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Low-Frequency Electromagnetic Backscattering from Tunnels

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Introduction

Low-frequency electromagnetic scattering from one or more tunnels in a lossy dielectric half-space is considered. The tunnel radii are assumed small compared to the wavelength of the electromagnetic field in the surrounding medium; a tunnel can thus be modeled as a thin scatterer, described by an equivalent impedance per unit length. We examine the normalized backscattering width for cases in which the air-ground interface is either smooth or rough.

Tunnel in a Lossy Half-Space

The tunnel is a circular-cylindrical region of free space situated in the half-space $z < 0$, of relative permittivity ϵ_r. The tunnel is parallel to the y-axis; its axis is located at $(x, z) = (0, -d)$ and its radius is a, which we assume to be small compared to the wavelength in the surrounding region. By virtue of this assumption, we can model the tunnel as a thin scatterer described by an equivalent impedance per unit length Z'_w. It can be shown that this equivalent impedance per unit length is

$$Z'_w = \frac{jk_0 Z_0}{\pi(k_0 a)^2(\epsilon_r - 1)} \tag{1}$$

in which k_0 is the free-space wavenumber and Z_0 is the intrinsic impedance of free space. The time dependence $\exp(j\omega t)$ is assumed. At the exterior wall of the tunnel, the electric field component parallel to the tunnel axis is equal to the impedance per unit length multiplied by the equivalent current I_0 carried by the scatterer. (Because the tunnel radius is electrically small, the axial electric field is essentially uniform around the tunnel wall.) Our approach to the scattering problem is to determine the field incident on the scatterer and the field excited by the (as yet unknown) equivalent current on it; we then impose the boundary condition at the tunnel wall to determine the equivalent current, and the solution is complete.

We assume that a perpendicularly-polarized plane electromagnetic wave is incident on the lower half-space from the free-space region $z > 0$. The incident electric field in the free-space region and the electric field transmitted into the lower medium are

$$E_{yi}(x, z) = E_0 e^{-jk_0(x \sin\theta_i - z\cos\theta_i)} \tag{2}$$

$$E_{yt}(x, z) = T(\theta_i) E_0 e^{-jk_0(x\sin\theta_i - z\sqrt{\epsilon_r - \sin^2\theta_i}\,)} \tag{3}$$

with θ_i denoting the angle of incidence; the transmission coefficient $T(\theta)$ is

$$T(\theta) = \frac{2\cos\theta}{\cos\theta + \sqrt{\epsilon_r - \sin^2\theta}} \tag{4}$$

Next consider the electric field created by a filamentary current I_0 at $(x, z) = (0, -d)$. This field can be expressed in the two regions as the Fourier integrals

$$E_{y>}(x, z) = \frac{jk_0 Z_0 I_0}{2\pi} \int_{-\infty}^{\infty} e^{-jk_x x - jk_{z0} z - jk_{zg} d} \frac{dk_x}{j(k_{z0} + k_{zg})} \tag{5}$$

$$E_{y<}(x, z) = \frac{jk_0 Z_0 I_0}{2\pi} \int_{-\infty}^{\infty} e^{-jk_x x} \left[\frac{k_{zg} - k_{z0}}{k_{zg} + k_{z0}} e^{jk_{zg}(z-d)} - e^{-jk_{zg}|z+d|} \right] \frac{dk_x}{2jk_{zg}} \tag{6}$$

where $k_{z0} = \sqrt{k_0^2 - k_x^2}$ and $k_{zg} = \sqrt{k_0^2 \epsilon_r - k_x^2}$. The second term in the integrand on the RHS of eq. (6) above yields the electric field directly radiated by the filamentary current, and the first term yields the field that is reflected back into the ground by the air-ground interface. The electric field radiated into the free-space region, evaluated in the far zone, is

$$E_{y>}(\rho, \theta) \sim -\frac{jk_0 Z_0 I_0}{\sqrt{8\pi jk_0 \rho}} T(\theta) e^{-jk_0 \rho - jk_0 d \sqrt{\epsilon_r - \sin^2 \theta}} \tag{7}$$

We define the normalized equivalent backscattering width $k_0 \ell$ as

$$k_0 \ell = \lim_{\rho \to \infty} 2\pi k_0 \rho \left. \frac{S_r}{S_i} \right|_{\theta = \theta_i} = \left| \frac{k_0 I_0 Z_0}{2E_0} \right|^2 \left| T(\theta_i) e^{-jk_0 d \sqrt{\epsilon_r - \sin^2 \theta_i}} \right|^2 \tag{8}$$

S_i and S_r denote the incident and backscattered power densities respectively. Now imposing the condition $E_y(0, -d + a) = Z_w' I_0$ and using eqs. (3) and (6), we obtain

$$I_0 = \frac{T(\theta_i) E_0 e^{-jk_0 d \sqrt{\epsilon_r - \sin^2 \theta_i}}}{jk_0 Z_0 \zeta_n} \tag{9}$$

with the normalized impedance ζ_n given by

$$\zeta_n = \frac{Z_w'}{jk_0 Z_0} + \frac{1}{4j} H_0^{(2)}(k_g a) + \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{-2jk_{zg} d} \left(\frac{k_{zg} - k_{z0}}{k_{zg} + k_{z0}} \right) \frac{dk_x}{2jk_{zg}} \tag{10}$$

with $k_g = k_0 \sqrt{\epsilon_r}$. The normalized scattering width $k_0 \ell$ is

$$k_0 \ell = \frac{|T(\theta_i)|^4}{|2\zeta_n|^2} \left| e^{-jk_0 d \sqrt{\epsilon_r - \sin^2 \theta_i}} \right|^4 \tag{11}$$

The relative permittivity of the ground is expressed in terms of a high-frequency relative permittivity $\epsilon_{r\infty}$ and a low-frequency conductivity σ_0 as [1]

$$\epsilon_r = \left(\sqrt{\epsilon_{r\infty}} + \sqrt{\sigma_0 Z_0 / (jk_0)} \right)^2 \tag{12}$$

Typical values of $\epsilon_{r\infty}$ are in the range 6 to 8, and σ_0 varies upward from 10^{-4} S/m for very dry soils. The value $\epsilon_{r\infty} = 7$ was used in the numerical results shown here. In Figure 1 we show the normalized scattering width of a tunnel of radius $a = 5$ m at depth $d = 50$ m as a function of frequency for incidence angles of 0°, 30°, and 60°. The low-frequency conductivity $\sigma_0 = 10^{-4}$ S/m. We note that the normalized scattering widths tend to increase with increasing frequency, up to a maximum value that occurs at approximately 6.25 MHz, and then drop off fairly rapidly as the frequency increases further. The normalized scattering width also decreases rapidly with increasing ground conductivity.

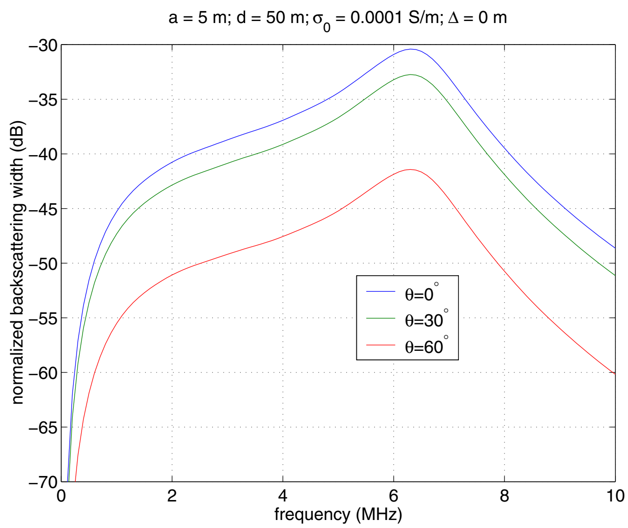

Figure 1: Normalized scattering width vs. frequency in very dry soil ($\sigma_0 = 10^{-4}$ S/m). Depth $d = 50$ m and radius $a = 5$ m; incidence angles $\theta_i = 0°$, $30°$, and $60°$.

Rough-Surface Effects on Tunnel Backscatter

We model the interface as a random-phase screen. This model is valid only when the standard deviation of the phase shift imposed by the screen is small, so that our analysis is valid for a "slightly rough" air-ground interface. The interface is now taken to be the surface $z = \Delta(x, y)$, where the random process $\Delta(x, y)$ is homogeneous and Gaussian, with expected value $\mathcal{E}\{\Delta(x, y)\} = 0$ and variance σ^2. Using this model we can show (see, e.g., [2]) that the coherent amplitude of the electric field of a plane wave transmitted across the screen is reduced by the factor

$$F = e^{-q^2(k_{x0},k_{y0})\sigma^2/2} \tag{13}$$

in which k_{x0} and k_{y0} are the transverse propagation constants of the plane wave and the function $q(\cdot)$ is given for the present problem by

$$q(k_{x0}, k_{y0}) = k_0 \Re \left\{ \sqrt{\epsilon_r - \sin^2\theta_i} - \cos\theta_i) \right\} \tag{14}$$

In the problem of backscattering from a buried target, the electromagnetic wave passes through the rough interface twice, so that the coherent backscattered electric-field amplitude is reduced by the factor F^2 and the backscattering width is reduced by the factor F^4.

A portion of the equivalent current induced on the buried tunnel results from reflection by the air-ground interface of the field that is radiated by the tunnel. The fact that this surface is rough is accommodated by including the following factor in the integrand of the integral in eq. (10):

$$\exp\left[-2k_0^2\sigma^2\Re^2\left\{\sqrt{\epsilon_r - k_x^2/k_0^2}\right\}\right] \tag{15}$$

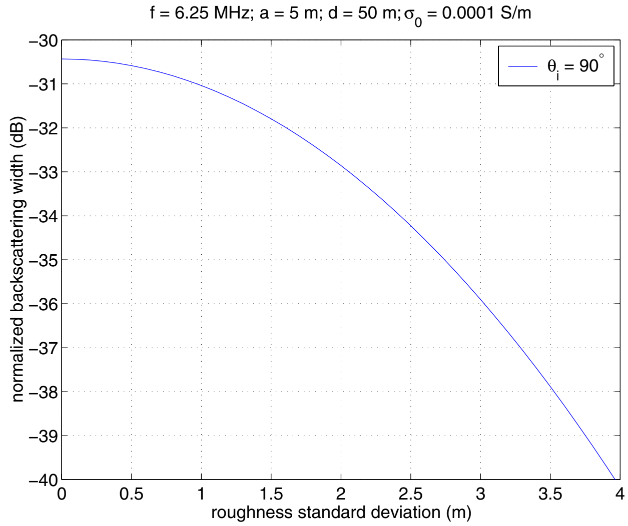

Figure 2: Normalized scattering width vs. σ in very dry soil ($\sigma_0 = 10^{-4}$ S/m). Depth $d = 50$ m and radius $a = 5$ m; $\theta_i = 0°$ and $f = 6.25$ MHz.

Computations of the backscattering width are performed as before with the modification described above made in eq. (10) and the overall result multiplied by the factor F^4. Figure 2 shows the normalized scattering width as a function of the standard deviation of the surface roughness for normal incidence when the frequency is 6.25 MHz. When the standard deviation reaches 4 m, the normalized backscattering width is reduced by approximately 10 dB from its value for a smooth interface.

Concluding Remarks

We have investigated the problem of low-frequency electromagnetic backscattering from a tunnel located in a lossy ground. The fact that the tunnel diameter is assumed to be small in comparison to the wavelength allows us to model the tunnel as a thin scatterer described by an equivalent impedance per unit length. We computed the electric field backscattered from a tunnel for the case of a smooth air-ground interface as a function of the signal frequency, tunnel depth and radius, incidence angle, and soil properties. Rough-surface effects were included by using a random phase-screen model for the rough interface.

References

[1] M. A. Messier, "The propagation of an electromagnetic impulse through soil: influence of frequency dependent parameters," Technical Report MRC-N-415, Mission Research Corporation, Santa Barbara, CA, 1980.

[2] Kendall F. Casey, "Rough-surface effects on subsurface target detection," Proceedings of SPIE, Vol. 4394, 2001.