Measurement of the Pseudoscalar Decay Constant $f_D$, Using Charm-Tagged Events in $e^+e^-$ Collisions at $\sqrt{s} = 10.58$ GeV

Using 230.2 fb$^{-1}$ of $e^+e^-$ annihilation data collected with the BABAR detector at and near the peak of the $T(4S)$ resonance, 489 ± 55 events containing the pure leptonic decay $D_s^+ \rightarrow \mu^+ \nu_\mu$ have been isolated in charm-tagged events. The ratio of partial widths $\Gamma(D_s^+ \rightarrow \mu^+ \nu_\mu)/\Gamma(D_s^+ \rightarrow \phi\pi^+)$ is measured to be 0.143 ± 0.018 ± 0.006 allowing a determination of the pseudoscalar decay constant $f_{D_s} = (283 ± 17 ± 7 ± 4) \text{ MeV}$. The errors are statistical, systematic, and from the $D_s^+ \rightarrow \phi\pi^+$ branching ratio, respectively.

PACS numbers: 13.20.He, 14.40.Nd, 14.60.Fg

Measurements of pure leptonic decays of charmed pseudoscalar mesons are of particular theoretical importance. They provide an unambiguous determination of the overlap of the wavefunctions of the heavy and light quarks within the meson, represented by a single decay constant ($f_M$) for each meson species ($M$). The partial width for a $D_s^+$ meson to decay to a single lepton flavor ($l$) and its accompanying neutrino ($\nu_l$), is given by

$$\Gamma(D_s^+ \rightarrow l^+\nu_l) = \frac{G_F^2|V_{cs}|^2}{8\pi} f_{D_s}^2 m_{D_s}^2 m_{D_s} \left(1 - \frac{m_l^2}{m_{D_s}^2}\right),$$

where $m_{D_s}$ and $m_l$ are the $D_s^+$ and lepton masses, respectively, $G_F$ is the Fermi constant, and $V_{cs}$ is the CKM matrix element giving the coupling of the weak charged current to the $c$ and $s$ quarks. The partial width is governed by two opposing terms in $m_l^2$. The first term reflects helicity suppression in the decay of the spin-0 meson, requiring the charged lepton to be in its unfavored helicity state. The second term is a phase-space factor. As a result, the ratio of $\tau : \mu : e$ decays is approximately 10 : 1 : 0.00002. Lattice calculations have resulted in $f_{D_s} = (249 ± 17) \text{ MeV}$ and a ratio $f_{D_s}/f_{D^0} = 1.24 ± 0.07$.[4] CLEO-c has recently measured a value for $f_{D_s} = (223 ± 17) \text{ MeV}$.[4]

We present herein the most precise measurement to date of the ratio $\Gamma(D_s^+ \rightarrow \mu^+\nu_\mu)/\Gamma(D_s^+ \rightarrow \phi\pi^+)$ and the decay constant $f_{D_s}$. The data (230.2 fb$^{-1}$) were collected with the BABAR detector at the asymmetric-energy $e^+e^-$ storage ring PEP-II at and below the $T(4S)$ resonance. The BABAR detector is described in detail elsewhere.[4] Briefly, the components used in this analysis are the tracking system composed of a five-layer silicon vertex detector and a 40-layer drift chamber (DCH), the Cherenkov detector (DIRC) for charged $\pi$–$K$ discrimination, the CsI(Tl) calorimeter (EMC) for photon and electron identification, and the 18-layer flux return (IFR) located outside the 1.5 T solenoid coil and instrumented with resistive plate chambers for muon identification and hadron rejection.

The analysis proceeds as follows. In order to measure $D_s^+ \rightarrow \mu^+\nu_\mu$, the decay chain $D_s^+ \rightarrow \phi D_s^+$, $D_s^+ \rightarrow \mu^+\nu_\mu$ is reconstructed from $D_s^+$ mesons produced in the hard fragmentation of continuum $e^+e^-$ events. The subsequent decay results in a photon, a high-momentum $D_s^+$ and daughter muon and neutrino, lying mostly in the same hemisphere of the event. Signal candidates are required to lie in the recoil of a fully reconstructed $D^0$, $D^+, D_s^+$, or $D^{*+}$ meson (the “tag”), wherein the tag flavor, and hence the expected charge of the signal muon, is uniquely determined. To eliminate signal from $B$ decays, the minimum tag momentum is chosen to be close to the kinematic limit for charm mesons arising from $B$ decays. Tagging in this manner significantly reduces backgrounds, while improving the missing mass resolution of the signal.

Tag candidates are reconstructed in the following modes: $D^0 \rightarrow K^–\pi^+\pi^–\pi^0$, $K^–\pi^+\pi^+\pi^0$, $D^+ \rightarrow K^–\pi^+\pi^+\pi^0$, $K^0_S\pi^+\pi^0$, $K^0_S\pi^+\pi^–\pi^0$, $K^0_S\pi^0\pi^+\pi^–K^+K^–\pi^0$, $D_s^+ \rightarrow K^0_SK^+\phi$, and $D^{*+} \rightarrow \pi^0D^0$, with $D^0 \rightarrow K^0_S\pi^+\pi^–\pi^0$, $K^0_SK^+K^–\pi^0$. Kaons are identified using information from the DCH and the DIRC. Requirements on the vertex probability of the tag decay products are imposed. For each tag mode a signal region and sideband regions in the tag mass distribution are defined. The signal region spans ±2 standard deviations ($\sigma_{tag}$) around the mean ($\mu_{tag}$), determined from fits to the tag mass distribution in data events. The sidebands extend from 3 to 6 $\sigma_{tag}$ on either side of $\mu_{tag}$ (Fig. 1).

For each event a single tag candidate is chosen and then used in the subsequent analysis. To pick this tag among multiple candidates within an event (there are 1.2 candidates on average in events with at least one candidate) modes of higher purity are preferred. In events where two tag candidates are reconstructed in the same mode, the quality of the vertex fit of the $D$ meson is used as a secondary criterion. After subtracting combinatorial background there are $5 \times 10^5$ charm tagged events with a muon amongst the recoiling particles.

The signature of the decay $D_s^+ \rightarrow \phi D_s^+$ is a narrow peak in the distribution of the mass difference $\Delta M = M(\mu\nu\gamma) - M(\mu\nu)$ at 143.5 MeV/$c^2$. The $D_s^+$ signal is reconstructed from a muon and a photon candidate in the recoil of the tag. Muons are identified as non-showering tracks penetrating the IFR. The muon must have a momentum of at least 1.2 GeV/$c$ in the center-of-mass (CM) frame and have a charge consistent with the tag flavor.
significantly separated from the signal. A requirement of correctly reconstructed \( D^0 \) is minimized, while the invariant mass of the neutrino-\( D^0 \) pair is required to be the known mass of the \( D^0 \). Studies of simulated decays of signal and background \( \ell \ell \) events show that the quantity \( p_{\text{corr}} = |p_{\mu \nu \ell}^\text{miss}| - |p_{\mu}^\text{mis} | \) is centered at 0 for signal decays, while for the \( \ell \ell \) events it peaks at a negative value significantly separated from the signal. A requirement \( p_{\text{corr}} > -0.06 \) GeV/c is imposed. To reduce contributions from background events where particles are lost along the beam pipe in the forward direction, a requirement on the neutrino CM polar angle \( \theta^\nu_\mu \) > 38° is made. The muon CM four-momentum \( p_{\mu}^\text{mis} \) is combined with \( p_{\mu}^\text{mis} \) to form the \( D^+ \) candidate. Unlike the signal \( D^+ \), a large number of random \( D^0 \) combinations have the muon candidate aligned with the \( D^+ \) flight direction. A requirement \( \cos(\alpha_{\mu,D^+}) < 0.90 \) is made on the angle between the muon direction in the \( D^+ \) frame and the \( D^+ \) flight direction in the CM frame. The \( D^0 \) candidate is then combined with a photon candidate to form the \( D^+ \). The CM momentum of correctly reconstructed \( D^+ \) is typically higher than that of random combinations; signal candidates are required to have \( |p_{D^+}^\text{mis}| > 3.55 \) GeV/c. The resulting signal detection efficiency in tagged events is \( \epsilon_{\text{Sig}} = 8.13 \% \).

The selection requirements on \( E_{\text{mis}}^{\text{corr}}, \alpha_{\mu,D^+}, p_{\text{corr}}, \theta^\nu_\mu \), and \( |p_{D^+}^\text{mis}| \) are optimized using simulation to maximize the significance \( s/\sqrt{s+b} \), where \( s \) and \( b \) are the signal and background yields expected in the data set. Backgrounds arise from several distinct sources. The first class of background are events \( e^+e^- \rightarrow f f \), where \( f = u, d, s, b, \) or \( \tau \), which do not contain a real charm tag. The contribution of these events is estimated from data using the tag sidebands. In addition there are events \( e^+e^- \rightarrow c\bar{c} \) where the tag is incorrectly reconstructed. Although these events potentially contain the signal decay, they are also subtracted using the tag sidebands. These two sources amount to \( \approx 42 \% \) of the background.

The second class of background events (\( \approx 26 \% \)) are correctly tagged \( c\bar{c} \) events with the recoil muon coming from a semileptonic charm decay or from \( \tau^+ \rightarrow \mu^+\nu_\tau\bar{\nu}_\tau \). This includes events \( D^0 \rightarrow \gamma D^+ \rightarrow \gamma \tau^+\nu_\tau \rightarrow \mu^+\nu_\tau\bar{\nu}_\tau \). To estimate the size and shape of this background contribution, the analysis is repeated, substituting a well-identified electron for the muon. Except for a small phase-space correction, the widths of weak charm decays into muons and electrons are assumed to be equal. QED effects such as bremsstrahlung \( (e^+ \rightarrow e^+\nu e^-) \) energy losses and photon conversion \( (\gamma \rightarrow e^+e^-) \), where the muon equivalents have a much lower rate, are explicitly removed. In particular, bremsstrahlung photons found in the vicinity of an electron track are combined with the track. The small number of events with an electron from a converted photon that survive the selection are suppressed by a photon conversion veto, using the vertex and the known radial distribution of the material in the detector. The muon selection efficiency as a function of momentum and direction is measured using \( e^+e^- \rightarrow \mu^+\mu^-\gamma \) events, while radiative Bhabha events are used to quantify the electron efficiency. The ratio of muon to electron efficiencies is applied as a weight to each electron event.

The remaining backgrounds are estimated from simulation. These include events (\( \approx 20 \% \)) with pure leptonic decays of a \( D_s^0 \) or \( D^+ \) meson, \( D^+_s \rightarrow \mu^+\nu_\mu \), where the \( D^+_s \) is produced either directly in \( c\bar{c} \) fragmentation or in decays of \( D^+_s \), excluding the signal decay chain. If the photon used in the reconstruction originates from a \( \pi^0 \) of a \( D^+_s \) decay, the \( \Delta M \) distribution peaks sharply around 70 MeV/c^2; otherwise it is flat. A small background (\( \approx 1 \% \)) arises from decays \( D^+_s \rightarrow \gamma D^+_s \rightarrow \gamma \tau^+\nu_\tau \), with \( \tau^+ \rightarrow \pi^+(\pi^0)\nu_\tau \) and the charged pion being misidentified as a muon. Its \( \Delta M \) distribution peaks close to that of the signal. Other backgrounds (\( \approx 10 \% \)) include signal events with an incorrectly chosen photon candidate, and hadronic \( c\bar{c} \) events with one of the final state hadrons,
usually a $\pi^+$ or a $K^+$, being misidentified as a muon. These backgrounds have a flat $\Delta M$ distribution.

Events that pass the signal selection are grouped into four sets, depending on whether the tag lies in the signal region or the sideband regions, and on whether the lepton is a muon or an electron (Fig. 2). For each lepton type the sideband $\Delta M$ distribution is subtracted. The electron distribution, scaled by the relative phase-space factor (0.97) appropriate to semileptonic charm meson decays and leptonic $\tau$ decays is then subtracted from the muon distribution. The resulting $\Delta M$ distribution is fitted with a function $(N_{\text{Sig}}f_{\text{Sig}} + N_{\text{Bkgd}}f_{\text{Bkgd}})(\Delta M)$, where $f_{\text{Sig}}$ and $f_{\text{Bkgd}}$ describe the simulated signal and background $\Delta M$ distributions. The function $f_{\text{Sig}}$ is a double Gaussian distribution. The function $f_{\text{Bkgd}}$ consists of a double and a single Gaussian distribution describing the two peaking background components, and a function describing the flat background component. The relative sizes of the background components, along with all parameters except $N_{\text{Sig}}$ and $N_{\text{Bkgd}}$ are fixed to the values estimated from simulation. The $\chi^2$ fit yields $N_{\text{Sig}} = 489 \pm 55 (\text{stat})$ signal events and has a fit probability of 8.9% (Fig. 4).

The branching fraction of $D_s^+ \rightarrow \mu^+\nu_\mu$ cannot be determined directly, since the production rate of $D_s^{(*)+}$ mesons in $c\bar{c}$ fragmentation is unknown. Instead the partial width ratio $\Gamma(D_s^+ \rightarrow \mu^+\nu_\mu)/\Gamma(D_s^{*+} \rightarrow \phi\pi^+)$ is measured by reconstructing $D_s^{*+} \rightarrow \gamma D_s^+ \rightarrow \phi\pi^+$ decays. The $D_s^+ \rightarrow \mu^+\nu_\mu$ branching fraction is evaluated using the measured branching fraction for $D_s^{*+} \rightarrow \phi\pi^+$.

Candidate $\phi$ mesons are reconstructed from two kaons of opposite charge. The $\phi$ candidates are combined with charged pions to form $D_s^+$ meson candidates. Both times a geometrically constrained fit is employed, and a minimum requirement on the fit quality is made. The $\phi$ and the $D_s^+$ candidate masses must lie within $2\sigma$ of their nominal values, obtained from fits to simulated events and data. Photon candidates are then combined with the $D_s^+$ to form $D_s^{*+}$ candidates. The same requirements on the CM photon energy and $D_s^{*+}$ momentum as in the $D_s^+ \rightarrow \mu^+\nu_\mu$ signal selection are made. The $D_s^{*+} \rightarrow \gamma D_s^+ \rightarrow \gamma\phi\pi^+$ selection efficiency in tagged events is $\epsilon_{\phi\pi} = 9.90\%$. Data events that pass the selection are grouped into two sets: the tag signal and sideband regions. After the tag sideband has been subtracted from the tag signal $\Delta M$ distribution, the remaining distribution is fitted with $(N_{\text{signal}}f_{\text{signal}} + N_{\text{background}}f_{\text{background}})(\Delta M)$, where $f_{\text{signal}}$ is a triple Gaussian, describing the simulated $D_s^{*+} \rightarrow \gamma D_s^+ \rightarrow \gamma\phi\pi^+$ signal, and $f_{\text{background}}$ consists of a broad Gaussian centered at 70 MeV/$c^2$ and a function describing the simulated background $\Delta M$ distributions. The Gaussian describes the background $D_s^{*+} \rightarrow \pi^0 D_s^+ \rightarrow \pi^0\phi\pi^+$ where the photon candidate originates from the $\pi^0$. The relative sizes of the background components, along with all parameters except $N_{\text{signal}}$ and $N_{\text{background}}$ are fixed to the values estimated from simulation. The $\chi^2$ fit yields $N_{\text{signal}} = 2093 \pm 99$ events and has a probability of 25.0% (Fig. 4). From simulation $48 \pm 23$ events $D_s^{*+} \rightarrow \gamma D_s^+ \rightarrow \gamma f_0(980)(K^+K^-)\pi^+$ are expected to contribute to the signal, where the error is mostly from the uncertainty in the $D_s^+ \rightarrow f_0(980)(K^+K^-)\pi^+$ branching ratio.

Precise knowledge of the efficiency of reconstructing the tag is not important, since it mostly cancels in the calculation of the partial width ratio. However, the presence of two charged kaons in $D_s^+ \rightarrow \phi\pi^+$ events leads to an increased number of random tag candidates, compared to $D_s^+ \rightarrow \mu^+\nu_\mu$ events, which decreases the chances that the correct tag is picked. The size of the correction for this effect to the efficiency ratio ($\epsilon_{\phi\pi}/\epsilon_{\text{signal}}$) is determined to be $-1.4\%$ in simulated events.

To measure the effect of a difference between the $D_s^{*+}$ momentum spectrum in simulated and data events, $D_s^{*+} \rightarrow \gamma D_s^+ \rightarrow \gamma\phi\pi^+$ events are selected in data with the $D_s^{*+}$ momentum requirement removed. The sample is purified by requiring the CM momentum of the charged
pion to be at least 0.8 GeV/c. The efficiency-corrected $D_s^+$ momentum distribution in data is compared to that of $D_s^+$ in simulated $D_s^+ \rightarrow \gamma D_s^+ \rightarrow \gamma \phi \pi^+$ events. A harder momentum spectrum is observed in data. The detection efficiencies for signal and $D_s^+ \rightarrow \gamma D_s^+ \rightarrow \gamma \phi \pi^+$ events are re-evaluated after weighting simulated events to match the $D_s^+$ momentum distribution measured in data. The correction to the efficiency ratio is +1.5 %.

With both corrections applied, the partial width ratio is determined to be $\Gamma_{\mu \nu}/\Gamma_{\phi \pi} = (N/\epsilon)_{\text{sig}}/(N/\epsilon)_{\text{sig}} \times B(\phi \rightarrow K^+K^-) = 0.143 \pm 0.018 \text{(stat)}$, with $B(\phi \rightarrow K^+K^-) = 49.1 \%$. \footnote{\((8)\)}

The combined systematic uncertainty due to the corrections applied, taken as half the size of each correction, is 1.0 %. The systematic error in the signal efficiency due to selection criteria insensitive to the $D_s^+$ momentum is evaluated using reconstructed $D^{*0} \rightarrow \gamma D^0 \rightarrow \gamma K^- \pi^+$ events. The conditions present in the signal are emulated by removing the charged pion, taken to represent the neutrino, from these events. The signal reconstruction and selection steps are repeated, and the selection efficiencies compared between simulated and data events. The assigned systematic uncertainty is 1.4 %. For the $D_s^+ \rightarrow \phi \pi^+$ selection, requirements on the $D_s^+$ and $\phi$ vertex fit probability contribute a systematic uncertainty of 0.7 %, estimated from comparisons of $D_s^+ \rightarrow \phi \pi^+$ events in simulation and data. Control samples of $e^+e^- \rightarrow \mu^+\mu^-\gamma$ and $D^{*+} \rightarrow \pi^+D^0 \rightarrow \pi^+K^-\pi^+$ events are used to measure the particle identification efficiencies of muons and charged kaons and pions in data, and to correct the simulated signal and $D_s^+ \rightarrow \gamma D_s^+ \rightarrow \gamma \phi \pi^+$ efficiencies. An uncertainty of 0.7 % is associated with these corrections, mainly due to the limited statistics of the control samples. The systematic uncertainties in the track reconstruction efficiency cancel partially in the $D_s^+ \rightarrow \mu^+\nu_\mu$ to $D_s^+ \rightarrow \phi \pi^+$ ratio and contribute 1.2 %. An additional uncertainty of 1.1 % is due to the statistical limitations of the simulated signal and $D_s^+ \rightarrow \phi \pi^+$ event samples.

Simulation studies are used to evaluate the systematic uncertainties arising from a possible inadequate parameterization of the signal (0.9 %) and background (2.3 %) shapes. Simulations are also used to determine the systematic uncertainty associated with the subtraction of the electron sample (0.4 %). The error on the branching ratio $B(\phi \rightarrow K^+K^-)$ is 1.2 %, the uncertainty on the $D_s^+ \rightarrow f_0(980)\pi^+$ background is 1.1 %. The total systematic uncertainty on $\Gamma(D_s^+ \rightarrow \mu^+\nu_\mu)/\Gamma(D_s^+ \rightarrow \phi \pi^+)$ is 3.9 %.

Using the BABAR average for the branching ratio $B(D_s^+ \rightarrow \phi \pi^+) = (4.71 \pm 0.46) \% \footnote{\((9)\)}$, we obtain the branching fraction $B(D_s^+ \rightarrow \mu^+\nu_\mu) = (6.74 \pm 0.83 \pm 0.66) \times 10^{-3}$ and the decay constant $f_{D_s} = (283 \pm 17 \pm 7 \pm 14) \text{ MeV}$. The first and second errors are statistical and systematic, respectively; the third is the uncertainty from $B(D_s^+ \rightarrow \phi \pi^+)$. The ratio of our value for $f_{D_s}$ to $f_D$ from the CLEO-c measurement, $f_{D_s}/f_D = 1.27 \pm 0.14$, is consistent with lattice QCD.

Using $B(D_s^+ \rightarrow \phi \pi^+)_{\text{PDG}} = (3.6 \pm 0.9) \% \footnote{\((10)\)}$, the branching fraction is $B(D_s^+ \rightarrow \mu^+\nu_\mu) = (5.15 \pm 0.63 \pm 0.20 \pm 1.29) \times 10^{-3}$ and the decay constant $f_{D_s} = (248 \pm 15 \pm 6 \pm 31) \text{ MeV}$.

We are grateful for the excellent luminosity and machine conditions provided by our PEP-II colleagues, and for the substantial dedicated effort from the computing organizations that support BABAR. The collaborating institutions wish to thank SLAC for its support and kind hospitality. This work is supported by DOE and NSF (USA), NSERC (Canada), IHEP (China), CEA and CNRS-IN2P3 (France), BMBF and DFG (Germany), INFN (Italy), FOM (The Netherlands), NFR (Norway), MIST (Russia), and PPARC (United Kingdom). Individuals have received support from the Marie Curie EIF (European Union) and the A. P. Sloan Foundation.

\begin{figure}[h]
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\includegraphics[width=\textwidth]{fig4.png}
\caption{\(\Delta M\) distribution of selected $D_s^+ \rightarrow \gamma D_s^+ \rightarrow \gamma \phi \pi^+$ events after the tag sideband is subtracted. The solid line is the fitted signal and background distribution \((N_{\text{sig}}f_{\text{sig}} + N_{\text{bkgd}}f_{\text{bkgd}})\), the dashed line is the background distribution \((N_{\text{bkgd}}f_{\text{bkgd}})\) alone.}
\end{figure}
091104 (2005).