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A STUDY OF DETACHED TURBULENCE PROMOTERS FOR INCREASING
MASS TRANSFER RATES IN AQUEOUS CHANNEL FLOW
(Thesis)

Jack Samuel Watson

A dissertation presented to the Graduate Council of the University of Tennessee in partial fulfillment of the requirements for the Degree of Doctor of Philosophy.

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A STUDY OF DETACHED TURBULENCE PROMOTERS FOR INCREASING
MASS TRANSFER RATES IN AQUEOUS CHANNEL FLOW

ABSTRACT

The effectiveness of detached cylindrical turbulence promoters in improving mass transfer rates in aqueous channel flow has been studied. A rectangular channel (0.49 by 2.5 in. in cross section) was employed at Reynolds numbers from 513 to 5800 (based upon the smaller channel axis). Promoters of three different diameter sizes ($1/16$ in., $3/32$ in., and $1/8$ in.) were tested with three different spacings of the promoter center line from the wall ($1/16$ in., $3/32$ in., and $1/8$ in.). The experimental technique involved measuring the rate of ferricyanide reduction at a polarized electrode (or test surface) with a large excess of supporting electrolyte (sodium hydroxide).

The promoters locally increased the mass transfer rate to as much as eight times the unpromoted rate; the average transfer rate over the entire test surface was increased as much as threefold. For a given increase in mass transfer rates, turbulence promoters required smaller increases in pumping power than would have been required by increased circulating rates in clear, or unpromoted, channels. The maximum promotion occurred at a Reynolds number near the laminar-turbulent transition point. The optimum promoter

spacing from the wall appeared to be approximately 1.5 diameters, and the optimum spacing between promoters appeared to be approximately 8.0 diameters.

Measurements were made of the fluctuating, as well as the mean (time-averaged) component of the mass transfer rate. Also, by using very small, isolated electrodes, measurements could be made which were related to the mean and fluctuating velocities very near the wall. A mathematical analysis was developed to describe the effects that velocity disturbances near the wall will have on the mean and fluctuating component of the mass transfer rate. Although the analysis could not be completely evaluated because of the unavailability of sufficient satisfactory data on disturbances frequencies and propagation velocities, some comparisons of the analysis with experimental data were possible and agreement in these cases was satisfactory.

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CHAPTER I

INTRODUCTION

When heat or mass is transferred between a solid surface and a flowing fluid, the major resistance limiting the transfer rate often occurs in the fluid near the surface. This resistance, called film resistance or concentration polarization, results from the zero velocity at the solid surface; thus, in this region, molecular diffusion is the predominant means of mass (or heat) transfer. The layer of fluid near the surface, where the velocity differs appreciably from that in the main stream, is often called the "hydrodynamic boundary layer," and the region where the concentration differs significantly from the main stream is called the "concentration boundary layer" or (usually when a linear concentration profile in the layer is assumed) the Nernst layer. The concept of a boundary layer of definite thickness is a simplification because neither the velocity nor the concentration reaches the main stream value at a definite thickness; however, the idea of a boundary layer of definite thickness is a convenient pictorial and mathematical approximation.

The objective of this study was to investigate methods of altering the hydrodynamic conditions near the solid surface (or within the concentration boundary layer) to promote higher mass transfer rates from the surface. Flow disturbances or eddy motions in the wake

behind obstructions located on or near the surface can enhance mixing of fluid near the wall with the main stream and improve mass or heat transfer rates.

The need for this particular study arose from concentration polarization problems in hyperfiltration (reverse osmosis), but the results are equally applicable to many heterogeneous mass or heat-transfer processes. In hyperfiltration processes, a saline solution is forced under pressure through a membrane which selectively passes the water and rejects the salts. The component of flow toward the membrane, coupled with salt rejections, results in an accumulation of salts near the membrane surface. The concentration of salt continues to increase until diffusive and convective fluxes toward the bulk stream equal the incoming flux. High concentrations of salt at the membrane surface hinder the hyperfiltration operation in any or all of three ways: (1) higher salt concentrations at the membrane surface require the system to overcome higher osmosis pressures and thereby increase energy costs; (2) salt leakage becomes greater; and (3) relatively insoluble salts, such as CaSO_4 , may precipitate and hinder flow through the membrane or even permanently damage the membrane.

The approach used in this investigation for increasing mass transfer rates involved suspending small cylindrical obstructions near (but slightly detached from) the transfer surface. In an earlier and continuing parallel study, Thomas (41)(42)(43) has shown that such obstructions may substantially increase the rate at which naphthalene

sublimes into a flowing air stream. The obstructions probably induce eddy motions, near the wall, which enhance mixing of the boundary layer fluid with the main stream.

This investigation was an extension of the one made by Thomas, and expanded his studies of detached turbulence promoters to include conditions of closed channel flow and high Schmidt numbers. Thomas' results cover a wide range of promoter geometries and flow rates, clearly demonstrating that detached promoters can significantly improve mass transfer rates. Thomas' studies, however, were limited to a system with a Schmidt number of approximately two (that of naphthalene in air) and to mass transfer through a developing boundary layer on a flat plate. Hyperfiltration systems involving sodium chloride in water have Schmidt numbers of several hundred and use closed channels with usually rectangular, circular, or annular geometries. This study, using developed channel flow and an aqueous system with a Schmidt number of 1580, closely resembles that of the salt systems important in hyperfiltration. Also, this investigation includes a study of instantaneous mass transfer rates. An effort was made to analyze the effects of eddy motions near the wall on both the mean (time-averaged) and instantaneous (fluctuating) mass transfer rates and to better understand the behavior and performance of detached promoters.

CHAPTER II

LITERATURE REVIEW

Mass Transfer Correlations

Heat or mass transfer to a fluid in laminar flow through a circular channel with no turbulence promotion was studied several years ago by Graetz (9). The laminar velocity profile was assumed to be completely developed before heat or mass transfer commenced. The wall temperature (or concentration) and fluid properties were assumed to be constant (independent of temperature or concentration). The solution that resulted was an infinite series, but could be approximated by (35):

$$N_{St} = 1.08 \left(N_{Sc} N_{Re} \frac{d}{L} \right)^{-2/3} \quad (1)$$

for $N_{Sc} N_{Re} \frac{d}{L} \geq 200$,

where

$$N_{St} = \text{Stanton number} = \frac{\bar{k}}{U_a} ,$$

$$N_{Sc} = \text{Schmidt number} = \frac{\nu}{D} ,$$

$$N_{Re} = \frac{U_a d}{\nu} ,$$

d = diameter of channel,

L = length of mass transfer section,

$\bar{k}$ = average mass transfer coefficient over the transfer reaction,

U_a = average velocity,

ν = kinematic viscosity,

D = diffusion coefficient.

Similar calculations were made numerically by Montgomery and Wiberlswas (24) for laminar heat (or mass) transfer in channels with rectangular cross sections. For a 5x1 channel and $N_{Re} N_{Sc} \frac{d_h}{L} \geq 100$, their results extrapolate to

$$N_{St} = 1.64 \left(N_{Sc} N_{Re} \frac{d_h}{L} \right)^{-2/3}, \quad (2)$$

where

d_h = hydraulic or equivalent diameter

$$= \frac{4 \times \text{cross section area of the channel}}{\text{perimeter of the channel}}.$$

With a constant wall temperature and high values of $N_{Re} N_{Sc} \frac{d_h}{L}$, experimental Stanton numbers were higher than the calculated results, but no explanations were given.

In turbulent flow, expressions for mass transfer rates become semiempirical and are based upon an analogy between the momentum and mass transfer. A commonly used expression is that proposed by Colburn (5):

$$N_{St} = \frac{f_f}{2} N_{Sc}^{-2/3}, \quad (3)$$

where

$$f_f = \text{friction factor} = \frac{2 g_c}{\rho U_a^2} \tau_w,$$

g_c = conversion factor, $32.17 \text{ (ft./sec}^2\text{.) } \left(\frac{\text{lb. mass}}{\text{lb. force}} \right)$,

ρ = density of fluid,

τ_w = shear stress at the wall.

Substituting an empirical expression for f_f , Equation 3 becomes

$$N_{St} = 0.023 N_{Sc}^{-2/3} N_{Re}^{-0.2} . \quad (4)$$

This relation is usually considered adequate for Reynolds numbers above 10,000 and for Schmidt numbers between 0.7 and at least 160.

For systems with higher Schmidt numbers, Deissler (6) developed an expression.

$$N_{St} = 0.079 N_{Sc}^{-3/4} f_f^{-1/2} . \quad (5)$$

The lower exponent on f_f results from the assumed velocity profile and the larger (in a negative sense) Schmidt number dependence results from the assumed variation of eddy diffusivity with distance from the wall.

The proper relation to use for systems with high Schmidt numbers is not yet clearly resolved, but is still undergoing investigation. Hubbard and Lightfoot (13) studied mass transfer rates in a rectangular channel using the ferricyanide-ferrocyanide oxidation-reduction system. By taking measurements at different temperatures and supporting electrolyte concentrations, they were able to vary the Schmidt number from 1700 to 30,000. Their Reynolds numbers (based upon the hydraulic diameter) ranged from 7000 to 60,000. At least at lower Reynolds numbers (10,000), their results show $-2/3$ power dependence on the

Schmidt number, as predicted by Colburn rather than $-3/4$ power dependence as predicted by Deissler. Harriott and Hamilton (10) studied Schmidt numbers up to 100,000 and also found a -0.654 (essentially $-2/3$) power dependence. However, Kiskinevsky, Denisova, and Parmenov (15) measured benzoic acid and naphthalene dissolution rates into water, aqueous solutions of glycerine, sucrose, or methanol. These systems had Schmidt numbers as high as 60,000, and their data followed a $-1/2$ power Schmidt number dependence.

In summary, one may conclude that, even at very high Schmidt numbers, the Colburn equation is at least as reliable as the Deissler equation. However, there is still considerable disagreement between investigators on both the Schmidt number and Reynolds number dependence. Hubbard and Lightfoot noted that although they found their data agreed better with the Colburn equation than with any of several correlations considered, there is evidence that all of these relations are too simple to describe the mass transfer process. For instance, they point out that the friction factor or Reynolds number dependence seems to be a function of Schmidt number. At moderately high Schmidt numbers, the Stanton number is approximately proportional to the friction factor; however, at still higher Schmidt numbers it appears to be proportional to the square root of the friction factor. More insight into the turbulent convection process will be needed before this problem can be resolved.

To a first approximation, the turbulent mass transfer correlations developed largely from circular channel data may be applied

to noncircular channels if the hydraulic diameter is used as the characteristic length. This is based upon the assumption that, near the wall, the velocity and concentration are only a function of the distance from the wall and not a function of the position around the channel perimeter. At high Reynolds numbers and high Schmidt numbers, this is an appropriate assumption. The studies of Hubbard and Lightfoot (13) mentioned earlier were made in a rectangular, rather than circular, channel, and the results agreed with the Colburn equation.

There are, however, two properties of rectangular channels which should be mentioned. First, turbulent flow in rectangular channels is accompanied by secondary flows (a component of the mean flow in a direction other than that of the channel axis) or roll cells that turn toward each corner. These can be detected in the turbulent velocity distribution measurements of Nikuradse (25); Hartnett, Koh, and McComas (11); and Leutheusser (18). The relative intensities of the secondary flows increase with decreasing Reynolds number, and thus, presumably, would be most significant at or just above the transition Reynolds number. At a Reynolds number of 75,000 and with a 2x1 channel, Hartnett, Koh, and McComas (11) reported the secondary velocities were as much as four per cent of the mean axial flow. Secondary flows are also affected by entrance conditions. Pletcher and McManus (26) studied entrance flow into a 3x1 channel and found more than two roll cells per corner. However, at 39.6 diameters downstream, the two-roll-cell-per-corner conditions of developed flow

had formed. Elsewhere, Bromfeld (4) showed that less than 30 diameters were required for developing the flow.

The second property, asymmetric heating, is not strictly a property of rectangular channels, but in rectangular channels, frequently only one side of the channel is heated, or as in the present experiment, mass transfer occurs from only one side of the channel. Sparrow, Lloyd, and Hixon (38) made heat transfer measurements in rectangular channels with all four channel walls at the same temperature (symmetrical heating) and with only one wall heated (the other walls were insulated). With only one wall heated, the heat transfer coefficient was less than that observed with symmetrical heating. The difference between symmetrical and asymmetrical heating decreased as the Reynolds number and/or Prandtl number increased. These observations have been confirmed by James, Martin, and Martin (14). At high Reynolds numbers and with symmetrically heated channels, the results of Sparrow, Lloyd, and Hixon correlate well with the Colburn equation. The studies of Hubbard and Lightfoot (13) which were mentioned earlier also correlated with the Colburn equation although the mass transfer did not occur from all surfaces. Their work, however, was done at high Schmidt numbers and apparently their results would have been only slightly different if mass transfer had occurred from all surfaces.

Turbulence Promotion

Several methods to enhance mass or heat transfer rates have been considered and tested in the past. A good survey of the work presented

before 1965 was prepared by Bergles and Morton (2). Several investigations discussed by Bergles and Morton used roughened transfer surfaces or fins (baffles) attached to the surface to add more transfer area, disrupt the flow, and enhance fluid mixing near the wall. In a few investigations the baffles or obstructions were slightly removed (or detached) from the surface. In others swirl flow was induced in the channel with twisted tapes or impellers, the transfer surface (or the fluid) was vibrated, or solid or liquid additives were placed in the fluid to promote higher heat or mass transfer rates. For many mass transfer processes (and especially for hyperfiltration), detached promoters appeared most promising; thus, they were chosen as the subject of this study. In many mass transfer processes, especially those involving membranes, roughened surfaces may be difficult to obtain. Fins or baffles in these instances probably will not only fail to provide additional transfer surface but will also cover or block some of the existing area. In hyperfiltration, stagnant regions near the base of surface fins can be especially undesirable. These regions may become highly polarized and not only fail to contribute to the mass transfer rate but may also contribute to the salt leakage rate.

Turbulence promoters which induce vortex or swirl flow in the channel are effective in increasing heat transfer rates when fluids are being heated or boiled in the channel. They are less effective, however, when the fluid is being cooled. In heating or boiling

processes, the centrifugal field generated from the vortex motion enhances the mixing of the main stream fluid with the fluid near the wall. However, in the cooling process, the fluid near the wall is more dense than the fluid in the channel center; thus the centrifugal field retards mixing. In some mass transfer processes, the fluid near the wall may be less dense, and swirl flow may significantly improve mass transfer rates. However, swirl flow is not likely to be helpful in hyperfiltration, where salt accumulates at the wall and increases the density.

Vibrations or solid additives may improve transfer rates, but they usually present other problems such as expensive handling equipment or scaling by the added particles. None of these systems at the present time appear attractive.

Bergles and Morton reviewed two studies of detached turbulence promoters. They chose to compare promoted and unpromoted (e.g. open or smooth channel) rates at the same pumping power. Koch (16) studied detached rings, disc baffles, and propeller-shaped inserts suspended in a circular channel (as well as attached twisted tapes and solid packing materials). Evans and Churchill (8) studied disc baffles and streamlined bodies of revolution located along the center line. Transfer rates with streamline bodies were at best only slightly higher with an empty channel at the same pumping power. And at higher Reynolds numbers they are less effective. Disc baffles and axially supported rings gave higher rates than empty channels at low Reynolds numbers (from transition up to 10^4 or 10^5). But the

promotion decreases with Reynolds number, and eventually at higher Reynolds numbers even these promoters do not compare well with empty-channel data. The rings appeared better than the discs and gave as much as 40 per cent higher rates than empty channels with the same pumping power.

Thomas (42)(43) studied the effects of cylinders or wires suspended near the outer edge of the boundary layer on a flat plate on the rate of naphthalene sublimation from the plate. Thomas used a wide range of flow rate and promoter sizes and spacings. He was able to obtain local mass transfer rates near the promoter four times the rates obtained at the same Reynolds numbers without promoters. And the average rate over a distance of 130 cylinder diameters downstream was 2.7 times that observed without promoters. The promotion was a strong function of velocity and separation of the promoter from the wall, and there appeared to be merit in grouping the promoters in pairs. The promotion behind paired promoters were noticeably different from those behind single promoters and suggested a "tuning phenomenon," where instabilities are excited in the boundary layer to give premature transition. Pressure gradients were measured along the plate with and without turbulence promoters, but the data were insufficient to determine the net (unrecovered) pressure loss resulting from the promoters. An upper limit on the pressure loss was estimated by calculating the drag on each cylinder as if it were alone in the stream. However, when cylinders are aligned with the plane of their axes parallel to the flow, the drag on the downstream cylinders has been

shown by Taylor (40) to be substantially reduced. These detached turbulence promoters appeared to be promising for many mass and heat transfer processes.

Detached promoters may improve mass transfer rates in two ways. First, the promoter may act as a constriction and increase the velocity and the velocity gradient near the wall. In most of the systems studied by Evans and Churchill, this may have been the principal mechanism by which the transfer rates were improved; it probably also contributed in the systems of Koch and of Thomas. The last author measured local mass transfer rates along the surface and noted very high local rates under each promoter which probably resulted from the constricted flow under the promoter.

Detached promoters may also generate flow disturbances or velocity fluctuations in their wake, which improve the mass transfer rate. The fluctuations may be damped and thus be effective for only a finite distance downstream, or in the case of a developing laminar boundary layer the disturbances may excite instabilities in the flow and promote a transition to turbulence that lasts indefinitely downstream. Thomas' data indicated that the promoters had, in many cases, effected an early transition from a laminar to a turbulent boundary layer.

The wake behind circular cylinders is better understood than those behind most promoter shapes; fortunately circular cylinders are of most interest to this study. A detailed review of the properties

of the wake behind circular cylinders is provided in a paper by Roshko (31). Briefly, at Reynolds numbers (based upon the cylinder diameter) greater than 40 or 50, vortices on the back side of the cylinder are alternately detached from opposite sides of the cylinder producing two rolls or streets of vortices. The frequency of vortex shedding is shown in Figure 1. The frequency is normalized and expressed as the Strouhal number, N_{Sd} , where $N_{Sd} = \frac{nd}{U_u}$, and n = shedding frequency (from one side of the cylinder). At high Reynolds numbers ($N_{Re} \geq 200$), the wake becomes turbulent in appearance, and a spectrum of frequencies becomes evident; however, a predominate frequency still remains. In this region the Strouhal number has an approximately constant value of 0.2.

The effects of small disturbances within laminar boundary layers have been discussed by Tollmien (45) and Schlichting (32). Near the leading edge of a flat plate (or for small Reynolds numbers based upon the laminar boundary layer thickness) all disturbances are damped. However, above a critical Reynolds number a selective amplification of certain frequencies occur. These analyses assume small disturbance amplitudes and then determine if the small disturbances will grow or will be damped. Schubauer and Skramstad (34) were able to confirm most of these results experimentally.

The effects of oscillations in the free stream were studied by Lighthill (20) and by Lin (22). The first study was limited to small disturbances, but Lin's approach remains valid for disturbances

Figure 1. Strouhal number versus Reynolds number.

comparable to the mean flow. Both treatments, however, only apply to high frequencies. The results describe velocity fluctuations at various positions within the boundary layer as a function of fluctuations in the free stream. The conditions required for flow reversal (fluctuations at a point larger than the mean flow at that point) are estimated. No claim is made that a flow reversal will cause transition, but, with steady laminar boundary layers, flow reversals result in boundary layer separation and more turbulent conditions.

The effect of velocity fluctuations in the free stream on mass transfer rates through laminar boundary layers was studied by Thomas (41). Naphthalene sublimations rates were measured as a function of turbulence level in the free stream (r.m.s. velocity fluctuation intensities in all directions divided by the mean velocity). For low turbulence levels, the mass transfer rates were essentially constant. However, a threshold occurred at a turbulence level of 2.8 per cent. Above this level, increases in fluctuation intensities in the free stream significantly improved the mass transfer rate.

Electrochemical Determination of Mass Transfer Rates

The experimental technique chosen for this study involved measuring the rate of the reduction of ferricyanide on a polarized electrode. As ferricyanide was reduced at the cathode, ferrocyanide was oxidized at the anode. There was no net consumption of chemicals or change in the composition of the solution. This system, like the

desalination processes of interest, has a high Schmidt number (with the solution concentrations and temperature used in this study, the Schmidt number was approximately 1580); thus the results should simulate the processes of interest. This electrochemical system was even more attractive because instantaneous, as well as mean (or time-averaged), rates could be measured. The instantaneous fluctuations in the mass transfer rates provide additional information on flow disturbances and on the way in which promoters increase mass transfer rates.

Polarization of the cathode or test surface was achieved by making the anode much larger than the cathode; then the reaction process at the cathode controlled the current. The reduction of ferricyanide is so rapid that diffusion to the electrode surface is rate-controlling. Diffusion and migration in electrolytes have been described by Tobias, Eissenberg, and Wilke (44). The migration of an ion in both electric and concentration fields may be expressed by the Nernst-Planck equation:

$$Q = D \left(\nabla C + \frac{ZC}{RT} \nabla \phi \right), \quad (6)$$

where

C = concentration of reacting ion, moles/l.,

Q = flux of the ion, moles/cm².-sec.,

D = diffusion coefficient, cm²./sec.,

Z = valence,

R = gas constant,

T = absolute temperature, $^{\circ}\text{K.}$,

F = Faraday's constant, coulombs/mole,

ϕ = electric potential, volts.

The first term in the expression represents the contribution of ordinary diffusion to the flux, and the second term represents the contribution of electromigration. A large concentration (relative to that of the reacting ion) of an inert electrolyte, sodium hydroxide, was maintained in the solution. This inert electrolyte does not react at the electrodes but alters the dielectric properties of the solution such that the potential will decrease smoothly across the region between the electrodes while the concentration drops sharply across the thin polarized layer near the cathode. Thus, the term containing the electric potential becomes relatively small and

$$Q \approx D \nabla C .$$

Then the current flowing between the electrodes is a measure of mass transfer rates governed by ordinary molecular diffusion. This current is sometimes called the "diffusion current."

Electrochemical techniques have often been used to study or to measure electrolyte diffusion coefficients or mobilities.

Polarization currents are measured for a given electrolyte, using well-defined electrode shapes and flow patterns. The rotating disc electrode is one system used in such studies, and the mass transfer rate has been related to the system parameters by Levich (19).

In recent years, however, several investigations have taken the opposite approach. An electrolyte system with well-known transport properties may be used to study mass transfer under hydrodynamic conditions that are more complicated or less well understood.

Lin, Denton, Gaskill, and Putman (21) used four electrochemical systems to study mass transfer rates at the surfaces in the annular region between two cylinders. These systems were: (1) reduction of ferricyanide in a sodium hydroxide solution on a copper electrode - $\text{Fe}(\text{CN})_6^{3-} + e^- \rightarrow \text{Fe}(\text{CN})_6^{4-}$; (2) quinone reduction in a buffered solution on a silver electrode - $\text{C}_6\text{H}_4\text{O}_2 + 2\text{H}^+ + 2e^- \rightarrow \text{C}_6\text{H}_4(\text{OH})_2$; (3) oxygen reduction in a sodium hydroxide solution on a silver electrode - $\text{O}_2 + 4e^- + 2\text{H}_2\text{O} \rightarrow 4\text{OH}^-$; and (4) oxidation of ferrocyanide in a sodium hydroxide solution on a stainless steel electrode - $\text{Fe}(\text{CN})_6^{4-} \rightarrow \text{Fe}(\text{CN})_6^{3-} + e^-$. All of these systems were employed successfully in the annular apparatus and gave results in agreement with existing data and correlations. However, the reduction of ferricyanide gave polarization curves that were essentially flat over the widest range of applied voltage and flow rate.

Eissenberg, Tobias, and Wilke (7) used the ferricyanide reduction and ferrocyanide oxidation systems to study mass transfer rates at the surface of rotating cylinders. The electrolyte system was essentially the same as that used by Lin, Denton, Gaskill, and Putman (21) except that nickel electrodes were used and the solution

composition was 0.01 to 0.2 M. in ferricyanide and ferrocyanide and 2 M. in sodium hydroxide. The system performed satisfactorily and gave results which were in agreement with mass transfer rates obtained by measuring the rate of dissolution of benzoic and cinnamic acids from rotating cylinders.

Shaw, Reiss, and Hanratty (37) used the ferricyanide system to study the mass transfer entry region in pipe flow. The mass transfer section was assembled from several nickel rings of various thickness. These rings were stacked with plastic (electrical) insulation between them. The insides of the rings or segments were ground smooth and flush with connecting plastic pipe. The current or average mass transfer rate from various lengths of mass transfer segments were studied. Again, this technique apparently gave very satisfactory results, which compared well with existing correlations and calculations.

Shaw (36) studied instantaneous rates of mass transfer to a small portion of a large mass transfer surface. Thus, instantaneous (as well as time-averaged) measurements were made effectively at points on an extended mass transfer area. The larger mass transfer area was a nickel anode. Point or local mass transfer rates were obtained by drilling small holes in the cathodes and inserting nickel wires which were electrically insulated (or isolated) from the rest of the cathode by a thin coating of epoxy resin that served to hold the wires in the holes. The wires were ground flush with the

cathode surface and adjusted to the same electrical potential as the cathode. Thus, the wire acted as part of the cathode, but the current drawn by the wire was isolated and measured. The root mean square (RMS) of the fluctuating component of the current was approximately 0.48 of the mean (time-averaged) component.

Reiss (27) and Reiss and Hanratty (28)(29) used the ferricyanide reduction system to measure mass transfer rates from very small isolated cathodes (small electrodes which were not part of a larger transfer surface). These electrodes were cross sections of 5- to 64-mil.-diam. wires embedded inside of a 1-in.-diam. pipe and ground flush with the surface. In the case of isolated electrodes, the concentration boundary layer begins at the forward edge of each electrode. If the electrodes are sufficiently small, the concentration boundary layer over the electrode will remain very thin, and flow within the boundary layer will be approximately laminar with a constant gradient. Assume that within the boundary layer, the concentration profile has the form:

$$\bar{C} = \frac{3}{2} \frac{y}{\delta} - \frac{1}{2} \left(\frac{y}{\delta} \right)^3, \quad (7)$$

where

$\bar{C}$ = concentration in the boundary layer (relative to that in the free stream),

y = distance from the wall,

δ = thickness of the concentration boundary layer.

Figure 2. Power spectral density measurements at the wall in clear or unpromoted channels.

The integral mass transfer equation reduces to

$$\frac{d\delta}{dx} = \frac{15}{2G} \frac{D}{\delta^2}, \quad (8)$$

where

x = distance in the direction of flow,

D = diffusion coefficient,

G = velocity gradient in the concentration boundary layer.

In terms of the mass transfer coefficient, k , this equation may be written as

$$\frac{dk}{dx} = \frac{20k^4}{9GD^2}. \quad (9)$$

Integrating this equation yields:

$$k = \left(\frac{3GD^2}{20x} \right)^{1/3}. \quad (10)$$

The average mass transfer coefficient, $\bar{k}$, over an effective electrode length, L_e , is obtained by multiplying Equation (10) by dx/L_e and integrating from 0 to L_e . This gives

$$\bar{k} = \frac{1}{L_e} \int_0^{L_e} k \, dx = \frac{3}{2} \left(\frac{3GD^2}{20L_e} \right)^{1/3}. \quad (11)$$

Thus the current or mass transfer rate will be proportional to the cube root of the velocity gradient at the wall, G . The fluctuating component of the current was found to be approximately one-tenth of the mean component, over the conditions studied (Reynolds numbers up to 34,000); the spectra of the fluctuations are shown in Figure 2. Longitudinal correlation coefficients were also reported.

Mitchell (23) refined the technique further by using very small rectangular, rather than circular, probes. These electrodes were made by embedding narrow strips of thin metal sheets in epoxy resin in the same way that Reiss embedded round wires. When the strips were ground flush with the pipe surface, the exposed metal cross section was rectangular. Since these electrodes were more sensitive to the gradient of the velocity component normal to the major axis of the electrode, this enabled him to distinguish between fluctuations in the velocity in the direction of mean flow and those normal to that direction. The results agree well with those of Reiss. The RMS of the fluctuating signal was approximately one-tenth of the mean signal, and the spectra (shown in Figure 2) closely approximated those measured by Reiss. The smaller rectangular probes, however, allowed more precise measurement of correlations and integral scales.

Mitchell analyzed the frequency response of polarized electrodes to fluctuations in the velocity gradient at the wall. For low-frequency disturbances, these signals can be interpreted as being proportional to the fluctuating component of the cube root of the wall velocity gradient, [Equation (11)]. However, the thin concentration boundary layer over the electrodes does have a finite response time, and high-frequency fluctuations are attenuated. In many respects, the boundary layer acts as an electrical capacitance. Mitchell's analysis arrived at the following result:

$$\frac{[(G')^2]^{1/2}}{\bar{G}} = 3 \left[\int_0^{\infty} \frac{\text{PSD}(n)}{|A|^2} dn \right]^{1/2} \frac{[(k')^2]^{1/2}}{\bar{k}}, \quad (12)$$

with

$$|A|^2 = \frac{1}{1 + 0.060 (L^+)^{\frac{4}{3}} J^2}, \quad (13)$$

where

G' = fluctuating component of the velocity gradient at the wall,

$\bar{G}$ = mean (time-averaged) velocity gradient at the wall,

k' = fluctuating component of the mass transfer coefficient,

$\bar{k}$ = mean mass transfer coefficient,

$\text{PSD}(n)$ = power spectral density function of the mass transfer coefficient,

$L^+ = L_e U^* / U_a$,

$J = 2\pi\nu (N_{Sc})^{1/3} / U^{*2}$,

n = frequency,

ν = kinematic viscosity,

U^* = friction velocity = $U_a \frac{f}{2}$,

$N_{Sc} = \text{Schmidt number} = \frac{\nu}{D}$,

D = diffusion coefficient,

f = friction factor = $\frac{2\nu}{U_a} \bar{G}$,

U_a = average velocity in the channel.

At high frequencies (n), J becomes large, and $|A|^2$ becomes small. Then $[(k')^2]^{1/2}/\bar{k}$ is less than $[(G')^2]^{1/2}/\bar{G}$; that is, high frequencies are attenuated. Note also that J increases with increasing values of the Schmidt number, and that L^+ increases with electrode length. Thus, to make measurements of velocity gradient disturbances with high frequencies, one needs a system with very small electrodes and reasonably low Schmidt numbers.

CHAPTER III

THEORY

Estimation of Concentration Fluctuations Near the Wall

As mentioned earlier, detached turbulence promoters may influence the mass transfer rate in two ways. First, the promoter may simply provide an obstruction, and the accelerated flow under the promoter may give locally higher velocities and, in turn, higher mass transfer rates. Second, the wake behind the promoter may contain disturbances (velocity fluctuations) that significantly influence the mass transfer rate for a substantial distance downstream. To achieve increased mass transfer rates with a nominal increase in pressure drop, the wake effect appears to be a more promising mechanism. Thus the theoretical portion of this study attempts to explore the relation between velocity disturbances near the wall and mass transfer rates.

The differential equation for mass transfer in a velocity field can be written (32):

$$\frac{\partial C}{\partial t} + U \frac{\partial C}{\partial x} + V \frac{\partial C}{\partial y} + W \frac{\partial C}{\partial z} = D \left(\frac{\partial^2 C}{\partial x^2} + \frac{\partial^2 C}{\partial y^2} + \frac{\partial^2 C}{\partial z^2} \right), \quad (14)$$

where

C = concentration,

D = diffusion coefficient,

U, V, W = components of velocity in the x, y, and z directions.

The velocity components will be considered positive if they are in the direction of the positive axis. To solve this equation, several assumptions, or simplifications, are required. First, assume that the problem is two-dimensional or that C is not a significant function of z . Let the x -axis be in the principal direction of flow, and parallel to the wall; let the y -axis be normal to the wall, with the origin ($y = 0$) on the wall. It will be convenient to write the concentration and the two remaining velocities as the sum of a mean and a fluctuating component:

$$C = \bar{C} + c, \quad (15)$$

$$U = \bar{U} + u, \quad (16)$$

$$V = \bar{V} + v. \quad (17)$$

Assume now that the fluid is incompressible and that the flow and mass transport are fully developed; that is, that $\bar{U}$ and $\bar{C}$ are independent of x . Then, using the time average of the continuity equation,

$$\frac{\partial \bar{U}}{\partial x} = 0 = -\frac{\partial \bar{V}}{\partial y}, \quad (18)$$

and the boundary condition that all velocity components vanish at the wall, it follows that $\bar{V}$ is zero. Equation (14) then becomes:

$$\frac{\partial c}{\partial t} + \bar{U} \frac{\partial c}{\partial x} + u \frac{\partial c}{\partial x} + v \frac{\partial \bar{C}}{\partial y} + v \frac{\partial c}{\partial y} = D \left(\frac{\partial^2 c}{\partial x^2} + \frac{\partial^2 \bar{C}}{\partial y^2} + \frac{\partial^2 c}{\partial y^2} \right). \quad (19)$$

If the fluctuating components of concentration and velocity are much smaller than the mean quantity, the second-order terms can be neglected. By retaining only first-order fluctuating terms, Equation (19) reduces to:

$$\frac{\partial c}{\partial t} + \bar{U} \frac{\partial c}{\partial x} + v \frac{\partial \bar{C}}{\partial y} = D \frac{\partial^2 c}{\partial y^2} . \quad (20)$$

This expression becomes dimensionless by dividing through by the bulk concentration (or setting it equal to unity) and by substituting as follows:

$$\tau = \frac{U_0}{\delta} t , \quad (21)$$

$$\xi = \frac{x}{\delta} , \quad (22)$$

$$\eta = \frac{y}{\delta} , \quad (23)$$

where

U_0 = reference velocity,

δ = thickness of the concentration boundary layer.

Then

$$\frac{\partial c}{\partial \tau} + \frac{\bar{U}}{U_0} \frac{\partial c}{\partial \xi} + \frac{v}{U_0} \frac{\partial \bar{C}}{\partial \eta} = \left(\frac{D}{U_0 \delta} \right) \frac{\partial^2 c}{\partial \eta^2} . \quad (24)$$

The concentration boundary layer is assumed to be thin enough that, over its thickness, the mean velocity profile is essentially linear. This assumption is appropriate for systems, such as the one of present interest, with very high Schmidt numbers. For the reference velocity, U_0 , the velocity at the outer edge of the concentration boundary layer (at $\eta = 1$) is chosen. Then

$$\frac{\bar{U}}{U_0} = \eta . \quad (25)$$

The fluctuating component of velocity, u , is also assumed to be linear within the concentration boundary layer.

$$\frac{u}{U_0} = B \eta g . \quad (26)$$

B is a constant which specifies the amplitude of the fluctuations (relative to the mean velocity), and g is an oscillating quantity which is a function of s alone. And

$$s = \xi - b\tau . \quad (27)$$

That is, g represents a traveling wave, and b is the normalized phase velocity (phase velocity divided by U_0). The v component of velocity is easily obtained from continuity.

$$\frac{\partial v}{\partial \eta} = - \frac{\partial u}{\partial \xi} . \quad (28)$$

Then

$$\frac{v}{U_0} = - \frac{1}{U_0} \int_0^\eta \frac{\partial u}{\partial \xi} d\eta = - B \frac{\partial g}{\partial \xi} \int_0^\eta \eta d\eta = - \frac{B\eta^2}{2} \frac{\partial g}{\partial \xi} = - \frac{B\eta^2}{2} \frac{dg}{ds} . \quad (29)$$

The fluctuating component of concentration is assumed to follow the velocity fluctuations and have the same frequencies, but the phases and intensities of concentration fluctuations may vary differently with distance from the wall. The concentration fluctuations are written as:

$$c = fg, \quad (30)$$

where

f = function of η .

Substituting these approximations into Equation (24) gives:

$$-bf \left(\frac{\partial g}{\partial s} \right) + \eta f \left(\frac{\partial g}{\partial s} \right) + \frac{B\eta^2}{2} \left(\frac{\partial g}{\partial s} \right) = \left(\frac{D}{U_o \delta} \right) g \left(\frac{\partial^2 f}{\partial \eta^2} \right) \quad (31)$$

Dividing this expression by

$$\left(\frac{\partial g}{\partial s} \right) \left(\frac{\partial^2 f}{\partial \eta^2} \right)$$

yields:

$$\frac{-bf + \eta f + \frac{B\eta^2}{2} \left(\frac{\partial \bar{C}}{\partial \eta} \right)}{\frac{\partial^2 f}{\partial \eta^2}} = \frac{g}{\frac{\partial g}{\partial s}} \left(\frac{D}{U_o \delta} \right) \quad (32)$$

Now the left side of the equation is a function of η alone, and the right side is a function of s alone. This is possible only when both sides are equal to a constant, which we will call β . The two variables are now separated to give two ordinary differential equations instead of a single partial differential equation.

$$\beta \frac{d^2 f}{d\eta^2} + (b - \eta)f - \frac{B\eta^2}{2} \left(\frac{d\bar{C}}{d\eta} \right) = 0, \quad (33)$$

and

$$\frac{dg}{ds} = \frac{g}{\beta} \left(\frac{D}{U_o \delta} \right) \quad (34)$$

Equation (34) is easily solved to give:

$$g = C_1 e^{\frac{Ds}{U_o \delta \beta}} \quad (35)$$

Now g is the imposed disturbance in the velocity gradient. Since this should be an oscillating function, β must be a purely imaginary constant. Then g becomes the sum of a real and an imaginary sine

function 90 degrees out of phase. A real constant, β_1 , is defined by:

$$\frac{\beta U_o \delta}{D} = -i\beta_1. \quad (36)$$

C_1 is set equal to unity since the magnitude of the disturbance is included in the constant B. (One could, however, equally well include the magnitude of the disturbance in g, or C_1 .) Then the velocity disturbance function is:

$$g = e^{is/\beta_1}. \quad (37)$$

Equation (38) shows that β_1 determines the frequency, ω , of the disturbance:

$$\omega = \frac{i}{\beta_1} = -\frac{iD}{U_o \delta \beta}. \quad (38)$$

The problem is to solve Equation (33) for disturbances with various frequencies (various values of β_1), amplitudes (B), and phase velocities (b). The resulting function, f, represents the amplitude of the concentration fluctuations at various positions, η , in the boundary layer. The equation is complex since it contains an imaginary coefficient, β , as well as real coefficients (all other coefficients). Then f is complex, with its real part representing the component of the concentration fluctuations that is in phase with the velocity fluctuations, and its imaginary part representing the component that is 90 degrees out of phase with the x-velocity, u, fluctuations. Both components (or alternatively, both the magnitude and the phase angle between the components) of f will be important in this analysis.

To solve Equation (33), one must assume an expression for $\bar{C}$ (or for $\frac{d\bar{C}}{d\eta}$). Hopefully, the resulting solution for f will not be greatly affected by small errors in the expression for $\bar{C}$. This approach has been useful in calculations with laminar boundary layer flow since the quantities of most interest (usually flux or shear at the wall) are not sensitive to errors in the assumed profile. Some difficulties that do appear with turbulent boundary layers will be discussed later. Assuming that $\bar{C}$ can be approximately described in the region $0 \leq \eta \leq 1$ by Equation (7),

$$\bar{C} = \frac{3}{2} \eta - \frac{1}{2} \eta^3, \quad (7)$$

Equation (33) can be rewritten as:

$$\beta \frac{d^2 f}{d\eta^2} + (b - \eta)f - \frac{3}{4} B \eta^2 (1 - \eta^2) = 0. \quad (39)$$

This is a second-order equation whose solution can be expressed as the sum of the general solution of the complimentary equation and a particular solution of the entire equation. The complimentary equation,

$$\beta \frac{d^2 f}{d\eta^2} + (b - \eta)f = 0, \quad (40)$$

is linear, and, thus, any number of solutions exist that are simple multiples of each other. That is, if $F_1(\eta)$ is a solution of the complimentary equation, [Equation (40)],

$$f_{\text{complimentary}} = C_2 F_1(\eta). \quad (41)$$

In this particular problem, the concentration fluctuations must be zero when the velocity gradient

fluctuations, denoted by B, are zero. To meet this condition C_2 was set equal to zero. Now note also that if Equation (33) is divided by B,

$$\beta \frac{d^2 f^*}{d\eta^2} + (b - \eta)f^* - \frac{3}{4} \eta^2 (1 - \eta^2) = 0, \quad (42)$$

where

$$f^* = \frac{f}{B}. \quad (43)$$

The particular solution of this equation was obtained as a Fourier series. The concentration fluctuations were interpreted as resulting from interactions of the fluctuating v component of velocity with the mean concentration gradient. Thus, when the mean concentration gradient becomes zero at the outer edge of the boundary layer, f^* should also be zero. This was one of the boundary conditions required to solve Equation (42). The other was simply that f^* is zero at $\eta = 0$ because the concentration is zero at the wall. Since f^* is an odd function, only $\sin(n\pi\eta)$ terms appeared in the solution. The problem was coded by J. V. Wilson for an automatic evaluation of both the coefficients and the function, using an IBM-7090 computer. Examples of the calculated concentration fluctuation amplitudes, f^* , are shown in Figures 3 and 4 as a function of η , the distance from the wall. All the curves in Figure 3 were calculated at the same phase velocity, $b = 1$, but for different dimensionless frequencies, σ (where $\sigma = |\frac{1}{\beta}|$). The curves in Figure 4 were obtained at the same frequency parameter, σ , but at different phase velocities, b . Both

Figure 3. Calculated concentration fluctuations, $b = 1.0$.

Figure 4. Calculated concentration fluctuations, $\sigma = 5$.

curves show only the magnitude or absolute value of f^* . The phase angles of the concentration fluctuations are shown in Figure 5 for several disturbance frequencies as a function of the distance from the wall.

In studying mass transfer rate fluctuations, the variable of interest is the gradient of the concentration fluctuation amplitude at the wall. Figure 6 shows the effect of a velocity gradient fluctuation with frequency σ and phase velocity on the concentration gradient fluctuations at the wall. The most interesting feature of these curves is the maximum that occurs for each curve. This indicates that certain frequencies of velocity fluctuations are most efficiently converted into mass transfer rate fluctuations. In limits of both extremely large and small values of σ , $\frac{\partial f^*}{\partial \eta} \Big|_{\eta=0} = 0$ must be zero; therefore a maximum is expected. At extremely high values of σ , the second-derivative term can be neglected, leaving only the following equation to be examined:

$$f^* \approx \frac{\frac{3}{4} \eta^2 (1 - \eta^2)}{b - \eta} \quad (44)$$

For very small values of η ,

$$f^* \approx \frac{3}{4} \frac{\eta^2}{b}, \quad (45)$$

and

$$\lim_{\eta \rightarrow 0} \frac{\partial f^*}{\partial \eta} = \lim_{\eta \rightarrow 0} \frac{3\eta}{2b} = 0. \quad (46)$$

Figure 5. Phase angles of concentration fluctuations relative to u-velocity fluctuations.

Figure 6. The gradient of concentration fluctuations at the wall.

Conversely, for very small values of σ , the differential equation may be approximated by:

$$\frac{\partial^2 f^*}{\partial \eta^2} = 0, \quad (47)$$

which has the solution

$$f^* = K_1 + K_2 \eta. \quad (48)$$

To fit the required boundary conditions, it is necessary that

$$f^*(0) = f^*(1) = 0, \quad (49)$$

$$K_1 = K_2 = 0, \quad (50)$$

and

$$\frac{\partial f^*}{\partial \eta} = 0. \quad (51)$$

Estimation of Mean Mass Transfer Rates

Now that an expression for the concentration fluctuations has been obtained, it would be desirable, if possible, to use this relation to calculate the effects of velocity gradient fluctuations on the mean or time-averaged mass transfer rate. Equation (7) is used to develop an expression for the mean mass transfer rate. Again, the flow is assumed to be two-dimensional and fully developed. Let U , V , and C have mean and fluctuating components as shown in Equation (19):

$$\frac{\partial c}{\partial t} + \bar{U} \frac{\partial c}{\partial x} + u \frac{\partial c}{\partial x} + v \frac{\partial \bar{C}}{\partial y} + v \frac{\partial c}{\partial y} = D \left(\frac{\partial^2 c}{\partial x^2} + \frac{\partial^2 \bar{C}}{\partial y^2} + \frac{\partial^2 c}{\partial y^2} \right). \quad (19)$$

By taking time averages of each term, Equation (19) reduces to:

$$\bar{u} \frac{\partial c}{\partial x} + \bar{v} \frac{\partial c}{\partial y} = D \frac{\partial^2 \bar{C}}{\partial y^2}. \quad (52)$$

Note that all the terms in the fluctuating analysis have been eliminated. By using continuity, this equation can be written as:

$$\frac{\partial}{\partial x} (\overline{uc}) + \frac{\partial}{\partial y} (\overline{vc}) = D \frac{\partial^2 \overline{C}}{\partial y^2} . \quad (53)$$

Since the transfer process was assumed to be completely developed, no mean (time-averaged) quantity can vary with x . Then the first term in Equation (53) is zero, or

$$\frac{\partial}{\partial y} (\overline{vc}) = D \frac{\partial^2 \overline{C}}{\partial y^2} . \quad (54)$$

Multiplying by δ/U_0 makes this expression dimensionless:

$$\frac{\partial}{\partial \eta} \left(\frac{\overline{vc}}{U_0} \right) = \left(\frac{D}{U_0 \delta} \right) \frac{\partial^2 \overline{C}}{\partial \eta^2} . \quad (55)$$

Integrating from 0 to η , the equation becomes:

$$\frac{D}{U_0 \delta} \left(\frac{\partial \overline{C}}{\partial \eta} \right)_0 = \frac{D}{U_0 \delta} \left(\frac{\partial \overline{C}}{\partial \eta} \right)_\eta - \left(\frac{\overline{vc}}{U_0} \right)_\eta + \left(\frac{\overline{vc}}{U_0} \right)_0 . \quad (56)$$

Here a problem results from the assumed expression for $\overline{C}$ and the boundary conditions for c (or f). These assumptions are inconsistent with the mass transfer equation, and the consequences are evident from Equation (56). The term on the left is a constant that is independent of η ; it is, in fact, the normalized mass transfer rate. The terms on the right are, however, functions of η . The assumed boundary conditions on both v and c at the wall require the last term to be zero. As η approaches unity, both terms on the right become zero, while the left side remains constant and nonzero. This inconsistency results because the assumption that $\overline{C}$ reaches a constant value at a short finite distance from the wall is not exact.

Actually, $\bar{C}$ continues to gradually increase until one reaches the center line of the channel, and, thus neither $\frac{\partial \bar{C}}{\partial \eta}$ nor c become zero except at the center line (it is not essential that c become zero even there). As the magnitude of the first term on the right side of Equation (56) decreases, the second term must likewise increase (v increases with distance from the wall); thus the sum of the two terms remains constant and equal to the left side of the equation.

Nevertheless, in the present analysis, it is essential to restrict the problem to the simplified condition where all of the concentration drop occurs over a thin boundary layer near the wall. Otherwise it would be necessary to take into account more complicated flow patterns which occur farther from the wall. In this study, Equation (56) was multiplied by $d\eta$ and integrated from zero to unity. This averages both sides of the equation. The use of this average value of the right side of Equation (56) for the mass transfer rate is a separate assumption (or, at best, an approximation) that is suggested as one means of forcing the preceding assumptions to conform with Equation (56).

After inserting the assumed expression for $\bar{C}$ [Equation (7)] and integrating, Equation (56) becomes:

$$\frac{3}{2} \frac{D}{U_0 \delta} = \frac{3}{2} \frac{D}{U_0 \delta} - \frac{1}{2} \frac{D}{U_0 \delta} - \int_0^1 \frac{\bar{vc}}{U_0} d\eta, \quad (57)$$

or

$$\frac{1}{2} \frac{D}{U_0 \delta} = - \int_0^1 \frac{\bar{vc}}{U_0} d\eta. \quad (58)$$

In Equation (29), v was written as:

$$\frac{v}{U_0} = -B \frac{\eta^2}{2} \frac{dg}{ds}, \quad (29)$$

and in Equation (30), the concentration fluctuations were written as:

$$c = fg = Bf^*g. \quad (30)$$

Substituting these relations into Equation (58) gives:

$$\int_0^1 \left(-B \frac{\eta^2}{2} \frac{dg}{ds} \right) (Bf^*g) d\eta = \frac{1}{2} \frac{D}{U_0 \delta}. \quad (59)$$

Substituting Equation (37) for g gives:

$$\int_0^1 \left(-B \frac{\eta^2}{2} w e^{i\omega s} \right) (Bf^* e^{i\omega s}) d\eta = \frac{1}{2} \frac{D}{U_0 \delta}. \quad (60)$$

This expression is valid for only one frequency, ω . In most actual mass transfer processes, however, u , v , and c have spectra of many frequencies. To rewrite Equation (60) in terms of a spectrum of velocity, u , frequencies, let

$$B_{\text{RMS}} = \frac{\sqrt{u^2}}{\bar{U}}, \text{ overall root-mean-square velocity disturbance}$$

and

$$S_{\omega} = \frac{\sqrt{[u(\omega)^2]}}{\sqrt{u^2}}, \text{ root-mean-square of velocity disturbance}$$

frequencies between ω and $\omega + d\omega$ divided
by overall root-mean-square disturbance.

Then

$$B_{\text{RMS}}^2 \int_0^1 \left[\int_0^\infty -i \frac{\eta^2}{2} \omega_1 S_{\omega_1} e^{i\omega_1 s} d\omega_1 \int_0^\infty f S_{\omega_2} e^{i\omega_2 s} d\omega_2 \right] d\eta = \frac{1}{2} \frac{D}{U_0 \delta} . \quad (61)$$

Changing the order of integration, this becomes:

$$-B_{\text{RMS}}^2 \int_0^\infty i\omega_1 S_{\omega_1} e^{i\omega_1 s} d\omega_1 \int_0^\infty F S_{\omega_2} e^{i\omega_2 s} d\omega_2 = \frac{D}{U_0 \delta} , \quad (62)$$

where

$$F = \int_0^1 \eta^2 f^* d\eta . \quad (63)$$

Of the quantities in Equation (62), only $e^{i\omega_1 s}$ and $e^{i\omega_2 s}$ depend on time [recall that from Equation (20), $s = \xi - b\tau$]; consequently, we can find the time average before performing the integrations over ω_1 and ω_2 . But

$$\begin{aligned} \overline{e^{i\omega_1 s} e^{i\omega_2 s}} &= \lim_{T \rightarrow \infty} \frac{1}{T} \int_0^T \overline{e^{i\omega_1 s} e^{-i\omega s}} ds = \delta(\omega_1 - \omega_2) \\ &= 0 \text{ if } \omega_1 \neq \omega_2 = 1 \text{ if } \omega_1 = \omega_2 . \end{aligned} \quad (64)$$

Hence Equation (62) becomes:

$$-B_{\text{RMS}}^2 \int_0^\infty \int_0^\infty i\omega_1 S_{\omega_1} F S_{\omega_2} \delta(\omega_1 - \omega_2) d\omega_1 d\omega_2 = \frac{D}{U_0 \delta} . \quad (65)$$

Integrating over ω_2 is equivalent to setting $S_{\omega_2} = S_{\omega_1}$. Then

$$-B_{\text{RMS}}^2 \int_0^\infty i\omega_1 S_{\omega_1}^2 F d\omega_1 = \frac{D}{U_0 \delta} . \quad (66)$$

Note that the right side of this equation is real. The left side appears to have both a real and an imaginary component. We are only interested in the real component of the left side of Equation (66). On the left side, F is multiplied by the imaginary number i ; thus the real part of this side of the equation depends upon the imaginary component of F (F_i). That is,

$$B_{\text{RMS}}^2 \int_0^{\infty} \omega_1 F_i S_{\omega_1}^2 d\omega_1 = \frac{D}{U_0 \delta} \quad (67)$$

From this expression, one can evaluate the mean mass transfer rate. The function F_i was calculated using the previous results for f^* ; F_i vs. σ is plotted in Figure 7. With Equation (67), only hydrodynamic properties of the system (i.e., intensities and spectra of velocities fluctuations) are needed to evaluate the mass transfer rate. Even in situations where all these data are not available, Equation (67) should be useful in the designing of present or future turbulence promoters.

Application of the Analysis to Unpromoted Turbulent Flow

To illustrate the use of Equation (67) and to demonstrate its approximate validity, the equation will be used to predict mass transfer rates in a circular channel with ordinary (no promoter) turbulence. Power spectral density measurements of mass, and heat transfer rate fluctuations from isolated sources with fully developed turbulent flow in circular channels, were shown in Figure 2, page 22. There are some obvious discrepancies in the results from various

Figure 7. F_i versus σ for several values of b .

investigations using different systems and Reynolds numbers, but the data approximately follow a single curve with the normalized coordinates shown. The principal differences between the various investigations depend upon whether the mass transfer or the heat transfer system was used. The mass transfer systems had high Schmidt numbers, and the electrical capacitance of the boundary layer may have sufficiently damped the higher-frequency fluctuations to cause the differences between the mass and heat transfer results. On the other hand, the boundary layer in some of the heat transfer experiments was sufficiently thick to extend past the laminar sublayer; thus the probes may have picked up disturbances that did not reach the wall. To simplify this analysis, these data are approximated by the rectangular curve shown in Figure 2, page 22, by dashed lines. The simplified spectrum has a constant amplitude, S , for frequencies less than, or equal to, N , and zero amplitude for frequencies greater than N . This maximum disturbance frequency, N , is approximately given by:

$$N \approx \frac{2U_a}{d}, \quad (68)$$

where

U_a = average channel velocity,

d = channel diameter.

This frequency is normalized by multiplying by $\frac{\delta}{U_o}$ [see Equation (21), (22), (27), and (38)]. The maximum frequency, or the limit of the integral in Equation (67) is $\frac{2U_a \delta}{dU_o}$. Also, since

$$\int_0^{\infty} S d\omega = 1 = \int_0^{\frac{2U_a \delta}{dU_o}} S d\omega = \frac{2U_a \delta}{dU_o} S , \quad (69)$$

$$S = \frac{dU_o}{2U_a \delta} . \quad (70)$$

Then

$$B_{RMS}^2 \frac{dU_o}{2U_a \delta} \int_0^{\frac{2U_a \delta}{dU_o}} \omega F_i d\omega = \frac{D}{U_o \delta} . \quad (71)$$

In Figure 7, page 46, F_i was plotted as a function of the dimensionless frequency σ (or $|\frac{1}{\beta}|$). From Equation (38), the dimensionless frequencies ω and σ are related in the following way:

$$\omega = \frac{D}{U_o \delta} \left| \frac{1}{\beta} \right| = \frac{D}{U_o \delta} \sigma . \quad (72)$$

Substituting σ for ω in Equation (71) gives:

$$\int_0^{\frac{2U_a \delta}{Dd}} \sigma F_i d\sigma = \frac{4U_a^2 \delta^3}{B_{RMS}^2 D^2 U_o} . \quad (73)$$

The friction factor may be defined as:

$$f_f = \frac{2\nu}{U_a^2} \left(\frac{d\bar{U}}{dy} \right)_{y=0} = \frac{2\nu}{U_a^2} \frac{U_o}{\delta} , \quad (74)$$

and then

$$U_o = \frac{8f_f U_a^2}{2\nu} . \quad (75)$$

Substituting for U_o , Equation (73) becomes:

$$\int_0^{\frac{2U_a \delta^2}{dD}} \sigma F_i d\sigma = \frac{8\delta^2 \nu}{B_{RMS}^2 D d^2 f_f} \quad (76)$$

The limit of the integral can be rewritten as:

$$\frac{2U_a \delta^2}{dD} = 2 \left(\frac{U_a d}{\nu} \right) \left(\frac{\nu}{D} \right) \left(\frac{\delta}{d} \right)^2 = 2 N_{Re} N_{Sc} \left(\frac{\delta}{d} \right)^2 \quad (77)$$

In a similar manner, the right side of Equation (76) can be rewritten as:

$$\frac{8\delta^2 \nu}{B_{RMS}^2 D d^2 f_f} = \left(\frac{8}{B_{RMS}^2 f_f} \right) N_{Sc} \left(\frac{\delta}{d} \right)^2 \quad (78)$$

Then Equation (76) becomes:

$$\int_0^{2 N_{Re} N_{Sc} \left(\frac{\delta}{d} \right)^2} \sigma F_i d\sigma = \frac{8}{B_{RMS}^2 f_f} N_{Sc} \left(\frac{\delta}{d} \right)^2 \quad (79)$$

This equation can be further transformed entirely into dimensionless quantities by noting that:

$$N_{St} = \frac{\frac{3}{2} \frac{D}{\delta}}{U_a} = \frac{3}{2} \frac{Dd}{\delta U_a} = \frac{3}{2} (N_{Sc} N_{Re})^{-1} \left(\frac{\delta}{d} \right)^{-1} \quad (80)$$

or

$$\frac{\delta}{d} = \frac{3}{2} (N_{St} N_{Sc} N_{Re})^{-1} \quad (81)$$

And then

$$\int_0^{\frac{9}{2} (N_{Sc} N_{Re})^{-1} N_{St}^{-2}} \sigma F_i d\sigma = \left(\frac{18}{B_{RMS}^2 f_f} \right) N_{Sc}^{-1} (N_{Re} N_{St})^{-2} . \quad (82)$$

To evaluate this equation, more hydrodynamic data are needed: the friction factor, f_f ; the root-mean-square velocity disturbance, B_{RMS} ; and the dimensionless propagation velocity, b . Any of several empirical expressions for the friction factor may be used. For this analysis, an equation developed by Blausis (3) was chosen:

$$f_f = 0.079 N_{Re}^{-1/4} . \quad (83)$$

In the development of this analysis, two possible modes of behavior for the dimensionless propagation velocity, b , were considered. First, it seemed possible for the propagation velocity at any point to be related to (probably proportional to) the mean velocity at that point. The analysis, as discussed thus far, has treated the propagation velocity as a constant that is independent of position in the boundary layer. If this first hypothesis were true, a mean or effective value of b should describe the mass transfer process. Since the local propagation velocity was expected to be less than the mean velocity at that point, the propagation velocity should always be less than U_0 , and the effective value of b should be less than unity. The second hypothesis considers the propagation velocity at the wall to be related to the mean velocity in the channel and, at least over the thickness of the concentration boundary layer, to be independent of distance from the wall. With

very thin concentration boundary layers, U_0 may be very small, thereby making b large. Furthermore, at a given mean channel velocity, b would be inversely proportional to the thickness of the boundary layer and thus related to the Schmidt number.

To evaluate the first hypothesis (b is a constant that is approximately equal to, or less than, unity and independent of Reynolds number and Schmidt number), F_i for $b = 1.0$ was approximated by segments of straight lines. The actual curve and the approximations are shown in Figure 8. B_{RMS}^2 was assumed to be equal to 0.1, as reported by Mitchell (23); and Blasius's equation [Equation (83)] was used to describe the friction factor. Equation (82) then gives the relationship between N_{St} , N_{Sc} , and N_{Re} , but the equation cannot be solved for the Stanton number explicitly. Thus, a trial-and-error procedure was used to evaluate the latter. The procedure was coded for, and calculated by, an IBM 7090 computer. The results, along with the curves predicted by the Colburn equation [Equation (4)], are shown in Figures 9 and 10. The predicted rates are sufficiently high that the analysis seems inapplicable with the first hypothesis. The results could be brought approximately into agreement with those representing the Colburn equation by using a higher value for b or a lower value for B_{RMS}^2 . With the assumption that the propagation velocity at any point is proportional to the mean velocity at that point, an effective b that is greater than unity seems inappropriate since the propagation velocity, in this case, would probably be less than the mean velocity.

Figure 8. Segmented approximation of F_i for $b = 1.0$.

Figure 9. Comparison of predicted Stanton numbers with those predicted by the Colburn equation at $N_{Sc} = 2000$, assuming $b = 1$.

Figure 10. Comparison of predicted Stanton numbers with those predicted by the Colburn equation at $N_{Re} = 10^4$ and 10^5 , assuming $b = 1$.

The value of 0.1 was selected for B_{RMS}^2 because Mitchell's results, which were based upon the same mass transfer system, seemed applicable here. Mitchell's results for B_{RMS}^2 are higher than those measured by Armistead (1) in heat transfer systems, but this difference does not appear sufficient to account for the high Stanton numbers predicted by this analysis. Thus, at the present time further evaluation of this hypothesis will not be made.

In using the second hypothesis (that the propagation velocity very near the wall is proportional to the mean stream velocity and independent of distance from the wall), note that, with systems having high Schmidt numbers (thin boundary layers), b will be large (much greater than unity). At high values of b , the calculated curves for F_i may be combined by plotting bF_i vs. $b\sigma$. The results of such an operation are shown in Figure 11. Assume that

$$\hat{\sigma} = b\sigma, \quad (84)$$

and

$$\hat{F}_i = bF_i. \quad (85)$$

Equation (82) can be rewritten in terms of these new variables as follows:

$$\int_0^{\frac{9}{2} (N_{\text{Sc}} N_{\text{Re}} b)^{-1} N_{\text{St}}^{-2}} \hat{\sigma} \hat{F}_i d\sigma = \frac{18 b^3}{B_{\text{RMS}}^2 f_f} N_{\text{Sc}}^{-1} (N_{\text{Re}} N_{\text{St}})^{-2}. \quad (86)$$

Figure 11. Calculated points and segmented approximations for F_i .

Assume that the propagation velocity divided by the mean stream velocity is some constant K , which is probably less than unity. Then

$$b = K \frac{U_a}{U_o} . \quad (87)$$

Substituting Equation (70) for U_o gives:

$$b = \frac{2 K v}{\delta f_f U_a} . \quad (88)$$

It is obvious from Equation (80) that

$$U_a \delta = \frac{3}{2} \frac{D}{N_{St}} ; \quad (89)$$

therefore,

$$b = \frac{4}{3} \frac{K}{f_f} N_{Sc} N_{St} . \quad (90)$$

Substituting Equation (90) into Equation (86) gives:

$$\int_0^{\frac{27}{8} N_{Sc}^{-2} N_{Re}^{-1} N_{St}^{-3} \frac{f_f}{K}} \hat{\sigma} \hat{F}_i d\sigma = \frac{128 K^3}{B_{RMS}^2 f_f^4} N_{Sc}^2 N_{Re}^{-1} N_{St} . \quad (91)$$

Again, the Stanton number is an implicit function of the Reynolds number, the Schmidt number, and the variable K . Before a solution can be obtained, however, K must be evaluated or estimated. Since no direct measurements of propagation velocities at the wall were known to the author, approximate values were inferred from other related measurements that were available. The power spectral density for axial velocity disturbances and the axial correlation of axial

disturbances are related in a manner described by Hinze (12). In describing homogeneous turbulence, Hinze assumed that the propagation, or phase, velocity was equal to the average bulk velocity and related the integral scale to the energy spectra of the turbulence. At the wall, the average velocity is zero; however, the arguments appear to hold here, also, provided that the disturbances travel down the channel at a velocity V_p . Thus the propagation velocity at the wall appears to be related to the ratio of the integral scale for axial disturbance correlations and the power spectral densities of the disturbances:

$$\lim_{n \rightarrow 0} \frac{\text{PSD}(n)}{\overline{u^2}} = \frac{4\Lambda_f}{V_p}, \quad (92)$$

where $\text{PSD}(n)dn$ is the fluctuation energy (in units of velocity squared) with frequencies between n and $n + dn$, $\overline{u^2}$ is the total fluctuation energy, Λ_f is the axial integral scale (for axial velocity fluctuations), and V_p is the propagation velocity. Mitchell (23) has reported both power spectral density measurements and axial integral scales for disturbances at the wall. These power spectral density measurements, along with the results of several other authors, are shown in Figure 2, page 22. At zero frequency, $[\text{PSD}(0)/\overline{u^2}][Ua/d]$ approximately 2.

The integral scales values reported by Mitchell (23) were fitted to the expression

$$\frac{\Lambda_f}{d} = .02N_{\text{Re}}^{1/4}. \quad (93)$$

Then the propagation velocity can be approximated by:

$$K = \frac{V_p}{U_a} = 0.04 N_{Re}^{1/4} \quad (94)$$

This expression is not likely to be valid outside the range of Mitchell's data. Note that, with a system having a very high Reynolds number, V_p would be larger than the free stream velocity, a situation that is highly improbable. Equation (94) will be considered valid only up to a Reynolds number of 60,000, which was the largest value studied by Mitchell.

By substituting Equation (93) for the propagation velocity and Blasius' equation for the friction factor, Equation (91) was solved by a trial-and-error procedure using the IBM 7090 computer. The resulting values of the Stanton number, along with the curves predicted by the Colburn equation, are shown in Figures 12 and 13. The calculated values agree reasonably well with those obtained from the Colburn equation, but more nearly approximated the trend suggested by Deissler. The Reynolds number dependence of the Stanton number is less than that of the friction factor dependence; the Schmidt number dependence is closer to the $-3/4$ power predicted by Deissler than to the $-2/3$ power predicted by Colburn.

It is gratifying that this analysis, with the required simplifications, can predict approximate turbulent mass transfer rates with no arbitrary constants. However, for proper evaluation, this model requires extensive, reliable hydrodynamic data on fluctuation

Figure 12. Comparison of predicted Stanton numbers with those predicted by the Colburn equation at $N_{Sc} = 2000$, assuming $K = 0.04$ $N_{Re}^{1/4}$.

Figure 13. Comparison of predicted Stanton numbers with those predicted by the Colburn equation at $N_{Re} = 10^4$ and 10^5 , assuming $K = 0.04 N_{Re}^{1/4}$.

intensities, fluctuation spectra, and propagation velocities. As noted earlier, propagation velocities can only be estimated indirectly from other measurements. The spectral measurements shown in Figure 1, page 15, illustrate the unreliability of the data for this parameter. The difference between the results from heat transfer and mass transfer systems is the most disturbing feature of this plot. The difference that occurs in the higher-frequency regions can be accounted for by the slower response of the mass transfer electrodes or by the fact that the thermal boundary layer over the heat transfer probes extended beyond the "laminar sublayer." Data from smaller probes in both systems would bring the two sets of measurements together.

CHAPTER IV

EXPERIMENTAL

Description of the Test Loop

The experimental equipment used in this study is shown in the schematic diagram in Figure 14 and in the photograph in Figure 15. A 1 M. sodium hydroxide solution 0.01 M. in both ferrocyanide and ferricyanide was pumped from the 6-ft³. hold tank to a vented head tank with an overflow port to maintain a constant level. The portion of solution not withdrawn through the overflow port flowed through the test section. The flow rate was measured with a sharp-edged orifice (different size openings were used to extend the operating range) and controlled with a Foxboro model 53P5F controller. A portion of the solution pumped from the hold tank was recycled back to the tank through a 2-ft². heat exchanger to remove the heat added to the system by the pump. Temperature control was obtained by manually adjusting the rate at which cooling water was fed to this heat exchanger.

The tanks were made of nickel, and all piping was of either nickel or plastic (PVC or Lucite). Two-inch-diameter pipe was used in the overflow line from the head tank, but all other piping was 1 in. (sched. 40). The pump was a Nash centrifugal pump (size 1-1/2 x 2) which was capable of delivering 75 ft. of head at no flow, or 120 gallons per minute at 70 ft. Both the pump body and the impeller were made of glass, and the shaft seal was made of graphite.

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Figure 14. Schematic of the equipment.

Figure 15. Photograph of the equipment.

The system was leak tested, and orifice calibrations were made with water. Sharp-edged orifices with 1/2-in. and 1/4-in. openings were used in the 1-in.-diam. nickel pipe leading from the test section. The orifices were calibrated by determining the time required for a specified change in the level in a catch tank of known diameter. Scatter of these measurements was approximately ± 5 per cent.

Test Section

The test section is shown in more detail in Figures 16 and 17. It consisted of a rectangular channel, 2.5 x 0.49 in., with an active transfer section 12 in. long. One side of the section contained the nickel test surface or cathode, and the other side contained another electrode or plate that was electrically connected to all the nickel piping and tanks in the system (approximately 20 ft². of total surface). All these surfaces were grounded and served as the polarizing electrode.

Mass Transfer Rate Measurements. The test surface, or nickel cathode, contained 20 holes that were 0.013 in. in diameter and filled with 0.010-in.-diam. nickel wires. These wires were used to measure instantaneous mass transfer rates at certain points on the surface. They were first given a thin coat of lacquer and then a thick coat of epoxy resin before they were inserted into the test plate. The epoxy hardened in the annular space between the wire and plate,

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Figure 16. Sketch of the test section.

Figure 17. Photograph of the test section.

providing electrical insulation. The wires were then ground flush with the plate, using successively finer grades of emery cloth, and were finally buffed with crocus cloth. A photomicrograph of a finished wire is shown in Figure 18. Electrical shorting of the wires to the plate presented a problem. If the wires were not approximately centered in the holes, it was easy, in the grinding step, to "smear" or "flake" the metal enough to make electrical contact with the plate. The preliminary coat of lacquer ensured that the wire did not actually touch the plate anywhere in the passage through the hole.

Isolated Electrodes. In another series of tests, the nickel cathode was replaced with a similar Lucite plate having embedded nickel wires, which are referred to as isolated electrodes. They are not part of a larger mass transfer surface; a fresh boundary layer starts at the leading edge of each electrode. If these electrodes are sufficiently small that the concentration boundary layer remains within the region very near the wall of predominantly laminar or viscous flow, the signals from these probes are proportional to the cube root of the velocity gradient at the wall [see Equation (11)]. Using isolated electrodes as well as electrodes embedded in a larger mass transfer surface provides information on how turbulence promoters affect the flow near the wall (as denoted by the velocity gradient at the wall) and how these disturbances affect the local mass transfer rates (both time-averaged and instantaneous values). In the discussion

Figure 18. Photograph showing the cross section of a finished wire electrode.

that follows, it is important to keep in mind that the signals from isolated electrodes represent fluctuations in the cube root of the velocity gradient and that signals from electrodes which are part of a larger transfer area represent local fluctuations in mass transfer rates.

Flow Development. The test section was preceded by a 5-in.-i.d. x 2-ft.-long calming section. A fine screen in the calming section reduced large-scale motions. Preceding the test surface, the test section had a sharp-edged (10 degrees) entrance region that was 4 in. long and made of inert plastic (PVC). At the test surface, the flow was not completely developed but only approximated far downstream conditions. For turbulent flow, as many as 30 hydraulic diameters may be required for complete development (4); this system had only 5. In laminar flow the entrance length may be approximated from calculations made for flow between infinite parallel planes (33). The entrance length is approximately $0.04\ell N_{Re}$, where ℓ is the separation between the planes and N_{Re} is the Reynolds number based upon ℓ . Thus, at a Reynolds number of 1000, an entrance length of 40ℓ would be required, whereas a length of only 8ℓ was available. Nevertheless, the flow through the test section was reproducible. Local and mean mass transfer rates without turbulence promoters were periodically determined throughout the experimental program. The differences in these results were no larger than the variations

normally encountered in data in a single experiment. Also, an idea of how closely the mean flow near the surface of the test section approached completed development may be obtained from Figure 19. This figure shows mean (time-averaged) currents or mass transfer rates from isolated electrodes located at various positions down the channel. These readings, as mentioned earlier, are proportional to the cube root of the velocity gradient at the wall. The curves measured with lower Reynolds numbers are essentially horizontal, indicating that the mean flow very near the wall is nearly developed, although, if measurements had been taken near the center of the channel, the continuing flow development might have been more evident. With systems having higher Reynolds numbers, however, a noticeable change in the wall gradient occurred over the first few inches of the test surface.

The most probable result of entrance effects or entrance disturbances in this study is a decrease in the apparent effectiveness of the turbulence promoters. The mass transfer rates observed with promoters were compared with those without promoters. Entrance disturbances will increase the transfer rates observed with no promoters; thus the promoters will probably appear less effective. This appears to apply even if the entrance disturbances and promoter disturbances are additive because the effect of a given increase in disturbance level decreased with increasing background disturbance level.

Figure 19. Mean current from isolated electrodes with no promoters.

Solution

The solutions used in this study were 1 M. in sodium hydroxide and approximately 0.01 M. in both potassium ferrocyanide and potassium ferricyanide. The exact ferricyanide concentration was determined by the ORNL Analytical Chemistry Division each day that data were taken; the ferrocyanide concentration was determined occasionally. The solution was maintained under a nitrogen atmosphere to avoid oxidation of the ferrocyanide. Nitrogen was bubbled through the caustic solution in the hold tank to remove dissolved oxygen before the ferrocyanide was added. Then a slow "bleed" of nitrogen into the hold tank was maintained to keep the solution under a nitrogen atmosphere. A new batch of solution was prepared every two weeks since the solution is known to decompose slowly (32), probably by the following scheme:

The decomposition is believed to be enhanced by light. Since the solutions in this study were kept in a darkened container (all-metal hold tank), the extent of decomposition was probably minor. No evidence of decomposition was ever observed in the system. Samples of solutions were intentionally left in glass (clear and amber) sample bottles to observe decomposition. After three to five days, a brown precipitate had formed in the clear bottles. Even after two weeks, no such precipitate was detected in the amber bottles or in the working solutions stored in the hold tank.

Instrumentation

In order to act as parts of the larger electrode surface, the embedded wires had to be maintained at an electrical potential identical to that of the remainder of the test surface. The scheme used to maintain the proper wire potential was suggested by S. J. Ball and involved using operational amplifiers to automatically null the difference between the potentials of the test plate and the wire. Figure 20 illustrates the technique. Since the grid of amplifier 1 must be essentially at ground, e_w must be equal to e_p . Furthermore, one can note that:

$$i_f = \frac{e_p}{R_2} - i_w, \quad (95)$$

and the output of amplifier 3 is:

$$V_3 = (1 + \frac{R_1}{R_2})e_p - e_p - i_f R_1 = \frac{R_1}{R_2} e_p - i_f R_1. \quad (96)$$

Substituting for i_f ,

$$V_3 = \frac{R_1}{R_2} e_p - e_p \frac{R_1}{R_2} + i_w R_1 = i_w R_1, \quad (97)$$

or

$$i_w = \frac{V_3}{R_1}. \quad (98)$$

This provides a convenient method for measuring the wire current, i_w . Figure 20 also shows the current being averaged over a time constant, T ; both the average and fluctuating components can be obtained.

The accuracy with which the wire is maintained at the potential of the test surface is determined by the uniformity of the input and

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Figure 20. Analog circuit for maintaining plate and wire electrodes at the same potential.

feedback resistors in amplifiers 1 and 2. By using an applied potential of 0.7 v., the components of a TR-5 analog computer (Electronics Associates, Inc.), which are rated to maintain potential to within ± 0.1 per cent, held the potential of the wires to within 0.7 mv. of that of the test plate. This accuracy was confirmed by readings of a sensitive digital voltmeter connected between the wires and the plate.

The root-mean-square fluctuating signals (output voltage from amplifier 5) were measured with a Flow Corporation model 12A1 random signal voltmeter. This instrument has a 16-sec. time constant and is thus useful in regions of relatively low frequencies. But, like all instruments of this type, its sensitivity eventually approaches zero as the frequency approaches zero. The actual response of the instrument was measured, and the results are shown in Figure 21. The instrument is accurate to as low as 1 cycle per second, and meaningful data can be obtained down to 0.5 cycles per second. However, for lower frequencies, the instrument quickly loses sensitivity.

A limited number of fluctuation signals were analyzed to determine their frequency spectra. About 20 min. of each signal was recorded on a Mnemotron model M200 tape recorder (with an Ampex 351 deck) and replayed to a spectral analyzer. A blank or background signal from the tape recorder was also played to the analyzer to subtract the low-frequency periodic signal generated by the electronic

Figure 21. Frequency response of the random signal voltmeter.

components of the system. The spectral analyzer was constructed to operate at certain fixed frequencies; for this study, 20 frequency settings between 0.14 and 30 cycles per second were used. The device has been described in detail in earlier publications (30).

CHAPTER V

RESULTS

Preliminary Tests

In preliminary tests, the total plate current was measured at several different applied potentials and over the range of flow rates that was expected to be covered in this study. These investigations were made to ensure that the test surface could be properly polarized and to determine a suitable operating potential. The results are shown in Figure 22. The left portion of the curve represents reduction of ferricyanide on the polarized test surface, and the right portion represents oxidation of ferrocyanide on the test surface. Note that only the reduction process gives a broad, level plateau before side reactions such as the hydrolysis of water cause the current to rise again. Other workers (21) have also reported a wider plateau for the reduction step. Two further tests were made to ensure that the test surface (not the anode) was polarized. In the first, the ferrocyanide concentration was doubled. Since this did not change the limiting current, the polarization apparently was occurring at the cathode or test surface. In the second test, polarization curves were measured with anodes or polarizing electrodes of different areas. Two anode areas were tested. The first area consisted of the usual anode, all nickel piping

Figure 22. Polarization curves for a solution that is 0.01 molar in ferricyanide and ferrocyanide and 1.0 molar in sodium hydroxide ($T = 24.5^{\circ}\text{C}.$).

and tanks plus the nickel plate opposite the test surface. The second area included only the piping and tanks without the nickel plate. The results are shown in Figure 23. The initial slopes of the two curves were different because the electrical resistance between the two electrodes was different. However, the limiting currents were identical, adding further evidence that the test surface was properly polarized.

The limiting currents were then measured, with no promoters present, as a function of the Reynolds number. These results are shown in Figure 24. Also shown for comparison are the turbulent curves predicted by the Colburn equation and by the Deissler equation (using the Blasius friction factors) for completely developed mass transfer. The experimental rates were about 30 to 40 per cent larger than those predicted by the Colburn or the Deissler equation. The differences between the observed mass transfer rates and the usual correlations occurred principally because neither the mass transfer rates nor the hydrodynamics were fully developed over the transfer surface. At the leading edge of a mass transfer surface, the transfer rates will be very high, much higher than those downstream where the concentration boundary layer is fully developed. Deissler calculated the effects of these locally higher rates upon the average rate over the entire surface; however, these calculations were made only for Reynolds numbers of 10,000 and above. As noted earlier, any entrance effects (hydrodynamic or mass transfer) will cause higher mass transfer

Figure 23. Effect of polarizing electrode area.

Figure 24. Mean mass transfer rate with no promoter.

rates to be measured when no promoters are used. The promoter effectiveness was determined by comparing the rates observed with promoters with those in Figure 24 (obtained without promoters). Thus one can see that entrance effects reduce the observed promotion and subsequently make this evaluation of detached turbulence promoters slightly conservative. Figure 24 was replotted periodically during the experiments and was always reproduced. These tests were made to detect any deposition that might foul the plate as well as to determine the reproducibility of the data.

Local Mass Transfer Rates

Once the system was shown to be capable of giving reproducible, meaningful results with an unobstructed channel, the effects of various promoters were investigated. Figures 25 through 28 show the relative promotion, $\bar{k}/\bar{k}_0$ (time-averaged or mean mass transfer rate, using the promoters, divided by the mean rate without the promoters) as a function of distance down the channel. These figures are representative of the data of this type and show the effects of single and paired promoters of two different diameters. (The remaining data of this type are shown in Appendix A. Data are presented for three different promoter diameters, 1/16, 3/32, and 1/8 in. For each of the first two diameters, spacings of the promoter center lines of 1/16, 3/32, and 1/8 in. from the wall were investigated. The effects of separation between paired promoters with separations from 1/4 to 1 in. are also shown.)

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Figure 25. Local mass transfer rate promotion from a $3/32$ -in. promoter $1/8$ in. from the wall.

Figure 26. Local mass transfer rate promotion from two 3/32-in. promoters 5/8 in. apart and 1/8 in. from the wall.

Figure 27. Local mass transfer rate promotion from a 1/16-in. promoter 1/16 in. from the wall.

Figure 28. Local mass transfer rate promotion from two 1/16-in. promoters 1/2 in. apart and 1/16 in. from the wall.

Overall Mass Transfer Rates

The overall effects of turbulence promoters on the total downstream plate areas are shown in Figure 29 through 41. These results were obtained by measuring the total current drawn by the test surface. We are interested in the transfer surface downstream of the promoter since any surface of the plate upstream should be essentially unaffected by the promoter. The arrangements of the promoters used in these studies are shown in Figure 42. Note that the first promoter was located $2\frac{3}{4}$ in. downstream from the start of the mass transfer surface, and that the first $2\frac{3}{4}$ in. of the 12-in. test surface was unpromoted. To estimate the promotion in the region downstream from the promoter by means of measurements of the total plate current, it was necessary to take the unpromoted area into account. A simple, but approximate, method was used to correct the total current measurements for the unpromoted upstream effect. Two principal assumptions were made. The flow was considered to be fully developed over the entire length of the mass transfer surface, and the concentration boundary layer was assumed to be always within the linear portion of the velocity profile. The first assumption was discussed in Chapter IV. The mean flow near the wall appeared to be essentially completely developed over the test surface except in systems having the highest Reynolds numbers. Even here the errors are not large. The high Schmidt number of the present system results in a thin concentration boundary layer. The Colburn equation implies that the ratio of the hydrodynamic to the concentration boundary layer thickness is roughly the cube root of the

Figure 29. Overall mass transfer rate promotion over 9 1/4-in. test section from a 1/16-in. promoter.

Figure 30. Overall mass transfer rate promotion over 9 1/4-in. test section from two 1/16-in. promoters 1/16 in. from the wall.

Figure 31. Overall mass transfer rate promotion over 9 1/4-in. test section from two 1/16-in. promoters 3/32 in. from the wall.

Figure 32. Overall mass transfer rate promotion over 9 1/4-in. test section from two 1/16-in. promoters 1/8 in. from the wall.

Figure 33. Overall mass transfer rate promotion over 9 1/4-in. test section from evenly spaced 1/16-in. promoters 1/16 in. from the wall.

Figure 34. Overall mass transfer rate promotion over 9 1/4-in. test section from pairs of 1/16-in. promoters 1/2 in. apart and 1/16 in. from the wall.

Figure 35. Overall mass transfer rate promotion over 9 1/4-in. test section from a 3/32-in. promoter.

Figure 36. Overall mass transfer rate promotion over 9 1/4-in. test section from two 3/32-in. promoters 1/8 in. from the wall.

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Figure 37. Overall mass transfer rate promotion over 9 1/4-in. test section from two 3/32-in. promoters 3/32 in. from the wall.

Figure 38. Overall mass transfer rate promotion over 9 1/4-in. test section from two 3/32-in. promoters 1/2 in. apart and 1/16 in. from the wall.

Figure 39. Overall mass transfer rate promotion over 9 1/4-in. test section from evenly spaced 3/32-in. promoters 1/8 in. from the wall.

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Figure 40. Overall mass transfer rate promotion over 9 1/4-in. test section from pairs of 3/32-in. promoters 1/8 in. from the wall.

Figure 41. Overall mass transfer rate promotion over 9 1/4-in. test section from 1/8-in. promoters 1/8 in. from the wall.

Figure 42. Arrangement of promoters.

Schmidt number. In this particular study, this ratio is 12. If the flow is completely developed, the thickness of the hydrodynamic boundary layer is essentially half the channel thickness. With the highest Reynolds number used in this study (5800 based upon the minor axis or channel thickness), the concentration boundary layer will remain near the linear portion of the velocity profile. Assume now a concentration profile with the form given in Equation (7):

$$\bar{C} = \frac{3}{2} \frac{y}{\delta} - \frac{1}{2} \left(\frac{y}{\delta} \right)^3. \quad (7)$$

The mean mass transfer rate can then be calculated from Equation (10):

$$\bar{k} = \left(\frac{3GD^2}{20x} \right)^{1/3}. \quad (10)$$

Let the first promoter be located at X_o , and let the mass transfer surface end at X_F . Then define

$$I_1 = \int_0^{X_o} \bar{k} dx = \left(\frac{3GD^2}{20} \right)^{1/3} \frac{2}{3} X_o^{2/3}, \quad (99)$$

and

$$I_2 = \int_0^{X_F} \bar{k} dx = \left(\frac{3GD^2}{20} \right)^{1/3} \frac{2}{3} X_F^{2/3}. \quad (100)$$

I_1 is the total overall mass transfer rate upstream of the promoter, and I_2 is the overall mass transfer rate from the entire surface with no promoter. Also, let I_p be the overall mass transfer rate from the entire surface with promoters. Then the relative promotion, P , over the distance X_o to X_p (downstream of the promoter) is:

$$P = \frac{I_p - I_1}{I_2 - I_1} \quad (101)$$

Dividing both numerator and denominator by I_2 , Equation (101) becomes:

$$P = \frac{\frac{I_p}{I_2} - \frac{I_1}{I_2}}{1 - \frac{I_1}{I_2}} \quad (102)$$

But since

$$\frac{I_1}{I_2} = \frac{X_o}{X_p}^{2/3}, \quad (103)$$

$$P = \frac{\frac{I_p}{I_2} - \frac{X_o}{X_F}^{2/3}}{1 - \frac{X_o}{X_F}^{2/3}} \quad (104)$$

In this particular study, $X_o = 2.75$ in. and $X_F = 12$ in. Using these values,

$$P = \frac{\frac{I_p}{I_2} - 0.375}{0.625} \quad (105)$$

In Figures 29 through 41, pages 91 through 103, the ratios of the observed promoted current (I_p) to the unpromoted current (I_2) were corrected, using this equation, to show how the promoters effected mass transfer downstream. With laminar flow the correction is valid, but above the transition point it becomes progressively less reliable as the eddy terms start to become significant. In systems with low

Reynolds numbers, the correction was considered sufficiently accurate for comparing promoter performances. The maximum correction was about 25 per cent (when $P = 3.0$); this correction was made very near the transition point, where the assumptions are still valid. As the Reynolds number was increased, the assumptions become less accurate, and the observed promotion (and thus the correction) becomes smaller; therefore the corrections should not cause errors in the reported promotion significantly greater than the approximately 5 per cent variation or scatter shown by the data.

Figures 29, 35, and 41, pages 91, 97, and 103, show the effects of single promoters, located $2 \frac{3}{4}$ in. from the leading edge of the test surface, on the mass transfer rate downstream. Figures 30, 31, 32, 36, 37, 39, and 41, pages 92, 93, 94, 98, 99, 101, and 103, show the effects of two promoters; in these instances, the first promoter was positioned $2 \frac{3}{4}$ in. from the leading edge of the test surface, and the second promoter was stationed a specified distance farther downstream. The effective promotion shown is that estimated over the area downstream from the first promoter. The distance from the wall refers to the separation of the center line of the promoter from the wall.

Figures 32 through 41, pages 94 through 103, show the effects of several promoters. Again, the first promoter was located $2 \frac{3}{4}$ in. from the leading edge of the test surface. When the promoters were "evenly spaced," the region downstream was divided approximately into

equal areas with a promoter positioned at the beginning of each area. When the promoters were "paired," the downstream region was again divided into approximately equal areas, with a pair of promoters located at the leading edge of each area. The exact arrangements of promoters in these studies are shown in Figure 42, page 104.

Fluctuating Component of Local Mass Transfer Rates

The root-mean-square (RMS) fluctuating components of the mass transfer rates are shown in Figures 43 through 46. These results were obtained from electrodes that were embedded in a larger mass transfer surface and represent local fluctuations in the transfer rate. To emphasize the effects of the promoters, the mass transfer rate fluctuations obtained with no promoters are shown as dashed curves.

Mean Current from Isolated Electrodes

The mean (time-averaged) current from isolated electrodes are shown in Figures 47 through 50 as a function of distance down the channel. The positions of the promoters are indicated. As discussed earlier in Chapter IV, page 63, the mean current from isolated electrodes is proportional to the cube root of the mean velocity gradient at the wall. Also, it can be recalled from Figure 20, page 76, that the mean signals from isolated electrodes in the channel with no promoters were essentially independent of the distance down the channel. The mean currents from these electrodes with no promoters are designated by the short horizontal marks in Figures 47 through 50 for each Reynolds number.

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Figure 43. RMS mass transfer rate fluctuations from a 1/16-in. promoter 1/8 in. from the wall.

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Figure 44. RMS mass transfer rate fluctuations from a $3/32$ -in. promoter $1/8$ in. from the wall.

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Figure 45. RMS mass transfer rate fluctuations from a 1/8-in. promoter 1/8 in. from the wall.

Figure 46. RMS mass transfer rate fluctuations from two 1/16-in. promoters 1/2 in. apart and 1/16 in. from the wall.

ORNL-DWG 66-10027

Figure 47. Mean current from isolated electrodes with a 1/16-in. promoter 1/16 in. from the wall.

ORNL-DWG 66-10028

Figure 48. Mean current from isolated electrodes with a $3/32$ -in. promoter $1/8$ in. from the wall.

Figure 49. Mean current from isolated electrodes with a 1/8-in. promoter 1/8 in. from the wall.

ORNL-DWG 66-10025

Figure 50. Mean current from isolated electrodes with two 3/32-in. promoters 1/8 in. from the wall.

Fluctuating Current from Isolated Electrodes

The root-mean-square (RMS) fluctuating components of the current from isolated electrodes are shown in Figures 51 through 55. Again, the promoter positions are specified, and the data obtained with no promoters are shown by dashed curves. For low-frequency disturbances, these signals can be interpreted as being proportional to the RMS fluctuating component of the cube root of the wall velocity gradient. However, the thin concentration boundary layer over the electrodes has a finite response time, and high-frequency fluctuations are attenuated as described in Chapter II, page 4.

Power Spectral Density Measurements from Isolated Electrodes

The spectra of some of the isolated electrode signals from this study are shown in Figures 56 through 59. Data were taken at three Reynolds numbers: one in the laminar region (730), one in the transition region (1850), and one in the turbulent region (5800). Both single and paired ($3/4$ in.) promoters were tested, but only one diameter ($3/32$ in.) was used. In Figures 56 through 59, the power density spectra are shown for various distances down the channel for each condition studied. To avoid confusion in these graphs, only the spectral curves are shown; these same curves, along with the data points, are shown in Appendix B. Note that Figures 56 through 59 also show both the approximate vortex shedding frequency from the promoters and the approximate frequency at which a signal will be damped 50 per cent by the slow response of the concentration boundary layer over the probes. The 50 per cent attenuation frequency was estimated from Equation (12).

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Figure 51. RMS current fluctuations from isolated electrodes with a 1/16-in. promoter 1/16 in. from the wall.

Figure 52. RMS current fluctuations from isolated electrodes with a 3/32-in. promoter 1/8 in. from the wall.

Figure 53. RMS current fluctuations from isolated electrodes with a 1/8-in. promoter 1/8 in. from the wall.

Figure 54. RMS current fluctuations from isolated electrodes with two $3/32$ -in. promoters $3/4$ in. apart and $1/8$ in. from the wall.

Figure 55. RMS current fluctuations from isolated electrodes with no promoters.

Figure 56. Power spectral density measurements from isolated electrodes with a $3/32$ -in. promoter $1/8$ in. from the wall, $N_{Re} = 5800$.

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Figure 57. Power spectral density measurements from isolated electrodes with a $3/32$ -in. promoter $1/8$ in. from the wall, $N_{Re} = 1835$.

Figure 58. Power spectral density measurements from isolated electrodes with two $\frac{3}{32}$ -in. promoters $\frac{3}{4}$ in. apart and $\frac{1}{8}$ in. from the wall, $N_{Re} = 1835$.

Figure 59. Power spectral density measurements from isolated electrodes with two $3/32$ -in. promoters $3/4$ in. apart and $1/8$ in. from the wall, $N_{Re} = 730$.

Measurements of power density spectra are illustrated in Figures 60 and 61 for Reynolds numbers of 1835 and 5800, respectively; no promoters were used. Similar measurements were made at a Reynolds number of 730; however, the signal at this Reynolds number was low and difficult to distinguish from instrumentation "noise."

Pressure Drop Measurements

Although promoters substantially increase mass transfer rates, they will not be practical unless the higher rates are achieved with only a moderate increase in pressure loss or pumping costs. Pressure drop measurements were not made for all the promoter arrangements studied, but enough data were collected to illustrate the general applicability of detached turbulence promoters. The pressure loss in most of the conditions studied can be estimated (or at least guessed) from the data reported. The pressure drop results are shown in Figures 62 through 64 and are expressed in terms of the friction factor as defined by Equation (74). The results obtained with various promoters, along with data taken using an empty channel, are shown. The line drawn through the empty-channel data is that reported by Hartnett, Koh, and McComas (11) for fully developed flow in a 5 x 1 rectangular channel. It agrees well with the experimental data. Most of the large variations occur when the value of f_f is low, since the pressure differences are low and difficult to measure.

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Figure 60. Power spectral density measurements with no promoter, $N_{\text{Re}} = 1835$.

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Figure 61. Power spectral density measurements with no promoter, $N_{Re} = 5800$.

Figure 62. Friction factor with 1/16-in. promoters 1/16 in. from the wall.

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Figure 63. Friction factor with 3/32-in. promoters 1/8 in. from the wall.

Figure 64. Friction factor with a 1/8-in. promoter 1/8 in. from the wall.

CHAPTER VI

DISCUSSION OF RESULTS

Summary of Local Mass Transfer Rates

Measurements of local promotion generally show sharp peaks just under each promoter (see Figures 25 through 28, pages 86 through 89, and Appendix A) followed by a broad region of promotion that persists for a substantial distance downstream. This behavior is similar to that observed by Thomas (42)(43). The sharp peak under each promoter probably represents the effect of the accelerated flow under the cylinder, while the promotion downstream represents the effect of the cylinder wake on the mass transfer rate. A promotion peak probably exists under every promoter, but in a few instances this peak is not obvious from the plots shown in Appendix A. The peaks are sharp; therefore, some of them were probably undetected because probes were not located in the right places. Since the first promoter was always located at the same position, several electrodes were concentrated in this region. The downstream promoters, however, were placed at various positions, and, of course, electrodes were not always in suitable positions to detect the peaks. Quantitatively, the promotion peaks in this study were larger than those reported by Thomas from studies in a gaseous system. The largest peak observed in the air-naphthalene studies was 4.4, but the peaks in the present study have been as high as 8 or more. These peaks are so sharp that

they probably contribute little to the overall promotion, which is thought to be due largely to the wake effect.

Overall Mass Transfer Rates

Threshold Reynolds Number for Promotion. Figures 30 and 33, pages 92 and 95, show no promotion at Reynolds numbers below a threshold value of 600 although sharp peaks are seen under the promoters. The channel Reynolds number of 600 corresponds to a cylinder Reynolds number of about 50, based upon the promoter diameter and the velocity at the promoter center line (estimated by assuming a parabolic profile). As noted in Chapter II, this is near the critical Reynolds number for vortex shedding when the cylinder is in the free stream. Taneda (39) has investigated the effect of wall proximity on the critical Reynolds number for vortex shedding and concluded that proximity to the walls increases the critical Reynolds number. However, his cylinders were always in the center of the channel and, to bring the cylinder nearer to the channel walls, both walls were gradually moved closer together and closer to the cylinder. No data are available on the critical Reynolds number when the cylinder is near only one wall, but it appears likely that it will be approximately the value at which mass transfer promotion starts to occur.

Note in Figures 29, 38, and 40, pages 91, 100, and 102, what happens when larger promoters are used or when the promoters are moved

farther from the wall. For a given flow rate or Reynolds number (based upon the channel thickness), increasing the cylinder diameter or moving the cylinder farther from the wall into regions of higher velocity increases the cylinder Reynolds number (based upon the cylinder diameter and local velocity). Thus, if both vortex shedding and mass transfer promotion start at a cylinder Reynolds number of 50, increasing either the cylinder diameter or distance from the wall will cause promotion to start at a lower channel Reynolds number (based upon channel thickness and average velocity). This conjecture is confirmed in Figures 38 and 39, pages 100 and 101, which correspond to larger promoters, and in Figures 29, 31, and 32, pages 91, 93, and 94, in which the 1/16-in. promoters were moved farther from the wall. In both instances the critical channel Reynolds number was reduced as predicted, but, since the critical values were below the range of the data, further quantitative checks on the preceding argument are not possible.

Also, when a large number of promoters are used in close proximity of one another, the critical Reynolds number is reduced (see Figure 34, page 96). This is believed to result because any disturbance caused by the first promoters (although the disturbance may not be regular vortex shedding) affects the likelihood of vortex shedding from downstream promoters. Taneda (39) showed that the critical Reynolds number for vortex shedding decreases as the turbulence level increases. He observed as much as a 40-fold decrease

in the critical Reynolds number with increased turbulence level. Thus, one may speculate that a long series of turbulence promoters is able to "amplify" small disturbances in the stream until, somewhere downstream, steady vortex shedding and significant promotion are achieved even at promoter Reynolds numbers below 40.

Reynolds Number for Maximum Promotion. The next apparent point to note about the overall promotion shown in Figures 29 through 41, pages 91 through 103, is that the maximum promotion always occurs at the same channel Reynolds numbers (approximately 1800 to 1900 when based upon the channel minor axis. This corresponds to a Reynolds number of about 3000, based upon the hydraulic diameter, which is near the turbulent transition point in 5 x 1 channels (11).

Estimated Optimum Promoter Spacings. Several cross plots of the data in Figures 29 through 41, pages 91 through 103, were made to estimate the optimum promoter arrangements. The maximum overall promotion, that observed at channel Reynolds numbers of 1900, was plotted as a function of the geometric parameters. The choice of this channel Reynolds number to make the comparisons was arbitrary, but the overall promotion curves are so similar in shape (except at low Reynolds numbers near the vortex shedding threshold) that the indicated optimum spacings are not greatly affected by the choice of this Reynolds number for comparison.

Figure 65 shows the overall promotion from a single cylinder as a function of distance from the wall. Only three points for each of two promoter diameters, plus a single point for a third, are shown; however, these limited data suggest that the best promotion is obtained when the center line of the cylinder is 1.3 to 1.5 promoter diameters from the wall.

Figures 66 and 67 show the effects on the overall promotion of the spacing between single pairs of 1/16- and 3/32-in. promoters. Both figures contain four or more points for each of two spacings from the wall and a single point for a third spacing. The curves suggest an optimum separation of between 6 and 8 diameters. Using concentric ring promoters in a circular channel, Koch (16) reported an optimum separation ratio of 10. This optimum spacing also corresponds to placing the second promoter near the position of maximum velocity fluctuation intensity, which was reported by Kovaszny (17) to be about seven diameters behind the first cylinder.

Effects of Large Numbers of Promoters. The maximum promotion (at $N_{Re} = 1900$) that can be obtained by using several promoters over the 9-1/4 in. test surface is summarized in Figure 68. Here, promotion is plotted as a function of the number and diameters of promoters used. Note that the first promoter gives substantially higher mass transfer rates but that each later addition provides a smaller improvement. Eventually, a limit appears to be reached at about a threefold improvement in the transfer rate.

Figure 65. Overall promotion over 9 1/4-in. test section from a single promoter, $N_{Re} = 1900$.

Figure 66. Overall promotion over 9 1/4-in. test section from two 1/16-in. promoters, $N_{Re} = 1900$.

Figure 67. Overall promotion over 9 1/4-in. test section from two 3/32-in. promoters, $N_{Re} = 1900$.

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Figure 68. The effect of number and diameter of promoters on overall promotion over 9 1/4-in. test section, $N_{Re} = 1900$.

Efficiency of Promoters Compared with Higher Velocities in Clear Channels

Figures 29 through 41, pages 91 through 103, show that detached turbulence promoters are capable of increasing mass transfer rates significantly. Increases of more than threefold were obtained; however, these increases alone do not mean that the promoters will be useful in all instances. In any real applications, all costs associated with the promoters (increased capital and pumping costs) will have to be compared with the alternatives, larger transfer surfaces or higher circulating rates. The consideration of capital costs is outside the scope of this study since the latter are dependent upon the actual process and membrane or transfer surface under consideration; pumping costs, on the other hand, are properties of the promoters and are covered in this study. The promoters are compared in a manner suggested by Thomas (43). The improvement in transfer rate may be related to the friction factor increase by:

$$\frac{\bar{k}_p}{\bar{k}} = \left(\frac{(f_f)_p}{f_f} \right)^{n_f}, \quad (106)$$

where the subscript p refers to promoted conditions and no subscript refers to the no-promoter or clear-channel conditions. The exponent n_f denotes how much a given change in pressure drop will increase the mass transfer rate. Obviously, a promoter that gives a high value for n_f is desired. In fully developed turbulent flow, an experimental value of n_f allows the comparison of the increase in pumping power

required to obtain a certain degree of promotion; using a promoter, with that required to obtain the same increase in mass transfer rate by increasing the circulation rate. For turbulent flow over a smooth surface, the Colburn equation [Equation (3)] states that:

$$N_{St} = \frac{f_f}{2} N_{Sc}^{-2/3} . \quad (3)$$

Substituting the definitions for the Stanton number and friction factor yields:

$$\frac{\bar{k}_2/U_2}{\bar{k}/U} = \frac{(\Delta P)_2}{(\Delta P)} \frac{U^2}{U_2^2} , \quad (107)$$

where the subscript 2 refers to a condition where the flow rate is increased to U_2 and the pressure loss becomes $(\Delta P)_2$. Then to obtain a given improvement in mass transfer rate, $\bar{k}_2/\bar{k}$, by increasing the circulation rate to U_2 , the resulting change in the pressure loss can be calculated from Equation (108):

$$\frac{\bar{k}_2}{\bar{k}} = \left(\frac{U_2}{U} \frac{\Delta P_2}{\Delta P} \right) \frac{U^2}{U_2^2} . \quad (108)$$

The term enclosed in parentheses is the ratio of the pumping power at the higher velocity, U_2 , to the pumping power at the lower velocity U . By using a turbulence promoter, the mass transfer promotion was evaluated at a constant velocity. Then, assuming the relation between k_p/k and $(f_f)_p/f_f$ to be that given in Equation (106), we obtain:

$$\frac{\bar{k}_p}{k} = \left[\frac{(f_f)_p}{f_f} \right]^{n_f} = \left(\frac{\Delta P_p / U^2}{\Delta P / U^2} \right)^{n_f} = \left(\frac{\Delta P_p}{\Delta P} \right)^{n_f} = \left(\frac{U \Delta P_p}{U \Delta P} \right)^{n_f}. \quad (109)$$

Now compare the increase in pumping power required to increase the mass transfer rate to some value $\bar{k}_p$ or $\bar{k}_2$, using promoters, with the increase required in the circulation rate to obtain the same value. This can be done by equating Equations (108) and (109):

$$\frac{\bar{k}_p}{k} = \frac{\bar{k}_2}{k} = \left(\frac{U \Delta P_p}{U \Delta P} \right)^{n_f} = \frac{U_2 \Delta P_2}{U \Delta P} \left(\frac{U}{U_2} \right)^2, \quad (110)$$

or

$$\frac{U \Delta P_p}{U \Delta P} = \left(\frac{U_2 \Delta P_2}{U \Delta P} \right)^{1/n_f} \left(\frac{U}{U_2} \right)^{2/n_f}. \quad (111)$$

Assume that for the smooth channel, the Blasius equation applies and that

$$f \propto N_{Re}^{-1/4}, \quad (112)$$

or

$$\frac{\Delta P}{U^2} \propto U^{-1/4}. \quad (113)$$

Then

$$\frac{U_2}{U} = \left(\frac{U_2 \Delta P_2}{U \Delta P} \right)^{4/11}. \quad (114)$$

Substituting this relation into Equation (110), one finds that

$$\frac{U \Delta P_p}{U \Delta P} = \left(\frac{U_2 \Delta P_2}{U \Delta P} \right)^{1/n_f} \left(\frac{U_2 \Delta P_2}{U \Delta P} \right)^{-8/11n_f} = \left(\frac{U_2 \Delta P_2}{U \Delta P} \right)^{3/11n_f}. \quad (115)$$

The increase in pumping power required by the turbulence promoter will be less than that required by increasing the circulation rate if the exponent $3/11n_f$ is less than unity. That is, the turbulence promoter improves the mass transfer rates with a smaller increase in pumping costs if

$$n_f > \frac{3}{11} = 0.273. \quad (116)$$

The values of the exponent n_f calculated for all of the conditions under which pressure drop data were taken are given in Figure 69. Note that all the data shown have exponents above the critical value of 0.273; thus, under these conditions the use of turbulence promoters is a more efficient means of increasing the mass transfer rate than simply increasing the circulation rate. However, the curves shown in Figure 69 have a negative slope at the higher Reynolds numbers shown; and at some even higher Reynolds number each curve will probably cross the $n_f = 0.273$ line. At that point, the promoter will become less desirable as a means of increasing the mass transfer rate.

Figure 69 also indicates that promoters with smaller diameters are more efficient than those with larger diameters. Note that the curves for the 1/16-in. promoter lie above the corresponding curves for the 3/32-in. promoter, which, in turn, lie slightly above the curve for the 1/8-in. promoter. It appears desirable to select a promoter diameter as small as is practical from a mechanical or

Figure 69. Exponent in Equation (106).

fabrication point of view, but also large enough that the Reynolds number, based upon the promoter diameter, will be greater than the vortex shedding threshold of 50.

There also appears to be some merit in pairing the promoters rather than spacing them evenly along the channel. The curves (in Figure 69) for paired promoters lie above the curves for single promoters with the same diameter.

Changes in Spectra Downstream

One of the objectives of this study was to obtain a better understanding of the mechanisms of turbulence promotion by studying the fluctuating, as well as the mean, components of mass transfer rates and velocities near the wall. The nature of the velocity fluctuations near the wall, or the gradient fluctuations, is illustrated in Figures 56 through 61, pages 123 through 126, 128, and 129), which show measurements of power spectral densities for isolated electrode signals. (It will be recalled that these signals from isolated electrodes are proportional to the cube root of the velocity gradient.) Note that these curves are smooth, giving no evidence of any selected frequency predominating near the wall. Of course, the spectral measurements were taken at fixed frequencies, and very sharp peaks between these points would not be detected. The spacings of the readings are shown in Appendix B; these curves show how sharp a peak would have to be to avoid detection. The limited frequency response of the electrodes (as indicated by the 50 per cent

attenuation frequencies marked on these figures) did not allow detection of vortex shedding frequencies if disturbances of these frequencies reached the wall. In general, only low frequencies were observed at the wall. At the estimated 50 per cent attenuation frequency, the levels of the spectra were reduced by a factor of 5 to 10 from those at lower frequencies. Since this steep reduction in the spectra with increasing frequency is not accounted for by properties of the electrodes, the drop in power spectral density with increasing frequency and other features of Figures 56 through 61, (pages 123 through 126, 128, and 129), are believed to represent properties of the system. However, it is impossible to determine what happens above the electrode attenuation frequencies, where all the vortex street shedding frequencies lie.

Spectra are reported for three Reynolds numbers: 730 (below the turbulent transition point), 1835 (near the transition point), and 5800 (in the turbulent region). There is a notable difference between the spectra observed in the turbulent region, and the spectra observed at or below the transition point. At a Reynolds number of 5800, the flow is ordinarily turbulent, and the spectra do not appear to change as the distance down the channel increases. Although the intensity of the fluctuations decays, the spectrum of the fluctuations remains essentially unchanged downstream of the promoter and apparently (within the frequency range where these electrodes were sensitive) is unaffected by the promoters. At lower

Reynolds numbers, the spectrum changes noticeably downstream. The higher-frequency fluctuations decay faster with distance downstream than those of lower frequencies. The decay of higher-frequency fluctuations is illustrated in Figure 70. Here $\sqrt{4(k_{\text{RMS}}^2)} / \text{PSD}(0)$ is plotted as a function of distance down the channel. As noted in Chapter III, this quantity is approximately equal to or related to the ratio of the propagation velocity to the macroscale of the disturbance. Note that at Reynolds numbers below the transition point, the ratio is a maximum near the cylinder and then decreases farther downstream. If the propagation velocity is assumed to remain constant behind the cylinder, then the integral scale Λ_x is a minimum just behind the cylinder and increases with distance down the channel. That is, the higher-frequency, or lower-wavelength, disturbances decay faster than the lower-frequency disturbances. Above the transition Reynolds number, only a slight change in the integral scale is evident.

Relative Fluctuation Intensities

Isolated Electrodes. Further evidence that, with Reynolds numbers above the transition point, the nature of velocity fluctuations changes only slightly with distance downstream is suggested in Figures 71 through 73. Here, the ratios of the root-mean-square of the fluctuating signal to the mean signal are plotted as a function of distance down the channel for several Reynolds numbers. One or more sharp peaks occur immediately behind each

Figure 70. Estimated macroscale behavior behind promoters.

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Figure 71. Relative mass transfer rate fluctuation intensities from isolated electrodes with no promoters.

Figure 72. Relative mass transfer rate fluctuation intensities from isolated electrodes with a 1/16-in. promoter 1/16 in. from the wall.

Figure 73. Relative mass transfer rate fluctuation intensities from isolated electrodes with a $3/32$ -in. promoter $1/8$ in. from the wall.

promoter; then, gradually the ratio "decays" or returns to a stable downstream condition. With Reynolds numbers below the transition point, the ratio apparently decays toward zero since the condition far downstream would constitute laminar flow. With Reynolds numbers above the transition point, this ratio decreases quickly to a value (approximately 0.075) that is independent of, or only slightly affected by, Reynolds number and distance downstream. The quick return of both the spectra and this ratio to a constant condition downstream is evidence that, at these Reynolds numbers, downstream from the promoter, the flow near the wall quickly approaches a condition which, in some ways, resembles "normal turbulent" flow. This is not readily apparent when the mean or fluctuating currents from isolated electrodes are considered singly because, under normally turbulent conditions, large dips and peaks occur with each signal (see Figures 47 through 54, pages 113 through 116 and 118 through 121) at the same positions downstream. One can speculate that, since these dips and peaks occur with the mean component as well as the fluctuating component, there may be a significant diversion of flow over the promoter. That is, although the promoter is separated from the wall, it may act somewhat as a baffle. The main flow downstream may then have a component striking each wall alternately at various positions until the wake is eventually damped.

Mass Transfer Rates. The fluctuating components of the mass transfer rates shown in Figures 43 and 44, pages 109 and 110, have the

same general appearance as the signals from isolated electrodes. Ratios of the fluctuating components to the mean components of the mass transfer rate are shown in Figures 74 through 76 as a function of the distance downstream. These curves resemble the corresponding curves from isolated electrodes. At Reynolds numbers below the transition point, this ratio decays downstream toward zero. At Reynolds numbers above transition, the ratio decays toward a fixed value, which, over the range of turbulent Reynolds numbers covered in this study, showed little or no dependence upon Reynolds number. This limiting ratio, however, seems to be somewhat higher than that observed with isolated electrodes. With mass transfer rates, $k_{\text{RMS}}/\bar{k}$ was approximately 0.22, but with isolated electrodes, $k_{\text{i-RMS}}/\bar{k}_{\text{i}}$ was approximately 0.075. Both of these values are less than the corresponding values reported by Shaw (36) and Mitchell (23) with ordinary turbulent flow in a 1-in. pipe (a ratio of 0.48 for mass transfer rates and 0.1 for isolated electrodes). The difference results principally because those authors attempted to correct their readings for the finite electrode size.

Comparison with Theory. The relation between these two signals (mass transfer rate fluctuations and velocity gradient fluctuations) was discussed in Chapter III. Two hypotheses were suggested that allowed (after several assumptions of simplifications were made) the mean and fluctuating components of the mass transfer

Figure 74. Relative mass transfer rate fluctuation intensities with a 1/16-in. promoter 1/8 in. from the wall.

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Figure 75. Relative mass transfer rate fluctuation intensities with a $3/32$ -in. promoter $1/8$ in. from the wall.

Figure 76. Relative mass transfer rate fluctuation intensities with a $1/8$ -in. promoter $1/8$ in. from the wall.

rate to be related to the mean and fluctuating components of the velocity gradient. The two hypotheses differed in how the disturbance propagation velocity was assumed to behave near the wall. These hypotheses are necessary only because direct experimental measurements of propagation velocity are not available. The first hypothesis assumed that the propagation velocity at every point was some fraction of the mean velocity at that point. If this were true, the effective propagation velocity over the concentration boundary layer should not exceed the maximum velocity over the concentration boundary layer, U_0 ; thus b should be less than, or equal to, unity. The second hypothesis assumed that the propagation velocity near the wall approached some fraction of the main stream velocity. Thus, with thin concentration boundary layers, b may be very large and may vary with the thickness of the boundary layer. The mean mass transfer rates predicted by the analysis, using the velocity gradient spectra and intensities reported by Mitchell for ordinary turbulent flow, were closer to experimental results when the second hypothesis was used. The propagation velocity, however, had to be inferred from Mitchell's measurements of spectra and longitudinal correlation since no direct measurements are available. The behavior of mass transfer rate fluctuations and velocity gradient fluctuations will be discussed in terms of the second hypothesis.

Now consider the observations just made from the decay of fluctuating signals from isolated electrodes and mass transfer rates in terms of this analysis. Under normally turbulent conditions, the

signals from isolated electrodes decayed quickly to a state where the ratio of the fluctuating component to the mean component of the current was essentially constant, and only slightly affected by the Reynolds number. In terms of the analysis in Chapter III, this means that the flow decays toward a state where B_{RMS} is a constant. Experimentally, we have seen that the ratio of the fluctuating component to the mean component of the mass transfer rate also approaches a constant, but higher, value. The mass transfer rate fluctuations may be written as:

$$k_{\text{RMS}} = \frac{D}{\delta b} \int_0^{\infty} B_{\text{RMS}}(\sigma) b \left(\frac{\partial f^*}{\partial \eta} \right)_{\eta=0} d\sigma, \quad (117)$$

where $B_{\text{RMS}}(\sigma)d\sigma$ is the root-mean-square intensity of velocity gradient fluctuations with frequencies between σ and $\sigma + d\sigma$. If the velocity gradient spectrum is approximately rectangular, as indicated in Figure 56, page 123, this can be simplified to:

$$N_{\text{St-RMS}} = \frac{k_{\text{RMS}}}{U_a} = \frac{B_{\text{RMS}} D}{\delta b U_a} \frac{1}{\sigma_{\text{max}}} \int_0^{\sigma_{\text{max}}} \left[b \left(\frac{\partial f^*}{\partial \eta} \right)_{\eta=0} \right] d\sigma. \quad (118)$$

Then the ratio of the fluctuating component to the mean component of the mass transfer rate is:

$$R = \frac{N_{\text{St-RMS}}}{N_{\text{St}}} = \frac{2}{3} \left(\frac{B_{\text{RMS}}}{b} \right) \frac{1}{\sigma_{\text{max}}} \int_0^{\sigma_{\text{max}}} \left[b \left(\frac{\partial f^*}{\partial \eta} \right)_{\eta=0} \right] d\sigma. \quad (119)$$

The normalized propagation velocity, b , can be written in terms of the wall friction factor at any point as:

$$b = \frac{4}{3} \frac{K}{f_f} N_{Sc} N_{St} . \quad (90)$$

Thus

$$R = \frac{f_f}{2K} \left(\frac{B_{RMS}}{N_{Sc} N_{St}} \right) \frac{1}{\sigma_{max}} \int_0^{\sigma_{max}} \left[b \left(\frac{\partial f}{\partial \eta} \right)_{\eta=0} \right] d\sigma . \quad (120)$$

If one considers a distance sufficiently far downstream or if one considers small disturbances that do not greatly affect the mean flow, the friction factor can be approximated from the Blasius equation, and K can be obtained from Equation (94), which resulted from Mitchell's measurements of spectra and correlation. Then

$$\frac{f_f}{K} \approx 2 N_{Re}^{-1/2} , \quad (121)$$

or

$$R_M \approx \left(\frac{B}{N_{Sc} N_{St} N_{Re}^{1/2}} \right) \frac{J_1}{\sigma_{max}} , \quad (122)$$

where

$$J_1(\sigma_{max}) = \int_0^{\sigma_{max}} \left[b \left(\frac{\partial f}{\partial \eta} \right)_{\eta=0}^* \right] d\sigma_{max} . \quad (123)$$

At higher values of σ_{max} , J_1 is approximately proportional to $\sigma_{max}^{1/2}$. [This is evident from Figure 11, page 56, because $b \left(\frac{\partial f}{\partial \eta} \right)_{\eta=0}^*$ varies as the $-1/2$ power of σ .] Also,

$$\sigma = bn \frac{\delta^2}{D} = \frac{3}{2} \frac{nvK}{\bar{G} N_{St}} . \quad (124)$$

Thus

$$\frac{J_1}{\sigma_{\max}} \propto \frac{N_{\max} K}{\bar{G} N_{St}}^{-1/2} , \quad (125)$$

or

$$R_M \propto \frac{B \bar{G}^{1/2}}{N_{Sc}^{1/2} N_{St}^{1/2} N_{Re}^{1/2} N_{\max}^{1/2} K^{1/2}} \quad (126)$$

Far downstream, or with rather small disturbances, assume that the flow resembles ordinary turbulent flow. That is,

$$G \propto N_{Re}^{1.8} , \quad (127)$$

$$N_{St} \propto N_{Re}^{-0.2} , \quad (128)$$

$$N_{\max} \propto N_{Re} , \quad (129)$$

$$K \propto N_{Re}^{1/4} . \quad (130)$$

Then

$$R \propto \frac{B_{RMS}}{N_{Sc}} N_{Re}^{-1/8} . \quad (131)$$

This suggests that the ratio of the fluctuating component to the mean component of the mass transfer rate at a distance far downstream should be a constant only slightly dependent upon Reynolds number, as shown by Figures 74 through 76, pages 156 through 158.

Decay of Fluctuations

Mass Transfer Rate Fluctuations. Now consider the decay of mass transfer rate fluctuations. The results shown in Figures 44 and 47, pages 110 and 113, are replotted in Figures 77 through 79. Only the data points in the region where the disturbances are decaying behind the promoters are plotted. The root-mean-square of the fluctuating component is shown on a log-log plot as a function of distance behind the promoter, divided by the Reynolds number (this ratio is proportional to the time required for the mean stream to travel the distance x behind the promoter). Shown in this manner, the curves for all Reynolds numbers converge into a single straight line with a slope of approximately -1. The same slope and behavior apply for all single promoters tested; however, with multiple promoters the correlation was not as good, probably because no single distance, x , could describe the distance behind both promoters. Note, however, that the x/N_{Re} plot also describes the decay of mass transfer rate fluctuations when no promoter was present. The disturbances in this case was generated by the channel entrance, and thus the total distance down the channel was used as the characteristic length.

Estimation of Decay of Isolated Electrode Fluctuation with Theory. From the analysis in Chapter III, we can make some estimates of the velocity disturbances near the wall that would cause this behavior in the mass transfer rate. In Equation (119), the mass transfer rate fluctuations were described by the relation:

Figure 77. The decay of mass transfer rate fluctuations behind a 1/16-in. promoter 1/8 in. from the wall.

Figure 78. The decay of mass transfer rate fluctuations behind a 3/32-in. promoter 1/8 in. from the wall.

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Figure 79. The decay of mass transfer rate fluctuations behind a 1/8-in. promoter 1/8 in. from the wall.

$$k_{\text{RMS}} = \left(\frac{D B_{\text{RMS}}}{\delta b} \right) \left(\frac{J_1}{\sigma_{\text{max}}} \right) = \left(\frac{J_1}{K U_a} \right) \left(\frac{J_1}{\sigma_{\text{max}}} \right) G_{\text{RMS}} , \quad (132)$$

where $k_{i\text{-RMS}}$ represents the root-mean-square of the fluctuating current from isolated electrodes. In Equation (125), $\frac{J_1}{\sigma_{\text{max}}}$ was shown to be proportional to:

$$\left(\frac{G N_{\text{St}}}{N_{\text{max}} K} \right)^{1/2} .$$

Thus

$$k_{\text{RMS}} \propto \left(\frac{D}{K U_a} \right) \left(\frac{G N_{\text{St}}}{N_{\text{RMS}} K} \right)^{1/2} G_{\text{RMS}} . \quad (133)$$

Under turbulent flow, the approximations given in Equations (127) through (130) may be assumed, and

$$\begin{aligned} \left(\frac{D}{K U_a} \right) \left(\frac{G N_{\text{St}}}{N_{\text{max}} K} \right)^{1/2} &\propto \left(\frac{1}{U_a^{0.25} U_a} \right) \left(\frac{U_a^{-0.2} U_a^{1.8}}{U_a U_a^{0.25}} \right)^{1/2} \\ &= \frac{1}{U_a} 1.075 \approx U_a^{-1} . \end{aligned} \quad (134)$$

Then, approximately,

$$G_{\text{RMS}} \propto U_a k_{\text{RMS}} , \quad (135)$$

and, if k_{RMS} is proportional to x/N_{Re} , then

$$G_{\text{RMS}} \propto \left(\frac{x}{N_{\text{Re}}} \right)^{-1} . \quad (136)$$

The signals from isolated electrodes are proportional to the cube root of the velocity gradient at the wall [see Equation (11)]. Then,

$$k_i = \bar{k}_i + k_i' \propto (G + G')^{1/3} \approx \bar{G}^{1/3} \left(1 - \frac{1}{3} \frac{G'}{\bar{G}}\right), \quad (137)$$

where the prime denotes the fluctuating component of G or k_i .

Equating the fluctuating portion of both sides of this equation gives:

$$k_i' \propto \frac{1}{3} \frac{G'}{\bar{G}^{2/3}} = \frac{1}{3} \left(\frac{G'}{\bar{G}}\right)^{2/3} (G')^{1/3}. \quad (138)$$

But the experimental data have shown that the ratio $(G_{\text{RMS}}/\bar{G})$ is approximately constant over most of the region behind the promoters and is independent of Reynolds number. Therefore,

$$k_{i\text{-RMS}} \propto (G_{\text{RMS}})^{1/3} \propto \frac{x}{N_{\text{Re}}^2}^{-1/3}. \quad (139)$$

The isolated electrode data downstream of single promoters are plotted in Figures 80 through 83 as a function of x/N_{Re}^2 . With these variables, the fluctuation data for all the Reynolds numbers tested above the transition point merge into a single curve. When the data were plotted simply as a function of x/N_{Re} , different curves for each Reynolds number were far more evident. Also, the slope of the curves in Figures 80 through 83 is approximately $-1/3$, as predicted by this simplified argument. Data from Reynolds numbers below the transition point decrease more sharply and do not fit this curve; however, this is not surprising since the Reynolds number and the x dependence of several terms in Equations (127) through (130) change

Figure 80. RMS current fluctuations from isolated electrodes behind a 1/16-in. promoter 1/8 in. from the wall.

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Figure 81. RMS current fluctuations from isolated electrodes behind a 3/32-in. promoter 1/8 in. from the wall.

Figure 82. RMS current fluctuations from isolated electrodes behind a 1/8-in. promoter 1/8 in. from the wall.

Figure 83. RMS current fluctuations from isolated electrodes with no promoter.

at the transition point.

The preceding arguments illustrate further how the analysis made in this study may be used to estimate the behavior of mass transfer rates from velocity data near the wall or, as in this particular example, velocity gradient properties may be estimated from mass transfer rate measurements. The particular decay curves shown in Figures 77 through 83, pages 164 through 166 and 169 through 172, illustrate a mode of decay that occurs over the distance downstream studied in this investigation. It would be presumptuous to assume that these plots may be extrapolated too far downstream (e.g., these plots would extrapolate eventually toward zero fluctuation intensity even for turbulent flow, and this is known not to be the case). However, in practical systems, turbulence promoters will probably be located reasonably close to each other, and the mode of decay just described will certainly be of interest.

Effects of Reynolds Number upon Promotion

At this point one can speculate about why the maximum promotion with detached promoters occurs near the transition point. Figures 47 through 54, pages 113 through 116 and 118 through 121, show that, below the transition point, the RMS intensity of the fluctuations increases with Reynolds number. Thus, in this region one should expect the promotion to increase with Reynolds number. Above the transition point, the relative intensities of the fluctuation are not so greatly affected by Reynolds numbers; however, the rate of

decay toward the far-downstream condition appears, in this region, to increase with Reynolds number (actually fluctuations decay approximately inversely with the square of the Reynolds number). That is, the distance over which the fluctuating intensities are significantly affected by the promoter decreases with increasing Reynolds number. Thus, the promotion in this region should decrease with increasing Reynolds number, and the maximum promotion should occur near the transition point.

CHAPTER VII

CONCLUSIONS

Detached turbulence promoters can substantially increase mass transfer rates in aqueous channel flow. For a given increase in mass transfer rates, turbulence promoters require smaller increases in pumping power than would be required by increased circulation rates in clear, or unpromoted, channels. The greatest increase in transfer rates occurs when detached promoters are used with flow near the laminar-turbulent transition point; here increases of more than threefold are observed. Substantial promotion persists at Reynolds numbers well above the transition point, but in this region promotion decreases with increasing Reynolds number. This decrease in promotion will probably limit the application of detached promoters to systems not requiring extremely high Reynolds numbers.

For maximum increases in the mass transfer rate, the center line of the promoter should be between 1 and 1.5 diameters from the wall, and the promoters should be spaced about 6 to 8 diameters apart. The pressure loss can be reduced, with only a slight loss in promotion, by removing every third promoter (giving alternate spacings of 8 and 16 diameters). The diameter of the promoter used should normally be as small as is practical from a mechanical viewpoint since the smaller promoters have relatively lower pressure

losses. However, it is also desirable (and necessary if only a few promoters are used) to use promoters large enough to ensure that the Reynolds number based upon the promoter diameter and the local velocity at the promoter center line will be greater than 50 (i.e., large enough to ensure steady vortex shedding).

In many respects the results from this study are similar to those reported earlier by Thomas (42)(43) for gaseous boundary layer flow. However, the promotion in aqueous channel flow is slightly higher, and there is no evidence of a sharp, tuned peak such as those observed earlier in the boundary layer studies.

Detached turbulence promoters generate disturbances in the velocity field near the wall which result in higher mass transfer rates. The frequencies of velocity disturbances observed in this study were low, and the intensity of these fluctuations dropped sharply with increasing frequency above a few cycles per second. However, the frequency response of the probes used in this study also fell sharply at frequencies only slightly higher than the measured disturbance spectra. Unfortunately, the frequencies of vortex shedding from the promoters were too high and could not be detected by the probes used in this study, even if disturbances with these frequencies reached the wall.

A mathematical model has been developed to predict the effects that fluctuations in the velocity field very near the wall

have on the mass transfer rate. The model considers a simplified two-dimensional flow near the wall and a fully developed concentration boundary layer. The analysis is limited to systems with high Schmidt numbers, where the concentration boundary layer is thin and is essentially contained within the linear portion of the hydrodynamic boundary layer. To properly evaluate the model, more hydrodynamic data are needed, particularly data on velocity fluctuation frequencies and phase velocities. However, where the comparison of predicted results with experimental data is possible, agreement is satisfactory. This analysis has been used to predict mass transfer rates in ordinary unpromoted turbulent flow, and the results agree reasonably well with traditional correlations, which, in turn, are known to show reasonable agreement with experimental data. However, the principal merit of this analysis, in comparison with the traditional correlations, is the ability to predict what happens when the velocity field is altered significantly from that observed in ordinary turbulent flow. Thus, this analysis could be very useful in turbulence promoter design, where one attempts to alter the velocity field near the wall to obtain improved mass transfer rates. For instance, the analysis indicates that higher-frequency velocity fluctuations near the wall contribute relatively more toward the mean mass transfer rate than do lower-frequency fluctuations; eventually, however, the effectiveness levels off, and further increases in frequency have little effect in improving the mass transfer rate. It is unfortunate

that the probes used in this study were not sensitive to frequencies as high as the vortex shedding frequencies, since changing the promoter diameter or vortex shedding frequency is a method one might consider to alter the velocity disturbance frequency near the wall. Although no data are available to confirm that velocity fluctuations at the vortex shedding frequency reach the wall, the data in this study indicated that smaller promoters, with higher vortex shedding frequencies, are more effective in promoting higher mass transfer rates, with only a small accompanying increase in pumping costs.

CHAPTER VIII

RECOMMENDATIONS

During this investigation, the need for more accurate data and some different types of measurements became apparent. These measurements were not always within the capability of the experimental equipment used in the present study. Thus, further investigations into these areas are recommended.

First, data on velocity fluctuations near the wall need to be extended to higher frequencies. Since higher-frequency disturbances appear to contribute relatively more toward the mean mass transfer rate, better spectral data are needed for frequencies above about U_a/d . Even in clear channels, there are substantial differences between data obtained using different probe sizes and Schmidt numbers. With turbulence promoters in the channel, data for this frequency range and above are especially needed. The probes in this study were never able, for instance, to detect disturbances at or near the vortex shedding frequencies. Subsequent studies should attempt to use smaller, isolated probes and possibly a system having a lower Schmidt number. Both changes increase the sensitivity of electrodes to higher frequencies, but it is also desirable to keep the Schmidt number high enough that the concentration boundary layer building up over the probe length will be predominantly within the linear region

of the hydrodynamic boundary layer (i.e., within the so-called "laminar sublayer").

Propagation velocities at the wall have never been determined directly, although, according to the analysis proposed in this study, this is an important parameter in determining mass transfer rates. The use in this study of indirect measurements to estimate the propagation velocity is not altogether satisfactory and direct measurements with space-time correlations of velocity fluctuations are recommended.

Measurements of the mean wall velocity gradient behind promoters with turbulent flow showed a cyclic pattern, suggesting that the main flow channel may be diverted over the promoter much as if it were a baffle. The flow, then, may cycle a few times alternately, having a mean component toward each wall, until the oscillations are damped further downstream. With only a few wall velocity gradient measurements to rely upon, this description is largely speculation, but diverted flows could be important enough from the standpoint of detached (or even attached) turbulence promoter performance to be investigated further. The basic phenomena can be confirmed by placing directionally sensitive hot wires in a channel, or possibly by simply making flow visualization studies. The cycling wake behind the promoters may be especially important in spacing the downstream promoters. One should keep in mind that in some processes,

such as hyperfiltration, transfer surfaces are likely to be on both sides of the channel; in that event, promoters would also be required on both sides.

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APPENDIX A
LOCAL MASS TRANSFER RATES

Figure 84. Local mass transfer rate promotion from two 1/16-in. promoters 1/4 in. apart and 1/16 in. from the wall.

Figure 85. Local mass transfer rate promotion from two 1/16-in. promoters 3/8 in. apart and 1/16 in. from the wall.

Figure 86. Local mass transfer rate promotion from two 1/16-in. promoters 5/8 in. apart and 1/16 in. from the wall.

Figure 87: Local mass transfer rate promotion from a 1/16-in. promoter 3/32 in. from the wall.

Figure 88. Local mass transfer rate promotion from two 1/16-in. promoters 1/4 in. apart and 3/32 in. from the wall.

Figure 89. Local mass transfer rate promotion from two 1/16-in. promoters 3/8 in. apart and 3/32 in. from the wall.

Figure 90. Local mass transfer rate promotion from two 1/16-in. promoters 1/2 in. apart and 3/32 in. from the wall.

Figure 91. Local mass transfer rate promotion from two 1/16-in. promoters 5/8 in. apart and 3/32 in. from the wall.

Figure 92. Local mass transfer rate promotion from two 1/16-in. promoters 3/4 in. apart and 3/32 in. from the wall.

Figure 93. Local mass transfer rate promotion from a 1/16-in. promoter 1/8 in. from the wall.

Figure 94. Local mass transfer rate promotion from a 3/32-in. promoter 1/16 in. from the wall.

Figure 95. Local mass transfer rate promotion from two $3/32$ -in. promoters $1/2$ in. apart and $1/16$ in. from the wall.

Figure 96. Local mass transfer rate promotion from a 3/32-in. promoter 3/32 in. from the wall.

Figure 97. Local mass transfer rate promotion from two $3/32$ -in. promoters $1/4$ in. apart and $3/32$ in. from the wall.

Figure 98. Local mass transfer rate promotion from two $3/32$ -in. promoters $3/8$ in. apart and $3/32$ in. from the wall.

Figure 99. Local mass transfer rate promotion from two $3/32$ -in. promoters $1/2$ in. apart and $3/32$ in. from the wall.

Figure 100. Local mass transfer rate promotion from two $3/32$ -in. promoters $5/8$ in. apart and $3/32$ in. from the wall.

Figure 101. Local mass transfer rate promotion from two $3/32$ -in. promoters $3/4$ in. apart and $3/32$ in. from the wall.

Figure 102. Local mass transfer rate promotion from two $3/32$ -in. promoters $1/4$ in. apart and $1/8$ in. from the wall.

Figure 103. Local mass transfer rate promotion from two $3/32$ -in. promoters $3/8$ in. apart and $1/8$ in. from the wall.

Figure 104. Local mass transfer rate promotion from two $3/32$ -in. promoters $1/2$ in. apart and $1/8$ in. from the wall.

Figure 105. Local mass transfer rate promotion from two $3/32$ -in. promoters $3/4$ in. apart and $1/8$ in. from the wall.

Figure 106. Local mass transfer rate promotion from a 1/8-in. promoter 1/8 in. from the wall.

APPENDIX B
POWER SPECTRAL DENSITY MEASUREMENTS

ORNL-DWG 67-4089

Figure 107. Power spectral density measurements from an isolated electrode $3/4$ in. behind a $3/32$ -in. promoter $1/8$ in. from the wall, $N_{\text{Re}} = 5800$.

Figure 108. Power spectral density measurements from an isolated electrode 1 1/4 in. behind a 3/32-in. promoter 1/8 in. from the wall, $N_{Re} = 5800$.

ORNL-DWG 67-4091

Figure 109. Power spectral density measurements from an isolated electrode $1\frac{1}{4}$ in. behind a $\frac{3}{32}$ -in. promoter $\frac{1}{8}$ in. from the wall, $N_{Re} = 5800$

ORNL-DWG 67-4092

Figure 110. Power spectral density measurements from an isolated electrode $2 \frac{3}{4}$ in. behind a $\frac{3}{32}$ -in. promoter $\frac{1}{8}$ in. from the wall, $N_{Re} = 5800$.

ORNL-DWG 67-4093

Figure 111. Power spectral density measurements from an isolated electrode $3 \frac{3}{4}$ in. behind a $\frac{3}{32}$ -in. promoter $\frac{1}{8}$ in. from the wall, $N_{\text{Re}} = 5800$.

ORNL-DWG 67-4094

Figure 112. Power spectral density measurements from an isolated electrode $7 \frac{3}{4}$ in. behind a $\frac{3}{32}$ -in. promoter $\frac{1}{8}$ in. from the wall, $N_{\text{Re}} = 5800$.

ORNL-DWG 67-4095

Figure 113. Power spectral density measurements from an isolated electrode $3/4$ in. behind a $3/32$ -in. promoter $1/8$ in. from the wall, $N_{Re} = 5800$.

ORNL-DWG 67-4096

Figure 114. Power spectral density measurements from an isolated electrode 1 1/4 in. behind a 3/32-in. promoter 1/8 in. from the wall, $N_{Re} = 5800$.

ORNL-DWG 67-4097

Figure 115. Power spectral density measurements from an isolated electrode $1 \frac{3}{4}$ in. behind a $\frac{3}{32}$ -in. promoter $\frac{1}{8}$ in. from the wall, $N_{Re} = 1835$.

ORNL-DWG 67-4098

Figure 116. Power spectral density measurements from an isolated electrode $2 \frac{3}{4}$ in. behind a $\frac{3}{32}$ -in. promoter $\frac{1}{8}$ in. from the wall, $N_{Re} = 1835$.

ORNL-DWG 67-4099

Figure 117. Power spectral density measurements from an isolated electrode $3 \frac{3}{4}$ in. behind a $\frac{3}{32}$ -in. promoter $\frac{1}{8}$ in. from the wall, $N_{\text{Re}} = 1835$.

ORNL DWG 67-5297

Figure 118. Power spectral density measurements from an isolated electrode $7 \frac{3}{4}$ in. behind a $\frac{3}{32}$ -in. promotor $\frac{1}{8}$ in. from the wall, $N_{\text{Re}} = 1835$.

ORNL-DWG 67-4122

Figure 119. Power spectral density measurements from an isolated electrode $3/4$ in. behind the first of two $3/32$ -in. promoters $1/8$ in. from the wall and $3/4$ in. apart, $N_{Re} = 1835$.

ORNL-DWG 67-4123

Figure 120. Power spectral density measurements from an isolated electrode 1 1/4 in. behind the first of two 3/32-in. promoters 1/8 in. from the wall and 3/4 in. apart, $N_{Re} = 1835$.

ORNL-DWG 67-4124

Figure 121. Power spectral density measurements from an isolated electrode 1 3/4 in. behind the first of two 3/32-in. promoters 1/8 in. from the wall and 3/4 in. apart, $N_{Re} = 1835$.

Figure 122. Power spectral density measurements from an isolated electrode $2 \frac{3}{4}$ in. behind the first of two $\frac{3}{32}$ -in. promoters $\frac{1}{8}$ in. from the wall and $\frac{3}{4}$ in. apart, $N_{Re} = 1835$.

Figure 123. Power spectral density measurements from an isolated electrode $3 \frac{3}{4}$ in. behind the first of two $\frac{3}{32}$ -in. promoters $\frac{1}{8}$ in. from the wall and $\frac{3}{4}$ in. apart, $N_{Re} = 1835$.

Figure 124. Power spectral density measurements from an isolated electrode $7 \frac{3}{4}$ in. behind the first of two $\frac{3}{32}$ -in. promoters $\frac{1}{8}$ in. from the wall and $\frac{3}{4}$ in. apart, $N_{Re} = 1835$.

ORNL-DWG 67-4128

Figure 125. Power spectral density measurements from an isolated electrode 1 1/4 in. behind the first of two 3/32-in. promoters 1/8 in. from the wall and 3/4 in. apart, $N_{Re} = 730$.

Figure 126. Power spectral density measurements from an isolated electrode 1 3/4 in. behind the first of two 3/32-in. promoters 1/8 in. from the wall and 3/4 in. apart, $N_{Re} = 730$.

Figure 127. Power spectral density measurements from an isolated electrode 2 3/4 in. behind the first of two 3/32-in. promoters 1/8 in. from the wall and 3/4 in. apart, $N_{Re} = 730$.

Figure 128. Power spectral density measurements from an isolated electrode $3\frac{3}{4}$ in. behind the first of two $\frac{3}{32}$ -in. promoters $\frac{1}{8}$ in. from the wall and $\frac{3}{4}$ in. apart, $N_{Re} = 730$.

ORNL-DWG 67-4132

Figure 129. Power spectral density measurements from an isolated electrode $7 \frac{3}{4}$ in. behind the first of two $\frac{3}{32}$ -in. promoters $\frac{1}{8}$ in. from the wall and $\frac{3}{4}$ in. apart, $N_{\text{Re}} = 730$.

Figure 130. Power spectral density measurements from an isolated electrode with no promoter, $N_{\text{Re}} = 5800$ and $x = 1 \frac{3}{4}$ in. behind the usual promoter location.

ORNL DWG 67-5299

Figure 131. Power spectral density measurements from an isolated electrode with no promoter, $N_{Re} = 5800$ and $x = 2 \frac{3}{4}$ in. behind the usual promoter location.

ORNL DWG 67-5300

Figure 132. Power spectral density measurements from an isolated electrode with no promoter, $N_{\text{Re}} = 5800$ and $x = 3 \frac{3}{4}$ in. behind the usual promoter location.

ORNL DWG 67-5301

Figure 133. Power spectral density measurements from an isolated electrode with no promoter, $N_{\text{Re}} = 1835$ and $x = 3/4$ in. behind the usual promoter location.

ORNL DWG 67-5302

Figure 134. Power spectral density measurements from an isolated electrode with no promoter, $N_{Re} = 1835$ and $x = 1 \frac{1}{4}$ in. behind the usual promoter location.

ORNL DWG 67-5303

Figure 135. Power spectral density measurements from an isolated electrode with no promotor, $N_{Re} = 1835$ and $x = 1 \frac{3}{4}$ in. behind the usual promotor location.

Figure 136. Power spectral density measurements from an isolated electrode with no promoter, $N_{\text{Re}} = 1835$ and $x = 2 \frac{3}{4}$ in. behind the usual promoter location.

Figure 137. Power spectral density measurements from an isolated electrode with no promoter, $N_{\text{Re}} = 1835$ and $x = 3 \frac{3}{4}$ in. behind the usual promoter location.

ORNL DWG 67-5306

Figure 138. Power spectral density measurements from an isolated electrode with no promoter, $N_{Re} = 1835$ and $x = 7 \frac{3}{4}$ in. behind the usual promoter location.

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APPENDIX C
SAMPLE CALCULATIONS

APPENDIX C

SAMPLE CALCULATIONS

Many of the results in this study could be reported directly or as simple ratios. Power spectral density calculations were made automatically on a CDC 1604 computer with a code written by D. Kryter especially for processing data from the spectra analyzer. However, several measurements were converted into more familiar quantities for evaluation; these are illustrated below.

Stanton Number

Define the following variables:

I = total plate current in ampere,

A_p = area of the test plate in cm^2 ,

C = concentration of ferricyanide in g.-moles/liter,

F = Faraday constant, 9.6×10^4 coul./g.-mole,

Q = mass flow rate in liters/sec.,

A_c = cross sectional area of the channel in cm^2 ,

$A_p = (12 \text{ in.}) (2.5 \text{ in.}) = 30 \text{ in}^2. = 193.5 \text{ cm}^2.$,

$A_c = (2.5 \text{ in.}) (0.49 \text{ in.}) = 1.22 \text{ in}^2. = 7.87 \text{ cm}^2.$,

k = overall mass transfer coefficient,

$$\begin{aligned}
 &= \frac{I}{(F)(A)(C)} = \frac{(I) \left(10^3 \frac{\text{cm}^3}{\text{liter}}\right)}{\left(9.6 \times 10^4 \frac{\text{coul.}}{\text{g.-mole}}\right) (193.5 \text{ cm}^2.) \left(C \frac{\text{g.-moles}}{\text{liter}}\right)} \\
 &= \frac{I}{(1.85 \times 10^5) C} \left(\frac{\text{cm.}}{\text{sec.}} \right) \left(\frac{\text{g.-moles}}{\text{liter}} \right)^{-1},
 \end{aligned}$$

$$U = \frac{(Q_M \text{ liters/sec.})(10^3, \text{ cm}^3/\text{liter})}{(A_c, \text{ cm}^2.)} = \frac{(10^3)(Q_M)}{7.87}, \frac{\text{cm.}}{\text{sec.}}$$

$$N_{St} = \frac{k}{U} = 4.24 \times 10^{-8} \left(\frac{1}{Q_M} \right).$$

Reynolds Number

$$\nu = \text{solution kinematic viscosity} = 10^{-5} \text{ ft}^2/\text{sec.}$$

$$= 9.27 \times 10^{-2} \text{ cm}^2/\text{sec.},$$

$$N_{Re} = \frac{U\ell}{\nu} = 1.36 \times 10^3 Q_M \ell,$$

where ℓ is the characteristic length.

If the characteristic length is assumed to be the channel minor axis,

$$\begin{aligned} N_{Re} &= (1.36 \times 10^3)(Q_M, \frac{\text{liters}}{\text{sec.}})(0.49 \text{ in.})(2.54 \frac{\text{cm.}}{\text{in.}}) \\ &= 1.69 \times 10^3 (Q_M \frac{\text{liters}}{\text{sec.}}). \end{aligned}$$

If the characteristic length is the hydraulic diameter,

$$d_h = \frac{(4)(2.5)(0.49)}{(2)(2.5 + 0.49)}, \text{ in.} = \frac{4.9}{5.98}, \text{ in.} = 0.82 \text{ in.} = 2.08 \text{ cm.},$$

$$N_{Re_h} = \left(\frac{0.82}{0.49} \right) N_{Re} = 2.83 \times 10^3 (Q_M, \frac{\text{liters}}{\text{sec.}}).$$

Friction Factor

$$f_f = \frac{2g_c d_h}{\rho U^2} \left(\frac{\Delta P}{L} \right),$$

where

$$g_c = \text{a constant, } 32 \text{ ft.-lbm./sec}^2\text{-lbf.},$$

$$\rho = \text{density, } \text{lbm./ft}^3. = 62.4 \text{ lbm./ft}^3.,$$

d_h = hydraulic diameter = 0.82 in. = 0.0684 ft.,

U = average velocity, ft./sec.,

ΔP = pressure drop, lbf./ft².,

L = distance between pressure taps, ft. = 0.835 ft.

The pressure drop was measured with an inclined manometer containing an indicating fluid with a density of 1.75 g./cm³. The manometer legs above this fluid were filled with water; therefore

$$\Delta P = (62.4)(1.75 - 1.00)\left(\frac{h}{12}\right) = 3.9 h, \text{ lbf./ft}^2.,$$

where h is the manometer reading in in.

In ft./sec,

$$U = \left(\frac{Q_M}{7.87 \times 10^{-3}}, \frac{\text{cm.}}{\text{sec.}} \right) \left(\frac{1}{12}, \frac{\text{ft.}}{\text{in.}} \right) \left(\frac{1}{2.54}, \frac{\text{in.}}{\text{cm.}} \right) \\ = 4.16Q, \text{ ft./sec.}$$

Then

$$f_f = \frac{(2)(32)(0.0684)(3.9)(h)}{(62.4)(4.16)^2(Q)^2(0.835)} = 0.019 \frac{h}{Q_M^2}.$$

LIST OF SYMBOLS

$ A $	defined by Equation (13),
A_c	cross-sectional area of the channel,
A_p	area of the test surface,
B	ratio of the velocity fluctuation intensity to the mean velocity,
B_{RMS}	ratio of the root-mean-square velocity fluctuations to the mean velocity,
b	dimensionless propagation velocity,
C	concentration,
C_1	a constant of integration,
C_2	a constant of integration,
c	fluctuating component of concentration,
D	diffusion coefficient,
d	diameter,
d_h	hydraulic diameter,
e_p	electric potential of the test surface,
F	integral defined by Equation (58),
F_i	imaginary component of F ,
$\hat{F}_i$	parameter defined by Equation (85),
F_1	a solution to Equation (40),
F	Faraday's constant,
f	intensity of concentration fluctuations,
f^*	relative concentration fluctuation intensities,
f_f	friction factor,
$(f_f)_p$	apparent friction factor with promoters,

G	velocity gradient,
G_{RMS}	root-mean-square of the fluctuating component of the velocity gradient,
g	an oscillating function describing a velocity disturbance,
g_c	conversion factor (32.17 ft.-lbm./sec ² .-lbf.),
h	manometer reading,
I	total plate current,
I_1	overall current upstream of the promoter,
I_2	overall current with no promoter,
I_p	overall current with promoters,
i	the square root of minus one,
i_f	feedback current to amplifier 1,
i_w	current from the test probe,
J	a parameter defined by Equation (13),
J_1	integral defined by Equation (123),
K	dimensionless propagation velocity,
K_1	constant of integration,
K_2	constant of integration,
k	mass transfer coefficient,
k_p	mass transfer coefficient with promoters,
L	length of the mass transfer surface,
L_e	effective length of the mass transfer surface,
L^+	dimensionless electrode length,
l	channel thickness,
N	maximum disturbance frequency,

N_{Sc}	Schmidt number,
N_{Re}	Reynolds number,
N_{St}	Stanton number,
n	frequency,
n_f	exponent in Equation (106),
P	relative promotion,
ΔP	pressure drop,
PSD	power spectral density,
Q	flux of an ion,
Q_M	mass flow rate,
R	gas constant,
R_1	feedback resistor,
R_2	input resistor,
R_M	ratio of the RMS and mean component of the mass transfer rate,
RMS	root-mean-square,
S_{ω}^2	power spectral density function for velocity fluctuations in the x-direction,
s	phase parameter,
T	temperature,
U	velocity in the x-direction,
U_0	velocity at the edge of the concentration boundary layer,
U_2	velocity at condition to give a specified transfer rate,
U_a	average velocity,
U^*	friction velocity,
u	fluctuating component of velocity in the x-direction,

V	velocity in the y-direction,
V_p	propagation velocity,
V_3	output voltage from amplifier 3,
v	fluctuating component of velocity in the y-direction,
w	velocity in the z-direction,
X_0	position of the first promoter,
X_F	length of the test surface,
x	distance in the direction of mean flow,
y	distance from the wall,
z	distance parallel to the wall but normal to the mean flow,
Z	valence,
β	imaginary parameter denoting the wavelength of a disturbance,
β_1	dimensionless wavelength,
δ	concentration boundary layer thickness,
η	dimensionless distance from the wall,
ξ	dimensionless distance down the channel,
Λ_f	macroscale for correlations in the x-direction of u-velocity fluctuations,
ρ	density,
σ	dimensionless frequency,
$\hat{\sigma}$	defined by Equation (84),
τ	dimensionless time,
τ_w	shear stress at the wall,
ω	dimensionless frequency,
$\overline{\quad}$	overbar denotes time average.

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