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UNDEVELOPED GEOTHERMAL FIELDS**

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June 16, 1975

CUMER 75-7

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ABSTRACT

A brief summary is given of the nondimensional equations and relevant parameters to be used for describing convection processes in the geothermal setting. Characteristic estimates of flow rates and pressure variation (beyond the hydrostatic value) are given. It is shown that the complete equation system can be reduced in complexity by neglecting terms which describe physically insignificant effects. The resulting equations are used to consider the onset of convection in a horizontal saturated porous slab when a realistic model of viscosity variation is used. It is shown that the classical constant viscosity prediction is in error both quantitatively and qualitatively. A calculation is made for the rate of convective upflow in a model fault zone. It is shown that under appropriate conditions it is possible to transport as much as $2 - 3 \times 10^6$ kg/day up a fault system with a horizontal cross-sectional area of only 0.3 km^2 .

1. INTRODUCTION

Efforts to produce accurate predictions of geothermal reservoir lifetime require realistic modeling of the physical processes occurring in the system. Particular emphasis must be placed on developing a sound understanding of the reservoir fluid dynamics for the purpose of describing the heat and mass transfer in the system. The results of such an analysis can then be used as a balance against system output due to exploitation in order to ascertain the productive capacity of the geothermal reservoir. A complementary value of fluid dynamical modeling relates to the physical characterization of the system and its correlation with geological analysis and geophysical measurements. At this point in time, geothermal systems are relatively poorly understood (in comparison say to oil fields) and hence the spectrum of geophysical measurements (resistivity, gravity, magnetics, self-potential, micro-seismics, magneto-tellurics, etc.) are difficult to interpret for purposes of locating accurately an exploitable geothermal field. Improved physical understanding of these systems should help to increase the reliability of prediction based on such measurements.

In order to develop quantitative models of fluid dynamical processes in liquid-dominated geothermal systems, it is absolutely essential to consider the details of geological setting and the actual physical properties of the liquid and rocks involved. From field data one requires input on faultzone locations, existence of aquifers, presence of intrusives, temperature and salinity distributions with depth, permeability and porosity values, possibility of extensive fracturing, rock types, etc. An examination of this type of information usually leads one to conclude that geothermal anomalies are relatively small-scale features ($O(10 \text{ km}^2)$)

within much larger regional structures. The scale seems to be related to local geological structure, in particular faulted regions, which act as high permeability vertical conduits of heated water from depth. High enthalpy fluids rising in these broken rock fault zones spread laterally whenever a horizontal aquifer is intersected. The result is a lateral convection of hot liquid which causes anomalously high heat flow at the surface above, over a region far larger than the actual upflow zone. A pictorial representation appears in Fig. 1. The essential point here is that local considerations should play a key role in determining the models to be used.

In this brief paper, we will discuss the equations appropriate for modeling liquid-dominated convecting systems. The nondimensionalized form of these equations is used to quantify heat and mass transfer processes. The parameters appearing are shown to be quite useful for characterizing the convective phenomena. An analysis of convection in a two-dimensional horizontal porous slab is considered to show the influence of the actual variable viscosity of water on the flow configuration. It is shown that results differ considerably from those of the classical constant viscosity analysis. Finally we consider a model of convection in a vertical fault zone. The results indicate that natural convection processes alone can produce mass-flow rates large enough to be of significance in the recharging of geothermal reservoirs.

2. MODELING EQUATIONS

The fluid dynamics of geothermal systems can, in general, be modeled in terms of flow in porous media. The porosity and concomitant permeability results from either intergranular properties or extensive distributions of discrete fractures or a combination of both. So long as the field data is used carefully to ascertain input values, the precise details should not be significant. The describing equations, whether derived from averaging approaches like that of Bear (1972) or from continuum mechanics arguments (Garg, et al., 1974) have the same form. In terms of nondimensional variables (defined below) the steady flow equations can be written in indicial form as:

$$(\rho q_\alpha)_{,\alpha} = 0 \quad , \quad q_i = \varphi v_i \quad (2.1)$$

$$D^2 \bar{R} \rho \varphi \frac{Dv_i}{Dt} = \frac{\varphi}{\tau} (T-1) \delta_{i3} - \frac{\mu \varphi}{k} q_i - \varphi p_{,i} + D (\varphi \sigma_{i\alpha})_{,\alpha} \quad (2.2)$$

$$R \rho \varphi \frac{DT}{Dt} = (\lambda_m T_{,\alpha})_{,\alpha} + M^2 P_r \left[\frac{q_\alpha \delta_{\alpha 3}}{D(\alpha' \Delta T')} + \frac{q_\alpha p_{,\alpha}}{D} + \varphi \sigma_{\alpha\beta} v_{\alpha,\beta} \right] \quad (2.3)$$

$$\rho = 1 - \alpha^* (T - 1) \quad (2.4)$$

The parameters are defined by

$$D = k'_0 / L'^2 \quad , \quad \bar{R} = g' \alpha'_0 \Delta T' L'^3 / \nu_o'^2 \quad (2.5a,b)$$

$$R = D \bar{R} C'_{po} \mu'_o / \lambda'_m \quad , \quad M^2 = q_R'^2 / C'_{po} T'_o \quad , \quad P_r = C'_{po} \lambda'_m / \mu'_o \quad (2.6 a,b,c)$$

$$q_R' = g' k'_0 \alpha'_0 \Delta T' / \nu_o' \quad , \quad \alpha^* = \alpha'_o T'_o \quad , \quad \tau = \Delta T' / T'_o \quad (2.7a,b,c)$$

where primes (') denotes dimensional quantities and the zero subscript denotes a specifically selected reference state. Here k' = permeability, L' = reference length (e.g., reservoir depth), g' = gravity constant, α' = thermal expansion coefficient of water, $\Delta T'$ = characteristic temperature difference, ν' = kinematic viscosity of water, C_p' = specific heat of water, μ' = dynamic viscosity of water, λ_m' = thermal conductivity of the mixture. In (2.1-4) the density (ρ) and temperature (T) are nondimensionalized by the reference state values ρ_0' and T_0' . The fluid velocity (v_i) is nondimensionalized by q_R' in (2.7a), the choice of which is described below. The pressure (p) represents the difference between the actual local pressure and the hydrostatic value based on the reference state density nondimensionalized with respect to $P_R' \equiv \rho_0' g' L' \alpha' \Delta T'$. In (2.2) σ_{ij} is the viscous stress tensor for the liquid. The derivation of (2.1-4) involves assumptions of constant specific heat and expansion coefficient and the simplified state equation shown in (2.4). For the range of temperature variation ($\approx 300^\circ\text{C}$) and pressure variation (≈ 300 bars) of interest in the geothermal environment these are reasonable assumptions.

The values of q_R' and p_R' are not known a priori and must be derived. These quantities are found by considering the basic physics of buoyancy-driven natural convection. This requires that there be an inherent balance between the physical phenomena represented by the first three terms on the right-hand side of (2.2); buoyant forces, the Darcy-like term and pressure gradients. (The values obtained could also be found by using the Buckingham Pi-Theorem). Quantitative estimates of q_R' and p_R' provide characteristic magnitudes of the vertical convection velocity and the pressure field (above and beyond hydrostatic) associated with the motion. If we consider data normally associated with active geothermal systems;

$$\bullet \quad \alpha' = 5 \times 10^{-4}/^{\circ}\text{C} \quad , \quad k' = 10^{-9} \text{ cm}^2 \quad , \quad v' = 2 \times 10^{-3} \text{ cm}^2/\text{s}$$

$$\Delta T' = 10^2 ^{\circ}\text{C} \quad (2.8)$$

we find that

$$q_R' \approx 1 \text{ cm/day} \quad , \quad p_R' \approx 10 \text{ bars} \quad (2.9)$$

The value of q_R' can be used to estimate vertical mass and energy flux due to convection in a geothermal system. For Wairakei where the active predevelopment area is about 15 km^2 (Mercer, 1973), we find a mass flux calculated from (2.9) of $1.75 \times 10^6 \text{ gm/sec}$. This compares with measured values of surface flux of $4.4 \times 10^5 \text{ gm/sec}$. The order of magnitude comparison is reasonable since it is likely that actual upflow area at Wairakei is considerably less than the surface area over which surface emanations appear. A similar comparison can be made on the basis of convective energy flux which is calculated from (2.9) to be $5 \times 10^{-3} \text{ cal/cm}^2\text{s}$ compared to $1 \times 10^{-3} \text{ cal/cm}^2\text{s}$ measured at Wairakei. It is to be noted that the typical conductive energy flux is about $10^{-6} \text{ cal/cm}^2\text{s}$ which is far smaller. These results suggest that the heat and mass transfer processes at Wairakei prior to development were essentially controlled by vertical convection of water from depth rather than by lateral ground water motion through the Waiora Formation assumed to be heated by unspecified sources from below.

The characteristic p_R' value in (2.9) is indicative of the deviation of the local pressure from the hydrostatic state. This substantial magnitude should be measurable in closed bore-holes in formations in which convective upwelling is occurring.

The describing differential equation (2.1) - (2.4) may be simplified to some extent by considering values of the parameters in (2.5) and (2.6)

for geothermal conditions. If we use the values in (2.8) and a typical reservoir depth $L' \approx 10^3 \text{ m}$ we find that $D \approx 10^{-19}$, $R \approx 10^{22}$, $R \approx 10^3$, $M^2 \approx 10^{-23}$ and $P_r \approx 1$. This suggests that the inertia and stress tensor terms in (2.2) are insignificant as are all the terms multiplied by M^2 in (2.3). The resulting simplified equations are used in the next two sections to describe model problems which help to elucidate several important physical features encountered in geothermal flows.

3. EFFECTS OF VARIABLE VISCOSITY ON CONVECTION

The equations appearing in section 2 admit a fluid viscosity that varies with temperature. This effect is particularly important in the geothermal setting where temperature variations of 400°C are possible. In this range, water viscosity decreases by a factor of about 15 between 5°C and 350°C (Keenen et al., 1969). We consider here a calculation for the onset of convection in a homogeneous saturated infinite horizontal porous slab with uniformly heated impermeable boundaries with a temperature difference $\Delta T'$. For simplicity it is assumed that $\rho \approx 1$, $\lambda_m \approx 1$, $k = 1$ and $\phi = \text{const.}$ Actual variation of the first two properties is at most about 25% for the temperature ranges used, compared to the factor of 15 in viscosity variation. It is this latter effect, on which we concentrate here. The viscosity variation is described by the empirical relation used by Wooding, 1957. If we neglect all the small terms discussed in section 2, and write $T = 1 - \tau y + \tau \theta(x, y)$ where $\theta \ll 1$, then (2.1-3) can be written as

$$\mu(T) \nabla^2 \Psi + \mu_y \Psi_y + \mu_x \Psi_x = -\theta_x \quad (3.1)$$

$$R[\Psi_y \theta_x - \Psi_x(1 - \theta_y)] = \nabla^2 \theta \quad (3.2)$$

where the stream function Ψ is defined in the standard manner with respect to the vertical and horizontal velocities. The appropriate boundary conditions have the form

$$\theta(x, 0) = \theta(x, -1) = \Psi_x(x, 0) = \Psi_x(x, -1) = 0$$

A linear stability analysis of (3.1) and (3.2), described in detail by Kassoy and Zebib, 1975, is used to find the value of R at which

convection begins and the associated wave number of the convection cell. The results are presented in Table 1 for the case when $T'_0 = 25^\circ\text{C}$. In the first row we see the classical result from the constant viscosity calculation (Lapwood, 1948). The critical Rayleigh number value of $4\pi^2$ will be valid for systems with $\Delta T'$ of a few degrees centigrade. Increasing $\Delta T'$ (or τ) is observed to cause a rather dramatic decrease in the critical value of R while the wave number increases only modestly. The results are expected on physical grounds since the hot fluid near the bottom boundary is considerably less viscous than elsewhere and hence becomes unstable sooner than a relatively cooler fluid.

In Fig. 2 the streamline pattern ($\Psi = \text{constant}$) is shown for $\tau = 0.84$. The x -scale is nondimensionalized by the wave length of the constant viscosity result. Since the wave length for the present result is slightly shorter, a small portion of the neighboring cell intrudes on the left. In distinction to the classical streamline pattern which is symmetric about $y = -0.5$, the present result shows a high density of streamlines near the hot bottom boundary. This implies that the relatively low viscosity water near the hot bottom boundary moves faster than the cooler water near the top surface. As a result of the larger velocities deep in the system, there is increased convection of energy and hence relatively larger temperature gradients. The latter effect may be observed in Fig. 3 where the disturbance isotherms are more closely spaced near the bottom. Once again the analogous classical result is symmetric about $y = -0.5$.

One may conclude from these results that an accurate convection flow description for systems with large temperature differences requires the inclusion of a variable viscosity relationship. Furthermore, the results

of this linear stability analysis suggest that qualitatively different types of flow patterns will be found when appropriate viscosity variation is included.

4. CONVECTIVE FLOW IN A POROUS CHANNEL

A vertical fault zone may act as a local high permeability conduit of heated fluid from depth. In this section, we consider an extremely simple model of this type of system in order to ascertain the magnitude of mass flow rates one could expect to find. Imagine a long narrow channel filled with a homogeneous porous material as shown in Fig. 4. The walls are impermeable and at a temperature that increases linearly with depth. In order to obtain an elementary analytical solution, we assume that the viscosity is constant while recognizing the inherent limitations of such an approximation. The describing mathematical system has the form

$$u_x + v_y = 0 \quad (4.1)$$

$$\mu u = -p_x + (T-1)/\tau, \quad \mu v = -p_y \quad (4.2a,b)$$

$$R \left[u T_x + v T_y \right] = \nabla^2 T \quad (4.3)$$

$$T(1, y_e) = 1 - \tau x, \quad T_y(x, 0) = 0 \quad (4.4)$$

$$\Psi(x, y_e) = \dot{m} y_e, \quad \Psi(x, 0) = 0 \quad (4.5)$$

where u and v are components of the specific discharge q , $y_e = y'_e/L' \ll 1$ is the nondimensional half width and $\dot{m}$ is the mass flux, nondimensionalized by $2\rho_0 y'_e q'_R$. The primary goal of the calculation is to find the net mass flux rate as a function of energy input into the system.

The complete details of the analysis found in Kassoy, 1975, will be summarized here. For the boundary conditions specified, there is a

so-called fully developed solution of the form

$$u = \dot{m}\alpha \frac{\cos \gamma y}{\sin \alpha}, \quad v = 0 \quad (4.6)$$

$$\theta = \mu \dot{m}\alpha \left[\frac{\cos \gamma y}{\sin \alpha} - \cot \alpha \right], \quad T = 1 - \tau x + \tau \theta \quad (4.7)$$

$$p_x = -x + \mu \dot{m}\alpha \cot \alpha$$

$$\alpha = \left(\frac{R}{\mu} \right)^{1/2} y_e \quad (4.9)$$

The velocity solution in (4.6) implies that for a sufficiently narrow channel, $0 < \alpha \leq \pi/2$, only upflow exists. However, for a wider system, $\pi/2 < \alpha < \pi$, there will be upflow in a central region of the channel, but downflow adjacent to the walls. This peculiar behavior results from an interaction of the pressure field in (4.8) and buoyant force represented by the third term in (4.2a). The case $\alpha = \pi$ will be shown below to correspond to a zero net mass flow ($\dot{m} = 0$). In this special case, we have a closed convection cell with adiabatic walls. For $\alpha > \pi$ the flow pattern involves multiple up and down flow regions.

The pressure field computed from (4.8) displays interesting behavior as well. For $0 < \alpha < \pi/2$ the local physical pressure is larger than the real hydrostatic head found at that depth; e.g. the system is over-pressured. However, for $\pi/2 < \alpha < \pi$ the system is actually under-pressured. Since the order of magnitude of this effect is as much as 10 bars, the effect should be measurable in a real system.

In order to find the value of $\dot{m}$, an overall energy balance is carried out for the system. The total energy flux across a plane at some x , and the energy loss by conduction at the walls between the

plane and the bottom of the channel at $x = -1$ are equated to the total nondimensional energy input Q_i at $x = -1$. It follows from this that

$$\dot{m} = \frac{\sin \alpha}{\mu \Omega(\alpha)} \left[\left\{ (\beta \sin \alpha)^2 + 2\mu \Omega(\alpha) \Gamma \right\}^{1/2} - \beta \sin \alpha \right] \quad (4.10)$$

$$\Omega(\alpha) = \alpha^2 - (\alpha/2) \sin 2\alpha$$

$$\Gamma = (Q_i - 1)/R$$

$$\beta = (1 + \tau)/\tau$$

In physical terms, Q_i is described by

$$Q_i = \frac{Q'_i / 2y'_e}{\lambda'_m \Delta T' / L'} \quad (4.11)$$

where the numerator represents the heat flux per unit area of channel entering the system (by convection and/or conduction) and the denominator represents the purely conductive flux that would occur in the saturated porous medium due to a temperature difference $\Delta T'$ between the top and bottom.

When $\alpha \rightarrow \pi$ it may be shown from (4.10) that

$$\dot{m} = \left[\frac{2(Q_i - 1)}{\mu R \tau^2} \right]^{1/2} \sin \alpha \quad (4.12)$$

which vanishes for $\alpha = \pi$ as mentioned earlier. It should be noted, however, that if (4.12) is used in (4.6-8) nonvanishing results are obtained for the velocity, temperature and pressure fields. In particular, the velocity field is symmetric with as much upflow in the central region as there is downflow adjacent to the walls.

A Geothermal Application

The actual mass flux up a fault zone of width $2y'_e$ can now be calculated in terms of $\dot{m}$ from (4.10) and its nondimensional definition

$$\dot{m}' = \frac{2\lambda'_m R^{1/2}}{C'_p} \alpha \dot{m} \quad (4.13)$$

The data used for the calculation includes

$$k' = 10^{-9} \text{ cm}^2, \quad v' = 2 \times 10^{-3} \text{ cm}^2/\text{s}, \quad \alpha' = 10^{-3}/^\circ\text{C}$$

$$L' = 1800 \text{ m}, \quad \lambda'_m = 3.25 \times 10^{-3} \text{ cal/cm s } ^\circ\text{C}$$

$$C'_p = 4.4 \text{ j/gm } ^\circ\text{C}, \quad \Delta T' = 50^\circ\text{C}$$

These values have been chosen to reflect, to some extent, conditions of the East Mesa Anomaly in the Imperial Valley (Black, 1975). The Rayleigh number R is conservatively estimated at $R = 10^3$. In Fig. 5 we show the actual upflow mass flux per day as a function of fault zone width for a fault whose hydrothermally active lateral extent is 10^3 m . For values of $2y'_e > 182 \text{ m}$ the corresponding $\alpha > \pi/2$ so that $\dot{m}$, which measures net upflow, has been replaced by the actual upflow calculated from

$$\dot{m}_{\text{up}} = \dot{m} / \sin \alpha, \quad \pi/2 < \alpha < \pi$$

The break in the curve at $2y'_e = 182 \text{ m}$ reflects the alteration in flow configuration at $\alpha = \pi/2$. Results are given for two values of Q_i . It is clear that the upflow of heated fluid can be substantial for the large Q_i values chosen. It should be noted that such values are not unreasonable for the fluid entering the deep "bottom" of our hypothetical fault zone will be carrying convective energy far in excess of even anomalous conductive heat fluxes.

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Table 1

τ	λ	R
0	π	$4\pi^2$
0.34	3.26	17.25
0.50	3.31	13.35
0.67	3.34	10.76
0.84	3.37	8.86

CAPTIONS

- Fig. 1 Conceptual model of vertical upflow in a fault zone intersecting an aquifer. The bulge in the temperature distribution results from lateral convection of hot water.
- Fig. 2 The streamline pattern for $\tau = 0.84$. Arrows indicate flow direction.
- Fig. 3 The disturbance isotherm pattern for $\tau = 0.84$.
- Fig. 4 The slender channel ($y'_e/L' \ll 1$) filled with porous material.
- Fig. 5 Actual mass upflow m'_{up} as a function of fault zone width $2y'_e$.

Figure 10.10: Contour plot of the function ψ .

Answer:

Fig. 3
State Map Per 1.0

Fig. 4. Thermal conductivity

Fig. 4