30.34</td>
<td>17.00</td>
</tr>
<tr>
<td>800</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>54.41</td>
<td></td>
<td>22.24</td>
<td>9.989</td>
<td>5.56</td>
</tr>
<tr>
<td>136.49</td>
<td></td>
<td></td>
<td>24.18</td>
<td>13.60</td>
</tr>
</tbody>
</table>

Table 4. PMMA Wafer Natural Frequencies \(f_1\) to \(f_5\) [Hz]
\((h\) thickness, \(l\) length\)
This work assesses the influence of elasticity on the response of a wafer and its features to megasonic waves. We studied an elastic wafer subjected to acoustic agitation in a LIGA development tank. The response is more complicated than that for a rigid wafer. Not only reflected waves, but also transmitted waves are generated because some of the incident wave energy passes through the wafer. In addition, supersonic bending waves are likely induced in the wafer because some of the incident wave energy is deposited in the wafer. Furthermore, the boundary shape and stiffness of the wafer and its feature columns determine natural modes and natural frequencies. These modes might interact with the incident waves to cause resonance. What does the influence of all these facts bring to LIGA development?

The megasonic frequency typically used in LIGA development is usually supercritical for a sufficiently thick wafer, but subcritical for wafer features having dimensions orders of magnitude smaller than the wafer. The critical bending frequency is determined by the bending stiffness and incident wave frequency and direction. When an incident frequency is supercritical, incident waves induce supersonic bending waves in the wafer. Since an acoustic wave frequency of 1 MHz is a supercritical frequency for a wafer thickness greater than 689 μm, this should usually be true of LIGA wafer. The incident wave energy can generally be channeled into three components, transmitted, reflected and energy deposited to the wafer, if damping (usually only 2-5%) is not included. How this energy is distributed among the three depends on the wafer bending stiffness and inertia, and hence depends on the wafer material, thickness and wave incidence angle.

Wafer bending stiffness and inertia result comprehensively from wafer thickness, wave frequency, and wave incidence angle. Large wafer thickness results in large wafer bending stiffness, while small wave incidence angle (not far from normal incidence) results in small wafer stiffness, and high incident wave frequency results in large wafer inertia. Mass controlled response in which wafer inertia dominates the response suppresses the fluid pressure and wafer particle velocity cooperation at the fluid-wafer interface, while stiffness controlled response promotes this cooperation at the interface. Coincidence angle is the angle at which the bending stiffness offsets inertia, resulting in transmitted waves almost as strong as incident waves, leaving almost no reflected waves, and no
energy deposition in the wafer. Coincidence angles exist only at supercritical incident frequencies because inertia is too weak to compete with stiffness at subcritical incident frequencies. As an example, the coincidence angle is 56° for 1-mm PMMA at 1 MHz waves. At this angle, incident wave energy is almost completely transmitted through the wafer. For other incidence angles, wafer response is mass controlled at smaller incidence angles and stiffness controlled at larger incidence angles. Therefore, when an incident frequency is subcritical as expected for LIGA wafer features, transmitted energy dominates and increases monotonically with incident angles while reflected energy and wafer energy are secondary and decrease monotonically with incident angles. When an incident frequency is supercritical as expected for LIGA wafers, however, transmitted energy does not dominate and does approach maximum at coincidence angles while reflected energy and wafer energy approach minimum at coincidence angles.

The sound reduction index, or refraction index, or transmission loss (not including damping) measures wave intensity loss in a decimal log scale. This loss is due to the energy carried away by reflection and deposited in the wafer. For example, at normal incidence, 1-MHz waves transmitted through a 1-mm wafer are reduced by a 9 dB loss. PMMA wafer-gained energy can be as large as 50% of the incident wave energy under 1 MHz acoustic waves, when incident wave angle is less than 40° for wafer thickness greater than 800 µm, or less than 30° for wafer thickness of 600 µm, because the wafer’s response is mass controlled, and incident wave energy flux impinges strongly to the wafer. This result agrees very well with the measurement by Bankert et al. In contrast, when the incident wave angle becomes large, or wafer becomes thin, then the wafer will not gain significant energy, because the wafer’s response is stiffness controlled, and the transmission loss is small.

The natural frequencies of a wafer are extremely low (< 100 Hz), so wafer resonance is not likely for incident megasonic waves. PMMA wafer feature columns have natural frequencies many times higher than that of the wafer, but the base natural frequencies are still low comparing with megasonic waves. Subsonic bending waves can be induced in wafer feature columns due to their finite boundaries. For a thicker and shorter feature column, resonance may occur.

Damping is a source of the transmission loss not considered here. However, interface damping and material damping usually contribute no more than 2–5% relative to the material stiffness that is included in our modeling. These mechanisms suppress the high frequency natural modes.

Acoustic absorption in fluid media is due to viscosity so that the wave energy dissipates into heat. The wave energy absorption coefficient in liquid is \( \alpha = (2\eta \omega^3)/(3 \rho_0 c^3) \), where \( \eta \) is viscosity. For water, \( \alpha \omega^2 = 24 \times 10^{-15} \ s^2/m \), which provides a reasonable estimate to the value appropriate for the liquid developer. This dissipation mechanism is very small for a 1 MHz acoustic wave, about 2.4% in a meter while the development tank height is much smaller, about 1/6 of a meter.
In summary, in LIGA development, transmitted waves produce acoustic motion of the developer on the backside of the wafer, especially for the coincidence case in which almost all incident wave energy transfers to the backside. This fluid motion should provide the same benefits as acoustic motion on the incident face. In addition, wafer bending waves cause wafer material to oscillate at high frequency, which may also promote the development process. Resonance occurring in wafer feature columns may also induce large relative movement of the fluid in feature cavities resulting in good agitation. This shaking of the features may also weaken feature bases. However, the likelihood of resonance is limited to columns that are relatively short and wide and, hence, least likely to be broken away from the substrate.

Since the LIGA development tank has a row of transducers working sequentially in time, the incidence angle of megasonic waves will change frequently during development, resulting in the frequently changed ratio of wafer stiffness and inertia. As a result, both wafer faces are almost continuously stimulated by incident waves and transmitted waves in which all mass controlled, stiffness controlled, and coincidence cases are involved. This time averaging of the effects of waves from both sides and at various angles helps to improve process uniformity across the wafer face, and reduces the sensitivity to wafer thickness, wafer inclination angle, and other details of a specific configuration.

ACKNOWLEDGMENTS

The present work was funded by Materials and Physics Models Project of the Sandia Accelerated Strategic Computing Initiative. Sandia is a multiprogram laboratory operated by Sandia Corporation, a Lockheed Martin Company, for the United States Department of Energy under contract DE-AC04-94AL85000. The author would like to thank R. H. Nilson and L. Hunter for their thoughtful discussion and review.
REFERENCES


<table>
<thead>
<tr>
<th>Distribution</th>
<th>MS 0835</th>
<th>J. M. McGlaun, 9140</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>MS 0824</td>
<td>W. L. Hermina, 9110</td>
</tr>
<tr>
<td></td>
<td>MS 0834</td>
<td>J. E. Johannes, 9114</td>
</tr>
<tr>
<td></td>
<td>MS 0826</td>
<td>S. N. Kempka, 9113</td>
</tr>
<tr>
<td></td>
<td>MS 0834</td>
<td>K. S. Chen, 9114</td>
</tr>
<tr>
<td></td>
<td>MS 9001</td>
<td>M. E. John, 8000</td>
</tr>
</tbody>
</table>

Attn: D. R. Henson, 8200 MS 9007
     R. H. Stulen, 8100, MS 9004
     W. J. McLean, 8300, MS 9054
     K. E. Washington, 8900, MS 9003

<table>
<thead>
<tr>
<th>Distribution</th>
<th>MS 9401</th>
<th>J. M. Goldsmith, 8751</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>MS 9401</td>
<td>L. L. Hunter, 8751</td>
</tr>
<tr>
<td></td>
<td>MS 9401</td>
<td>M. E. Malinowski, 8751</td>
</tr>
<tr>
<td></td>
<td>MS 9401</td>
<td>S. Mrowka, 8751</td>
</tr>
<tr>
<td></td>
<td>MS 9042</td>
<td>C. D. Moen, 8752</td>
</tr>
<tr>
<td></td>
<td>MS 9042</td>
<td>G. H. Evans, 8752</td>
</tr>
<tr>
<td></td>
<td>MS 9042</td>
<td>S. K. Griffiths, 8752</td>
</tr>
<tr>
<td></td>
<td>MS 9042</td>
<td>W. G. Houf, 8752</td>
</tr>
<tr>
<td></td>
<td>MS 9042</td>
<td>R. S. Larson, 8752</td>
</tr>
<tr>
<td></td>
<td>MS 9042</td>
<td>R. H. Nilson, 8752</td>
</tr>
<tr>
<td></td>
<td>MS 9042</td>
<td>A. Ting, 8752</td>
</tr>
<tr>
<td></td>
<td>MS 9042</td>
<td>W. S. Winters, 8752</td>
</tr>
<tr>
<td></td>
<td>MS 9401</td>
<td>G. Aigeldinger, 8753</td>
</tr>
<tr>
<td></td>
<td>MS 9401</td>
<td>M. A. Hekmaty, 8753</td>
</tr>
<tr>
<td></td>
<td>MS 9401</td>
<td>D. R. Boehme, 8753</td>
</tr>
<tr>
<td></td>
<td>MS 9401</td>
<td>J. T. Hachman, 8753</td>
</tr>
<tr>
<td></td>
<td>MS 9401</td>
<td>A. M. Morales, 8753</td>
</tr>
<tr>
<td></td>
<td>MS 9401</td>
<td>F. J. Pantenburg, 8753</td>
</tr>
<tr>
<td></td>
<td>MS 9401</td>
<td>D. M. Skala, 8753</td>
</tr>
<tr>
<td></td>
<td>MS 9401</td>
<td>T. I. Wallow, 8753</td>
</tr>
<tr>
<td></td>
<td>MS 9405</td>
<td>J. M. Hruby, 8700</td>
</tr>
<tr>
<td></td>
<td>MS 9405</td>
<td>K. L. Wilson, 8770</td>
</tr>
</tbody>
</table>

Attn: W. C. Replogle, 8771, MS 9409
     C. H. Cadden, 8772, MS 9402
     J. C. F. Wang, 8773, MS 9403
     P. A. Spence, 8774, MS 9042

<table>
<thead>
<tr>
<th>Distribution</th>
<th>MS 9404</th>
<th>G. D. Kubiak, 8750</th>
</tr>
</thead>
</table>

Attn: J. R. Garcia, 8754, MS 9404

<table>
<thead>
<tr>
<th>Distribution</th>
<th>MS 9161</th>
<th>W. R. Even, 8760</th>
</tr>
</thead>
</table>

Attn: D. L. Medlin, 8761, MS 9161
     T. J. Shepodd, 8762, MS 9403
     E. P. Chen, 8763, MS 9161

<table>
<thead>
<tr>
<th>Distribution</th>
<th>MS 9018</th>
<th>Central Technical Files, 8945-1</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>MS 0899</td>
<td>Technical Library, 9616</td>
</tr>
<tr>
<td></td>
<td>MS 9021</td>
<td>Classification Office, 8511, for Technical Library, MS 0899, 9616</td>
</tr>
</tbody>
</table>

DOE/OSTI via URL